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Research article

TOPSIS method based on correlation coefficient for solving decision-making problems with intuitionistic fuzzy soft set information

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Abstract: The theory of intuitionistic fuzzy soft set (IFSS) is an extension of the soft set theory which is utilized to precise the deficiency, indeterminacy, and uncertainty of the evaluation while making decisions. The conspicuous characteristic of this mathematical concept is that it considers two distinctive sorts of information, namely the membership and non-membership degrees. The present paper partitioned into two folds: (i) to define the correlation measures for IFSSs; (ii) to introduce the Technique for Order of Preference by Similarity to Ideal Solution(TOPSIS) for IFSS information. Further, few properties identified with these measures are examined thoroughly. In view of these techniques, an approach is presented to solve decision-making problems by utilizing the proposed TOPSIS method based on correlation measures. At last, an illustrative example is enlightened to demonstrate the appropriateness of the proposed approach. Also, its suitability and attainability are checked by contrasting its outcomes and the prevailing methodologies results.

Keywords: intuitionistic fuzzy soft sets; correlation coefficient; TOPSIS method; decision-making
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1. Introduction

Multiple attribute decision making (MADM) is the necessary context of the decision making science whose aim is to recognize the most exceptional targets among the feasible ones. In real decision making, the person needs to furnish the evaluation of the given choices by various types of evaluation conditions such as crisp numbers, intervals, etc. However, in many cases, it is difficult for a person to opt for a suitable one due to the presence of several kinds of uncertainties in the data, which may occur due to a lack of knowledge or human error. Accordingly, to quantify such risks and to examine the process, a large-scale family of the theory such as fuzzy set (FS) [1] and its extensions as intuitionistic FS (IFS) [2], cubic intuitionistic fuzzy set [3], interval-valued IFS [4], linguistic interval-valued IFS

[5], are appropriated by the researchers. In all these theories, an object is judged by an expert in terms of their two membership degrees such that their sum can't exceed one. Since its presence, various researchers have presented their ideas to illuminate decision-making (DM) problems by using aggregation operators or information measures [6–11]. For instance, Xu and Yager [12] performed the weighted geometric operators for IFSs. Garg [9, 10] offered interactive weighted aggregation operators with Einstein t-norm. Ali et al. [7] presented a graphical method to rank the intuitionistic fuzzy values using the uncertainty index and entropy. Liu et al. [11] presented aggregation operators based on the centroid transformations of intuitionistic fuzzy value. Garg and Kumar [13] presented new similarity measures for IFSs based on the connection number of set pair analysis.

Among all these different ideas, one is to locate the most appropriate alternative by making utilization of correlation measures which assumes a noteworthy job in statistical and designing applications as it furnishes us with an estimation of the interdependency of two factors. In the statistical investigation, the correlation measures assume as a critical part to degree the linear relationship between two factors, while in the FS hypothesis the correlation measure decides the degree of reliance between two FSs. As a vital content in fuzzy mathematics, the relationship between IFSs has also gained many considerations for their wide application in the real world, such as pattern recognition, decision making, and market expectation. A few strategies to calculate the correlation coefficient between IFSs have been proposed and investigated in recent years. Towards this direction, the correlation coefficients for FSs have been, firstly, characterized by Hung and Wu [14]. The correlation coefficients of IFSs have been firstly introduced by Gerstenkorn and Manko [15] which is afterward examined by Szmidth and Kacprzyk [16]. The research on correlation measure for interval-valued IFSs has been done by Bustince and Burillo [17]. Garg [18] investigated the correlation measures for Pythagorean FSs. Garg [19] proposed the correlation coefficients for the intuitionistic multiplicative sets and connected them to fathom the issues of pattern recognition. Garg and Kumar [20] presented the correlation coefficient for IFSs by using the connection number of the set pair analysis.

However, apart from that technique for order preference with respect to the similarity to the ideal solution (TOPSIS), created by Hwang and Yoon [21], is a notable technique to fathom DM issues. The objective of this technique is to select the best alternative which obtains the minimum distance from the positive ideal solution. After their presence, various endeavors are made by the analysts to utilize the TOPSIS strategy in fuzzy and IFS environment. For instance, Szmidt and Kacprzyk [22] characterized the idea of a distance measure between the IFSs. Hung and Yang [23] introduced the similarity measures between the two distinctive IFSs based on Hausdorff distance. Duegenci [24] exhibited a distance measure for interval-valued IFS (IVIFS) and their applications to DM with inadequate weight data. Garg [25] introduced the TOPSIS method along with an improved score function for IVIFSs. Mohammadi et al. [26] presented a gray relational analysis and TOPSIS approach to solving the DM problems. Garg et al. [27] proposed an entropy measure under the IFS environment and applied it to illustrate DM issues. Biswas and Kumar [28] investigated an integrated TOPSIS method to solve decision-making problems with IVIFS information. Li [29] presented a nonlinear programming methodology based on TOPSIS methodology with IVIFS information. Garg and Arora [30] extended Li [29] approach to the interval-valued intuitionistic fuzzy soft set environment. In Wang and Chen[31], Gupta et al. [32], authors developed a group decision-making method under the IVIFS environment by integrating extended TOPSIS and linear programming methods. Kumar and Garg [33, 34] presented TOPSIS methodology to solve MADM problems by utilizing a connection number of the set pair analysis theory. Zhang et al. [35] presented a TOPSIS approach based on the covering-based variable precision intuitionistic fuzzy rough set to solve the decision-making problems.

Although the prevailing theories are widely used by the researchers, they have restrictions because of their inadequacy over the parameterizations tool and consequently the decision-maker(s) can't give an accurate decision. To overwhelm these disadvantages, Molodtsov [36] collaborated the soft set (SS) theory in which ratings are given on specific parameters. Maji et al. [37, 38] extended this theory by joining it with existing FS and IFS theoretical approach and developed with the idea of fuzzy soft set (FSS) and intuitionistic fuzzy soft set (IFSS) respectively. In IFSS theory, the preferences of the experts are given over a set of parameters. For instance, in case an individual needs to buy a laptop then according to the IFS theory the information is captured corresponding to a single parameter on two classifications- one is acceptance degree and the other is degree of rejection, whereas in IFSS, a deep insight about the specifications of laptop are considered such as "price", "processor", "RAM", "screen size". In IFSS theory, the information is gathered based on more than one parameter which makes it more generalized than IFS theory. Since their appearance, some basic properties [39], entropy measures [40, 41], distance and similarity measures [42, 43] are proposed by several researchers using the soft set features. To aggregate the IFSS information, Arora and Garg [44, 45], firstly, established an averaging and geometric aggregation, as well as, the prioritized averaging and geometric aggregation operators to solve the MADM problems by aggregating the different intuitionistic fuzzy soft numbers (IFSNs). Later on, Garg and Arora [46] presented the concept of generalized and group generalized IFSS and connected it to illuminate MADM problems. Feng et al. [47] presented an application of the decision-making problems using the generalized intuitionistic fuzzy soft sets. Arora and Garg [48] describes an approach for solving decision-making problems with correlation measures having dual hesitant fuzzy soft set information. Garg and Arora [49] presented some generalized Archimedean t-norm based aggregation operators for IFSS information. In [50, 51], authors have defined the similarity measures for the pairs of the IFSSs. Petchimuthu et al. [52] defined the generalized product of the fuzzy soft matrices to solve the decision-making problems. Zhan et al. [53] presented the relationships among rough sets, soft sets, and hemirings and hence introduced the concept of soft rough hemirings. Garg and Arora [54] presented a Bonferroni mean aggregation operators by considering the influence of the two parameters into the analysis. Zhan et al. [55] introduced the Z-soft fuzzy rough set employing three uncertain models: soft sets, rough sets, and fuzzy sets. Garg and Arora [56] investigated distance measures for dual hesitant fuzzy FSSs and applied them to solve the MADM problems. Hu et al. [57] presented a similarity measure of IFSSs and applied it to solve the group decision-making model for medical diagnosis. Currently, research on the hypothetical and application perspectives of SSs and its different extensions are advancing quickly.

Considering the versatility of IFSS and the importance of the TOPSIS method, this paper extends the TOPSIS strategy to the IFSS environment, where the components are given in terms of intuitionistic fuzzy soft numbers. To address it completely, in the present paper, we develop novel correlation coefficients to measure the degree of dependency of two or more IFSSs. The several properties of the proposed measure are investigated. Further, in contrast to the classical TOPSIS strategy which is based on a separate degree, this paper applies the proposed correlation measure to set up the comparative index of the closeness coefficient. The presented method has been illustrated with a numerical example and the advantages, the superiorities of the method is shown over the classical TOPSIS method. To reach the target precisely, we extended the given TOPSIS approach for solving the MADM problems. Holding all the above tips in mind, the main objective of the present work is listed as

(i) to define some new correlation measures for given numbers under the IFSS environment.

(ii) to develop an algorithm to determine the MADM problems based on the extended TOPSIS approach.

(iii) to test the presented approach with a numerical example.

The correlation measures for IFSS are defined for the pairs of IFSSs which are also utilized to calculate the interrelation as well as the extent of dependency of one element over the other. Since the existing intuitionistic fuzzy set is a special case of the considered IFSS and hence the proposed measure is more generalized than the existing ones. Apart from this, the major assets of the presented TOPSIS method over the others as in the basic TOPSIS method, different distance, and similarity measures are usually involved to compute the closeness coefficient. However, in our presented TOPSIS method, the correlation coefficient is utilized to calculate the closeness coefficient. The main advantage of taking the correlation coefficient is that it preserves the linear relationship among the elements under consideration.

The rest of the text is summarized as. Section 2 presents a basic concept related to soft sets, FSS and IFSS. In Section 3, we define new correlation measures for the IFSSs and studied their properties. In Section 4, an extended TOPSIS method is presented based on the proposed correlation coefficient. In Section 5, we present an algorithm, based on the proposed TOPSIS method, to solve the MADM problem and illustrate with a numerical example. Further, the outcomes of the proposed algorithm are compared with the existing approaches to justify the proficiency of the proposed work. Finally, a concrete conclusion is given in Section 6.

2. Preliminaries

In this section, we discuss some basic terms associated with soft set theory. Let $\mathbf{E}$ be a set of parameters and $\mathbf{U}$ be the set of experts.

Definition 2.1. [36] A pair $(\mathbf{F},\mathbf{E})$ is called as soft set, if $\mathbf{F}$ is a map defined as $\mathbf{F}:\mathbf{E}\to\mathbf{K}^{\mathbf{U}}$ where $\mathbf{K}^{\mathbf{U}}$ is a set of all subsets of $\mathbf{U}$.

Definition 2.2. [36] Let $A, B\subset\mathbf{E}$ and $(\mathbf{F},A)$, $(\mathbf{G},B)$ be two soft sets over $\mathbf{U}$. Then, the basic operations over them are stated as

1) $(\mathbf{F},A)\subseteq(\mathbf{G},B)$ if $A\subseteq B$ and $\mathbf{F}(e)\subseteq\mathbf{G}(e)$, $\forall e\in A$.

2) $(\mathbf{F},A)=(\mathbf{G},B)$ if $(\mathbf{F},A)\subseteq(\mathbf{G},B)$ and $(\mathbf{G},B)\subseteq(\mathbf{F},A)$.

3) Complement: $(\mathbf{F},A)^c=(\mathbf{F}^c,A)$, where $\mathbf{F}^c: A\to\mathbf{K}^{\mathbf{U}}$ defined as $\mathbf{F}^c(e)=\mathbf{U}-\mathbf{F}(e)$, $\forall e\in A$.

Definition 2.3. [37] A map $\mathbf{F}:\mathbf{E}\to\mathbf{F}^{\mathbf{U}}$ is called fuzzy soft set defined as

$$\mathbf{F}_{u_i}(e_j)=\Big\{(u_i,\zeta_j(u_i))\mid u_i\in\mathbf{U}\Big\}, \tag{2.1}$$

where $\mathbf{F}^{\mathbf{U}}$ be a set of all fuzzy subsets of $\mathbf{U}$ and $\zeta_j(u_i)$ is acceptance degree of an expert u_i over parameter $e_j\in\mathbf{E}$.

Definition 2.4. [37] For $A, B \subset \mathbf{E}$ and $(\mathbf{F}, A)$, $(\mathbf{G}, B)$ be any two fuzzy soft sets over $\mathbf{U}$, then

1) $(\mathbf{F}, A) \subseteq (\mathbf{G}, B)$ if, $A \subset B$ and $\mathbf{F}(e) \leq \mathbf{G}(e)$ for each $e \in A$.

2) $(\mathbf{F}, A) = (\mathbf{G}, B)$ are equal if $(\mathbf{F}, A) \subseteq (\mathbf{G}, B)$ and $(\mathbf{G}, B) \subseteq (\mathbf{F}, A)$.

3) Complement: $(\mathbf{F}^c, A)$ where for each $e \in A$, $\mathbf{F}^c(e) = 1 - \mathbf{F}(e)$.

Definition 2.5. [38] A mapping $\mathbf{F} : \mathbf{E} \to \mathbf{IF}^{\mathbf{U}}$ is called as intuitionistic fuzzy soft set defined as

$$\mathbf{F}_{u_i}(e_j) = \left\{(u_i, \zeta_j(u_i), \vartheta_j(u_i)) \mid u_i \in \mathbf{U}\right\}, \tag{2.2}$$

where $\mathbf{IF}^{\mathbf{U}}$ is the intuitionistic fuzzy subsets of $\mathbf{U}$ and ζ_j and ϑ_j are "acceptance degree" and "rejection degree" respectively, with $0 \leq \zeta_j(u_i), \vartheta_j(u_i), \zeta_j(u_i) + \vartheta_j(u_i) \leq 1$ for all $u_i \in \mathbf{U}$. For simplicity, we denote the pair of $\mathbf{F}_{u_i}(e_j)$ as $\beta_{ij} = (\zeta_{ij}, \vartheta_{ij})$ or $(\mathbf{F}, \mathbf{E}) = (\zeta_{ij}, \vartheta_{ij})$ and called as an intuitionistic fuzzy soft number (IFSN).

Definition 2.6. [45] For an IFSN $(\mathbf{F}, \mathbf{E}) = (\zeta, \vartheta)$, a score function is defined as

$$\mathrm{Sc}(\mathbf{F}, \mathbf{E}) = \zeta - \vartheta, \tag{2.3}$$

while an accuracy function is

$$H(\mathbf{F}, \mathbf{E}) = \zeta + \vartheta \tag{2.4}$$

Definition 2.7. [45] An order relation to compare the two IFSNs $(\mathbf{F}, \mathbf{E})$ and $(\mathbf{G}, \mathbf{E})$, denoted by $(\mathbf{F}, \mathbf{E}) \succ (\mathbf{G}, \mathbf{E})$, holds if either of the condition holds:

1) $\mathrm{Sc}(\mathbf{F}, \mathbf{E}) > \mathrm{Sc}(\mathbf{G}, \mathbf{E})$;

2) $\mathrm{Sc}(\mathbf{F}, \mathbf{E}) = \mathrm{Sc}(\mathbf{G}, \mathbf{E})$ and $H(\mathbf{F}, \mathbf{E}) > H(\mathbf{G}, \mathbf{E})$.

Here "$\succ$" represent "preferred to".

3. Correlation coefficients for IFSSs

Let $\mathbf{E} = \{e_1, e_2, \ldots, e_m\}$ be a set of parameters and $\mathbf{U} = \{u_1, u_2, \ldots, u_n\}$ be the set of experts. In this section, we propose some correlation coefficients under the IFSS environment which are summarized as follows:

Definition 3.1. Let $(\mathbf{J}, \mathbf{E}) = \{u_i, (\zeta_{\mathbf{J}_j}(u_i), \vartheta_{\mathbf{J}_j}(u_i)) \mid u_i \in \mathbf{U}\}$ and $(\mathbf{K}, \mathbf{E}) = \{u_i, (\zeta_{\mathbf{K}_j}(u_i), \vartheta_{\mathbf{K}_j}(u_i)) \mid u_i \in \mathbf{U}\}$ be two IFSSs defined over a set of parameters $\mathbf{E} = \{e_1, e_2, \ldots, e_m\}$. Then, the informational intuitionistic energies of two IFSSs $(\mathbf{J}, \mathbf{E})$ and $(\mathbf{K}, \mathbf{E})$ are described as:

$$E_{\mathrm{IFSS}}(\mathbf{J}, \mathbf{E}) = \sum_{j=1}^{m} \sum_{i=1}^{n} \left(\zeta^2_{\mathbf{J}_j}(u_i) + \vartheta^2_{\mathbf{J}_j}(u_i)\right) \tag{3.1}$$

$$E_{\mathrm{IFSS}}(\mathbf{K}, \mathbf{E}) = \sum_{j=1}^{m} \sum_{i=1}^{n} \left(\zeta^2_{\mathbf{K}_j}(u_i) + \vartheta^2_{\mathbf{K}_j}(u_i)\right) \tag{3.2}$$

The correlation of two IFSSs $(\mathbf{J},\mathbf{E})$ and $(\mathbf{K},\mathbf{E})$ can be described as:

$$C_{IFSS}((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})) = \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{J}_j}(u_i)\zeta_{\mathbf{K}_j}(u_i) + \vartheta_{\mathbf{J}_j}(u_i)\vartheta_{\mathbf{K}_j}(u_i)\right) \tag{3.3}$$

It can be observed that the following properties hold for the correlation of IFSSs.

(i) $C_{IFSS}((\mathbf{J},\mathbf{E}),(\mathbf{J},\mathbf{E})) = E_{IFSS}(\mathbf{J},\mathbf{E})$;

(ii) $C_{IFSS}((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})) = C_{IFSS}((\mathbf{K},\mathbf{E}),(\mathbf{J},\mathbf{E}))$.

Definition 3.2. The correlation coefficient between two IFSSs $(\mathbf{J},\mathbf{E})$ and $(\mathbf{K},\mathbf{E})$ is given by $\rho_1((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E}))$ and is described as:

$$\begin{aligned}\rho_1((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})) &= \frac{C_{IFSS}((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E}))}{\sqrt{E_{\text{IFSS}}(\mathbf{J},\mathbf{E}).E_{\text{IFSS}}(\mathbf{K},\mathbf{E})}} \\ &= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{J}_j}(u_i)\zeta_{\mathbf{K}_j}(u_i) + \vartheta_{\mathbf{J}_j}(u_i)\vartheta_{\mathbf{K}_j}(u_i)\right)}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta^2_{\mathbf{J}_j}(u_i) + \vartheta^2_{\mathbf{J}_j}(u_i)\right)}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta^2_{\mathbf{K}_j}(u_i) + \vartheta^2_{\mathbf{K}_j}(u_i)\right)}}\end{aligned} \tag{3.4}$$

Theorem 3.1. The correlation coefficient between two IFSSs $(\mathbf{J},\mathbf{E})$ and $(\mathbf{K},\mathbf{E})$ stated in Eq. (3.4), fulfill the properties as follows:

(P1) $0 \le \rho_1((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})) \le 1$;

(P2) $\rho_1((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})) = \rho_1((\mathbf{K},\mathbf{E}),(\mathbf{J},\mathbf{E}))$;

(P3) If $(\mathbf{J},\mathbf{E}) = (\mathbf{K},\mathbf{E})$, i.e. if $\forall i, j, \zeta_{\mathbf{J}_j}(u_i) = \zeta_{\mathbf{K}_j}(u_i)$ and $\vartheta_{\mathbf{J}_j}(u_i) = \vartheta_{\mathbf{K}_j}(u_i)$, then $\rho_1((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})) = 1$.

Proof. For two IFSSs $(\mathbf{J},\mathbf{E})$ and $(\mathbf{K},\mathbf{E})$, we have

(P1) The inequality $\rho_1((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})) \ge 0$ is obvious, so we just need to prove $\rho_1((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})) \le 1$.

$$\begin{aligned}&C_{IFSS}((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})) \\ =\ & \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{J}_j}(u_i)\zeta_{\mathbf{K}_j}(u_i) + \vartheta_{\mathbf{J}_j}(u_i)\vartheta_{\mathbf{K}_j}(u_i)\right) \\ =\ & \sum_{j=1}^{m}\left(\zeta_{\mathbf{J}_j}(u_1)\zeta_{\mathbf{K}_j}(u_1) + \vartheta_{\mathbf{J}_j}(u_1)\vartheta_{\mathbf{K}_j}(u_1)\right) + \sum_{j=1}^{m}\left(\zeta_{\mathbf{J}_j}(u_2)\zeta_{\mathbf{K}_j}(u_2) + \vartheta_{\mathbf{J}_j}(u_2)\vartheta_{\mathbf{K}_j}(u_2)\right) \\ & + \ldots\ldots + \sum_{j=1}^{m}\left(\zeta_{\mathbf{J}_j}(u_n)\zeta_{\mathbf{K}_j}(u_n) + \vartheta_{\mathbf{J}_j}(u_n)\vartheta_{\mathbf{K}_j}(u_n)\right) \\ =\ & \left\{\begin{array}{l}\left(\zeta_{\mathbf{J}_1}(u_1)\zeta_{\mathbf{K}_1}(u_1) + \vartheta_{\mathbf{J}_1}(u_1)\vartheta_{\mathbf{K}_1}(u_1)\right) + \left(\zeta_{\mathbf{J}_2}(u_1)\zeta_{\mathbf{K}_2}(u_1) + \vartheta_{\mathbf{J}_2}(u_1)\vartheta_{\mathbf{K}_2}(u_1)\right) \\ + \ldots\ldots + \left(\zeta_{\mathbf{J}_m}(u_1)\zeta_{\mathbf{K}_m}(u_1) + \vartheta_{\mathbf{J}_m}(u_1)\vartheta_{\mathbf{K}_m}(u_1)\right)\end{array}\right\} \\ +\ & \left\{\begin{array}{l}\left(\zeta_{\mathbf{J}_1}(u_2)\zeta_{\mathbf{K}_1}(u_2) + \vartheta_{\mathbf{J}_1}(u_2)\vartheta_{\mathbf{K}_1}(u_2)\right) + \left(\zeta_{\mathbf{J}_2}(u_2)\zeta_{\mathbf{K}_2}(u_2) + \vartheta_{\mathbf{J}_2}(u_2)\vartheta_{\mathbf{K}_2}(u_2)\right) \\ + \ldots\ldots + \left(\zeta_{\mathbf{J}_m}(u_2)\zeta_{\mathbf{K}_m}(u_2) + \vartheta_{\mathbf{J}_m}(u_2)\vartheta_{\mathbf{K}_m}(u_2)\right)\}\end{array}\right\}\end{aligned}$$

$$
\begin{aligned}
&+ \quad \vdots \\
&\quad\ \vdots \\
&+ \left\{\begin{array}{l} \left(\zeta_{\mathbf{J}_1}(u_n)\zeta_{\mathbf{K}_1}(u_n) + \vartheta_{\mathbf{J}_1}(u_n)\vartheta_{\mathbf{K}_1}(u_n)\right) + \left(\zeta_{\mathbf{J}_2}(u_n)\zeta_{\mathbf{K}_2}(u_n) + \vartheta_{\mathbf{J}_2}(u_n)\vartheta_{\mathbf{K}_2}(u_n)\right) \\ + \ldots\ldots + \left(\zeta_{\mathbf{J}_m}(u_n)\zeta_{\mathbf{K}_m}(u_n) + \vartheta_{\mathbf{J}_m}(u_n)\vartheta_{\mathbf{K}_m}(u_n)\right)\} \end{array}\right\} \\
&= \sum_{j=1}^{m}\left(\zeta_{\mathbf{J}_j}(u_1)\zeta_{\mathbf{K}_j}(u_1) + \zeta_{\mathbf{J}_j}(u_2)\zeta_{\mathbf{K}_j}(u_2) + \ldots + \zeta_{\mathbf{J}_j}(u_n)\zeta_{\mathbf{K}_j}(u_n)\right) \\
&+ \sum_{j=1}^{m}\left(\vartheta_{\mathbf{J}_j}(u_1)\vartheta_{\mathbf{K}_j}(u_1) + \vartheta_{\mathbf{J}_j}(u_2)\vartheta_{\mathbf{K}_j}(u_2) + \ldots + \vartheta_{\mathbf{J}_j}(u_n)\vartheta_{\mathbf{K}_j}(u_n)\right)
\end{aligned}
$$

By applying Cauchy-Schwarz inequality:

$$(x_1y_1 + x_2y_2 + \ldots + x_ny_n)^2 \leq \left(x_1^2 + x_2^2 + \ldots + x_n^2\right)\left(y_1^2 + y_2^2 + \ldots + y_n^2\right)$$

we have

$$
\begin{aligned}
\left(C_{IFSS}((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E}))\right)^2 &\leq \left\{\begin{array}{l} \sum_{j=1}^{m}\left(\zeta_{\mathbf{J}_j}^2(u_1) + \zeta_{\mathbf{J}_j}^2(u_2) + \ldots + \zeta_{\mathbf{J}_j}^2(u_n)\right) \\ + \sum_{j=1}^{m}\left(\vartheta_{\mathbf{J}_j}^2(u_1) + \vartheta_{\mathbf{J}_j}^2(u_2) + \ldots + \vartheta_{\mathbf{J}_j}^2(u_n)\right) \end{array}\right\} \\
&\quad \times \left\{\begin{array}{l} \sum_{j=1}^{m}\left(\zeta_{\mathbf{K}_j}^2(u_1) + \zeta_{\mathbf{K}_j}^2(u_2) + \ldots + \zeta_{\mathbf{K}_j}^2(u_n)\right) \\ + \sum_{j=1}^{m}\left(\vartheta_{\mathbf{K}_j}^2(u_1) + \vartheta_{\mathbf{K}_j}^2(u_2) + \ldots + \vartheta_{\mathbf{K}_j}^2(u_n)\right) \end{array}\right\} \\
&= \left\{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{J}_j}^2(u_i) + \vartheta_{\mathbf{J}_j}^2(u_i)\right)\right\} \times \left\{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{K}_j}^2(u_i) + \vartheta_{\mathbf{K}_j}^2(u_i)\right)\right\} \\
&= E_{IFSS}((\mathbf{J},\mathbf{E})).E_{IFSS}((\mathbf{K},\mathbf{E}))
\end{aligned}
$$

Therefore, $\left(C_{IFSS}((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E}))\right)^2 \leq E_{IFSS}((\mathbf{J},\mathbf{E})).E_{IFSS}((\mathbf{K},\mathbf{E}))$. Thus, from Eq. (3.4), we get $\rho_1\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) \leq 1$ and hence $0 \leq \rho_1\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) \leq 1$.

(P2) It is obvious so omitted here.

(P3) As we have $\zeta_{\mathbf{J}_j}(u_i) = \zeta_{\mathbf{K}_j}(u_i)$ and $\vartheta_{\mathbf{J}_j}(u_i) = \vartheta_{\mathbf{K}_j}(u_i), \forall i, j$, then from Eq. (3.4), we get

$$\rho_1\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{K}_j}(u_i)\zeta_{\mathbf{K}_j}(u_i) + \vartheta_{\mathbf{K}_j}(u_i)\vartheta_{\mathbf{K}_j}(u_i)\right)}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{K}_j}^2(u_i) + \vartheta_{\mathbf{K}_j}^2(u_i)\right)}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{K}_j}^2(u_i) + \vartheta_{\mathbf{K}_j}^2(u_i)\right)}}$$

$$= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{K}_j}^2(u_i)+\vartheta_{\mathbf{K}_j}^2(u_i)\right)}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{K}_j}^2(u_i)+\vartheta_{\mathbf{K}_j}^2(u_i)\right)}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{K}_j}^2(u_i)+\vartheta_{\mathbf{K}_j}^2(u_i)\right)}}$$
$$= 1$$

Thus, $\rho_1\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) = 1$ and hence the result.

□

Definition 3.3. For two IFSSs $(\mathbf{J},\mathbf{E})$ and $(\mathbf{K},\mathbf{E})$, the correlation coefficient between them is described as:

$$\begin{aligned}\rho_2\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) &= \frac{C_{IFSS}\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right)}{\max\left\{E_{\mathrm{IFSS}}(\mathbf{J},\mathbf{E}),E_{\mathrm{IFSS}}(\mathbf{K},\mathbf{E})\right\}}\\ &= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{J}_j}(u_i)\zeta_{\mathbf{K}_j}(u_i)+\vartheta_{\mathbf{J}_j}(u_i)\vartheta_{\mathbf{K}_j}(u_i)\right)}{\max\left\{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{J}_j}^2(u_i)+\vartheta_{\mathbf{J}_j}^2(u_i)\right),\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\mathbf{K}_j}^2(u_i)+\vartheta_{\mathbf{K}_j}^2(u_i)\right)\right\}}\end{aligned} \tag{3.5}$$

Theorem 3.2. The correlation coefficient between two IFSSs $(\mathbf{J},\mathbf{E})$ and $(\mathbf{K},\mathbf{E})$ denoted as $\rho_2\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right)$ stated in Eq. (3.5), fulfill the properties as follows:

(P1) $0 \le \rho_2\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) \le 1$;

(P2) $\rho_2\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) = \rho_2\left((\mathbf{K},\mathbf{E}),(\mathbf{J},\mathbf{E})\right)$;

(P3) If $(\mathbf{J},\mathbf{E}) = (\mathbf{K},\mathbf{E})$, i.e. if $\forall i, j, \zeta_{\mathbf{J}_j}(u_i) = \zeta_{\mathbf{K}_j}(u_i)$ and $\vartheta_{\mathbf{J}_j}(u_i) = \vartheta_{\mathbf{K}_j}(u_i)$, then $\rho_2\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) = 1$.

Proof. (P1) Since $(\mathbf{J},\mathbf{E})$ and $(\mathbf{K},\mathbf{E})$ are the IFSSs therefore, the inequality $\rho_2\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) \ge 0$ is obvious. The only need is to prove $\rho_2\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) \le 1$, which we can easily prove by applying Cauchy-Schwarz inequality:

$$\sum_{i=1}^{n} x_i y_i \le \sqrt{\left(\sum_{i=1}^{n} x_i^2\right)\cdot\left(\sum_{i=1}^{n} y_i^2\right)} \tag{3.6}$$

with equality if and only if the two vectors $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ are linearly dependent. Also, from Eq. (3.6), we have

$$\sum_{i=1}^{n} x_i y_i \le \sqrt{\left(\sum_{i=1}^{n} x_i^2\right)\cdot\left(\sum_{i=1}^{n} y_i^2\right)} \le \sqrt{\left(\max\left\{\sum_{i=1}^{n} x_i^2, \sum_{i=1}^{n} y_i^2\right\}\right)^2} = \max\left\{\sum_{i=1}^{n} x_i^2, \sum_{i=1}^{n} y_i^2\right\}$$

and hence, by Eq. (3.5), we have $\rho_2\left((\mathbf{J},\mathbf{E}),(\mathbf{K},\mathbf{E})\right) \le 1$. The proof of remaining parts are similar as in Theorem 3.1.

□

However, in numerous viable circumstances, the distinctive set can take diverse weights and in this way, weight η_i for the expert u_i and ξ_j for the parameter e_j, ought to be taken under consideration. Let $\eta = (\eta_1, \eta_2, \ldots, \eta_n)$ be the weight vector for the experts $u_i, (i = 1, 2, \ldots, n)$ such that $\eta_i > 0$, $\sum_{i=1}^n \eta_i = 1$ and let the weight vector of the parameters $e_j, (j = 1, 2, \ldots, m)$ is $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$ with $\xi > 0, \sum_{j=1}^m \xi_j = 1$. Further, the above-defined correlation coefficient ρ_1, ρ_2 are extended to the weighted correlation coefficients between the IFSSs which are as follows:

Definition 3.4. For two IFSSs $(\mathbf{J}, \mathbf{E})$ and $(\mathbf{K}, \mathbf{E})$, the weighted correlation coefficient is described as:

$$\begin{aligned} \rho_3\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right) &= \frac{C_{WIFSS}((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E}))}{\sqrt{E_{\text{WIFSS}}(\mathbf{J}, \mathbf{E}).E_{\text{WIFSS}}(\mathbf{K}, \mathbf{E})}} \\ &= \frac{\sum\limits_{j=1}^{m} \xi_j \left(\sum\limits_{i=1}^{n} \eta_i \left(\zeta_{\mathbf{J}_j}(u_i)\zeta_{\mathbf{K}_j}(u_i) + \vartheta_{\mathbf{J}_j}(u_i)\vartheta_{\mathbf{K}_j}(u_i) \right) \right)}{\sqrt{\sum\limits_{j=1}^{m} \xi_j \left(\sum\limits_{i=1}^{n} \eta_i \left(\zeta^2_{\mathbf{J}_j}(u_i) + \vartheta^2_{\mathbf{J}_j}(u_i) \right) \right)} \sqrt{\sum\limits_{j=1}^{m} \xi_j \left(\sum\limits_{i=1}^{n} \eta_i \left(\zeta^2_{\mathbf{K}_j}(u_i) + \vartheta^2_{\mathbf{K}_j}(u_i) \right) \right)}} \end{aligned} \tag{3.7}$$

Remark 3.1. If we take $\eta = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$ and $\xi = \left(\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}\right)$, then the correlation coefficient $\rho_3\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right)$ reduces to $\rho_1\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right)$ define in Eq. (3.4).

Theorem 3.3. For two IFSSs $(\mathbf{J}, \mathbf{E})$ and $(\mathbf{K}, \mathbf{E})$, the correlation coefficient $\rho_3\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right)$ fulfill the properties as follows:

(P1) $0 \leq \rho_3\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right) \leq 1$;

(P2) $\rho_3\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right) = \rho_3\left((\mathbf{K}, \mathbf{E}), (\mathbf{J}, \mathbf{E})\right)$;

(P3) If $(\mathbf{J}, \mathbf{E}) = (\mathbf{K}, \mathbf{E})$, i.e. if $\forall i, j, \zeta_{\mathbf{J}_j}(u_i) = \zeta_{\mathbf{K}_j}(u_i)$ and $\vartheta_{\mathbf{J}_j}(u_i) = \vartheta_{\mathbf{K}_j}(u_i)$, then $\rho_3\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right) = 1$.

Proof. Proof is same as Theorem 3.1. □

Definition 3.5. For two IFSSs $(\mathbf{J}, \mathbf{E})$ and $(\mathbf{K}, \mathbf{E})$, the weighted correlation coefficient is described as:

$$\begin{aligned} \rho_4\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right) &= \frac{C_{WIFSS}((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E}))}{\max\left\{E_{\text{WIFSS}}(\mathbf{J}, \mathbf{E}).E_{\text{WIFSS}}(\mathbf{K}, \mathbf{E})\right\}} \\ &= \frac{\sum\limits_{j=1}^{m} \xi_j \left(\sum\limits_{i=1}^{n} \eta_i \left(\zeta_{\mathbf{J}_j}(u_i)\zeta_{\mathbf{K}_j}(u_i) + \vartheta_{\mathbf{J}_j}(u_i)\vartheta_{\mathbf{K}_j}(u_i) \right) \right)}{\max\left\{ \sum\limits_{j=1}^{m} \xi_j \left(\sum\limits_{i=1}^{n} \eta_i \left(\zeta^2_{\mathbf{J}_j}(u_i) + \vartheta^2_{\mathbf{J}_j}(u_i) \right) \right), \sum\limits_{j=1}^{m} \xi_j \left(\sum\limits_{i=1}^{n} \eta_i \left(\zeta^2_{\mathbf{K}_j}(u_i) + \vartheta^2_{\mathbf{K}_j}(u_i) \right) \right) \right\}} \end{aligned} \tag{3.8}$$

Remark 3.2. If we take $\eta = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$ and $\xi = \left(\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}\right)$, then the correlation coefficient $\rho_4\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right)$ reduces to $\rho_2\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right)$ define in Eq. (3.5).

Theorem 3.4. The correlation coefficient $\rho_4\left((\mathbf{J}, \mathbf{E}), (\mathbf{K}, \mathbf{E})\right)$ defined in Eq. (3.8) fulfills same properties as those in Theorem 3.3.

Proof. Proof is same as Theorem 3.2. □

From above defined correlation coefficients, we can easily examine that the correlation coefficient given in Eq. (3.4) and (3.7), utilize the geometric mean between the information energies of the IFSSs $(\mathbf{J}, \mathbf{E})$ and $(\mathbf{K}, \mathbf{E})$. On the other hand, the correlation coefficients defined in Eqs. (3.5) and (3.8), consider the greatest between them. For the idealistic evaluators, they tend to utilize the correlation coefficients characterized by Eq. (3.4) and (3.7). As opposed to the positive thinking evaluators, the cynical evaluators have a tendency to apply the correlation coefficients characterized by Eqs. (3.5) and (3.8).

4. TOPSIS approach for solving DM problems based on correlation coefficient

This section presents a technique for solving DM problems based on the proposed correlation coefficient by extending the TOPSIS method for the IFSS environment.

The TOPSIS method is firstly introduced by Hwang and Yoon [21] and utilized to affirm the order of the assessment objects concerning the positive and negative ideal solutions of DM issues. This method depends on the fact that the best choice should have the minimum distance from the positive ideal solution and maximum distance from the negative ideal solution. A TOPSIS technique is displayed in which the correlation measure is used to perceive the positive and negative ideals along with the ranking of the choices. In most of the previous TOPSIS techniques, distinctive type of distance and comparability measures are used to discover the closeness coefficient. If close things are connected, at that point far off things, albeit less related, will be connected as well and in distinctive ways reflecting their integration versus isolation within the information investigation process. The advantage of taking correlation coefficient instead of taking distance or similarity measure in TOPSIS method is that correlation measure preserves the linear relationship among the factors under consideration. The algorithm to choose the most excellent option by utilizing TOPSIS method based on proposed correlation coefficient is described as follows:

4.1. Description of DM problem

Consider a DM problem in which evaluation is done to select the best choice from a set of distinct choices $B = \{B^{(1)}, B^{(2)}, \ldots, B^{(b)}\}$. The ratings of the alternatives $B^{(z)} (z = 1, 2, \ldots, b)$ are given by a team of experts $\{u_1, u_2, \ldots, u_n\}$ by taking certain parameters $\{e_1, e_2, \ldots, e_m\}$ under consideration. The preferences of each alternative $B^{(z)} (z = 1, 2, \ldots, b)$ are described as IFSNs which is stated as $B^{(z)}_{ij} = \left(u_i, \zeta^{(z)}_{B_j}(u_i), \vartheta^{(z)}_{B_j}(u_i)\right)$ where $\zeta^{(z)}_{B_j}(u_i)$ and $\vartheta^{(z)}_{B_j}(u_i)$ indicate the degree of acceptance and rejection respectively, given by u_i over parameter e_j for an alternative $B^{(z)}$. For simplicity, we can write $B^{(z)}_{ij}$ as $\beta^{(z)}_{ij} = \left(\zeta^{(z)}_{ij}, \vartheta^{(z)}_{ij}\right), (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$. Therefore, a DM problem with IFSS information can be written in the form of decision matrix as $B^{(z)} = \left(u_i, \zeta^{(z)}_{ij}(u_i), \vartheta^{(z)}_{ij}(u_i)\right)_{n\times m}$.

$$\left(B^{(z)}, \mathbf{E}\right) = \begin{array}{c} \\ u_1 \\ u_2 \\ \vdots \\ u_n \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & \cdots & e_m \end{array} \\ \begin{pmatrix} \left(\zeta^{(z)}_{11}, \vartheta^{(z)}_{11}\right) & \left(\zeta^{(z)}_{12}, \vartheta^{(z)}_{12}\right) & \cdots & \left(\zeta^{(z)}_{1m}, \vartheta^{(z)}_{1m}\right) \\ \left(\zeta^{(z)}_{21}, \vartheta^{(z)}_{21}\right) & \left(\zeta^{(z)}_{22}, \vartheta^{(z)}_{22}\right) & \cdots & \left(\zeta^{(z)}_{2m}, \vartheta^{(z)}_{2m}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\zeta^{(z)}_{n1}, \vartheta^{(z)}_{n1}\right) & \left(\zeta^{(z)}_{n2}, \vartheta^{(z)}_{n2}\right) & \cdots & \left(\zeta^{(z)}_{nm}, \vartheta^{(z)}_{nm}\right) \end{pmatrix} \end{array}$$

4.2. Determine the PIA and NIA for IFSSs

As the data is given in the form of IFSNs so the positive ideal alternative (PIA) and negative ideal alternative (NIA) can be taken as 1 and 0, respectively. Thus, the rating values of PIA and NIA can be formatted as $r^+ = (1,0)_{n\times m}$ and $r^- = (0,1)_{n\times m}$, respectively. Of these, it can be observed that r^+ and r^- are complementary to one another. However, in case, we take these fixed reference points, at that point any change in alternatives does not effect on the ranking of the alternatives. Therefore, these points can easily be amplified to the circumstances in which these reference points are not settled a priori. To address this, a decision-maker can characterize them as

$$\beta^+ = (\zeta^+, \vartheta^+)_{n\times m} = \left(\zeta_{ij}^{(h)}, \vartheta_{ij}^{(h)}\right)_{n\times m} \tag{4.1}$$

$$\beta^- = (\zeta^-, \vartheta^-)_{n\times m} = \left(\zeta_{ij}^{(g)}, \vartheta_{ij}^{(g)}\right)_{n\times m} \tag{4.2}$$

where h and g, respectively, are the indices at which correlation coefficient with positive ideal r^+ is maximum and minimum with r^- for all i, j.

4.3. Compute the correlation coefficient

The correlation coefficient between each alternative $B^{(z)}, (z = 1, 2, \ldots, b)$ and PIA can be computed as follows:

$$\rho\left(\beta_{ij}^{(z)}, \beta^+\right) = \frac{C_{IFSS}\left(\beta_{ij}^{(z)}, \beta^+\right)}{\sqrt{E_{IFSS}\left(\beta_{ij}^{(z)}\right) \cdot E_{IFSS}\left(\beta^+\right)}} \tag{4.3}$$

Similarly, the correlation coefficient between each alternative $B^{(z)}, (z = 1, 2, \ldots, b)$ and NIA can be calculated as follows:

$$\rho\left(\beta_{ij}^{(z)}, \beta^-\right) = \frac{C_{IFSS}\left(\beta_{ij}^{(z)}, \beta^-\right)}{\sqrt{E_{IFSS}\left(\beta_{ij}^{(z)}\right) \cdot E_{IFSS}\left(\beta^-\right)}} \tag{4.4}$$

4.4. Calculate the relative closeness coefficient

The closeness coefficient corresponding to each alternative can be obtained by utilizing the values of correlation coefficient between each alternative and PIA as well as NIA. For each alternative $B^{(z)}, (z = 1, 2, \ldots, b)$, the relative closeness coefficient is defined as:

$$R^{(z)} = \frac{K\left(\beta_{ij}^{(z)}, \beta^-\right)}{K\left(\beta_{ij}^{(z)}, \beta^+\right) + K\left(\beta_{ij}^{(z)}, \beta^-\right)} \tag{4.5}$$

where $K\left(\beta_{ij}^{(z)}, \beta^-\right) = 1 - \rho\left(\beta_{ij}^{(z)}, \beta^-\right)$ and $K\left(\beta_{ij}^{(z)}, \beta^+\right) = 1 - \rho\left(\beta_{ij}^{(z)}, \beta^+\right)$ for all i, j.

To analyze how linearly the alternatives are correlated with the ideal alternatives with respect to the closeness coefficient, we need a computational property, which is $K(\beta_{ij}^{(z)}, \beta^-) = 1 - \rho(\beta_{ij}^{(z)}, \beta^-)$ and $K(\beta_{ij}^{(z)}, \beta^+) = 1 - \rho(\beta_{ij}^{(z)}, \beta^+)$. From this property, it can be easily seen that if the correlation coefficient between any alternative and positive ideal is 1 i.e. if the alternative is highly correlated with PIA then their corresponding closeness coefficient value is also 1, which is the maximum. It shows that closeness coefficient of an alternative is maximum when it has maximum distant form NIA. Also, we have $0 \leq K(\beta_{ij}^{(z)}, \beta^-) \leq K(\beta_{ij}^{(z)}, \beta^+) + K(\beta_{ij}^{(z)}, \beta^-)$ and hence, for all z, $0 \leq R^{(z)} \leq 1$.

5. Proposed approach and its application

In this section, a methodology is exhibited to solve the of DM issues by using proposed TOPSIS technique based on correlation coefficients.

5.1. Proposed approach

A set of b alternatives $B = \{B^{(1)}, B^{(2)}, \ldots, B^{(b)}\}$ is considered for evaluation under the experts $\mathbf{U} = \{u_1, u_2, \ldots, u_n\}$ with weights $\eta = (\eta_1, \eta_2, \ldots, \eta_n)^T$ such that $\eta_i > 0$ and $\sum_{i=1}^{n} \eta_i = 1$. The judgement is given on the set of parameters $\mathbf{E} = \{e_1, e_2, \ldots, e_m\}$ having weights $\xi = (\xi_1, \xi_2, \ldots, \xi_m)^T$ such that $\xi_j > 0$ and $\sum_{j=1}^{m} \xi_j = 1$. The evaluation of alternative $B^{(z)}, (z = 1, 2, \ldots, b)$ by the experts $u_i, (i = 1, 2, \ldots, n)$ on the parameters e_j, $(j = 1, 2, \ldots, m)$ are given as IFSNs denoted by $\beta_{ij}^{(z)} = \left(\zeta_{ij}^{(z)}, \vartheta_{ij}^{(z)}\right)$ such that $0 \leq \zeta_{ij}^{(z)}, \vartheta_{ij}^{(z)} \leq 1$ and $\zeta_{ij}^{(z)} + \vartheta_{ij}^{(z)} \leq 1$. Then, the procedure for choosing the finest alternative(s) by utilizing the proposed operators are summarized in the following steps:

Step 1: Collect the data related to each alternative $B^{(z)}; z = 1, 2, \ldots, b$ and summarized in a matrix as follows:

$$\left(B^{(z)}, \mathbf{E}\right) = \begin{array}{c} \\ u_1 \\ u_2 \\ \vdots \\ u_n \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & \ldots & e_m \end{array} \\ \begin{pmatrix} \beta_{11}^{(z)} & \beta_{12}^{(z)} & \ldots & \beta_{1m}^{(z)} \\ \beta_{21}^{(z)} & \beta_{22}^{(z)} & \ldots & \beta_{2m}^{(z)} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n1}^{(z)} & \beta_{n2}^{(z)} & \ldots & \beta_{nm}^{(z)} \end{pmatrix} \end{array}$$

Step 2: Construct the weighted decision matrix $\overline{B}^{(z)} = \left(\overline{\beta}_{ij}^{(z)}\right)$, where

$$\begin{aligned} \overline{\beta}_{ij}^{(z)} &= \xi_j \eta_i \beta_{ij}^{(z)} \\ &= \left(1 - \left(\left(1 - \zeta_{ij}^{(z)}\right)^{\eta_i}\right)^{\xi_j}, \left(\left(\vartheta_{ij}^{(z)}\right)^{\eta_i}\right)^{\xi_j}\right) \\ &= \left(\overline{\zeta}_{ij}^{(z)}, \overline{\vartheta}_{ij}^{(z)}\right) \end{aligned} \tag{5.1}$$

where η_i is the weight vector for ith expert, ξ_j is the weight vector for jth parameter and $\overline{B}^{(z)} = \left(\overline{\beta}_{ij}^{(z)}\right)_{n \times m}$. The matrices so obtained are stated as weighted decision matrices.

Step 3: The correlation coefficient of each value of $\overline{\beta}_{ij}^{(z)}$ with the perfect positive ideal $r^+ = (1, 0)$ is obtained as:

$$\rho_1\left(\overline{\beta}_{ij}^{(z)}, r^+\right) = \frac{C_{\text{IFSS}}\left(\overline{\beta}_{ij}^{(z)}, r^+\right)}{\sqrt{E_{IFSS}(\overline{\beta}_{ij}^{(z)}) \cdot E_{IFSS}(r^+)}} \tag{5.2}$$

or

$$\rho_2\left(\overline{\beta}_{ij}^{(z)}, r^+\right) = \frac{C_{\text{IFSS}}\left(\overline{\beta}_{ij}^{(z)}, r^+\right)}{\max\left\{E_{IFSS}(\overline{\beta}_{ij}^{(z)}), E_{IFSS}(r^+)\right\}} \tag{5.3}$$

By applying these, we get a decision matrix corresponding to each alternative and this matrix is called as correlation coefficient matrix and is denoted by $\phi^{(z)} = \left(\phi_{ij}^{(z)}\right)_{n\times m}, (z = 1, 2, \ldots, b)$. Here $\phi_{ij}^{(z)}$ is the correlation coefficient between the value $\overline{\beta}_{ij}^{(z)}$ and the positive ideal r^+.

Step 4: Find the indices h_{ij} and g_{ij} for each expert u_i and parameter e_j from the correlation coefficient matrices $\phi^{(z)}, (z = 1, 2, \ldots, b)$ such that $h_{ij} = \arg\max_z\{\phi_{ij}^{(z)}\}$ and $g_{ij} = \arg\min_z\{\phi_{ij}^{(z)}\}$. Based on these indices, determine the PIA β^+ and NIA β^- as

$$\beta^+ = (\zeta^+, \vartheta^+)_{n\times m} = \left(\overline{\zeta}_{ij}^{(h_{ij})}, \overline{\vartheta}_{ij}^{(h_{ij})}\right) \tag{5.4}$$

$$\text{and } \beta^- = (\zeta^-, \vartheta^-)_{n\times m} = \left(\overline{\zeta}_{ij}^{(g_{ij})}, \overline{\vartheta}_{ij}^{(g_{ij})}\right) \tag{5.5}$$

Step 5: Compute the correlation coefficient between weighted decision matrices $\overline{B}^{(z)}, (z = 1, 2, \ldots, b)$ of each alternative $B^{(z)}, (z = 1, 2, \ldots, b)$ and the PIA β^+ by utilizing either ρ_1 or ρ_2 proposed correlation coefficients as:

$$\begin{aligned} p^{(z)} &= \rho_1\left(\overline{B}^{(z)}, \beta^+\right) \\ &= \frac{C_{\text{IFSS}}\left(\overline{B}^{(z)}, \beta^+\right)}{\sqrt{E_{\text{IFSS}}(\overline{B}^{(z)}) \cdot E_{\text{IFSS}}(\beta^+)}} \\ &= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\overline{\zeta}_{ij}^{(z)}\zeta^+ + \overline{\vartheta}_{ij}^{(z)}\vartheta^+\right)}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\overline{\zeta}_{ij}^{(z)}\right)^2 + \left(\overline{\vartheta}_{ij}^{(z)}\right)^2\right)}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta^+\right)^2 + \left(\vartheta^+\right)^2\right)}} \end{aligned} \tag{5.6}$$

$$\begin{aligned} \text{or } p^{(z)} &= \rho_2\left(\overline{B}^{(z)}, \beta^+\right) \\ &= \frac{C_{\text{IFSS}}\left(\overline{B}^{(z)}, \beta^+\right)}{\max\left\{E_{\text{IFSS}}(\overline{B}^{(z)}), E_{\text{IFSS}}(\beta^+)\right\}} \\ &= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\overline{\zeta}_{ij}^{(z)}\zeta^+ + \overline{\vartheta}_{ij}^{(z)}\vartheta^+\right)}{\max\left\{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\overline{\zeta}_{ij}^{(z)}\right)^2 + \left(\overline{\vartheta}_{ij}^{(z)}\right)^2\right), \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta^+\right)^2 + \left(\vartheta^+\right)^2\right)\right\}} \end{aligned} \tag{5.7}$$

Step 6: Compute the correlation coefficient between weighted decision matrices $\overline{B}^{(z)}, (z = 1, 2, \ldots, b)$

and the NIA β^- as:

$$\begin{aligned} n^{(z)} &= \rho_1\left(\overline{B}^{(z)}, \beta^-\right) \\ &= \frac{C_{\text{IFSS}}\left(\overline{B}^{(z)}, \beta^-\right)}{\sqrt{E_{\text{IFSS}}(\overline{B}^{(z)}) \cdot E_{\text{IFSS}}(\beta^-)}} \\ &= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\overline{\zeta}_{ij}^{(z)}\zeta^- + \overline{\vartheta}_{ij}^{(z)}\vartheta^-\right)}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\overline{\zeta}_{ij}^{(z)}\right)^2 + \left(\overline{\vartheta}_{ij}^{(z)}\right)^2\right)}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta^-\right)^2 + \left(\vartheta^-\right)^2\right)}} \end{aligned} \tag{5.8}$$

$$\begin{aligned} \text{or } n^{(z)} &= \rho_2\left(\overline{B}^{(z)}, \beta^-\right) \\ &= \frac{C_{\text{IFSS}}\left(\overline{B}^{(z)}, \beta^-\right)}{\max\left\{E_{\text{IFSS}}(\overline{B}^{(z)}), E_{\text{IFSS}}(\beta^-)\right\}} \\ &= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\overline{\zeta}_{ij}^{(z)}\zeta^- + \overline{\vartheta}_{ij}^{(z)}\vartheta^-\right)}{\max\left\{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\overline{\zeta}_{ij}^{(z)}\right)^2 + \left(\overline{\vartheta}_{ij}^{(z)}\right)^2\right), \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta^-\right)^2 + \left(\vartheta^-\right)^2\right)\right\}} \end{aligned} \tag{5.9}$$

Step 7: Obtain the closeness coefficient for each alternative $B^{(z)}, (z = 1, 2, \ldots, b)$ as:

$$R^{(z)} = \frac{K\left(\overline{B}^{(z)}, \beta^-\right)}{K\left(\overline{B}^{(z)}, \beta^+\right) + K\left(\overline{B}^{(z)}, \beta^-\right)} \tag{5.10}$$

where $K\left(\overline{B}^{(z)}, \beta^-\right) = 1 - n^{(z)}$ and $K\left(\overline{B}^{(z)}, \beta^+\right) = 1 - p^{(z)}$.

Step 8: By utilizing descending value of $R^{(z)}$ give ranking to the alternatives $B^{(z)}(z = 1, 2, \ldots, b)$ and then find the desired one(s).

5.2. Illustrative example

In this section, an illustrative example for solving DM problem based upon outsourcing supplier selection problem is presented to describe the application of proposed approaches.

In the following, we discuss the IT Outsourcing Provider Selection problem. Millennium Semiconductors(MS), established in October 1995, is an ISO 9001 − 2015 association with a circulation of electronic parts as its center aptitude. This driving wholesaler of electronic segments in India is synonymous with advancement and today, it is a standout amongst the most rumored name in the market. MS has set up in nearly every locale of India with catering more than 1500 clients in all sections from the recent two decades. It engages in investigation and improvement, production and promoting of items, for example, full shading ultra high shine LED epitaxial items, chips, compound sun oriented cells and high control concentrating sunlight based items. The branch offices of MS is situated in Delhi, Bangalore, Hyderabad, Ahmedabad, Chennai and Mumbai in India and Overseas workplaces in Singapore and Shenzhen(China).

MS contributes to the extraordinary larger part of labor and financial resources to its center competition rather than IT. It's the outsourcing of IT is a better choice for MS as of its absence of the ability to do it efficiently. Therefore, MS selects the following outsourcing providers(alternatives): Tata Constancy services ($B^{(1)}$), Infosys ($B^{(2)}$), Wipro ($B^{(3)}$), HCL ($B^{(4)}$). Now, to find the more suitable or best outsourcing provider among the above choices, MS hires a team of five experts $\{u_1, u_2, u_3, u_4, u_5\}$ having weight vectors $(0.2, 0.3, 0.1, 0.3, 0.1)$. These experts evaluate each provider against the five parameters: Design development (e_1), Quality of product (e_2), Delivery Time (e_3), Risk factor (e_4) and Cost (e_5) having weights $(0.15, 0.25, 0.10, 0.30, 0.20)$. Each expert assesses its rating values for each provider over the parameters in terms of IFSNs. The procedure for selecting the best alternative by applying the proposed approach is listed as:

Step 1: The ratings given by the experts for each provider are summarized in Table 1.

Table 1. Decision matrices for each alternative in terms of IFSNs.

	For alternative $B^{(1)}$					For alternative $B^{(2)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	(0.7, 0.2)	(0.6, 0.3)	(0.8, 0.1)	(0.6, 0.2)	(0.7, 0.2)	(0.9, 0.1)	(0.7, 0.3)	(0.6, 0.1)	(0.4, 0.3)	(0.9, 0.1)
u_2	(0.5, 0.1)	(0.7, 0.1)	(0.9, 0.1)	(0.5, 0.3)	(0.6, 0.3)	(0.7, 0.2)	(0.8, 0.2)	(0.7, 0.1)	(0.5, 0.2)	(0.8, 0.2)
u_3	(0.6, 0.2)	(0.5, 0.4)	(0.7, 0.2)	(0.5, 0.4)	(0.7, 0.1)	(0.8, 0.1)	(0.6, 0.3)	(0.5, 0.3)	(0.6, 0.1)	(0.8, 0.1)
u_4	(0.7, 0.3)	(0.7, 0.2)	(0.7, 0.3)	(0.4, 0.3)	(0.6, 0.1)	(0.7, 0.3)	(0.9, 0.1)	(0.7, 0.2)	(0.5, 0.3)	(0.7, 0.1)
u_5	(0.5, 0.4)	(0.5, 0.5)	(0.6, 0.1)	(0.5, 0.1)	(0.8, 0.1)	(0.6, 0.1)	(0.7, 0.1)	(0.6, 0.3)	(0.4, 0.5)	(0.6, 0.2)
	For alternative $B^{(3)}$					For alternative $B^{(4)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	(0.8, 0.2)	(0.7, 0.3)	(0.9, 0.1)	(0.7, 0.2)	(0.8, 0.1)	(0.9, 0.1)	(0.4, 0.3)	(0.5, 0.2)	(0.6, 0.2)	(0.4, 0.3)
u_2	(0.7, 0.1)	(0.6, 0.2)	(0.6, 0.1)	(0.6, 0.3)	(0.7, 0.2)	(0.7, 0.2)	(0.6, 0.2)	(0.6, 0.4)	(0.7, 0.2)	(0.5, 0.4)
u_3	(0.6, 0.2)	(0.8, 0.1)	(0.5, 0.5)	(0.5, 0.4)	(0.6, 0.3)	(0.6, 0.1)	(0.7, 0.1)	(0.7, 0.3)	(0.7, 0.1)	(0.6, 0.3)
u_4	(0.5, 0.4)	(0.6, 0.3)	(0.8, 0.1)	(0.6, 0.2)	(0.9, 0.1)	(0.8, 0.2)	(0.7, 0.2)	(0.4, 0.6)	(0.5, 0.4)	(0.5, 0.2)
u_5	(0.8, 0.1)	(0.8, 0.2)	(0.7, 0.3)	(0.7, 0.1)	(0.8, 0.2)	(0.7, 0.1)	(0.6, 0.3)	(0.5, 0.4)	(0.6, 0.3)	(0.6, 0.2)

Step 2: The weighted decision matrices $\overline{B}^{(z)}, (z = 1, 2, 3, 4)$ are computed by utilizing Eq. (5.1). The results of it are summarized in Table 2.

Table 2. Weighted decision matrices $\overline{B}$ for each alternative.

	For alternative $B^{(1)}$					For alternative $B^{(2)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	(0.0355, 0.9529)	(0.0448, 0.9416)	(0.0317, 0.9550)	(0.0535, 0.9079)	(0.0470, 0.9377)	(0.0667, 0.9333)	(0.0584, 0.9416)	(0.0182, 0.9550)	(0.0302, 0.9303)	(0.0880, 0.9120)
u_2	(0.0307, 0.9016)	(0.0863, 0.8414)	(0.0667, 0.9333)	(0.0605, 0.8973)	(0.0535, 0.9303)	(0.0527, 0.9301)	(0.1137, 0.8863)	(0.0355, 0.9333)	(0.0605, 0.8652)	(0.0921, 0.9079)
u_3	(0.0137, 0.9761)	(0.0172, 0.9774)	(0.0120, 0.9840)	(0.0206, 0.9729)	(0.0238, 0.9550)	(0.0239, 0.9661)	(0.0226, 0.9703)	(0.0069, 0.9880)	(0.0271, 0.9333)	(0.0317, 0.9550)
u_4	(0.0527, 0.9473)	(0.0863, 0.8863)	(0.0355, 0.9645)	(0.0449, 0.8973)	(0.0535, 0.8710)	(0.0527, 0.9473)	(0.1586, 0.8414)	(0.0355, 0.9529)	(0.0605, 0.8973)	(0.0697, 0.8710)
u_5	(0.0103, 0.9863)	(0.0172, 0.9828)	(0.0091, 0.9772)	(0.0206, 0.9333)	(0.0317, 0.9550)	(0.0137, 0.9661)	(0.0297, 0.9441)	(0.0091, 0.9880)	(0.0152, 0.9794)	(0.0182, 0.9683)
	For alternative $B^{(3)}$					For alternative $B^{(4)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	(0.0471, 0.9529)	(0.0584, 0.9416)	(0.0450, 0.9550)	(0.0697, 0.9079)	(0.0623, 0.9120)	(0.0667, 0.9333)	(0.0252, 0.9416)	(0.0138, 0.9683)	(0.0535, 0.9079)	(0.0202, 0.9530)
u_2	(0.0137, 0.9016)	(0.0664, 0.8863)	(0.0271, 0.9333)	(0.0792, 0.8973)	(0.0697, 0.9079)	(0.0527, 0.9301)	(0.0664, 0.8863)	(0.0271, 0.9729)	(0.1027, 0.8652)	(0.0407, 0.9465)
u_3	(0.0137, 0.9761)	(0.0394, 0.9441)	(0.0069, 0.9931)	(0.0206, 0.9729)	(0.0182, 0.9762)	(0.0137, 0.9661)	(0.0297, 0.9441)	(0.0120, 0.9880)	(0.0355, 0.9333)	(0.0182, 0.9762)
u_4	(0.0307, 0.9596)	(0.0664, 0.9137)	(0.0471, 0.9333)	(0.0792, 0.8652)	(0.1290, 0.8710)	(0.0699, 0.9301)	(0.0863, 0.8863)	(0.0152, 0.9848)	(0.0605, 0.9208)	(0.0407, 0.9079)
u_5	(0.0239, 0.9661)	(0.0394, 0.9606)	(0.0120, 0.9880)	(0.0355, 0.9333)	(0.0317, 0.9683)	(0.0179, 0.9661)	(0.0226, 0.9703)	(0.0069, 0.9909)	(0.0271, 0.9645)	(0.0182, 0.9683)

Step 3: By utilizing Eq. (5.2) to determine the correlation coefficients between each alternative $B^{(z)}, (z = 1, 2, 3, 4)$ and the positive ideal r^+, we get

$$\phi^{(1)} = \begin{bmatrix} 0.0372 & 0.0475 & 0.0332 & 0.0588 & 0.0501 \\ 0.0340 & 0.1021 & 0.0713 & 0.0672 & 0.0574 \\ 0.0140 & 0.0176 & 0.0122 & 0.0211 & 0.0249 \\ 0.0556 & 0.0970 & 0.0368 & 0.0500 & 0.0613 \\ 0.0105 & 0.0175 & 0.0093 & 0.0220 & 0.0332 \end{bmatrix}; \phi^{(2)} = \begin{bmatrix} 0.0713 & 0.0619 & 0.0190 & 0.0324 & 0.0960 \\ 0.0566 & 0.1273 & 0.0380 & 0.0697 & 0.1009 \\ 0.0247 & 0.0233 & 0.0070 & 0.0290 & 0.0332 \\ 0.0556 & 0.1852 & 0.0372 & 0.0672 & 0.0798 \\ 0.0141 & 0.0314 & 0.0092 & 0.0155 & 0.0187 \end{bmatrix}$$

$$\phi^{(3)} = \begin{bmatrix} 0.0494 & 0.0619 & 0.0471 & 0.0765 & 0.0682 \\ 0.0584 & 0.0747 & 0.0290 & 0.0879 & 0.0765 \\ 0.0140 & 0.0417 & 0.0070 & 0.0211 & 0.0186 \\ 0.0320 & 0.0725 & 0.0504 & 0.0911 & 0.1466 \\ 0.0247 & 0.0410 & 0.0121 & 0.0380 & 0.0327 \end{bmatrix}; \phi^{(4)} = \begin{bmatrix} 0.0713 & 0.0268 & 0.0142 & 0.0588 & 0.0212 \\ 0.0566 & 0.0747 & 0.0279 & 0.1179 & 0.0430 \\ 0.0141 & 0.0314 & 0.0121 & 0.0380 & 0.0186 \\ 0.0749 & 0.0970 & 0.0154 & 0.0655 & 0.0448 \\ 0.0185 & 0.0233 & 0.0070 & 0.0281 & 0.0187 \end{bmatrix}$$

Step 4: By utilizing Eqs. (5.4) and (5.5), the PIA and NIA are computed as:

$$\beta^+ = \begin{bmatrix} (0.0667, 0.9333) & (0.0584, 0.9416) & (0.0450, 0.9550) & (0.0697, 0.9079) & (0.0880, 0.9120) \\ (0.0527, 0.9016) & (0.1137, 0.8863) & (0.0667, 0.9333) & (0.1027, 0.8652) & (0.0921, 0.9079) \\ (0.0239, 0.9661) & (0.0394, 0.9441) & (0.0120, 0.9840) & (0.0355, 0.9333) & (0.0317, 0.9550) \\ (0.0699, 0.9301) & (0.1586, 0.8414) & (0.0471, 0.9333) & (0.0792, 0.8652) & (0.1290, 0.8710) \\ (0.0239, 0.9661) & (0.0394, 0.9606) & (0.0120, 0.9880) & (0.0355, 0.9333) & (0.0317, 0.9550) \end{bmatrix}$$

$$\text{and} \quad \beta^- = \begin{bmatrix} (0.0355, 0.9529) & (0.0252, 0.9416) & (0.0138, 0.9683) & (0.03020.9303) & (0.0202, 0.9530) \\ (0.0307, 0.9016) & (0.0664, 0.8863) & (0.0271, 0.9729) & (0.0605, 0.8973) & (0.0407, 0.9465) \\ (0.0137, 0.9761) & (0.0172, 0.9774) & (0.0069, 0.9931) & (0.0206, 0.9729) & (0.0182, 0.9762) \\ (0.0307, 0.9596) & (0.0664, 0.9137) & (0.0152, 0.9848) & (0.0449, 0.8973) & (0.0407, 0.9079) \\ (0.0103, 0.9863) & (0.0172, 0.9828) & (0.0069, 0.9909) & (0.01520.9794) & ,(0.0182, 0.9683) \end{bmatrix}$$

Step 5: By using Eq. (5.6), the correlation coefficients between each alternative and the PIA β^+ are computed and get $p^{(1)} = 0.99926$, $p^{(2)} = 0.99964$, $p^{(3)} = 0.99944$ and $p^{(4)} = 0.99911$.

Step 6: By using Eq. (5.8), the correlation coefficients between each alternative and the NIA β^- are computed and get $n^{(1)} = 0.99973$, $n^{(2)} = 0.99921$, $n^{(3)} = 0.99937$ and $n^{(4)} = 0.99970$.

Step 7: The closeness coefficient of alternatives $B^{(z)}, (z = 1, 2, 3, 4)$ are computed by using Eq. (5.10) and the results are obtained as:

$$R^{(1)} = 0.26298; \quad R^{(2)} = 0.68716; \quad R^{(3)} = 0.527702; \quad R^{(4)} = 0.25051.$$

Step 8: Since $R^{(2)} > R^{(3)} > R^{(1)} > R^{(4)}$ and hence the given alternatives are ranked as $B^{(2)} \succ B^{(3)} \succ B^{(1)} \succ B^{(4)}$. Therefore, the best alternative is obtained as $B^{(2)}$.

5.3. Comparative Studies

To justify the predominance of our methodology, this section comprises the comparative investigation of the proposed approach with the prevailing methodologies [44, 45, 50, 51]. The computational procedure and the obtained results so described as follows:

1) *Comparison with the score function of Arora and Garg [44]:*

For any IFSN $(\mathbf{F}, \mathbf{E}) = (\zeta, \vartheta)$, Arora and Garg [44] defined the score function as

$$S(\mathbf{F}, \mathbf{E}) = \zeta - \vartheta$$

To implement the TOPSIS approach based on this score function, we utilize the information as presented in Table 2. Based on this table, we compute the score matrix between each alternative $B^{(z)}$ and the positive ideal r^+ and hence the PIA β^+ and NIA β^- are computed as

$$\beta^+ = \begin{bmatrix} (0.0667, 0.9333) & (0.0584, 0.9416) & (0.0450, 0.9550) & (0.0697, 0.9079) & (0.0880, 0.9120) \\ (0.0527, 0.9016) & (0.0863, 0.8414) & (0.0667, 0.9333) & (0.1027, 0.8652) & (0.0921, 0.9079) \\ (0.0239, 0.9661) & (0.0394, 0.9441) & (0.0120, 0.9840) & (0.0355, 0.9333) & (0.0317, 0.9550) \\ (0.0699, 0.9301) & (0.1586, 0.8414) & (0.0471, 0.9333) & (0.0792, 0.8652) & (0.1290, 0.8710) \\ (0.0239, 0.9661) & (0.0297, 0.9441) & (0.0091, 0.9772) & (0.0355, 0.9333) & (0.0317, 0.9550) \end{bmatrix}$$

$$\text{and} \quad \beta^- = \begin{bmatrix} (0.0355, 0.9529) & (0.0252, 0.9416) & (0.0138, 0.9683) & (0.03020.9303) & (0.0202, 0.9530) \\ (0.0527, 0.9301) & (0.0664, 0.8863) & (0.0271, 0.9729) & (0.0605, 0.8973) & (0.0407, 0.9465) \\ (0.0137, 0.9761) & (0.0172, 0.9774) & (0.0069, 0.9931) & (0.0206, 0.9729) & (0.0182, 0.9762) \\ (0.0307, 0.9596) & (0.0664, 0.9137) & (0.0152, 0.9848) & (0.0605, 0.9208) & (0.0407, 0.9079) \\ (0.0103, 0.9863) & (0.0172, 0.9828) & (0.0069, 0.9909) & (0.0152, 0.9794) & ,(0.0182, 0.9683) \end{bmatrix}$$

Now, the correlation coefficient of alternatives with PIA are computed as $p^{(1)} = 0.99935$, $p^{(2)} = 0.99961$, $p^{(3)} = 0.99945$ and $p^{(4)} = 0.99914$, while correlation coefficients with NIA are $n^{(1)} = 0.99972$, $n^{(2)} = 0.99927$, $n^{(3)} = 0.99937$ and $n^{(4)} = 0.99976$. Finally, the relative closeness coefficient of the given alternatives is obtained as

$$R^{(1)} = 0.29818; \quad R^{(2)} = 0.65022; \quad R^{(3)} = 0.53415; \quad R^{(4)} = 0.21257.$$

Thus, the ranking order is $B^{(2)} \succ B^{(3)} \succ B^{(1)} \succ B^{(4)}$ and hence the best alternative is $B^{(2)}$, which is same as that of proposed approach.

2) *Comparison with the score function of Arora and Garg [45]:*

For any IFSN $(\mathbf{F}, \mathbf{E}) = (\zeta, \vartheta)$, Arora and Garg [45] defined the score function as

$$S(\mathbf{F}, \mathbf{E}) = \frac{1 + \zeta - \vartheta}{2}$$

To compare the decision making process with this score function, we compute the score matrix between each alternative $B^{(z)}$ and the positive ideal r^+ and hence the PIA β^+ and NIA β^- are computed as

$$\beta^+ = \begin{bmatrix} (0.0667, 0.9333) & (0.0584, 0.9416) & (0.0450, 0.9550) & (0.0697, 0.9079) & (0.0880, 0.9120) \\ (0.0527, 0.9016) & (0.0863, 0.8414) & (0.0667, 0.9333) & (0.1027, 0.8652) & (0.0921, 0.9079) \\ (0.0239, 0.9661) & (0.0394, 0.9441) & (0.0120, 0.9840) & (0.0355, 0.9333) & (0.0317, 0.9550) \\ (0.0699, 0.9301) & (0.1586, 0.8414) & (0.0471, 0.9333) & (0.0792, 0.8652) & (0.1290, 0.8710) \\ (0.0239, 0.9661) & (0.0297, 0.9441) & (0.0091, 0.9772) & (0.0355, 0.9333) & (0.0317, 0.9550) \end{bmatrix}$$

$$\text{and} \quad \beta^{-} = \begin{bmatrix} (0.0355, 0.9529) & (0.0252, 0.9416) & (0.0138, 0.9683) & (0.03020.9303) & (0.0202, 0.9530) \\ (0.0527, 0.9301) & (0.0664, 0.8863) & (0.0271, 0.9729) & (0.0605, 0.8973) & (0.0407, 0.9465) \\ (0.0137, 0.9761) & (0.0172, 0.9774) & (0.0069, 0.9931) & (0.0206, 0.9729) & (0.0182, 0.9762) \\ (0.0307, 0.9596) & (0.0664, 0.9137) & (0.0152, 0.9848) & (0.0605, 0.9208) & (0.0407, 0.9079) \\ (0.0103, 0.9863) & (0.0172, 0.9828) & (0.0069, 0.9909) & (0.0152, 0.9794) & ,(0.0182, 0.9683) \end{bmatrix}$$

By using PIA and NIA, the closeness coefficients are obtained as

$$R^{(1)} = 0.29818; \quad R^{(2)} = 0.65022; \quad R^{(3)} = 0.53415; \quad R^{(4)} = 0.21257.$$

Hence, the ranking order of the alternative is $B^{(2)} \succ B^{(3)} \succ B^{(1)} \succ B^{(4)}$ and the best alternative is $B^{(2)}$.

3) *Comparison with similarity measure proposed by Sarala and Suganya [50]:*

For IFSSs, Sarala and Suganya [50] defined the similarity measure as

$$S((\mathbf{F}, \mathbf{E}), (\mathbf{K}, \mathbf{E})) = 1 - \frac{1}{2mn} \sum_{j=1}^{m} \sum_{i=1}^{n} \left[| \zeta_{\mathbf{F}_j}(u_i) - \zeta_{\mathbf{K}_j}(u_i) | + | \vartheta_{\mathbf{F}_j}(u_i) - \vartheta_{\mathbf{K}_j}(u_i) | \right]$$

To implement the TOPSIS approach based on this similarity measure, we have taken the weighted decision matrix as given in Table 2. Now, based on this table, we compute the similarity matrices between each alternative and the positive ideal r^{+} and get

$$\phi^{(1)} = \begin{bmatrix} 0.0413 & 0.0516 & 0.0383 & 0.0728 & 0.0547 \\ 0.0646 & 0.1225 & 0.0667 & 0.0816 & 0.0616 \\ 0.0188 & 0.0199 & 0.0140 & 0.0238 & 0.0344 \\ 0.0527 & 0.1000 & 0.0355 & 0.0738 & 0.0913 \\ 0.0120 & 0.0172 & 0.0159 & 0.0437 & 0.0383 \end{bmatrix}; \phi^{(2)} = \begin{bmatrix} 0.0667 & 0.0584 & 0.0316 & 0.0499 & 0.0880 \\ 0.0613 & 0.1137 & 0.0511 & 0.0977 & 0.0921 \\ 0.0289 & 0.0261 & 0.0094 & 0.0469 & 0.0383 \\ 0.0527 & 0.1586 & 0.0413 & 0.0816 & 0.0994 \\ 0.0238 & 0.0428 & 0.0105 & 0.0179 & 0.0249 \end{bmatrix}$$

$$\phi^{(3)} = \begin{bmatrix} 0.0471 & 0.0584 & 0.0450 & 0.0809 & 0.0752 \\ 0.0756 & 0.0901 & 0.0469 & 0.0909 & 0.0809 \\ 0.0188 & 0.0477 & 0.0069 & 0.0238 & 0.0210 \\ 0.0356 & 0.0764 & 0.0569 & 0.1070 & 0.1290 \\ 0.0289 & 0.0394 & 0.0120 & 0.0511 & 0.0317 \end{bmatrix}; \phi^{(4)} = \begin{bmatrix} 0.0667 & 0.0418 & 0.0227 & 0.0728 & 0.0336 \\ 0.0613 & 0.0901 & 0.0271 & 0.1188 & 0.0471 \\ 0.0238 & 0.0428 & 0.0120 & 0.0511 & 0.0210 \\ 0.0699 & 0.1000 & 0.0152 & 0.0698 & 0.0664 \\ 0.0259 & 0.0261 & 0.0080 & 0.0313 & 0.0249 \end{bmatrix}$$

Based on this ratings, we compute the PIA and NIA and hence the correlation coefficients between the given alternatives and PIA are computed as $p^{(1)} = 0.55378$, $p^{(2)} = 0.69324$, $p^{(3)} = 0.67131$ and $p^{(4)} = 0.47709$, while correlation coefficients with NIA are $n^{(1)} = 0.64577$, $n^{(2)} = 0.51651$, $n^{(3)} = 0.55318$ and $n^{(4)} = 0.76003$. The relative closeness coefficients of each alternative are computed as

$$R^{(1)} = 0.44253; \quad R^{(2)} = 0.61182; \quad R^{(3)} = 0.57616; \quad R^{(4)} = 0.31455.$$

Hence the ranking is $B^{(2)} \succ B^{(3)} \succ B^{(1)} \succ B^{(4)}$ and the best alternative is $B^{(2)}$.

4) *Comparison with similarity measure defined by Muthukumar and Krishnan [51]:*

Muthukumar and Krishnan [51] defined the similarity measure for a pair of IFSS as

$$S((\mathbf{F},\mathbf{E}),(\mathbf{K},\mathbf{E})) = \frac{\sum\left[\zeta_{\mathbf{F}}(e_i)\cdot\zeta_{\mathbf{K}}(e_i)+\vartheta_{\mathbf{F}}(e_i)\cdot\vartheta_{\mathbf{K}}(e_i)\right]}{\sum\left[\max(\zeta^2_{\mathbf{F}}(e_i),\zeta^2_{\mathbf{K}}(e_i))+\max(\vartheta^2_{\mathbf{F}}(e_i),\vartheta^2_{\mathbf{K}}(e_i))\right]}$$

By using this similarity measure, the matrices between each alternative and the positive ideal r^+ and get

$$\phi^{(1)} = \begin{bmatrix} 0.0186 & 0.0237 & 0.0166 & 0.0293 & 0.0250 \\ 0.0169 & 0.0506 & 0.0357 & 0.0335 & 0.0287 \\ 0.0070 & 0.0088 & 0.0061 & 0.0106 & 0.0124 \\ 0.0278 & 0.0484 & 0.0184 & 0.0249 & 0.0304 \\ 0.0052 & 0.0087 & 0.0047 & 0.0110 & 0.0166 \end{bmatrix}; \phi^{(2)} = \begin{bmatrix} 0.0357 & 0.0310 & 0.0095 & 0.0162 & 0.0480 \\ 0.0283 & 0.0637 & 0.0190 & 0.0346 & 0.0505 \\ 0.0123 & 0.0117 & 0.0035 & 0.0145 & 0.0166 \\ 0.0278 & 0.0929 & 0.0186 & 0.0335 & 0.0396 \\ 0.0071 & 0.0157 & 0.0046 & 0.0078 & 0.0094 \end{bmatrix}$$

$$\phi^{(3)} = \begin{bmatrix} 0.0247 & 0.0310 & 0.0235 & 0.0382 & 0.0340 \\ 0.0291 & 0.0372 & 0.0145 & 0.0439 & 0.0382 \\ 0.0070 & 0.0209 & 0.0035 & 0.0106 & 0.0093 \\ 0.0160 & 0.0362 & 0.0252 & 0.0453 & 0.0734 \\ 0.0123 & 0.0205 & 0.0061 & 0.0190 & 0.0163 \end{bmatrix}; \phi^{(4)} = \begin{bmatrix} 0.0357 & 0.0134 & 0.0071 & 0.0293 & 0.0106 \\ 0.0283 & 0.0372 & 0.0139 & 0.0587 & 0.0215 \\ 0.0071 & 0.0157 & 0.0061 & 0.0190 & 0.0093 \\ 0.0375 & 0.0484 & 0.0077 & 0.0327 & 0.0223 \\ 0.0093 & 0.0117 & 0.0035 & 0.0140 & 0.0094 \end{bmatrix}$$

and hence computed the PIA β^+ and NIA β^-. By using this information, the similarity between each alternative and PIA as well as NIA are obtained as, $p^{(1)} = 0.51418$, $p^{(2)} = 0.72021$, $p^{(3)} = 0.70004$, $p^{(4)} = 0.49318$ and $n^{(1)} = 0.69060$, $n^{(2)} = 0.49521$, $n^{(3)} = 0.56044$, $n^{(4)} = 0.71519$. Hence, the relative closeness coefficient degrees of the considered alternatives are computed as

$$R^{(1)} = 0.38907; \quad R^{(2)} = 0.64338; \quad R^{(3)} = 0.59438; \quad R^{(4)} = 0.35977.$$

and therefore, the ranking order is $B^{(2)} \succ B^{(3)} \succ B^{(1)} \succ B^{(4)}$ which implies that the best alternative is $B^{(2)}$ and coincides with the proposed approach ones.

From these studies and the comparative analysis, we conclude that the results computed through the presented approach coincide with the existing studies, and hence it is conservative. However, concerning the decision-making process, the main advantage of the proposed approach over the existing decision-making approaches is that they contain much more information to handle the uncertainties in the data. In it, information related to the object is expressed more precisely and objectively and hence is a useful tool for handling the imprecise and ambiguous information during the decision-making process. Furthermore, it is noted from the study that the computational procedure of the proposed approach is different from the existing approaches. This is due to the determination of the PIA and NIA. In the former approaches, these ideals, PIA and NIA, are computed based on the given decision matrices or by taking the extreme values only. Thus, the influence of rating values corresponding to each parameter doesn't impact other parameters and hence there is a certain loss of information during the process. On the other hand, in the proposed approach, these ideals are computed based on the impact of the maximum correlation coefficient on the given alternative ratings, and hence there is certainly less loss of information during the process. Finally, the strength of the correlation measures for each of these ideals is taken from the rating observation and hence compute the correlation between them to find the optimal solution. The advantage of the stated TOPSIS method with the stated correlation measures over the existing ones is that it not only taken into the account the degree of discrimination but also takes the degree of similarity between the observation, to avoid the decision only based on the negative reasons.

6. Conclusion

The key contribution of the work can be summarized below.

1) The examined study employs the IFSS to handle the inadequate, vague and conflicting data by considering membership degree, non-membership degree over the set of the parameters.

2) This paper offers new correlation measures to compute the degree of reliance between the two or more IFSSs. The various properties of the measures are also investigated in detail. The various existing correlation measures under the intuitionistic fuzzy set environment can be treated as a special case of the proposed measures by setting only one parameter in the analysis.

3) An extended TOPSIS method based on the proposed correlation measures has been introduced by considering the set of parameters and the experts. The advantages of the stated method are that it not only taken into account the degree of discrimination but also takes the degree of similarity between the observation, to avoid any short decision. Also, in the proposed approach, the ideal alternatives i.e., PIA (β^+) and NIA (β^-) are computed based on the index of the maximum correlation coefficient matrix of the given alternative ratings from its positive $r^+ = (1, 0)$. By using these PIA and NIA, correlation indices and hence define the relative closeness coefficient degree to rank the alternatives.

4) To demonstrate the performance of the stated algorithm, a numerical example is given and compare their results with the existing studies [44, 45, 50, 51]. It is concluded from this study that the proposed work gives more reasonable ways to handle the fuzzy information to solve the practical problems.

In the future, we shall lengthen the application of the proposed measures to the diverse fuzzy environment as well as different fields of application such as emerging decision problems, brain hemorrhage, risk evaluation, etc [58–60].

Conflict of Interest

The authors declare no conflicts of interest.

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