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Research article

Generalized iterative method for the solution of linear and nonlinear fractional differential equations with composite fractional derivative operator

Krunal B. Kachhia[1] **and Jyotindra C. Prajapati**[2,*]

[1] Department of Mathematical Sciences, P. D. Patel Institute of Applied Sciences, Charotar University of Science and Technology (CHARUSAT), Changa, Anand-388421, Gujarat, India

[2] Department of Mathematics, Sardar Patel University, Vallabh Vidyanagar-388120, Gujarat, India

* **Correspondence:** Email: drjyotindra18@gmail.com.

Abstract: In present paper, we introduced generalized iterative method to solve linear and nonlinear fractional differential equations with composite fractional derivative operator. Linear/nonlinear fractional diffusion-wave equations, time-fractional diffusion equation, time fractional Navier-Stokes equation have been solved by using generalized iterative method. The graphical representations of the approximate analytical solutions of the fractional differential equations were provided.

Keywords: composite fractional derivative; fractional diffusion-wave equation; Navier-Stokes equation; fractional Schrödinger equation; Mittag-Leffler function
Mathematics Subject Classification: Primary: 34A08, 35R11, 26A33; Secondary: 65J15

1. Introduction

The function space $C_\alpha, \alpha \in \mathbb{R}$ (Dimovski [1]) is defined as follows

Definition 1.1. *A real function $f(x), x > 0$ is said to be in space $C_\alpha, \alpha \in \mathbb{R}$, if there exist a real number $p(> \alpha)$, such that $f(x) = x^p f_1(x)$ where $f_1(x) \in C[0, \infty)$.*

Clearly, C_α is a vector space and the set of spaces C_α is ordered by inclusion according to

$$C_\alpha \subset C_\delta \Leftrightarrow \alpha \geq \delta.$$

Definition 1.2. *A real function $f(x), x > 0$ is said to be in space $C_\alpha^m, m \in \mathbb{N} \cup \{0\}$, if $f^{(m)} \in C_\alpha$.*

The left-sided Riemann-Liouville fractional integral of order $\alpha, \alpha > 0$ (Kilbas *et al.* [2]) defined as

$$I_t^{\alpha} f(x,t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(x,\tau)\, d\tau, \quad t > 0. \tag{1.1}$$

The left sided Caputo partial fractional derivative of f with respect to t, $f \in C_{-1}^m, m \in \mathbb{N} \cup \{0\}$ (Kilbas *et al.* [2]) defined as

$$D_t^{\alpha} f(x,t) = \begin{cases} \frac{\partial^m}{\partial t^m} f(x,t), & (\mu = m = 0), \\ I_t^{m-\alpha} \frac{\partial^m}{\partial t^m} f(x,t) & (m-1 < \alpha < m,\ m \in \mathbb{N}). \end{cases}$$

Note that,

$$I_t^{\alpha} D_t^{\alpha} f(x,t) = f(x,t) - \sum_{k=0}^{m-1} \frac{\partial^m f}{\partial t^m}(x,0) \frac{t^k}{k!},\ m-1 < \alpha < m,\ m \in \mathbb{N},$$

$$I_t^{\alpha} t^{\nu} = \frac{\Gamma(\nu+1)}{\Gamma(\alpha+\nu+1)} t^{\alpha+\nu}.$$

The composite fractional derivative of order α and β (Hilfer [3]) defined as

$$(D^{(\alpha,\beta)} y)(x) = (I^{\beta(n-\alpha)} \frac{d^n}{dx^n} (I^{(1-\beta)(n-\alpha)} y))(x), \tag{1.2}$$

where $x > 0$, $\alpha, \beta \in \mathbb{R}$, $n-1 < \alpha < n \in \mathbb{N}$ and $0 \leq \beta \leq 1$. The order β allows to interpolate continuously from the Riemman-Liouville case $D^{(\alpha,0)} = D^{\alpha}$ to the Liouville-Caputo case $D^{(\alpha,1)} = D_*^{\alpha}$.

The composite fractional derivative appeared in the theoretical modeling of broadband dielectric relaxation spectroscopy for glasses (Hilfer [4]). Kachhia and Prajapati [5] used composite fractional derivative to study heat transfer through diathermanous materials. Saxena *et al.* ([6, 7]) obtained analytical solution of some non-linear equations with composite fractional derivatives. Tomovski *et al.* [8] studied fractional diffusion equation with composite fractional derivatives. Ali and Malik [9] studied Hilfer fractional advection–diffusion using variational iteration method.

The composite fractional derivative (1.2) is not defined on the whole space C_{γ} (Hilfer *et al.* [10]).

Definition 1.3. *A function $y \in C_{-1}$ is said to be in the space Ω_{-1}^{μ}, if $D^{(\alpha,\beta)} y \in C_{-1}$ for all* $0 \leq \alpha \leq \mu,\ 0 \leq \beta \leq 1$.

The following study (Hilfer *et al.* [10]) is useful for further study.

Theorem 1.4. *If $y \in \Omega_{-1}^{\alpha}, n-1 < \alpha \leq n \in \mathbb{N}$, then Riemann-Liouville fractional integral (1.1) of composite fractional derivative (1.2) is given by*

$$(I_x^{\alpha} D^{(\alpha,\beta)} y)(x) = y(x) - y_{\alpha,\beta}(x), x > 0,$$

where,

$$y_{\alpha,\beta}(x) = \sum_{k=0}^{n-1} \frac{x^{k-n+\alpha-\beta\alpha+\beta n}}{\Gamma(k-n+\alpha-\beta\alpha+\beta n+1)} \lim_{x\to 0+} \frac{d^k}{dx^k} (I^{(1-\beta)(n-\alpha)} y)(x), x > 0.$$

The two parameter Mittag-Leffler function (Wiman [11]) defined as

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+\beta)},\ \alpha, \beta \in \mathbb{C},\ Re(\alpha) > 0 \tag{1.3}$$

Generalized Mittag-Leffler functions are introduced by Shukla and Prajapati ([12–14]).

2. The new iterative method

Daftardar-Gejji and Jafari [15] have considered the following nonlinear functional equation

$$u(\bar{x},t) = f(\bar{x},t) + L(u(\bar{x},t)) + N(u(\bar{x},t)), \tag{2.1}$$

where N is a nonlinear function and L is a linear function of u from a Banach space $B \rightarrow B$ and f is a known function, $\bar{x} = (x_1, x_2, ..., x_n)$. Eq (2.1) is assumed to have a solution of the form

$$u = \sum_{i=0}^{\infty} u_i \tag{2.2}$$

since L is linear,

$$L\left(\sum_{i=0}^{\infty} u_i\right) = \sum_{i=0}^{\infty} L(u_i)$$

the nonlinear operator N is decomposed (Daftardar-Gejji and Jafari [15]) as

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}.$$

Equation (2.2) can be written as

$$\sum_{i=1}^{\infty} u_i = f + \sum_{i=0}^{\infty} L(u_i) + N(u_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}.$$

The recurrence relation is defined as

$$u_0 = f$$

$$u_1 = L(u_0) + N(u_0)$$

$$u_{m+1} = L(u_m) + N(u_0 + u_1 + ... + u_m) - N(u_0 + u_1 + ... + u_{m-1}), m = 1, 2, ...$$

hence,

$$\sum_{i=1}^{m+1} u_i = L\left(\sum_{i=0}^{m} u_i\right) + N\left(\sum_{i=0}^{m} u_i\right)$$

and

$$\sum_{i=0}^{\infty} u_i = L\left(\sum_{i=0}^{\infty} u_i\right) + N\left(\sum_{i=0}^{\infty} u_i\right).$$

The k-term approximate solution of (2.1) is given by $u = u_0 + u_1 + ... + u_{k-1}$.

Many applications of the iterative method are given by Bhalekar and Daftardar-Gejji [16] and Daftardar-Gejji and Bhalekar [17].

3. The generalized iterative method for fractional initial value problem with composite fractional derivatives

We consider the following fractional initial value problem, for $\bar{x} \in \mathbb{R}^n$

$$D_t^{(\alpha,\beta)} u(\bar{x},t) = \sum_{i=1}^{n} a_i D_{\bar{x}_i}^{\beta_i} u(\bar{x},t) + A(u(\bar{x},t)),\ t > 0,\ m-1 < \alpha \le m, \tag{3.1}$$

$$\frac{\partial^k}{\partial t^k}(I_t^{(1-\beta)(m-\alpha)} u(\bar{x},0)) = h_k(\bar{x}),\ 0 \le k \le m-1,\ m = 1,2,...,1 < \beta_i \le 2, \tag{3.2}$$

where a_i are constants, $A(u)$ is non-linear function of u and h_k are functions of $\bar{x}$. Applying I_t^{α} on (3.1) and using (3.2) in the light of Theorem 1.4, we get

$$u(\bar{x},t) = \sum_{k=0}^{m-1} h_k(\bar{x}) \frac{t^{k-m+\alpha-\beta\alpha+\beta m}}{\Gamma(k-m+\alpha-\beta\alpha+\beta m+1)} + I_t^{\alpha}\left(\sum_{i=1}^{n} a_i D_{\bar{x}_i}^{\beta_i} u(\bar{x},t)\right) + I_t^{\alpha}(A(u)). \tag{3.3}$$

Equation (3.3) can be written as in the form (2.1) with

$$f = \sum_{k=0}^{m-1} h_k(\bar{x}) \frac{t^{k-m+\alpha-\beta\alpha+\beta m}}{\Gamma(k-m+\alpha-\beta\alpha+\beta m+1)},\ L(u) = I_t^{\alpha}\left(\sum_{i=1}^{n} a_i D_{\bar{x}_i}^{\beta_i} u(\bar{x},t)\right) \text{ and } N(u) = I_t^{\alpha}(A(u))$$

Now, the recurrence relation can be defined as

$$u_0 = f$$

$$u_1 = L(u_0) + N(u_0)$$

$$u_{m+1} = L(u_m) + N(u_0 + u_1 + ... + u_m) - N(u_0 + u_1 + ... + u_{m-1}), m = 1,2,...$$

hence,

$$\sum_{i=1}^{m+1} u_i = L\left(\sum_{i=0}^{m} u_i\right) + N\left(\sum_{i=0}^{m} u_i\right)$$

and

$$\sum_{i=0}^{\infty} u_i = L\left(\sum_{i=0}^{\infty} u_i\right) + N\left(\sum_{i=0}^{\infty} u_i\right).$$

The k-term approximate solution of (2.1) is given by $u = u_0 + u_1 + ... + u_{k-1}$.

Remark 3.1. *Observed that by taking $\beta = 1$ in the modified iterative method, it reduces to the iterative method given by Daftardar-Gejji and Jafari [15].*

4. Concrete examples

Example 4.1. *(Figure 1) Consider the time-fractional diffusion equation with composite fractional derivative,*

$$D_t^{(\alpha,\beta)} u(x,t) = \frac{\partial^2 u}{\partial x^2},\ t > 0,\ x \in \mathbb{R},\ 0 < \alpha \le 1,\ 0 \le \beta \le 1, \tag{4.1}$$

$$I_t^{(1-\beta)(1-\alpha)} u(x,0) = e^{-x} \tag{4.2}$$

This system of equations gives,

$$u(x,t) = e^{-x}\frac{t^{\alpha+\beta-\alpha\beta-1}}{\Gamma(\alpha+\beta-\alpha\beta)} + I_t^{\alpha}\left(\frac{\partial^2 u}{\partial x^2}\right).$$

In view of the new iterative method, we have

$$L(u) = I_t^{\alpha}\left(\frac{\partial^2 u}{\partial x^2}\right) \text{ and } N(u) = 0.$$

We get a recurrence relation

$$u_0 = e^{-x}\frac{t^{\alpha+\beta-\alpha\beta-1}}{\Gamma(\alpha+\beta-\alpha\beta)},$$

$$u_1 = L(u_0) + N(u_0) = e^{-x}\frac{t^{2\alpha+\beta-\alpha\beta-1}}{\Gamma(2\alpha+\beta-\alpha\beta)}, ...$$

finally, we get

$$u_k = e^{-x}\frac{t^{k\alpha+\alpha+\beta-\alpha\beta-1}}{\Gamma(k\alpha+\alpha+\beta-\alpha\beta)}, k = 0, 1, 2, ...$$

Using definition of Mittag-Leffler function (1.3), the solution of (4.1) and (4.2) can be written as

$$u(x,t) = u_0 + u_1 + u_2 + ... = e^{-x}t^{\alpha+\beta-\alpha\beta-1}E_{\alpha,\alpha+\beta-\alpha\beta}(t^{\alpha}).$$

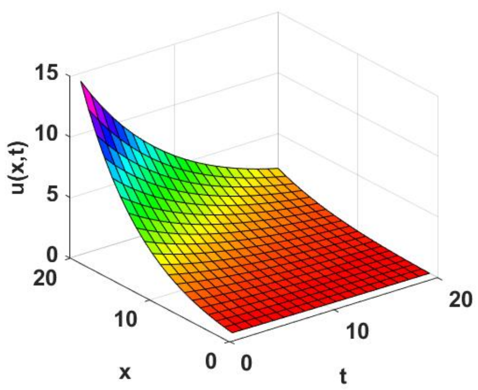

Figure 1. Example 4.1 with $\alpha = 0.932, \beta = 0.125$.

Example 4.2. *(Figure 2) Consider the time-fractional wave equation with composite fractional derivative*

$$D_t^{(\alpha,\beta)}u(x,t) = k\frac{\partial^2 u}{\partial x^2},\ t > 0,\ x \in \mathbb{R},\ 1 < \alpha \leq 2,\ 0 \leq \beta \leq 1, \tag{4.3}$$

$$I_t^{(1-\beta)(2-\alpha)}u(x,0) = x^2 \tag{4.4}$$

and

$$\frac{d}{dt}(I_t^{(1-\beta)(2-\alpha)}u(x,0)) = 1 \tag{4.5}$$

The above system of Eqs (4.3)–(4.5), leads to

$$u(x,t) = x^2\frac{t^{\alpha+2\beta-\alpha\beta-2}}{\Gamma(\alpha+2\beta-\alpha\beta-1)} + \frac{t^{\alpha+2\beta-\alpha\beta-1}}{\Gamma(\alpha+2\beta-\alpha\beta)} + I_t^{\alpha}\left(k\frac{\partial^2 u}{\partial x^2}\right)$$

Applying the new iterative method, we have

$$L(u) = I_t^{\alpha}\left(k\frac{\partial^2 u}{\partial x^2}\right) \text{ and } N(u) = 0.$$

$$u_0 = x^2\frac{t^{\alpha+2\beta-\alpha\beta-2}}{\Gamma(\alpha+2\beta-\alpha\beta-1)} + \frac{t^{\alpha+2\beta-\alpha\beta-1}}{\Gamma(\alpha+2\beta-\alpha\beta)}$$

$$u_1 = L(u_0) + N(u_0) = 2k\frac{t^{2\alpha+2\beta-\alpha\beta-1}}{\Gamma(2\alpha+2\beta-\alpha\beta-1)},$$

$$u_2 = L(u_1) + N(u_0+u_1) - N(u_0) = 0, u_3 = 0, ...$$

We arrived at

$$u(x,t) = u_0 + u_1 + u_2 + ... = x^2\frac{t^{\alpha+2\beta-\alpha\beta-2}}{\Gamma(\alpha+2\beta-\alpha\beta-1)} + \frac{t^{\alpha+2\beta-\alpha\beta-1}}{\Gamma(\alpha+2\beta-\alpha\beta)} + 2k\frac{t^{2\alpha+2\beta-\alpha\beta-1}}{\Gamma(2\alpha+2\beta-\alpha\beta-1)}$$

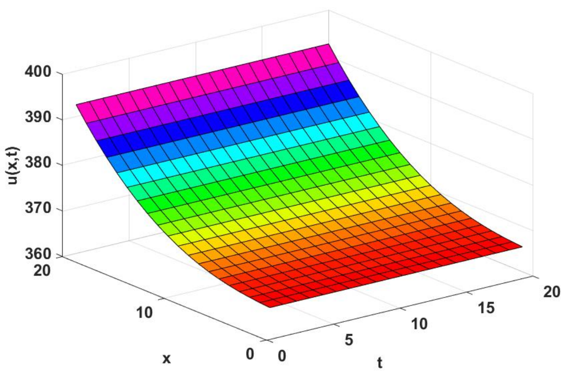

Figure 2. Example 4.2 with $\alpha = 1.733, \beta = 0.259$ and $k = 3$.

Example 4.3. *(Figure 3) Consider the following time fractional Navier-Stokes equation (Chaurasia and Kumar [18])*

$$D_t^{(\alpha,\beta)}u(r,t) = 1 + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right),\ t > 0,\ r \in \mathbb{R},\ 0 < \alpha \le 1,\ 0 \le \beta \le 1, \tag{4.6}$$

$$I_t^{(1-\beta)(1-\alpha)}u(x,0) = R^2 - r^2 \tag{4.7}$$

This initial value problem can be written as

$$u(r,t) = (R^2 - r^2)\frac{t^{\alpha+\beta-\alpha\beta-1}}{\Gamma(\alpha+\beta-\alpha\beta)} + I_t^{\alpha}\left(\mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)\right) + \frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

The new iterative method algorithm gives,

$$L(u) = I_t^{\alpha}\left(\mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)\right) \text{ and } N(u) = 0,$$

from which, we get

$$u_0 = (R^2 - r^2)\frac{t^{\alpha+\beta-\alpha\beta-1}}{\Gamma(\alpha+\beta-\alpha\beta)} + \frac{t^{\alpha}}{\Gamma(\alpha+1)},$$
$$u_1 = L(u_0) + N(u_0) = -4\mu\frac{t^{2\alpha+\beta-\alpha\beta-1}}{\Gamma(2\alpha+\beta-\alpha\beta)} + \frac{t^{\alpha}}{\Gamma(\alpha+1)},$$
$$u_2 = L(u_1) + N(u_0+u_1) - N(u_0) = 0, u_3 = 0, ...$$

here,

$$u(r,t) = u_0 + u_1 + u_2 + ... = (R^2 - r^2)\frac{t^{\alpha+\beta-\alpha\beta-1}}{\Gamma(\alpha+\beta-\alpha\beta)} - 4\mu\frac{t^{2\alpha+\beta-\alpha\beta-1}}{\Gamma(2\alpha+\beta-\alpha\beta)} + \frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

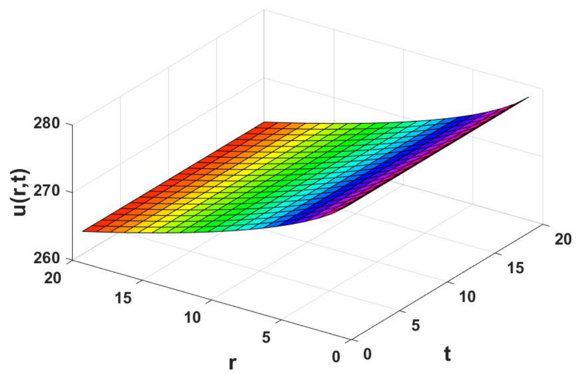

Figure 3. Example 4.3 with $\alpha = 0.657, \beta = 0.743$, $r = 2$, $R = 4$ and $\mu = 3$.

Example 4.4. *(Figure 4) Consider the time-fractional diffusion equation with composite fractional derivative*

$$D_t^{(\alpha,\beta)}u(x,t) = \frac{\partial^2 u}{\partial x^2} + 2u(\bar{x},t),\ t > 0,\ x \in \mathbb{R},\ 0 < \alpha \leq 1,\ 0 \leq \beta \leq 1, \tag{4.8}$$

$$I_t^{(1-\beta)(1-\alpha)}u(x,0) = \sin x \tag{4.9}$$

This system of equations reduces to

$$u(x,t) = \sin x\frac{t^{\alpha+\beta-\alpha\beta-1}}{\Gamma(\alpha+\beta-\alpha\beta)} + I_t^{\alpha}\left(\frac{\partial^2 u}{\partial x^2}\right) + I_t^{\alpha}(2u). \tag{4.10}$$

The new iterative method algorithm gives,

$$L(u) = I_t^{\alpha}\left(\frac{\partial^2 u}{\partial x^2} + 2u\right) \text{ and } N(u) = 0.$$

In view of new iterative method, we get recurrence relation

$$u_0 = \sin x \frac{t^{\alpha+\beta-\alpha\beta-1}}{\Gamma(\alpha+\beta-\alpha\beta)},$$

$$u_1 = L(u_0) + N(u_0) = \sin x \frac{t^{2\alpha+\beta-\alpha\beta-1}}{\Gamma(2\alpha+\beta-\alpha\beta)},$$

$$u_2 = L(u_1) + N(u_0+u_1) - N(u_0) = \sin x \frac{t^{3\alpha+\beta-\alpha\beta-1}}{\Gamma(3\alpha+\beta-\alpha\beta)}, ...$$

this leads to

$$u_k = \sin x \frac{t^{k\alpha+\alpha+\beta-\alpha\beta-1}}{\Gamma(k\alpha+\alpha+\beta-\alpha\beta)}, k = 0, 1, 2, ...$$

Using definition of Mittag-Leffler function (1.3), the solution of (4.8) is

$$u(x,t) = u_0 + u_1 + u_2 + ... = \sin x \; t^{\alpha+\beta-\alpha\beta-1} E_{\alpha,\alpha+\beta-\alpha\beta}(t^{\alpha}). \tag{4.11}$$

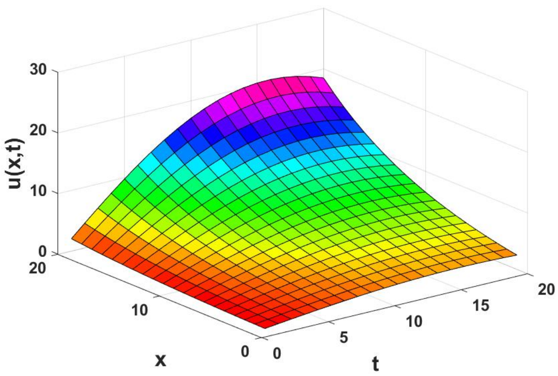

Figure 4. Example 4.4 with $\alpha = 0.357, \beta = 0.715$.

Example 4.5. *(Figure 5) Consider the non-linear time-fractional wave equation with composite fractional derivative*

$$D_t^{(\alpha,\beta)} u(x,t) = \frac{\partial^2 u}{\partial x^2} + 2(u(x,t))^2, \; t > 0, \; x \in \mathbb{R}, \; 1 < \alpha \le 2, \; 0 \le \beta \le 1, \tag{4.12}$$

$$I_t^{(1-\beta)(1-\alpha)} u(x,0) = 0, \; \frac{d}{dt}(I_t^{(1-\beta)(1-\alpha)} u(x,0)) = x^2 \tag{4.13}$$

This system of equations reduces to

$$u(x,t) = x^2 \frac{t^{\alpha+2\beta-\alpha\beta-1}}{\Gamma(\alpha+2\beta-\alpha\beta)} + I_t^{\alpha}\left(\frac{\partial^2 u}{\partial x^2}\right) + I_t^{\alpha}(2u^2). \tag{4.14}$$

In view of new iterative method, we have

$$L(u) = I_t^{\alpha}\left(\frac{\partial^2 u}{\partial x^2}\right) \text{ and } N(u) = I_t^{\alpha}(2u^2).$$

We get recurrence relation

$$u_0 = x^2 \frac{t^{\alpha+2\beta-\alpha\beta-1}}{\Gamma(\alpha+2\beta-\alpha\beta)},$$

$$L(u_0) = I_t^{\alpha}\left(x^2 \frac{t^{\alpha+2\beta-\alpha\beta-1}}{\Gamma(\alpha+2\beta-\alpha\beta)}\right) = 2\frac{t^{2\alpha+2\beta-\alpha\beta-1}}{\Gamma(2\alpha+2\beta-\alpha\beta)},$$

$$N(u_0) = I_t^{\alpha}(2u_0^2) = I_t^{\alpha}\left(2x^4 \frac{t^{2\alpha+4\beta-2\alpha\beta-2}}{(\Gamma(\alpha+2\beta-\alpha\beta))^2}\right) = \frac{2x^4\Gamma(2\alpha+4\beta-2\alpha\beta-1)t^{3\alpha+4\beta-2\alpha\beta-2}}{(\Gamma(\alpha+2\beta-\alpha\beta))^2\Gamma(3\alpha+4\beta-2\alpha\beta-1)},$$

we get

$$u_1 = L(u_0) + N(u_0) = \frac{2t^{2\alpha+2\beta-\alpha\beta-1}}{\Gamma(2\alpha+2\beta-\alpha\beta)} + \frac{2x^4\Gamma(2\alpha+4\beta-2\alpha\beta-1)t^{3\alpha+4\beta-2\alpha\beta-2}}{(\Gamma(\alpha+2\beta-\alpha\beta))^2\Gamma(3\alpha+4\beta-2\alpha\beta-1)},$$

this leads to

$$\begin{aligned} u(x,t) &= u_0 + u_1 \\ &= \frac{x^2 t^{\alpha+2\beta-\alpha\beta-1}}{\Gamma(\alpha+2\beta-\alpha\beta)} + \frac{2t^{2\alpha+2\beta-\alpha\beta-1}}{\Gamma(2\alpha+2\beta-\alpha\beta)} + \frac{2x^4\Gamma(2\alpha+4\beta-2\alpha\beta-1)t^{3\alpha+4\beta-2\alpha\beta-2}}{(\Gamma(\alpha+2\beta-\alpha\beta))^2\Gamma(3\alpha+4\beta-2\alpha\beta-1)} \end{aligned}$$

is a two term solution of (4.12)–(4.13).

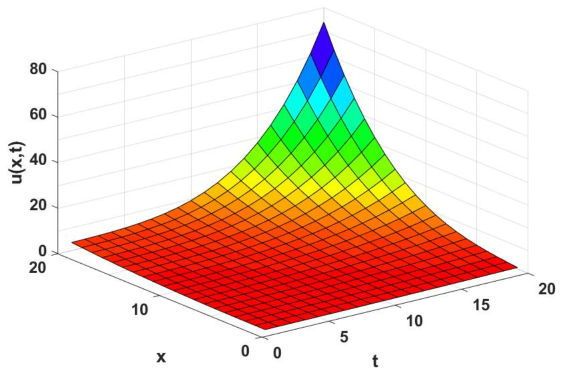

Figure 5. Example 4.5 with $\alpha = 1.57, \beta = 0.65$.

5. Conclusions

The generalized iterative method for solving functional equations with composite fractional derivatives has been derived. Examples deal with linear and nonlinear fractional differential equations with composite fractional derivative operator viz. heat equation, wave equation and Navier-Stokes equation. This method is also applicable for computer algorithms. We obtained the solution of linear and non linear differential equations in form of convergent series without any type of conventions. This method is also works well when the solution for integer order is not known. The behavior of solutions of the fractional differential equation were provided graphically as well. MATLAB is useful too for computations in this paper. Hence we ensured that present algorithm is reliable and powerful for obtaining solutions for different classes of linear and nonlinear fractional differential equations with composite fractional derivatives.

Conflict of interest

The authors declare no conflict of interest in this paper.

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