Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 5(4): 2909–2922.
DOI: 10.3934/math.2020188
Received: 3 February 2020
Accepted: 9 March 2020
Published: 19 March 2020

Research article

H_∞ filter design for a class of delayed Hamiltonian systems with fading channel and sensor saturation

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Abstract: This technical note is concerned with the problem of H_∞ filtering for a class of time delay nonlinear Hamiltonian systems with wireless network communication. The fading channel phenomenon and sensor saturation in the output measurements are considered. A H_∞ filter model is constructed to solve the issue of state estimation for the Hamiltonian systems with time varying delay in the state. Some sufficient conditions are proposed to obtain effective filter gain and achieve the H_∞ performance for the augmented system consisted of the Hamiltonian system and the filter. Simulation results illustrate the validity of the main results.

Keywords: delayed Hamiltonian systems; filter; fading channel; sensor saturation
Mathematics Subject Classification: 35Q93

1. Introduction

As a special class of nonlinear systems, port-Hamiltonian system has distinct structural feature and is easier to perform stability analysis than general nonlinear systems. The superiority of Hamiltonian systems is that Hamilton function in the system can be used as a candidate of Lyapunov function, and it can provide a more convenient way for system performance analysis. Many researches closely related to Hamiltonian systems can be found in recent years [1–5]. Some control synthesis problems based on Hamiltonian systems framework have been well studied [6–8]. Considering the inevitable time-delay phenomenon [9–12], some studies on the time-delay Hamiltonian system have also been investigated [13–16]. Besides, the issue of state estimation for Hamiltonian system has also received some attentions. For example, passivity-based approach is proposed in [17, 18] in order to obtain full-order observer for Hamiltonian systems. Output feedback stabilization based on observer method is concerned in [19] for a class of delayed Hamiltonian systems with external disturbances.

As we all know, to study state estimation and filtering technology has important significance in control field, such as target tracking and signal processing. Up to now, plenty of results have been reported in the literature on the state estimation problem for linear or nonlinear systems [20–24]. Among these results, the Kalman filtering scheme is proposed in [20] for the case that the dynamic system and its external noise information are completely known. In [21], H_∞ filtering was firstly introduced where the input signal is assumed to be energy limited. Compared to Kalman filter, H_∞ filter does not require too much known prior statistics of external noises. Up to now, H_∞ filter design has become a hot research topic in complex dynamic systems analysis, such as two-dimensional systems [22], time-delay systems [23], stochastic uncertain systems [24], delta operator systems [25], etc. However, the problem of H_∞ filter design for continuous Hamiltonian systems has not been studied yet.

On the other hand, with the rapid development of wireless communication technology, networked control system is becoming increasingly popular due to its strong applicability in factory automation, vehicle automated driving and other fields [26–28]. In practical applications, limited channel capacity always bring incomplete measurements which appear as the networked induced phenomenon. Comparing to the plenty of existing results on missing measurements, the fading channel problem in the wireless network communication systems has not yet received adequate research attentions. The existing studies show that the main causes of fading channel are some special physical factors such as reflection, diffraction and scattering [29, 30]. If the phenomenon of fading measurement can't be handled carefully, it will inevitably affect the stability of the systems. So far, some efforts on networked systems with fading channel have been made [31–34]. In these works, state estimator or H_∞ filter has been designed, feedback stabilization has been solved for the systems with fading channels.

In the process of communication signal transmission in networked system, another phenomenon that occurs frequently due to the physical limitations of system components is called saturation. Such nonlinear characteristic of sensor saturation not only degrades system performance, but also causes instability of the system more severely. Even worse, this feature is almost impossible to eliminate. By now, much progress has been made on filter/estimator design for systems with sensor saturation [35–37]. In [35], the fault estimation problem for a class of uncertain time-varying stochastic systems with randomly occurring fault and sensor saturations is investigated. A performance index function is proposed in [36] to deal with actuator saturation under denial-of-service attacks by an iterative adaptive dynamic programming algorithm. Nevertheless, as for filter design of Hamiltonian system with sensor saturation, to the authors' best knowledge, there is seldom relevant research reported.

Considering the factors discussed above, we can conclude that H_∞ filtering problems have achieved some preliminary results for linear and nonlinear systems. But when it comes to fading channel and sensor saturation for Hamiltonian systems, although it is important in control analysis, the H_∞ filtering problems have not been fully studied. In this paper, we will focus on this problem and propose some new results. The main contributions in this paper lie in two aspects: (1) a measurement output model with fading channel and sensor saturation is proposed for Hamiltonian systems with wireless network communication; (2) the filter design problem is, for the first time, solved for time delay Hamiltonian system considering network induced phenomenon.

The remainder of this paper is organized as follows. The considered class of Hamiltonian system is introduced and some preliminaries are presented in Section 2. In Section 3, the main results of this paper are presented, and H_∞ filtering performance analysis of the augmented system is given. Furthermore, some sufficient conditions are given to obtain appropriate filter gain. Two simulation examples are given in Section 4 to illustrate the validity of the main results. Finally, we conclude this paper in Section 5.

2. Problem formulation and preliminaries

Consider the following time varying delay Hamiltonian system

$$\begin{cases} \dot{x}(t) = A\nabla H(x(t)) + B\nabla H(x(t-d(t))) + Lv_1(t) \\ z(t) = M\nabla H(x(t)) \end{cases} \tag{2.1}$$

where $x(t) = \begin{bmatrix} x_1(t) & \cdots & x_n(t) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^n$ stands for the system state vector, $z(t) = \begin{bmatrix} z_1(t) & \cdots & z_p(t) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^p$ represents the output to be estimated; $v_1(t) \in \mathbb{R}^s$ represents the disturbance input and belongs to $\mathcal{L}_2([0,\infty),\mathbb{R}^s)$; $\nabla H(x(t))$ is the gradient of $H(x(t))$, $\nabla H(x) := \partial H(x)/\partial x$; the Hamiltonian $H(x(t)) : \mathbb{R}^n \mapsto \mathbb{R}$ is smooth which represents the total energy, $H(x) \geq 0, H(0) = 0$; $A \in \mathbb{R}^{n\times n}$ is the structure matrix of the Hamiltonian system with $A^{\mathrm{T}} + A \leq 0$; B, L and M are constant matrices with appropriate dimensions. $d(t)$ denotes the time-varying delay, and satisfies the following assumptions:

$$0 \leq d(t) \leq \tau, \;\; \dot{d}(t) \leq \mu < 1, \tag{2.2}$$

where τ and μ are scalars.

The fading channel and sensor saturation are considered in the measurement model in this paper. The actual measurement output is described as

$$y(t) = C(t)N\nabla H(x(t)) + \sigma(E\nabla H(x(t))) + Dv_2(t), \tag{2.3}$$

where $y(t) = \begin{bmatrix} y_1(t) & \cdots & y_q(t) \end{bmatrix} \in \mathbb{R}^q$ is the actual measurement output; $v_2(t) \in \mathbb{R}^q$ stands for the measurement noise and belongs to $\mathcal{L}_2([0,\infty),\mathbb{R}^q)$; N, E and D are known real matrix with appropriate dimensions; N is a full rank matrix; $C(t) = \mathrm{diag}\{\alpha_1(t),\cdots,\alpha_q(t)\} \in \mathbb{R}^{q\times q}$ denotes the memoryless multiplicative fading channel. $\alpha_i(t)$ is independently and identically distributed for each channel, and its probability density function is distributed on the interval $[0,1]$ with $\mathbb{E}\{\alpha_{\mathrm{i}}(\mathrm{t})\} = \bar{\alpha}_{\mathrm{i}}, \mathrm{Var}\{\alpha_{\mathrm{i}}(\mathrm{t})\} = \sigma_{\mathrm{i}}^2$, where $\bar{\alpha}_i$ represents the mathematical expectation of $\alpha_i(t)$ and σ_i^2 is the variance of $\alpha_i(t)$. Both $\bar{\alpha}_i$ and σ_i^2 are known constants.

Remark 1. *In wireless network systems, signals are transmitted between the various sensors through the channels. These measurement signals experience stochastic fading or information losses during the transmissions. This random fading phenomenon is manifested in Eq* 2.3, *where the random variable $\alpha_i(t)$ obeys any probability distribution over the interval* $[0,1]$. *The probability of the attenuation coefficient $\alpha_i(t)$ can be estimated by statistical tests in practical applications. Moreover, in addition to the random fading phenomenon, data error or packet loss in some digital network systems can also be expressed by* (2.3).

The saturation function $\sigma(\cdot)$ is defined as

$$\sigma(\vartheta) = \begin{bmatrix} \sigma(\vartheta_1) & \cdots & \sigma(\vartheta_q) \end{bmatrix}^{\mathrm{T}}, \quad \forall \vartheta \in \mathbb{R}^q$$

where $\sigma(\vartheta_i) = \mathrm{sign}(\vartheta_{\mathrm{i}})\min\{1,|\vartheta_{\mathrm{i}}|\}$ for each $i = 1,\cdots,q$. Here, the notation of "sign" denotes the signum function, ϑ is the saturation level. There exists scalar $0 < b_i < 1$ such that

$$[\sigma(\vartheta_i) - b_i\vartheta_i]\,[\sigma(\vartheta_i) - \vartheta_i] \leqslant 0. \tag{2.4}$$

For system (2.1), the filter is constructed as the following structure:

$$\begin{cases} \dot{\hat{x}}(t) = A\nabla H(\hat{x}(t)) + B\nabla H(\hat{x}(t-d(t))) + K[y(t) - \bar{C}N\nabla H(\hat{x}(t))] \\ \hat{z}(t) = M\nabla H(\hat{x}(t)), \end{cases} \tag{2.5}$$

where $\hat{x}(t) \in \mathbb{R}^n$ represents the estimate of the state, $\hat{z}(t) \in \mathbb{R}^p$ stands for the estimate of the output $z(t)$, $\bar{C} = \text{diag}\{\bar{\alpha}_1, \cdots, \bar{\alpha}_q\}$, and K is the filter gain to be determined.

Letting $\eta(t) = [x^{\mathrm{T}}(t),\ \hat{x}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\tilde{z}(t) = [z^{\mathrm{T}}(t),\ \hat{z}^{\mathrm{T}}(t)]^{\mathrm{T}}$, we get the following augmented system:

$$\begin{cases} \dot{\eta}(t) = A^*\nabla H(\eta(t)) + B^*\nabla H(\eta(t-d(t))) + K^*\sigma(EE_1\nabla H(\eta(t))) + L^*v(t) \\ \tilde{z}(t) = M^*\nabla H(\eta(t)), \end{cases} \tag{2.6}$$

where

$$A^* = \begin{bmatrix} A & 0 \\ KC(t)N & A - K\bar{C}N \end{bmatrix},\ B^* = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix},\ L^* = \begin{bmatrix} L & 0 \\ 0 & KD \end{bmatrix},$$

$$K^* = \begin{bmatrix} 0 \\ K \end{bmatrix},\ E_1 = \begin{bmatrix} I \\ 0 \end{bmatrix}^{\mathrm{T}},\ M^* = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}^{\mathrm{T}},\ v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix},$$

$$\nabla H(\eta(t)) = \begin{bmatrix} \nabla^{\mathrm{T}} H(x(t)) \\ \nabla^{\mathrm{T}} H(\hat{x}(t)) \end{bmatrix},\ \nabla H(\eta(t-d(t))) = \begin{bmatrix} \nabla H(x(t-d(t))) \\ \nabla H(\hat{x}(t-d(t))) \end{bmatrix}.$$

According to (2.4), the saturation function $\sigma(\cdot)$ satisfies

$$[\sigma(E_1E\nabla H(\eta(t))) - \Lambda E_1E\nabla H(\eta(t))]^{\mathrm{T}} \cdot [\sigma(E_1E\nabla H(\eta(t))) - E_1E\nabla H(\eta(t))] \leqslant 0, \tag{2.7}$$

where $\Lambda = \text{diag}\{b_1, \cdots, b_q\}$.

We give the following assumption which will be used in the following proofs.

Assumption 1. *The Hamilton function $H(\eta(t))$ and its gradient $\nabla H(\eta(t))$ satisfies*

1) $\varepsilon_1(\|\eta(t)\|) \leq H(\eta(t)) \leq \varepsilon_2(\|\eta(t)\|)$,

2) $\iota_1(\|\eta(t)\|) \leq \nabla^{\mathrm{T}} H(\eta(t))\nabla H(\eta(t)) \leq \iota_2(\|\eta(t)\|)$,

where $\varepsilon_1, \varepsilon_2, \iota_1, \iota_2$ are class-$\mathcal{K}$ functions.

Our objective of this paper is to design a filter (2.5) for the delayed Hamiltonian system with fading channel and sensor saturation. More specifically, we need to find out the filtering gain matrix K such that the augmented system (2.6) satisfies the following requirements simultaneously:

i) The augmented system (2.6) with $v(t) = 0$ is asymptotically stable in the mean square;

ii) For a given scalar $\gamma > 0$, under the zero-initial condition, the system (2.6) with $v(t) \neq 0$ is $\mathcal{L}_2$ gain output stability, i.e.,

$$\|\tilde{z}(t)\|_2 \leq \gamma\|v(t)\|_2,\ \ \forall v(t) \neq 0. \tag{2.8}$$

3. Main results

In this section, let us investigate the filtering performance and filter design problems for system (2.6). Firstly, we shall propose the following filtering performance criterion for the delayed Hamiltonian systems (2.1) and (2.3) with fading channel and sensor saturation.

Theorem 1. *Given scalar $\mu < 1$ and a time-varying delay $d(t)$ satisfying (2.2), if there exist matrices $Q = Q^{\mathrm{T}} > 0$, $Y = Y^{\mathrm{T}}$ with proper dimensions and filtering gain K such that*

$$\Phi_1 = \begin{bmatrix} \Pi_1 & B^* - (Y/2) & -Y & \Pi_2 \\ * & -(1-\mu)Q & Y/2 & 0 \\ * & * & Y & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{3.1}$$

holds, then the augmented system (2.6) is mean-square asymptotically stable with $v(t) = 0$, where

$$\Pi_1 = \bar{A}^* + \bar{A}^{*\mathrm{T}} + Q + Y - E^{\mathrm{T}} E_1^{\mathrm{T}} \Lambda^{\mathrm{T}} E_1 E,$$

$$\Pi_2 = (K^{*\mathrm{T}} + K^* + E^{\mathrm{T}} E_1^{\mathrm{T}} \Lambda^{\mathrm{T}} + E_1 E)/2,$$

and

$$\bar{A}^* = \begin{bmatrix} A & 0 \\ K\bar{C}N & A - K\bar{C}N \end{bmatrix}.$$

Proof: Choose a Lyapunov functional candidate as follows:

$$V(t) = 2H(\eta(t)) + \int_{t-d(t)}^{t} \nabla^{\mathrm{T}} H(\eta(\theta)) Q \nabla H(\eta(\theta)) \mathrm{d}\theta. \tag{3.2}$$

Due to Assumption 1, we have

$$2\varepsilon_1(\|\eta(t)\|) \leq V(t) \leq 2\varepsilon_2(\|\eta(t)\|) + \tau\pi\iota_2(\eta(t)),$$

where $\pi = \lambda_{max}(Q)$, $\lambda_{max}(\cdot)$ (respectively, $\lambda_{min}(\cdot)$) means the largest (respectively, smallest) eigenvalue.

Thus, the derivative of $V(t)$ along the trajectory of system (2.6) is given as

$$\begin{aligned}
\frac{\mathrm{d}(\mathbb{E}[V(t)])}{\mathrm{dt}} &= \mathbb{E}[\mathcal{L}V(t)] \\
&= \mathbb{E}[2\nabla^{\mathrm{T}} H(\eta(t))\dot{\eta}(t) + \nabla^{\mathrm{T}} H(\eta(t)) Q \nabla H(\eta(t)) \\
&\quad -(1-\dot{d}(t))\nabla^{\mathrm{T}} H(\eta(t-d(t))) Q \nabla H(\eta(t-d(t)))] \\
&= \mathbb{E}[\nabla^{\mathrm{T}} H(\eta(t))(A^* + A^{*\mathrm{T}} + Q)\nabla H(\eta(t)) + 2\nabla^{\mathrm{T}} H(\eta(t)) B^* \nabla H(\eta(t-d(t))) \\
&\quad +2\nabla^{\mathrm{T}} H(\eta(t)) K^* \sigma(EE_1 \nabla H(\eta(t))) - (1-\dot{d}(t))\nabla^{\mathrm{T}} H(\eta(t-d(t))) Q \nabla H(\eta(t-d(t)))] \\
&= \nabla^{\mathrm{T}} H(\eta(t))(\bar{A}^* + \bar{A}^{*\mathrm{T}} + Q)\nabla H(\eta(t)) + 2\nabla^{\mathrm{T}} H(\eta(t)) B^* \nabla H(\eta(t-d(t))) \\
&\quad +2\nabla^{\mathrm{T}} H(\eta(t)) K^* \sigma(EE_1 \nabla H(\eta(t))) - (1-\dot{d}(t))\nabla^{\mathrm{T}} H(\eta(t-d(t))) Q \nabla H(\eta(t-d(t))) \\
&\leq \nabla^{\mathrm{T}} H(\eta(t))(\bar{A}^* + \bar{A}^{*\mathrm{T}} + Q)\nabla H(\eta(t)) + 2\nabla^{\mathrm{T}} H(\eta(t)) B^* \nabla H(\eta(t-d(t))) \\
&\quad +2\nabla^{\mathrm{T}} H(\eta(t)) K^* \sigma(EE_1 \nabla H(\eta(t))) - (1-\mu)\nabla^{\mathrm{T}} H(\eta(t-d(t))) Q \nabla H(\eta(t-d(t))),
\end{aligned} \tag{3.3}$$

where the last inequality is based on (2.2).

It is easy to know that, for any symmetric matrix Y with proper dimensions, the following equation is true

$$[\nabla^{\mathrm{T}} H(\eta(t)) - \int_{t-d(t)}^{t} [\nabla^{\mathrm{T}} H(\eta(s))]' \mathrm{d}s] Y [\nabla H(\eta(t)) - \nabla H(\eta(t-d(t))) - \int_{t-d(t)}^{t} [\nabla H(\eta(s))]' \mathrm{d}s] \equiv 0, \tag{3.4}$$

where $[\ \cdot\ ]'$ represents the derivative of the matrix in parentheses.

Furthermore, (3.3) can be rewritten as

$$\begin{aligned}
\frac{\mathrm{d}(\mathbb{E}[V(t)])}{\mathrm{dt}} \leq\ & \nabla^{\mathrm{T}}H(\eta(t))(\bar{A}^{*}+\bar{A}^{*\mathrm{T}}+Q+Y)\nabla H(\eta(t))+2\nabla^{\mathrm{T}}H(\eta(t))B^{*}\nabla H(\eta(t-d(t))) \\
& -\nabla^{\mathrm{T}}H(\eta(t))Y\nabla H(\eta(t-d(t)))+2\nabla^{\mathrm{T}}H(\eta(t))K^{*}\sigma(EE_1\nabla H(\eta(t))) \\
& -2\nabla^{\mathrm{T}}H(\eta(t))Y\int_{t-d(t)}^{t}[\nabla H(\eta(s))]'\mathrm{ds}-(1-\mu)\nabla^{\mathrm{T}}\mathrm{H}(\eta(\mathrm{t}-\mathrm{d}(\mathrm{t}))) \\
& \cdot Q\nabla H(\eta(t-d(t)))+\int_{t-d(t)}^{t}[\nabla^{\mathrm{T}}H(\eta(s))]'\mathrm{ds}Y\nabla H(\eta(t-d(t))) \\
& +\int_{t-d(t)}^{t}[\nabla^{\mathrm{T}}H(\eta(s))]'\mathrm{ds}Y\int_{t-d(t)}^{t}[\nabla H(\eta(s))]'\mathrm{ds}. \qquad (3.5)
\end{aligned}$$

From (2.7), the above formula can be rewritten as the following formula

$$\begin{aligned}
\frac{\mathrm{d}(\mathbb{E}[V(t)])}{\mathrm{dt}} \leq\ & \nabla^{\mathrm{T}}H(\eta(t))(\bar{A}^{*}+\bar{A}^{*\mathrm{T}}+Q+Y)\nabla H(\eta(t))+2\nabla^{\mathrm{T}}H(\eta(t))B^{*}\nabla H(\eta(t-d(t))) \\
& -\nabla^{\mathrm{T}}H(\eta(t))Y\nabla H(\eta(t-d(t)))+2\nabla^{\mathrm{T}}H(\eta(t))K^{*}\sigma(EE_1\nabla H(\eta(t))) \\
& -2\nabla^{\mathrm{T}}H(\eta(t))Y\int_{t-d(t)}^{t}[\nabla H(\eta(s))]'\mathrm{ds}-(1-\mu)\nabla^{\mathrm{T}}H(\eta(t-d(t)))Q\nabla H(\eta(t-d(t))) \\
& +\int_{t-d(t)}^{t}[\nabla^{\mathrm{T}}H(\eta(s))]'\mathrm{ds}Y\nabla H(\eta(t-d(t)))+\int_{t-d(t)}^{t}[\nabla^{\mathrm{T}}H(\eta(s))]'\mathrm{ds}Y\int_{t-d(t)}^{t}[\nabla H(\eta(s))]'\mathrm{ds} \\
& -\sigma^{\mathrm{T}}(E_1E\nabla H(\eta(t)))\sigma(E_1E\nabla H(\eta(t)))+\nabla^{\mathrm{T}}H(\eta(t))E^{\mathrm{T}}E_1^{\mathrm{T}}\Lambda^{\mathrm{T}}\sigma(E_1E\nabla H(\eta(t))) \\
& +\sigma^{\mathrm{T}}(E_1E\nabla H(\eta(t)))E_1E\nabla H(\eta(t))-\nabla^{\mathrm{T}}H(\eta(t))E^{\mathrm{T}}E_1^{\mathrm{T}}\Lambda^{\mathrm{T}}E_1E\nabla H(\eta(t)) \\
=\ & \xi_1^{\mathrm{T}}\Phi_1\xi_1, \qquad (3.6)
\end{aligned}$$

where

$$\xi_1=\begin{bmatrix}\nabla H(\eta(t)) \\ \nabla H(\eta(t-d(t))) \\ \int_{t-d(t)}^{t}[\nabla H(\eta(s))]'\mathrm{ds} \\ \sigma(E_1E\nabla H(\eta(t)))\end{bmatrix}.$$

According to Assumption 1 and (3.1), we have

$$\frac{\mathrm{d}(\mathbb{E}[V(t)])}{\mathrm{dt}} \leq \xi_1^{\mathrm{T}}\Phi_1\xi_1 \leq -\varphi\|\nabla H(\eta(t))\|^2 \leq -\varphi\iota_1(\|\eta(t)\|)<0, \qquad (3.7)$$

where $\varphi=\lambda_{min}(-\Phi_1)$.

By Lyapunov-Krasovskii stability theorem, we can conclude that the augmented system (2.6) is mean-square asymptotically stable with $v(t)=0$. The proof is completed. □

Theorem 1 shows that the system is asymptotically stable in mean square without external disturbance. Next, we prove that the system satisfies $\mathcal{L}_2$ gain output stability in the presence of external disturbance.

Theorem 2. *Given H_∞ performance index $\gamma>0$ and a known scalar $\mu<1$, the system (2.6) satisfies:*

1) asymptotically stable in the mean square with $v(t)=0$;

2) for all $v(t) \neq 0$, under the zero-initial condition, the system (2.6) satisfies $\mathcal{L}_2$ gain output stability if there exist matrices $Q = Q^{\mathrm{T}} > 0$, $Y = Y^{\mathrm{T}}$ and filter gain K such that

$$\Phi_2 = \begin{bmatrix} \bar{\Pi}_1 & B^* - (Y/2) & -Y & \bar{\Pi}_2 & L^* \\ * & -(1-\mu)Q & Y/2 & 0 & 0 \\ * & * & Y & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{3.8}$$

where

$$\begin{aligned} \bar{\Pi}_1 &= A^* + A^{*\mathrm{T}} + Q + Y - E^{\mathrm{T}} E_1^{\mathrm{T}} \Lambda^{\mathrm{T}} E_1 E + M^{*\mathrm{T}} M^*, \\ \bar{\Pi}_2 &= (K^{*\mathrm{T}} + K^* + E^{\mathrm{T}} E_1^{\mathrm{T}} \Lambda^{\mathrm{T}} + E_1 E)/2. \end{aligned}$$

Proof: According to Theorem 1, the system (2.6) with $v(t) = 0$ is mean-square asymptotically stable. Next we prove that for all $v(t) \neq 0$, under the zero initial condition, the system (2.6) satisfies $\mathcal{L}_2$ gain output stability.

Consider a Lyapunov functional candidate which is defined in (3.2). The derivative of $V(t)$ along the trajectory of system (2.6) satisfies:

$$\begin{aligned} \frac{\mathrm{d}(\mathbb{E}[V(t)])}{\mathrm{dt}} \leq\ & \nabla^{\mathrm{T}} H(\eta(t))(\bar{A}^* + \bar{A}^{*\mathrm{T}} + Q + Y - E_1^{\mathrm{T}} E^{\mathrm{T}} \Lambda^{\mathrm{T}} E E_1)\nabla H(\eta(t)) \\ & + 2\nabla^{\mathrm{T}} H(\eta(t)) B^* \nabla H(\eta(t-d(t))) - \nabla^{\mathrm{T}} H(\eta(t)) Y \nabla H(\eta(t-d(t))) \\ & - 2\nabla^{\mathrm{T}} H(\eta(t)) Y \int_{t-d(t)}^{t} [\nabla H(\eta(s))]' \mathrm{ds} - (1-\mu)\nabla^{\mathrm{T}} H(\eta(t-d(t))) Q \nabla H(\eta(t-d(t))) \\ & + \int_{t-d(t)}^{t} [\nabla^{\mathrm{T}} H(\eta(s))]' \mathrm{ds} Y \nabla H(\eta(t-d(t))) + \int_{t-d(t)}^{t} [\nabla^{\mathrm{T}} H(\eta(s))]' \mathrm{ds} Y \int_{t-d(t)}^{t} [\nabla H(\eta(s))]' \mathrm{ds} \\ & - \sigma^{\mathrm{T}}(E_1 E \nabla H(\eta(t))) \sigma(E_1 E \nabla H(\eta(t))) + \nabla^{\mathrm{T}} H(\eta(t)) E^{\mathrm{T}} E_1^{\mathrm{T}} \Lambda^{\mathrm{T}} \sigma(E_1 E \nabla H(\eta(t))) \\ & + \sigma^{\mathrm{T}}(E_1 E \nabla H(\eta(t))) E_1 E \nabla H(\eta(t)) - \nabla^{\mathrm{T}} H(\eta(t)) E^{\mathrm{T}} E_1^{\mathrm{T}} \Lambda^{\mathrm{T}} E_1 E \nabla H(\eta(t)) \\ & + 2\nabla^{\mathrm{T}} H(\eta(t)) L^* v(t) + \gamma^2 v^{\mathrm{T}}(t) v(t) - \gamma^2 v^{\mathrm{T}}(t) v(t) + \tilde{z}^{\mathrm{T}}(t)\tilde{z}(t) - \tilde{z}^{\mathrm{T}}(t)\tilde{z}(t) \\ =\ & \xi_2^{\mathrm{T}} \Phi_2 \xi_2 + \gamma^2 v^{\mathrm{T}}(t) v(t) - \tilde{z}^{\mathrm{T}}(t)\tilde{z}(t), \end{aligned} \tag{3.9}$$

where

$$\xi_2 = \begin{bmatrix} \nabla H(\eta(t)) \\ \nabla H(\eta(t-d(t))) \\ \int_{t-d(t)}^{t} [\nabla H(\eta(s))]' \mathrm{ds} \\ \sigma(E_1 E \nabla H(\eta(t))) \\ v(t) \end{bmatrix}.$$

From (3.8), we obtain

$$\frac{\mathrm{d}(\mathbb{E}[V(t)])}{\mathrm{dt}} \leq \gamma^2 v^{\mathrm{T}}(t) v(t) - \tilde{z}^{\mathrm{T}}(t)\tilde{z}(t). \tag{3.10}$$

Furthermore, simultaneously integrating both sides of (3.10) from $[0, T]$, we have

$$\mathbb{E}[V(T)] - \mathbb{E}[V(0)] \leq \gamma^2 \int_0^T v^{\mathrm{T}}(t) v(t) \mathrm{dt} - \int_0^T \tilde{z}^{\mathrm{T}}(t)\tilde{z}(t) \mathrm{dt}.$$

Since $\mathbb{E}[V(T)] - \mathbb{E}[V(0)] \geq 0$, then

$$\int_0^T \tilde{z}^{\mathrm{T}}(t)\tilde{z}(t)\mathrm{d}t \leq \gamma^2 \int_0^T v^{\mathrm{T}}(t)v(t)\mathrm{d}t. \tag{3.11}$$

According to the definition of $\mathcal{L}_2$ norm, there is $\|\tilde{z}(t)\|_2 \leq \gamma\|v(t)\|_2$. This completes the proof. □

Remark 2. *There are two main difficulties in the theoretical derivation of this paper. The first difficulty is how to establish the measurement output model. Due to the particularity of the Hamiltonian system, its measurement output model is different from the general ones. Considering fully the structural properties of the system and combining with the existing literatures on the sensor model, we introduce a constant matrix N with suitable dimension in constructing the sensor model. The second difficulty is how to solve the random term in the channel attenuation coefficient. For overcoming this difficulty, we use the mean value in the stability derivation using Lyapunov-Krasovsill functional theorem.*

Remark 3. *Specifically, when the Hamiltonian in the Hamiltonian system (2.1) is* $H(x) = \frac{1}{2}x^2$, *then the system can be expressed as*

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t - d(t)) + Lv_1(t) \\ z(t) = Mx(t). \end{cases} \tag{3.12}$$

Obviously, it is a linear system formulation. We may achieve the H_∞ *filter result for system (3.12) according to Theorem 1 and Theorem 2. It can be seen that the main results of this paper can also applicable for linear systems in the form of (3.12).*

4. Illustrative examples

In this section, our main purpose is to demonstrate the feasibility and validity of the method proposed in this paper. Two examples are given below.

Example 1. *Consider the Hamiltonian systems described by (2.1) with the following parameters:*

$$M = \begin{bmatrix} -0.3 & 0.2 \\ 0.6 & -0.7 \end{bmatrix}, \quad L = \begin{bmatrix} 0.8 & 0.5 \\ 0.3 & 0.6 \end{bmatrix}, \quad A = \begin{bmatrix} -0.63 & -0.7 \\ 0 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.15 \end{bmatrix},$$

and the Hamiltonian H(x) is given as $H(x) = \frac{1}{2}x_1^2 + 1 - \cos x_2$. *It can be seen that the above nonlinear functions satisfy Assumption 1 with* $\varepsilon_2 = 2$ *and* $\iota_2 = 1$. *The time-varying delay is taken as* $d(t) = (\pi + 2\arctan t)/6$ *from which we have* $\mu = 0.33$ *and* $\tau = 1.04$.

In this example, the intensity of the measurement noise is given by $D = [0.5 \;\; 0.7]^{\mathrm{T}}$, *the constant matrices are represented by*

$$N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0.3 & 0.2 \\ 0.4 & 0.2 \end{bmatrix}.$$

The information received by each sensor is transmitted over the network with fading channel. The channel attenuation coefficient of each sensor channel can be obtained by statistical experiments, and the measurement signal has the following probability density function

$$p_t^1(s) = \begin{cases} 0.05, & s = 0 \\ 0.10, & s = 0.5 \\ 0.85, & s = 1 \end{cases}, \quad p_t^2(s) = \begin{cases} 0.06, & s = 0 \\ 0.30, & s = 0.5 \\ 0.60, & s = 1 \end{cases}$$

with expectation $\bar{\alpha}_1 = 0.9$ and variance $\sigma_1^2 = 0.065$, expectation $\bar{\alpha}_2 = 0.75$ and variance $\sigma_2^2 = 0.124$.

The H_∞ performance attenuation level is set as $\gamma = 0.6$. We solve the LMI (3.8) via Matlab and obtain the filter gain

$$K = \begin{bmatrix} 1.3635 & -0.7680 \\ -0.9993 & 0.4183 \end{bmatrix}.$$

In the simulation, the external disturbance inputs are selected as $v_1(t) = \sin(10t/3)$, $v_2(t) = \cos(10t/3)$. The initial state of the system and its estimate are chosen as $x(0) = [0.4,\ 0.4]^{\mathrm{T}}$ and $\hat{x}(0) = [0.5,\ 0.5]^{\mathrm{T}}$. Simulation results for this example are shown in Figures 1–3. The states of the Hamiltonian system x_i $(i = 1, 2)$ and their estimates are shown in Figures 1 and 2. The output of the Hamiltonian system z and its estimates are depicted in Figure 3. The simulation results show that the proposed filters can guarantee the asymptotically stability and $\mathcal{L}_2$ gain output stability of delayed Hamiltonian system under fading channel and sensor saturation.

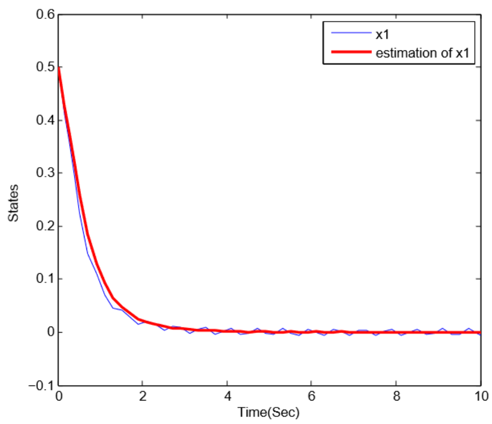

Figure 1. State trajectory of x_1 and its estimate.

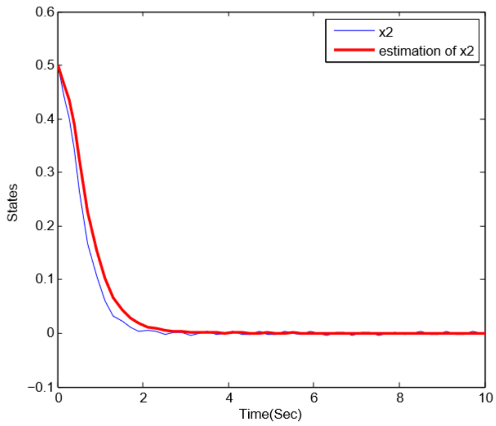

Figure 2. State trajectory of x_2 and its estimate.

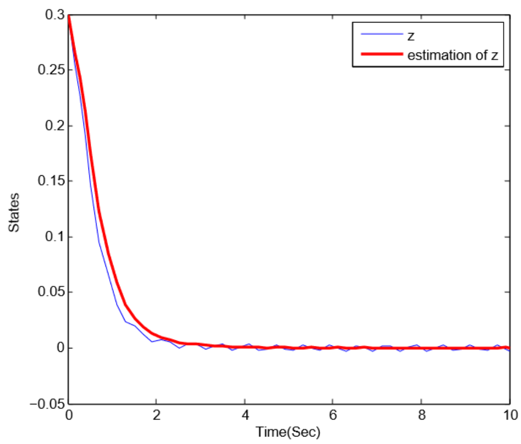

Figure 3. Output z and its estimate.

Example 2. *The selected system model in this example is a test runs of an aircraft which is powered by two F-404 engines. We intend to track this aircraft through wireless communication, which is affected by fading channel and sensor saturation. In practical applications, the probability $C(t)$ can be determined by statistical tests. Consider the system model (2.1) in [32] and [38], where the nominal system matrix A is linearized models from the F-404 aircraft engine system:*

$$A = \begin{bmatrix} -0.9158 & -0.5380 & -0.2698 \\ 0.1368 & -1.5070 & -0.3625 \\ 0.2869 & 0.1304 & -1.1990 \end{bmatrix}.$$

Each channel measurement attenuation coefficient is chosen as follows

$$p_t^1(s) = \begin{cases} 0.05, & s = 0 \\ 0.10, & s = 0.5 \\ 0.85, & s = 1 \end{cases}, \quad p_t^2(s) = \begin{cases} 0.06, & s = 0 \\ 0.30, & s = 0.5 \\ 0.60, & s = 1 \end{cases}, \quad p_t^3(s) = \begin{cases} 0.04, & s = 0 \\ 0.10, & s = 0.5 \\ 0.75, & s = 1 \end{cases}.$$

Through the above probability density functions, we can obtained the mathematical expectation $\bar{\gamma}_i$ and variance $\sigma_i^2 (i = 1, 2, 3)$ are $0.9, 0.75, 0.8$ and $0.065, 0.124, 0.248$, respectively.

As discussed in [32], almost all aircraft engine systems are subject to uncontrolled external forces. These external disturbances, such as gravity gradients, wind gusts, sensor and actuator noise can affect the stability and reduce the control performance of the system. In this example, the intension of measurement noise is given by $D = \begin{bmatrix} 0.1 & 0.2 & 0.2 \end{bmatrix}^{\mathrm{T}}$.

In this model, a series of path points (x_1, x_2, x_3) are used to track the position of the aircraft, x_1 and x_2 represent the horizontal positions, and x_3 represents the altitude of the aircraft. Now, we design the time-varying filter in the form of (2.5) in the network communication environment to track the state of the F-404 aircraft engine system. Actually, when we model aircraft engine system, fading channel and sensor saturation must be taken into account. The corresponding parameters are given as follows:

$$M = \begin{bmatrix} 0.3 & 0.2 & 0.7 \\ 0.8 & 0.7 & 0.5 \\ 0.8 & 0.83 & 0.6 \end{bmatrix}, \quad L = \begin{bmatrix} 0.8 \\ 0.6 \\ 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.75 & 0 \\ 0 & 0 & -0.86 \end{bmatrix},$$

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0.27 & 0.16 & 0.32 \\ 0.23 & 0.14 & 0.21 \\ 0.27 & 0.31 & 0.14 \end{bmatrix},$$

and select the Hamilton function as

$$H(x) = \frac{1}{2}x_1^2 + 1 - \cos x_2 + \frac{1}{2}x_3^2,$$

In this example, the H_∞ performance attenuation level is taken as $\gamma = 0.7$. We solve the LMI (3.8) via MATLAB and get the filter gain

$$K = \begin{bmatrix} -0.4083 & 0.2026 & 0.2878 \\ 0.1954 & -0.7588 & 0.1719 \\ 0.2608 & 0.1630 & -0.6488 \end{bmatrix}.$$

The exogenous disturbance inputs are selected as $v_1(t) = \sin(10t/3)$, $v_2(t) = \cos(10t/3)$, which are same as in Example 1. The initial states of the system and its estimate are selected as $x(0) = [0.4,\ 0.4,\ 0,4]^{\mathrm{T}}$ and $\hat{x}(0) = [0.5,\ 0.5,\ 0.5]^{\mathrm{T}}$. Simulation results for this example are shown in Figures 4–7. The states of the system $x_i(i = 1, 2, 3)$ and their estimates are shown in Figures 4–6. The output of the system z and its estimates are presented in Figure 7. The simulation results show that the filters designed according to Theorem 1 and Theorem 2 can accurately estimate the system states and make the augmented system $\mathcal{L}_2$ gain output stability in the presence of fading channel and sensor saturation.

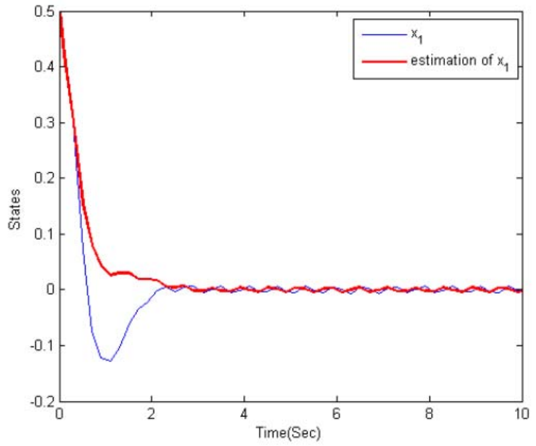

Figure 4. State trajectory of x_1 and its estimate.

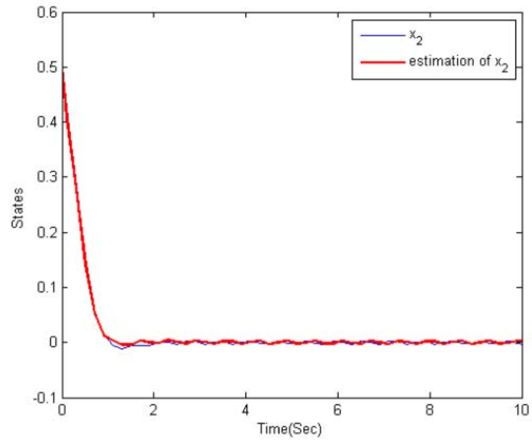

Figure 5. State trajectory of x_2 and its estimate.

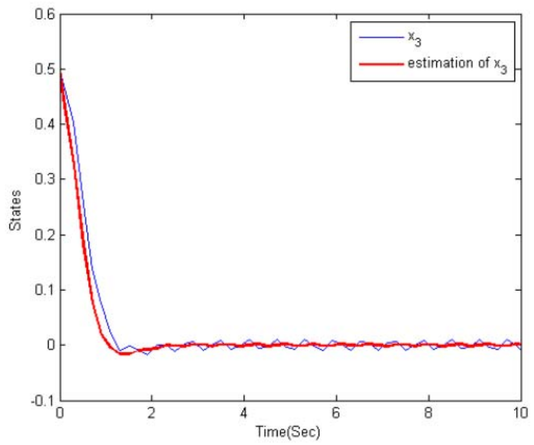

Figure 6. State trajectory of x_3 and its estimate.

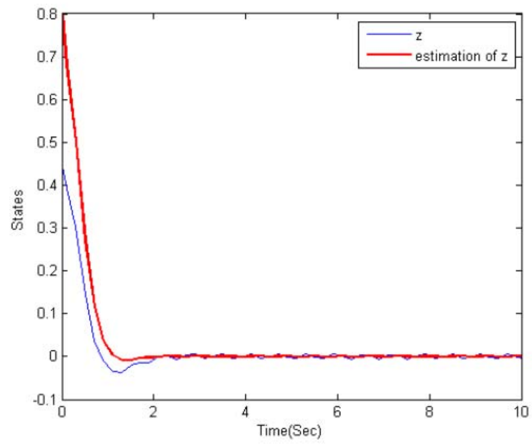

Figure 7. Output z and its estimate.

5. Conclusions

In this paper, we have dealt with the H_∞ filtering design problem for a class of delayed Hamiltonian systems with fading channel and sensor saturation. In this issue, a sensor model with fading channel and sensor saturation are established. By Lyapunov-Krasovskii stability theorem, some sufficient conditions have been proposed to ensure that the augmented system satisfies $\mathcal{L}_2$ gain output stability under zero-initial condition. At the end of this paper, two illustrative examples give prominence to the validity of H_∞ filtering technology presented in this paper. A possible future research may be feedback stabilization of nonlinear Hamiltonian systems with fading noisy channel.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (G61703232), and in part by the Natural Science Foundation of Shandong Province (ZR2017MF068, ZR2017QF013).

Conflict of interest

The authors state that there is no conflict of interest.

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