Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 5(3): 2780–2800.
DOI: 10.3934/math.2020179
Received: 24 December 2019
Accepted: 05 March 2020
Published: 17 March 2020

Research article

Adaptive neural networks event-triggered fault-tolerant consensus control for a class of nonlinear multi-agent systems

Zuo Wang[1], Hong Xue[2,*], Yingnan Pan[1] and Hongjing Liang[1]

[1] School of Engineering, Bohai University, Jinzhou 121013, Liaoning, China
[2] School of Mathematics and Physics, Bohai University, Jinzhou 121013, Liaoning, China

* **Correspondence:** Email: xuehong19870101@163.com.

Abstract: This paper studies the event-triggered fault-tolerant control problem of nonlinear multi-agent systems. The goal is to ensure the stability of event-based sampling multi-agent systems when the actuator faults occurs. The neural networks approximate property is used to approximate unknown ideal control parameters, which can reduce the exact requirements of control parameters. Based on the states information of neighboring agents, a distributed fault-tolerant consensus controller is designed for the leaderless multi-agent systems. Moreover, an event-triggered mechanism with special definition of event-triggered error is applied to reduce the amount of communications. In addition, the Zeno behaviour is avoided. By using Lyapunov stability theory, it is proved that all signals are bounded in the closed-loop systems. Finally, a numerical simulation result is presented to prove the effectiveness of the proposed method.

Keywords: fault-tolerant control; event-triggered mechanism; multi-agent systems; neural networks; lyapunov stability theory
Mathematics Subject Classification: 93B52, 93C95, 93D05

1. Introduction

In the past decades, the adaptive consensus control problem of multi-agent systems has attracted increasing attention due to its wide range of applications in industrial and military fields, such as exploration robots, surface vehicles formation and so on [1–6]. Generally, the consensus problem can be divided leader-following consensus problem [7] and leaderless consensus problem [8]. The leader-following consensus problem similar to tracking control [9–11]. The purpose of the consensus problem is that these agents can reach synchronization. To achieve this goal, a proper distributed control protocol is designed based on the local information of agents and its neighbors. According to different working conditions, the consensus problem has been widely studied. In [12], the consensus problem of discrete-time systems with nonlinear dynamics was studied. The problem of communication noises was studied in [13]. In order to realize the consensus control of multi-agent systems, the adaptive control method was applied to the research of multi-agent systems [14]. Adaptive consensus control technology refers to the system adjusting itself according to the change of environment so that its behavior can achieve the best or at least allowable characteristics and functions in the new or changed environment. That is to say, it is an online adjustment mode, which cause unnecessary waste of communication resources. To overcome this disadvantage, more and more researchers have begun to research the event-triggered adaptive control technology.

In event-triggered control (ETC) scheme, when the event-triggered condition is satisfied, the information is transmitted. Generally speaking, the event-triggered condition include a predefined threshold and event-triggered error [15–19]. In general, based on the actual demands, the threshold is set. If the event-triggered error exceeds the threshold, the information is allowed to transmit. In this way, the burden of communication is reduced. In [20], the ETC problem was researched for uncertain nonlinear systems, and three different event-triggered conditions were designed of controller update. In [21], an ETC technology was studied for a simple single system. In [22], the technology in literature [21] had been improved, a distributed ETC scheme was proposed, which means that the trigger mechanism was extended to each agent system. In [23], the event-triggered tracking consensus control problem of nonlinear systems with unknown disturbances was researched. The fuzzy adaptive distributed ETC protocol was proposed for uncertain systems in [24]. With the development of science and technology, there has been a better development on how to use event-triggered mechanism to solve the communication burden. Based on describe the motivation, the methods of algorithmic synthesis, the technical challenges, and their application in distributed control, the development of the event-triggered mechanism of average consensus was introduced, and the event-triggered network system control problem was studied in [25]. In [26], the event-triggered coverage control problem was studied for asynchronous multi-agent systems, and a completely asynchronous communication sensing solution was proposed by the agent to decide when to push information to others in the networks. Based on ETC scheme, the global stabilization problem of k-valued logical control networks was studied in [27]. However, most schemes consider the fault-free system model in the above results.

In many practical systems, various faults may suddenly occur in the process of system operation [28–30]. Therefore, the effectiveness of components cannot reach the ideal goal. The actual performance of the systems may decline or instability, when the faults occur. In order to guarantee stability and safety of the systems, the some fault-tolerant control (FTC) methods have been developed. Based on fuzzy systems and sliding-mode methods, the problems of fault estimation and fault-tolerant control for stochastic systems with sensor faults were studied in [31]. The problem of sensor failures was researched for nonlinear pure-feedback systems, and an adaptive fuzzy fault-tolerant control method was proposed by the parameter separation technology in [32]. In [33], an adaptive fault-tolerant consensus control method was developed based on local filter to estimate the unmeasurable states for multi-agent systems. In [34], Fault tolerant control and event-triggered mechanism were considered at the same time in nonlinear systems. In order to compensate for actuator fault and uncertainty of systems, a robust adaptive decentralized FTC scheme based on neural network was proposed for interconnected systems in [35]. An observer-based fault-tolerant controller was designed by the T-S fuzzy and delta operator methods in [36]. Hence, it is meaningful that the fault-tolerant control problem is investigated. Moreover, because of the unknown dynamics existed in the actual system models, the neural networks (NNs) and fuzzy control have been widely studied [37–41]. The Lyapunov stability theorem is often used to prove that the signals of systems are bounded in different control environments, such as discrete-time systems [42], impulsive systems [43], semi-markov jump systems [44], pure-feedback interconnected nonlinear systems [45], and so on.

In this paper, the fault-tolerant leaderless consensus control problem is considered for nonlinear multi-agent systems. First, this paper attempts to apply a special event-triggered error definition to reduce the amount of communications in multi-agent systems. Then, a fault-tolerant control scheme is designed by using model reference control method and approximate property of neural networks to unknown functions, which reduces the exact requirements of controller parameters. At last, an event-triggered controller is designed to ensure the stability of multi-agent systems, where adaptive law is updated by event-triggered control method.

The rest of the structure is as follows. The proposes the graph theory and the NNs approximation method are given in Section II. Section III give the controller design. Stability analysis of multi-agent formations is presented in Section IV. In Section V, it is proved that the ETC scheme avoid zeno behavior. A simulation result is given in Section VI. Section VII summarize the conclusion.

2. Preliminaries

2.1. Graph theory

The identify matrix is described as I_N. Let $\mathcal{G}$=$\{\mathcal{V},\mathcal{E}\}$ be an undirected graph, where $\mathcal{V} = \{1, 2, ..., N\}$, $N \geq 1$ and $\mathcal{E}$=$\{(i, j)|i \neq j, i, j \in \mathcal{V}\}$ are described as a node set and an edge set, respectively. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is associated with $\mathcal{G}$, a_{ij}=1 if the agent i can receive the information from agent j, otherwise a_{ij}=0. The degree matrix $\mathcal{D}$=diag$\{d_1, ..., d_N\}$, where d_i=$\sum_{j=1}^{j=N} a_{ij}$. Define $\mathcal{L}$=$\mathcal{D}$-$\mathcal{A}$ is Laplacian matrix of $\mathcal{G}$.

For a vector $s = (s_1, ..., s_N)^T$, $||s||$ represents the 2-norm of s. Let matrix $W = [w_1, w_2; w_3, w_4]$, the vectorization of the matrix W is defined $vec(W) = [w_1, w_2, w_3, w_4]^T$, and $vec(W)^T vec(W) = tr\{W^T W\}$. For a square matrix $E \in \mathbb{R}^{n\times n}$, the minimum eigenvalue of E is defined as $\lambda_{min}(E)$, and the maximum eigenvalue of E is defined as $\lambda_{max}(E)$. The Z^+ is difference of Z in a moment. The $\otimes$ is Kronecker product.

2.2. Function approximation

In recent years, radial basis function NNs are employed to deal with unknown dynamics of systems [46]. Defined a compact set Ω_α, the unknown function $F(\alpha) \in \Omega_\alpha$, there exists a constant $\omega^{*T}\xi(\alpha)$ satisfying the following form

$$\sup_{\alpha\in\Omega_\alpha} |\omega^{*T}\xi(\alpha) - F(\alpha)| \leq \delta, \tag{2.1}$$

where $\alpha \in \Re^N$ is the input variable, δ is arbitrary positive constant, $\omega^* \in \Re^N$ is the ideal NNs weight vector, and $\xi(\alpha)$ is a smooth basis vector.

Based on the NNs approximation property, an unknown function $F(\alpha)$ can be represented in the following form

$$F(\alpha) = \omega^{*T}\xi(\alpha) + \delta(\alpha), \tag{2.2}$$

where $\delta(\alpha)$ is the smallest approximation error and $\delta(\alpha) \leq \bar{\delta}$. Throughout this paper, we define $\xi(x) = [\xi_1(x_1)^T, ..., \xi_N(x_N)^T]^T$, and $\delta(x) = [\delta_1(x_1), ..., \delta_N(x_N)]^T$. There exists $\|\xi(x)\| < \bar{\xi}$ and $\|\delta(x)\| < \bar{\delta}$, in which $\bar{\xi}$ and $\bar{\delta}$ are positive constants.

2.3. Systems description

The nonlinear multi-agent systems are considered, and its mathematical model can be expressed as

$$\dot{x}_i\,(t) = A_0 x_i(t) + f_{0i}(x_i) + g(x_i)u_{i_i}, \tag{2.3}$$

where $x_i \in \mathbb{R}^m$ is state of the i-th agent, and $x = [x_1^T, x_2^T, ..., x_N^T]^T$. A_0 is a known constant matrix with compatible dimension. The $f_{0i} \in f_0 = [f_{01}^T, ..., f_{0N}^T]^T$ is smooth continuous nonlinear function. u_i is the controller of each agent, and u=$[u_1^T, u_2^T, ..., u_N^T]^T$. $g(x_i) \in g(x) = [g(x_1)^T, ..., g(x_N)^T]^T$ is gain coefficient. For the multi-agent systems (2.3) have following assumption.

Assumption 1. *System (2.3) is controllable, and the nonlinear function $f_2(x)$ can be linearizable. $g(x)$ is the control gain matrix, and $g(x)$ is bounded for all x, satisfying $\|g(x)\| \leq g_{max}$ and $g_{max} > 0$.*

In this paper, the actuator fault is considered, the multi-agent systems model can be rewritten as

$$\dot{x}_i\,(t) = A_1 x_i(t) + f_{1i}(x_i) + g(x_i)p_i u_i, \tag{2.4}$$

where p_i=diag$(\rho_1, ..., \rho_N)$ is the actuator effectiveness factor of each agent with $\rho_i \in (0, 1)$, and p=diag$(p_1, ..., p_N)$. A_1 is a known constant matrix with compatible dimension. The $f_{1i} \in f_1 = [f_{11}^T, ..., f_{1N}^T]^T$ is smooth continuous nonlinear function. It is assumed that the ideal controller $u_i^* = Q(x)$ enables the multi-agent systems to achieve the desired performances. If the controller $u_i^* = Q(x)$ is added to the fault multi-agent systems, the systems model can be rewritten as

$$\dot{x}_i\,(t) = A_2 x_i(t) + f_{2i}(x_i), \tag{2.5}$$

where A_2 is a known constant matrix with compatible dimension, and the $f_{2i} \in f_2 = [f_{21}^T, ..., f_{2N}^T]^T$ is smooth continuous nonlinear function. There are the following assumptions for equality (2.2) and system (2.5).

Assumption 2. *For the multi-agent systems (2.5), the positive matrices P and Q satisfy the following form*

$$P(A_2 \otimes I_N) + (A_2 \otimes I_N)^T P + \sigma {l_f}^2 I + \frac{1}{\sigma} PP \leq -Q, \tag{2.6}$$

where σ represents an appropriate positive constant, I is defined as the identity matrix.

Assumption 3. *For nonlinear smooth bounded function functions $f_2(x)$ and $\xi(x)$, there exist constant l_f and l_ξ satisfying*

$$\begin{aligned} \|f_2(x) - f_2(y)\| &\leq l_f\|x - y\|, \\ \|\xi(x) - \xi(y)\| &\leq l_\xi\|x - y\|, \end{aligned} \tag{2.7}$$

where l_f and l_ξ are Lipschitz constants.

2.4. Stability theoretics

An impulsive dynamical system is considered [47], it is defined as

$$\begin{aligned} \dot{X} =& F_c(X), X(0) = X_0, X \in \mathcal{F} \subset \mathcal{I}, X \notin \mathcal{J}, \\ \Delta X =& F_d(X) = \Delta X(t^+) - \Delta X(t), X \in \mathcal{J} \subset \mathcal{I}, \end{aligned} \tag{2.8}$$

where $X \in \mathcal{I}$ is the state vector of the system, and $\mathcal{I}$ is an open set with $0 \in \mathcal{I}$. $\mathcal{F}$ and $\mathcal{J}$ are the flow and the jump sets, respectively. $\Delta X(t^+) = \underset{a\to 0}{lim} X(t + a)$. The functions $F_c(X)$ represents continuous dynamics of the impulsive dynamical system, and the functions $F_d(X)$ is reset dynamics of the impulsive dynamical system.

Remark 1. *In this paper, because the event-triggered sampling mechanism is used, the influence of sampling impulse should be considered when discussing the stability. And, the various impulsive theories have been widely used [48].*

Lemma 1. *[49] It is assumed that the function $V(X)$ is continuously differentiable. $M(*)$ and $O(*)$ are continuous functions with initial value being 0, such that $V(X)$, $M(*)$ and $O(*)$ satisfy the following forms*

$$M(\|X\|) \le V(X) \le O(\|X\|),\ X \in \mathcal{I}, \tag{2.9}$$

$$\frac{\partial V(X)}{\partial X} F_c(X) < 0, X \in \mathcal{I}, X \notin \mathcal{J},\ \|x\| > \chi, \tag{2.10}$$

$$V(X + F_d(X)) - V(X) \le 0,\ X \in \mathcal{I}, X \in \mathcal{J},\ \|x\| > \chi, \tag{2.11}$$

where χ is a positive constant. If the above equations are satisfied, the system state is locally ultimately bounded.

3. Controller and event-triggered mechanism design

3.1. Controller design

Based on the NNs approximate property (2.2), the ideal controller u_i^* can be shown as

$$u_i^* = \sum_{j\in\mathcal{E}} a_{ij}(x_i(t) - x_j(t)) + \omega_i^T \xi(x_i) + \delta(x_i), \tag{3.1}$$

According to the above analysis, the actual controller can be designed in the following form

$$u_i = H(t) \sum_{j\in\mathcal{E}} a_{ij}(x_i(t_k) - x_j(t_k)) + \hat{\omega}^T \xi(x_k), \tag{3.2}$$

where $H(t) = \exp(-\tau(t - t_k))$. The $\hat{\omega}_i$ is estimation of ω_i, and $\hat{\omega}_i \in \hat{\omega}^T = [\hat{\omega}_1^T, ..., \hat{\omega}_N^T]^T$, $\omega^T = [\omega_1^T, ..., \omega_N^T]^T$.

Remark 2. *Because the multi-agent systems exist interconnections among agents, the structure of the controller (3.1) is different from the one in [34, 47]. In this way, the technology is further introduced into the field of multi-agent.*

At the time $t = t_k$, the event-triggered condition is triggered, so the $\hat{\omega}_i$ is updated. The next triggered time is defined as t_{k+1}. Until the next triggered time t_{k+1}, the $\hat{\omega}_i$ is held. The time interval $(t_k, t_{k+1}]$ represents the occurrence of an event triggered. Therefore, the adaptive update law is designed as

$$\begin{aligned} \hat{\omega}_i^+ &= \nu\hat{\omega}_i - \frac{a_e\xi(x_i)e_i^T B_{1i}}{b_e + \|e_i\|^2} - \frac{a_{\tilde{x}}\xi(x_i)\tilde{x}_i^T B_{2i}}{b_{\tilde{x}} + \|\tilde{x}_i\|^2},\ t = t_k \\ \dot{\hat{\omega}}_i &= 0,\ t \in (t_k, t_{k+1}] \end{aligned} \tag{3.3}$$

where $0 < \nu < 1$, $a_e > 0$, $a_{\tilde{x}} > 0$, b_e and $b_{\tilde{x}}$ are small positive constants, $B_{1i} \in B_1$ =diag$[B_{11}, ..., B_{1N}]$ and $B_{2i} \in B_2$ =diag$[B_{21}, ..., B_{2N}]$ are nonzero matrices with appropriate dimension. e_i is event-triggered error.

Remark 3. *The b_e and $b_{\tilde{x}}$ are defined as small positive constants. They exist to avoid the denominator of equality (3.3) equal to* 0.

Let $\tilde{x}_i = x_i - \hat{x}_i$, where $\hat{x}_i$ is reference dynamics with the following form

$$\begin{aligned} \hat{x}_i^+ &= x_i(t),\ t = t_k \\ \dot{\hat{x}}_i &= A_2\hat{x}_i(t)_i + f_{2i}(\hat{x}_i(t)),\ t \in (t_k, t_{k+1}] \end{aligned} \tag{3.4}$$

Next, the $\tilde{\omega}_i = \omega_i - \hat{\omega}_i$ denotes the estimation error, and $\tilde{\omega} = [\tilde{\omega}_1^T, ..., \tilde{\omega}_N^T]^T$. The estimation error dynamics are obtained as

$$\begin{aligned} \tilde{\omega}_i^+ &= \omega_i - \hat{\omega}_i^+ \\ &= \omega_i - \nu\hat{\omega}_i + W_{e_i}\xi(x_i)e_i^T B_{1i} + W_{\tilde{x}_i}\xi(x_i)\tilde{x}_i^T B_{2i} \\ &= \tilde{\omega}_i + \Delta\tilde{\omega}_i,\ t = t_k \\ \dot{\tilde{\omega}}_i &= 0,\ t \in (t_k, t_{k+1}] \end{aligned} \tag{3.5}$$

where $\Delta\tilde{\omega}_i = (1 - \nu)\hat{\omega}_i + W_{e_i}\xi(x_i)e_i^T B_{1i} + W_{\tilde{x}_i}\xi(x_i)\tilde{x}_i^T B_{2i}$, $W_{e_i} = \frac{a_e}{b_e+\|e_i\|^2}$, $W_{\tilde{x}_i} = \frac{a_{\tilde{x}}}{b_{\tilde{x}}+\|\tilde{x}_i\|^2}$, $W_{e_i} \in W_e = [W_{e_1}, ..., W_{e_N}]^T$ and $W_{\tilde{x}_i} \in W_{\tilde{x}} = [W_{\tilde{x}_1}, ..., W_{\tilde{x}_N}]^T$.

3.2. Event-triggered mechanism design

In this section, the event-triggered mechanism is introduced [34]. First, e_i represents event-triggered error. Define the $e_i = x_i(t) - x_{ik}$, where $e_i \in e = [e_1^T, ..., e_N^T]^T$, $x_{ik} = x_i(t_k)\exp(-\tau(t - t_k))$. Then, considering the stability of the multi-agent systems, the design of event-triggered condition is as follow

$$\|e_i(t)\| \geq \begin{cases} \zeta_e\|x_i\|, & \|x_i\| > \bar{B}_X \\ \zeta_e(\|x_i\| + \bar{B}_X), & \|x_i\| \leq \bar{B}_X \end{cases} \tag{3.6}$$

where $\bar{B}_X$ is a small positive constant. The $\bar{B}_X$ can avoid the frequent occurrence of events, when the $\|x\|$ is too small. In order to ensure the stability of the multi-agent systems, ζ_e is designed as follows

$$\zeta_e = \frac{K_\zeta}{2g_{max}\|P\|(\|L \otimes I_n\| + l_\xi\|\hat{\omega}\|)}, \tag{3.7}$$

where K_ζ is a constant, and $0 < K_\zeta < \frac{1}{\zeta}\lambda_{min}(Q)$. The constant $\zeta > 0$.

Remark 4. *The event-triggered error e_i has been designed, and a negative exponential function is added to the event-triggered error. According to the characteristics of the negative exponential function, the event-triggered mechanism avoided no triggering after a long period of system stability. Because $\xi(x)$ is a smooth continuous, l_ξ donot equal to zero. In multi-agent systems, the design of event-triggered scheme needs to consider the interconnections among agents. Although $\|\hat{\omega}\|$ is equal to zero, it is guaranteed that the denominator of equality (3.7) is not equal to zero. Differ in [34], the parameter selection needs to consider another form when the 2-norm of weight vector estimation equal to zero. In other words, the denominator of ζ_e does not appear to be equal to zero in this paper.*

4. System stability analysis

4.1. Impulsive dynamical model

According to the multi-agent systems (2.4) and controller (3.2), the multi-agent systems can be rewritten as

$$\begin{aligned} \dot{x}_i = {} & A_1 x_i + f_{1i}(x_i) + g_i(x) p_i(H(t) \sum_{j \in \mathcal{E}} a_{ij}(x_i(t_k) - x_j(t_k)) \\ & + \hat{\omega}^T \xi(x_k)), \ t \in (t_k, t_{k+1}] \end{aligned} \tag{4.1}$$

Furthermore, by the definition of Laplacian matrix, one has

$$\dot{x} = (A_1 \otimes I_n)x + f_1(x) + g(x)p((L \otimes I_n)x_k + \hat{\omega}^T \xi(x_k)), \ t \in (t_k, t_{k+1}] \tag{4.2}$$

Add and subtracting $g(x)pu^*$ yields

$$\begin{aligned} \dot{x} = {} & (A_1 \otimes I_n)x + f_1(x) + g(x)p((L \otimes I_n)x_k \\ & + \hat{\omega}^T \xi(x_k) - u^* + u^*), \ t \in (t_k, t_{k+1}] \end{aligned} \tag{4.3}$$

Using the ideal controller (8) and system (4) , we have

$$\begin{aligned} \dot{x} = {} & (A_2 \otimes I_n)x + f_2(x) + g(x)p((L \otimes I_n)x_k + \hat{\omega}^T \xi(x_k) \\ & - (L \otimes I_n)x - \omega^T \xi(x) - \delta(x)), \ t \in (t_k, t_{k+1}] \end{aligned} \tag{4.4}$$

According to $\tilde{\omega} = \omega - \hat{\omega}$, one obtains

$$\begin{aligned} \dot{x} = {} & (A_2 \otimes I_n)x + f_2(x) + g(x)p(L \otimes I_n)(x_k - x) + g(x)p(\hat{\omega}^T \xi(x_k) \\ & - \hat{\omega}^T \xi(x)) - g(x)p(\tilde{\omega}^T \xi(x) + \delta(x)), \ t \in (t_k, t_{k+1}] \end{aligned} \tag{4.5}$$

Define the state error $\tilde{x} = x - \hat{x}$. According to the above equation, the following equation can be obtained

$$\begin{aligned} \dot{\tilde{x}} = {} & (A_2 \otimes I_n)\tilde{x} + f_2(x) - f_2(\hat{x}) + g(x)p((L \otimes I_n)(x_k - x)) \\ & + g(x)p(\hat{\omega}^T \xi(x_k) - \hat{\omega}^T \xi(x) - g(x)p(\tilde{\omega}^T \xi(x) + \delta(x)), \ t \in (t_k, t_{k+1}] \\ \Delta \tilde{x} = {} & \hat{x}(t) - x(t), \ t = t_k \end{aligned} \tag{4.6}$$

Define a sign $\psi = [x^T, x_k^T, \tilde{x}^T, vec(\hat{\omega})^T]^T \in \mathcal{I}$, where ψ is an augmented vector. Then, the closed-loop impulsive dynamical is obtained as

$$\begin{aligned} \dot{\psi} &= \begin{bmatrix} \dot{x} \\ -\tau x(t_k)\, \mathrm{e}^{-\tau(t-t_k)} \\ \dot{\tilde{x}} \\ 0 \end{bmatrix}, \quad t \in (t_k, t_{k+1}] \\ \Delta\psi &= \begin{bmatrix} 0 \\ e(t) \\ -\tilde{x} \\ \Gamma \end{bmatrix}, \quad t = t_k \end{aligned} \tag{4.7}$$

where $\Gamma = vec((1-\nu)\hat{\omega} + W_e \xi(x) e^T B_1 + W_{\tilde{x}} \xi(x) \tilde{x}^T B_2)$.

4.2. Stability analysis

In this section, the stability is established via Lyapunov function. Firstly, The estimation error $\tilde{\omega}$ needs to be bounded, so the following Lemma is given.

Lemma 2. *Consider the multi-agent systems (2.3) and the controller (3.1) expressed as the the impulsive system (4.6), and the adaptive update law is (3.5). Let Assumptions 1-3 be satisfied, and the initial $\hat{\omega}(0)$ in a compact set. Therefore, the estimated error $\tilde{\omega}$ is bounded by selecting the appropriate constant.*

Proof : In impulsive systems, the stability of continuous dynamics and stability of jump dynamics need to be considered, respectively. The function $V_w = tr\{\tilde{\omega}^T \tilde{\omega}\}$ is defined as Lyapunov function for the impulsive dynamics (3.5). The $\hat{\omega}_i$ is held in the continuous part of impulsive dynamics, so $\dot{V}_w = 0$ for $t \in (t_k, t_{k+1}]$. Further, the adaptive estimation error $\tilde{\omega}$ is bounded.

Then, the $\hat{\omega}_i$ is updated when the event-triggered condition is triggered. The stability of jump part is considered. According to the above analysis, we know that

$$\begin{aligned} \Delta V_w &= tr\{\tilde{\omega}^{+T} \tilde{\omega}^{+}\} - tr\{\tilde{\omega}^T \tilde{\omega}\}, \\ &= tr\{(\tilde{\omega} + \Delta\tilde{\omega})^T (\tilde{\omega} + \Delta\tilde{\omega})\} - tr\{\tilde{\omega}^T \tilde{\omega}\}, \end{aligned} \tag{4.8}$$

Using the equality (3.5), we can get

$$
\begin{aligned}
\Delta V_w = & tr\{(\tilde{\omega} + (1-\nu)(\omega - \tilde{\omega}) + W_e\xi(x)e^T B_1 + W_{\tilde{x}}\xi(x)\tilde{x}^T B_2)^T(\tilde{\omega} + \\
& (1-\nu)(\omega - \tilde{\omega}) + W_e\xi(x)e^T B_1 + W_{\tilde{x}}\xi(x)\tilde{x}^T B_2)\} - tr\{\tilde{\omega}^T\tilde{\omega}\}, \\
= & tr\{\tilde{\omega}^T\tilde{\omega} + (1-\nu)\tilde{\omega}^T\omega - (1-\nu)\tilde{\omega}^T\tilde{\omega} + W_e\tilde{\omega}^T\xi(x)e^T B_1 \\
& + W_{\tilde{x}}\tilde{\omega}^T\xi(x)\tilde{x}^T B_2 + (1-\nu)\omega^T\tilde{\omega} + (1-\nu)^2\omega^T\omega - (1-\nu)^2\omega^T\tilde{\omega} \\
& + (1-\nu)W_e\omega^T\xi(x)e^T B_1 + (1-\nu)W_{\tilde{x}}\omega^T\xi(x)\tilde{x}^T B_2 - (1-\nu)\tilde{\omega}^T\tilde{\omega} \\
& - (1-\nu)^2\tilde{\omega}^T\omega + (1-\nu)^2\tilde{\omega}^T\tilde{\omega} - (1-\nu)W_e\tilde{\omega}^T\xi(x)e^T B_1 \\
& - (1-\nu)W_{\tilde{x}}\tilde{\omega}^T\xi(x)\tilde{x}^T B_2 + W_e B_1^T e\xi(x)^T\tilde{\omega} + (1-\nu)W_e B_1^T e\xi(x)^T\omega \\
& - (1-\nu)W_e B_1^T e\xi(x)^T\tilde{\omega} + W_e^2 B_1^T e\xi(x)^T\xi(x)e^T B_1 \\
& + W_e W_{\tilde{x}} B_1^T e\xi(x)^T\xi(x)\tilde{x}^T B_2 + W_{\tilde{x}} B_2^T\tilde{x}\xi(x)^T\tilde{\omega} \\
& + (1-\nu)W_{\tilde{x}} B_2^T\tilde{x}\xi(x)^T\omega - (1-\nu)W_{\tilde{x}} B_2^T\tilde{x}\xi(x)^T\tilde{\omega} \\
& + W_e W_{\tilde{x}} B_2^T\tilde{x}\xi(x)^T\xi(x)e^T B_1 + W_{\tilde{x}}^2 B_2^T\tilde{x}\xi(x)^T\xi(x)\tilde{x}^T B_2\} - tr\{\tilde{\omega}^T\tilde{\omega}\}, \\
= & tr\{-(1-\nu)^2\tilde{\omega}^T\tilde{\omega} + 2(\nu - \nu^2)\omega^T\tilde{\omega} + 2\nu W_e B_1^T e\xi(x)^T\tilde{\omega} \\
& + 2\nu W_{\tilde{x}} B_2^T\tilde{x}\xi(x)^T\tilde{\omega} + (1-\nu)^2\omega^T\omega + 2(1-\nu)W_e\omega^T\xi(x)e^T B_1 \\
& + 2(1-\nu)W_{\tilde{x}}\omega^T\xi(x)\tilde{x}^T B_2 + 2W_e W_{\tilde{x}} B_1^T e\xi(x)^T\xi(x)\tilde{x}^T B_2 \\
& + W_e^2 B_1^T e\xi(x)^T\xi(x)e^T B_1 + W_{\tilde{x}}^2 B_2^T\tilde{x}\xi(x)^T\xi(x)\tilde{x}^T B_2\},
\end{aligned}
\tag{4.9}
$$

According to the equality (3.3), the $0 \le W_{\tilde{x}}\|\tilde{x}\| < 1$ and $0 \le W_e\|e\| < 1$ is obtained. We get

$$
\begin{aligned}
\Delta V_w \le & -(1-\nu)^2\|\tilde{\omega}\|^2 + 2(\nu - \nu^2)\|\omega\|\|\tilde{\omega}\| + 2\nu\bar{\xi}\|B_1\|\|\tilde{\omega}\| \\
& + 2\nu\bar{\xi}\|B_2\|\|\tilde{\omega}\| + (1-\nu)^2\|\omega\|^2 + 2(1-\nu)\bar{\xi}\|\omega\|\|B_1\| \\
& + 2(1-\nu)\bar{\xi}\|\omega\|\|B_2\| + 2\bar{\xi}^2\|B_1\|\|B_2\| + \bar{\xi}^2\|B_1\|^2 + \bar{\xi}^2\|B_2\|^2, \\
\Delta V_w \le & -(1-\nu)^2\|\tilde{\omega}\|^2 + (2(\nu - \nu^2)\|\omega\| + 2\nu\bar{\xi}\|B_1\| + 2\nu\bar{\xi}\|B_2\|)\|\tilde{\omega}\| \\
& + (1-\nu)^2\|\omega\|^2 + 2(1-\nu)\bar{\xi}\|\omega\|(\|B_1\| + \|B_2\|) + \bar{\xi}^2(\|B_1\| + \|B_2\|)^2,
\end{aligned}
\tag{4.10}
$$

Combined with the above inequalities and $0 < \nu < 1$, we can get

$$
\Delta V_w \le -\eta_1\|\tilde{\omega}\|^2 + \eta_2\|\tilde{\omega}\| + \eta_3, \tag{4.11}
$$

where

$$
\begin{aligned}
\eta_1 &= (1-\nu)^2, \\
\eta_2 &= 2(\nu - \nu^2)\|\omega\| + 2\nu\bar{\xi}\|B_1\| + 2\nu\bar{\xi}\|B_2\|, \\
\eta_3 &= (1-\nu)^2\|\omega\|^2 + 2(1-\nu)\bar{\xi}\|\omega\|(\|B_1\| + \|B_2\|) + \bar{\xi}^2(\|B_1\| + \|B_2\|)^2,
\end{aligned}
\tag{4.12}
$$

And, inequality (4.11) can be rewritten as follows

$$
\begin{aligned}
\Delta V_w &\le -\frac{\eta_1}{2}\|\tilde{\omega}\|^2 - (\sqrt{\frac{\eta_1}{2}}\|\tilde{\omega}\| - \frac{\eta_2}{\sqrt{2\eta_1}})^2 + \eta_4, \\
\Delta V_w &\le -\frac{\eta_1}{2}\|\tilde{\omega}\|^2 + \eta_4,
\end{aligned}
\tag{4.13}
$$

where $\eta_4 = \eta_3 + (\eta_2{}^2/2\eta_1)$. It is found that ΔV_w is negative when $\|\tilde{\omega}\|^2 > 2\eta_4/\eta_1$. In the triggered instant t_k, the estimated error $\tilde{\omega}$ is ultimately bounded.

According to the above analysis, it is proved that the error $\tilde{\omega}$ is locally ultimately bounded. Then, the stability is illustrated in the following theorem.

Theorem 1. *Consider the multi-agent systems (2.4), the controller (3.1), the event-triggered condition (3.6), and adaptive update law (3.3), suppose that the initial augmentation state $\psi(0) \in \mathcal{I}$. If the Assumptions 1–3 are satisfied. Then, the augmented state ψ is locally ultimately bounded.*

Proof: In order to prove the stability of system (4.7), the Lyapunov function is constructed as

$$V_f(\psi) = V_x + V_{x_k} + V_{\tilde{x}} + V_w, \tag{4.14}$$

where $V_x = x^T P x$, $V_{x_k} = x_k{}^T x_k$, $V_{\tilde{x}} = \tilde{x}^T P \tilde{x}$ and $V_w = tr\{\tilde{\omega}^T \tilde{\omega}\}$. P is a positive matrix satisfying the Assumption 3.

In the first step, the continuous part of the dynamical (4.7) is considered for $t \in (t_k, t_{k+1}]$. The time derivative of the Lyapunov function (4.14) can be expressed in the following four parts.

We know that the $\dot{V}_x$ can be described as

$$\begin{aligned}\dot{V}_x = {} & x^T P((A_2 \otimes I_n)x + f_2(x) + g(x)p(L \otimes I_n)(x_k - x) \\ & + g(x)p(\hat{\omega}^T \xi(x_k) - \hat{\omega}^T \xi(x)) - g(x)p(\tilde{\omega}^T \xi(x) + \delta(x))) \\ & + ((A_2 \otimes I_n)x + f_2(x) + g(x)pL(x_k - x) \\ & + g(x)p(\hat{\omega}^T \xi(x_k) - \hat{\omega}^T \xi(x)) - g(x)p(\tilde{\omega}^T \xi(x) + \delta(x)))^T Px, \\ = {} & x^T (P(A_2 \otimes I_n) + (A_2 \otimes I_n)^T P)x + 2x^T P f_2(x) \\ & + 2x^T P g(x)p((L \otimes I_n)(x_k - x)) - 2x^T P g(x)p(\tilde{\omega}^T \xi(x) + \delta(x)) \\ & + 2x^T P g(x)p(\hat{\omega}^T \xi(x_k) - \hat{\omega}^T \xi(x)),\end{aligned} \tag{4.15}$$

Using the Assumption 2 and Young's inequality , one has

$$\begin{aligned}\dot{V}_x \le {} & x^T (P(A_2 \otimes I_n) + (A_2 \otimes I_n)^T P + \sigma l_f{}^2 I + \frac{1}{\sigma} PP)x \\ & + 2x^T P g(x)p((L \otimes I_n)(x_k - x)) + 2x^T P g(x)p(\hat{\omega}^T \xi(x_k) - \hat{\omega}^T \xi(x)) \\ & - 2x^T P g(x)p(\tilde{\omega}^T \xi(x) + \delta(x)),\end{aligned} \tag{4.16}$$

Using Assumption 3, it is rewritten as

$$\begin{aligned}\dot{V}_x \le {} & -x^T Q x + 2x^T P g(x)p((L \otimes I_n)(x_k - x)) \\ & + 2x^T P g(x)p(\hat{\omega}^T \xi(x_k) - \hat{\omega}^T \xi(x)) - 2x^T P g(x)p(\tilde{\omega}^T \xi(x) + \delta(x)),\end{aligned} \tag{4.17}$$

It is fact that $0 < \|p\| < 1$, Lipschitz condition (6) of Assumption 2, and the $g(x)$ is bounded, we can get

$$\begin{aligned}\dot{V}_x \le {} & -\lambda_{min}(Q)\|x\|^2 + 2g_{max}\|P\|(\|\tilde{\omega}\|\bar{\xi} + \bar{\delta})\|x\| \\ & + (2g_{max}\|P\|\|x\|(\|L \otimes I_n\| + l_\xi\|\hat{\omega}\|))\|e\|,\end{aligned} \tag{4.18}$$

Consider the event-triggered condition (3.6), the $\|e\| \leq \zeta_e(\|x\| + \bar{B}_X)$ is given. Therefore, we can obtained

$$\dot{V}_x \leq -(\lambda_{min}(Q) - K_\zeta)\|x\|^2 + 2(g_{max}\|P\|(\|\tilde{\omega}\|\bar{\xi} + \bar{\delta}) + K_\zeta \bar{B}_X)\|x\|, \tag{4.19}$$

The derivative of $\dot{V}_{x_k}$ is given as

$$\dot{V}_{x_k} = -2\tau x(t_k)^2 \exp(-\tau(t - t_k)) \leq 0, \tag{4.20}$$

Next, the derivative of $V_{\tilde{x}}$ is given as follows

$$\begin{aligned} \dot{V}_{\tilde{x}} = \tilde{x}^T P((A_2 \otimes I_n)\tilde{x} + f_2(x) - f_2(\hat{x}) + g(x)p((L \otimes I_n)(x_k - x)) \\ + g(x)p(\hat{\omega}^T\xi(x_k) - \hat{\omega}^T\xi(x)) - g(x)p(\tilde{\omega}^T\xi(x) + \delta(x))) \\ + ((A_2 \otimes I_n)\tilde{x} + f_2(x) - f_2(\hat{x}) + g(x)p((L \otimes I_n)(x_k - x)) \\ + g(x)p(\hat{\omega}^T\xi(x_k) - \hat{\omega}^T\xi(x)) - g(x)p(\tilde{\omega}^T\xi(x) + \delta(x)))^T P\tilde{x}, \end{aligned} \tag{4.21}$$

Similar to (4.16)-(4.18), we can obtain

$$\begin{aligned} \dot{V}_{\tilde{x}} \leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2g_{max}\|P\|(\|\tilde{\omega}\|\bar{\xi} + \bar{\delta})\|\tilde{x}\| \\ + 2g_{max}\|P\|\|\tilde{x}\|(\|L \otimes I_n\| + l_\xi\|\hat{\omega}\|)\|e\|, \end{aligned} \tag{4.22}$$

Based on the event-triggered condition (3.6) and Young's inequality, the following inequality holds

$$\begin{aligned} \dot{V}_{\tilde{x}} &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + K_\zeta\|x\|\|\tilde{x}\| \\ &\quad + 2(g_{max}\|P\|(\|\tilde{\omega}\|\bar{\xi} + \bar{\delta}) + K_\zeta\bar{B}_X)\|\tilde{x}\|, \\ &\leq -(\lambda_{min}(Q) - \beta K_\zeta)\|\tilde{x}\|^2 \\ &\quad + 2(g_{max}\|P\|(\|\tilde{\omega}\|\bar{\xi} + \bar{\delta}) + K_\zeta\bar{B}_X)\|\tilde{x}\| + \frac{1}{\beta}K_\zeta\|x\|, \end{aligned} \tag{4.23}$$

where β is a positive constant, and the equality $\beta^2 = \beta + 1$ is satisfied.

Because the $\hat{\omega}_i$ is held in the event-triggered time intervals, we can get $\dot{V}_w = 0$. Therefore, one has

$$\begin{aligned} \dot{V}_f(\psi) &\leq -(\lambda_{min}(Q) - \frac{\beta + 1}{\beta}K_\zeta)\|x\|^2 \\ &\quad + 2(g_{max}\|P\|(\|\tilde{\omega}\|\bar{\xi} + \bar{\delta}) + K_\zeta\bar{B}_X)\|x\| \\ &\quad - (\lambda_{min}(Q) - \beta K_\zeta)\|\tilde{x}\|^2 \\ &\quad + 2(g_{max}\|P\|(\|\tilde{\omega}\|\bar{\xi} + \bar{\delta}) + K_\zeta\bar{B}_X)\|\tilde{x}\|, \end{aligned} \tag{4.24}$$

According to β is solution of the $\beta^2 = \beta + 1$ and $\beta > 0$, we can get

$$\dot{V}_f(\psi) \leq -\theta_1\|\bar{X}\|^2 + \theta_2\|\bar{X}\|, \tag{4.25}$$

where $\bar{X} = [\|x\|, \|\tilde{x}\|]^T$, $\theta_1 = \lambda_{min}(Q) - \beta K_\zeta$ and $\theta_2 = 2\sqrt{2}(g_{max}\|P\|(\|\tilde{\omega}\|\bar{\xi} + \bar{\delta}) + K_\zeta\bar{B}_X)$. According to Lemma 2,we know that $\tilde{\omega}$ is bounded, we can conclude that the θ_2 is bounded. It is fact that $\dot{V}_f(\psi)$ is less than zero when $\|\bar{X}\| > \bar{G}_{\bar{X}} = \theta_2/\theta_1$ is satisfied, and it means that the multi-agent systems state $x(t)$ and the state error $\tilde{x}$ are locally ultimately bounded for $t \in (t_k, t_{k+1}]$.

Consider the jump part of the dynamical (4.7) in $t = t_k$, the Lyapunov function is defined as (4.14). The difference is obtained as follow form

$$\Delta V_f(\psi) = \Delta V_f(\psi^+) - \Delta V_f(\psi), \tag{4.26}$$

Using $\Delta\psi = \psi^+ - \psi$ and the dynamics (4.7), we have

$$\begin{aligned}\Delta V_f(\psi) &= x^{+T} P x^+ + {x_k}^{+T} x_k^+ + \tilde{x}^{+T} P \tilde{x}^+ + tr\{\tilde{\omega}^{+T}\tilde{\omega}^+\} \\ &\quad - x^T P x - {x_k}^T x_k - \tilde{x}^T P \tilde{x} - tr\{\tilde{\omega}^T \tilde{\omega}\}, \\ &= x^T x - {x_k}^T x_k - \tilde{x}^T P \tilde{x} + tr\{\tilde{\omega}^{+T}\tilde{\omega}^+\} - tr\{\tilde{\omega}^T\tilde{\omega}\},\end{aligned} \tag{4.27}$$

For any $t \in (t_k, t_{k+1}]$, it is fact that the $x(t)$ is bounded. Consider the inequality (4.11), one obtains

$$\begin{aligned}\Delta V_f(\psi) &\leq -\|x_k\|^2 - \eta_1\|\tilde{\omega}\|^2 + \eta_2\|\tilde{\omega}\| + \eta_3 + \eta_x, \\ &\leq -\|x_k\|^2 - \eta_1(\|\tilde{\omega}\| - \frac{\eta_2}{2\eta_1})^2 + \frac{4\eta_1(\eta_3 + \eta_x) + {\eta_2}^2}{4\eta_1},\end{aligned} \tag{4.28}$$

where η_x is bounded for $\|x\|^2$ and $\eta_x < \|\bar{X}\|^2$.

Consider the above analysis, $\Delta V_f(\psi)$ is negative when $\|x_k\| > G_{x_k}$ or $\|\tilde{\omega}\| > G_{\tilde{\omega}}$, where

$$G_{x_k} = \sqrt{\frac{4\eta_1(\eta_3 + \eta_x) + \eta_2^2}{4\eta_1}}, \tag{4.29}$$

$$G_{\tilde{\omega}} = \frac{\eta_2 + \sqrt{4\eta_1(\eta_3 + \eta_x) + \eta_2^2}}{2\eta_1}, \tag{4.30}$$

Further, the state ψ is locally ultimately bounded in $t = t_k$. Because of $x^+ = x$ and $\|\tilde{x}^+\| \leq \|\tilde{x}\|$ for $t = t_k$, the $\|\bar{X}\| \leq \bar{G}_{\bar{X}}$ in $t = t_k$. we can conclude that G_{x_k} and $G_{\tilde{\omega}}$ converge to $\bar{G}_{x_k}$ and $\bar{G}_{\tilde{\omega}}$, where

$$\bar{G}_{x_k} = \sqrt{\frac{4\eta_1\left(\eta_3 + \bar{G}_{\bar{X}}^2\right) + \eta_2^2}{4\eta_1}}, \tag{4.31}$$

$$\bar{G}_{\tilde{\omega}} = \frac{\eta_2 + \sqrt{4\eta_1\left(\eta_3 + \bar{G}_{\bar{X}}^2\right) + \eta_2^2}}{2\eta_1}, \tag{4.32}$$

The $\hat{\omega}$ remains constant, and $\|x_k\|$ is monotone decreasing function on $t \in (t_k, t_{k+1}]$. If $\|\psi\| \geq \sqrt{\bar{G}_{\bar{X}}^2 + \bar{G}_{x_k}^2 + \bar{G}_{\tilde{\omega}}^2}$, the Lemma 1 is satisfied. Then, it can be proven that all signals of the multi-agent systems are locally ultimately bounded.

5. The event-triggered time interval

In this section, it is proven that the event-triggered time interval is a nonzero positive constant in the following theorem. Therefore, the event-triggered time interval is a lower bound.

Theorem 2. *Consider the controller (3.1) and the event-triggered condition (3.6) are proposed in this paper. Let Assumptions 1-3 be satisfied and the initial $\hat{\omega}(0)$ is in a compact set. Then, the event-triggered time interval T_k is a positive scalar with nonzero lower bound.*

In the event-triggered time interval $t \in (t_k, t_{k+1}]$, the following equalities hold

$$\begin{aligned} e &= x(t) - x_k = x(t) - x(t_k)\exp(-\tau(t-t_k)), \\ \dot{e} &= \dot{x}(t) + \tau x(t_k)\exp(-\tau(t-t_k)), \end{aligned} \tag{5.1}$$

Based on the system dynamics (4.4) and (4.6), one has

$$\begin{aligned} \dot{e} =& (A_2 \otimes I_n)x + f_2(x) + g(x)p(L \otimes I_n)(x_k - x) + g(x)p(\hat{\omega}^T\xi(x_k) - \hat{\omega}^T\xi(x)) \\ &- g(x)p(\tilde{\omega}^T\xi(x) + \delta(x)) + \tau x(t_k)\exp(-\tau(t-t_k)), \\ =& (A_2 \otimes I_n)x - (A_2 \otimes I_n)x_k + (A_2 \otimes I_n)x_k \\ &+ f_2(x) - f_2(x_k) + f_2(x_k) + g(x)p(L \otimes I_n)(x_k - x) \\ &+ g(x)p(\hat{\omega}^T\xi(x_k) - \hat{\omega}^T\xi(x)) \\ &- g(x)p(\tilde{\omega}^T\xi(x) + \delta(x)) + \tau x(t_k)\exp(-\tau(t-t_k)), \\ =& (A_2 \otimes I_n)e + (\tau I_n + A_2 \otimes I_n)x_k \\ &+ f_2(x) - f_2(x_k) + f_2(x_k) + g(x)p(L \otimes I_n)(x_k - x) \\ &+ g(x)p(\hat{\omega}^T\xi(x_k) - \hat{\omega}^T\xi(x)) \\ &- g(x)p(\tilde{\omega}^T\xi(x) + \delta(x)), \end{aligned} \tag{5.2}$$

Therefore, one obtains

$$\begin{aligned} \|\dot{e}\| \le& \|(A_2 \otimes I_n)\|\|e\| + \|(\tau I_n + A_2 \otimes I_n)\|\|x_k\| + \|f_2(x) - f_2(x_k)\| \\ &+ \|f_2(x_k)\| + g_{max}\|p\|\|L \otimes I_n\|\|e\| + g_{max}\|p\|\|(\hat{\omega}^T\xi(x_k) - \hat{\omega}^T\xi(x))\| \\ &+ g_{max}\|p\|\|(\tilde{\omega}^T\xi(x) + \delta(x))\|, \\ \le& (\|(A_2 \otimes I_n)\| + g_{max}\|L \otimes I_n\| + l_f)\|e\| + (\|\tau I_n + A_2 \otimes I_n\| + l_f)\|x_k\| \\ &+ g_{max}(2\|\hat{\omega}\|\bar{\xi} + \|\tilde{\omega}\|\bar{\xi} + \bar{\delta}), \\ \le& (\|(A_2 \otimes I_n)\| + g_{max}\|L \otimes I_n\| + l_f)\|e\| + \Gamma_e, \end{aligned} \tag{5.3}$$

where $\Gamma_e = (\|\tau I_n + A_2 \otimes I_n\| + l_f)\|x_k\| + g_{max}(2\|\hat{\omega}\|\bar{\xi} + \|\tilde{\omega}\|\bar{\xi} + \bar{\delta})$.

Consider the comparison lemma in [50] and $e(t_k) = 0$, one has

$$\begin{aligned} \|e\| &\le \int_{t_k}^{t} exp((\|(A_2 \otimes I_n)\| + g_{max}\|L \otimes I_n\| + l_f)(t-s))\Gamma_e ds, \\ &\le \frac{\Gamma_e}{\Gamma_1}(exp(\Gamma_1(t-t_k)) - 1), \end{aligned} \tag{5.4}$$

where $\Gamma_1 = \|(A_2 \otimes I_n)\| + g_{max}\|L \otimes I_n\| + l_f$

Based on the event-triggered condition (3.6), the $\|e\| \ge \zeta_e \bar{B}_x$ in the trigger instant $t = t_k$. The $T_k = t_{k+1} - t_k$ represents the k-th event-triggered time interval. Further, we can get

$$\zeta_e \bar{B}_x \le \frac{\Gamma_e}{\Gamma_1}(exp(\Gamma_1 T_k) - 1), \tag{5.5}$$

which means that

$$T_k \ge \frac{1}{\Gamma_1}\ln(\frac{\zeta_e \bar{B}_x \Gamma_1}{\Gamma_e} + 1) > 0, \tag{5.6}$$

Therefore, the event-triggered time interval is a lower bound. In other words, there is no Zeno behavior.

6. Simulation

In this section, we prove the theoretical results by numerical examples. We consider the leaderless multi-agent systems with four follower nodes in Figure 1. The each agent's dynamic is given as follows

$$\dot{x}_i(t) = A_1 x_i(t) + f_{1i}(x_i) + g(x_i) p_i u_i, \tag{6.1}$$

where $x_i = [x_{i1}, x_{i2}]^T$, $u_i = [u_{i1}, u_{i2}]^T$, $i = 1, \ldots, 4$. The matrices A_1, $f_{1i}(x_i)$, $g(x_i)$, and p_i are chosen as follows

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad f_{1i}(x_i) = \begin{bmatrix} -2x_{i2} + x_{i1} \\ -2x_{i1} + x_{i2} \end{bmatrix}, \\ g(x_i) &= \begin{bmatrix} \frac{-1}{cos^2 x_{i2} - 2} & \frac{1+cos x_{i2}}{cos^2 x_{i2} - 2} \\ \frac{1+cos x_{i2}}{cos^2 x_{i2} - 2} & \frac{-3-2cos x_{i2}}{cos^2 x_{i2} - 2} \end{bmatrix}, \quad p_i = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.7 \end{bmatrix}, \end{aligned} \tag{6.2}$$

We assume the ideal controller $u_i^* = Q(x)$, so the each agent's dynamic is rewritten as

$$\dot{x}_i(t) = A_2 x_i(t) + f_{2i}(x_i), \tag{6.3}$$

where A_2 =diag$\{-20, -25\}$ and $f_{2i} = [4x_{i1}, 5x_{xi2}]^T$.

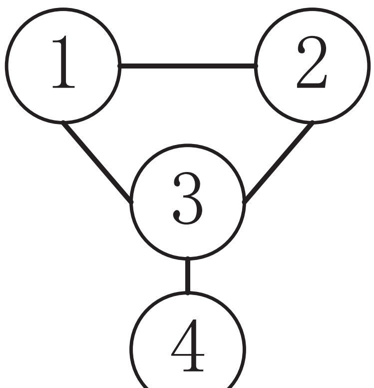

Figure 1. Communication topology.

The constants are chosen as P =diag$(1/60, 1/60)$, $l_f = 2.4$, $l_\xi = 2.4$, $\sigma = 1$, $\nu = 0.0015$, $a_e = 5$, $a_{\tilde{x}} = 5$, $b_{\tilde{x}} = 1$, $b_e = 1$, $K_\zeta = 0.09$, $\tau = 0.81$, B_{1i} =diag$(1, 1)$ and B_{2i} =diag$(1, 1)$. We choose the initial values as $x_1 = [5, 2.5]^T$, $x_2 = [3, 2]^T$, $x_3 = [1, 1.21]^T$, $x_4 = [4, 3]^T$. Figures 2–7 show the simulation results. The Figure 2 depicts the responses of the system state vector x_{11}, x_{21}, x_{31} and x_{41}. The Figure 3 depicts the responses of the system state vector x_{12}, x_{22}, x_{32} and x_{42}. The above figures show the leaderless consensus. The event-triggered time intervals are presented in Figures 4–7.

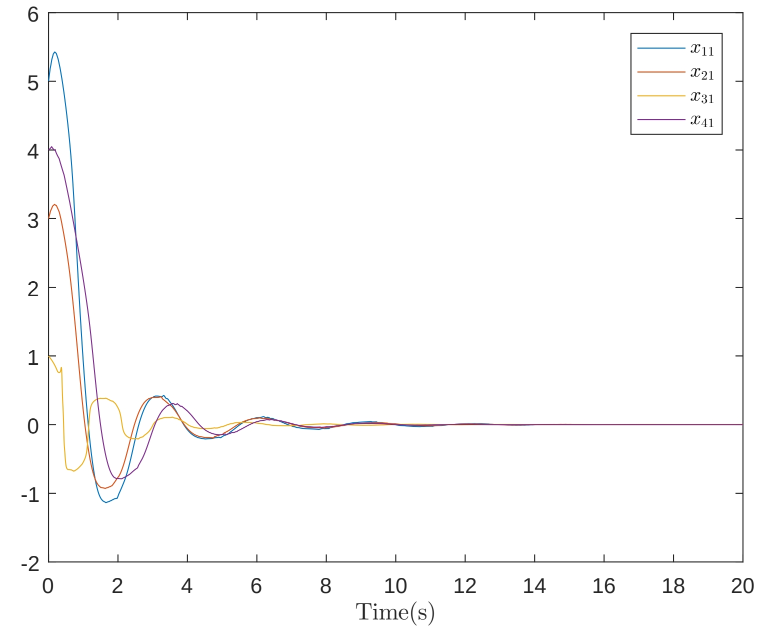

Figure 2. Responses of the state x_{11}, x_{21}, x_{31} and x_{41}.

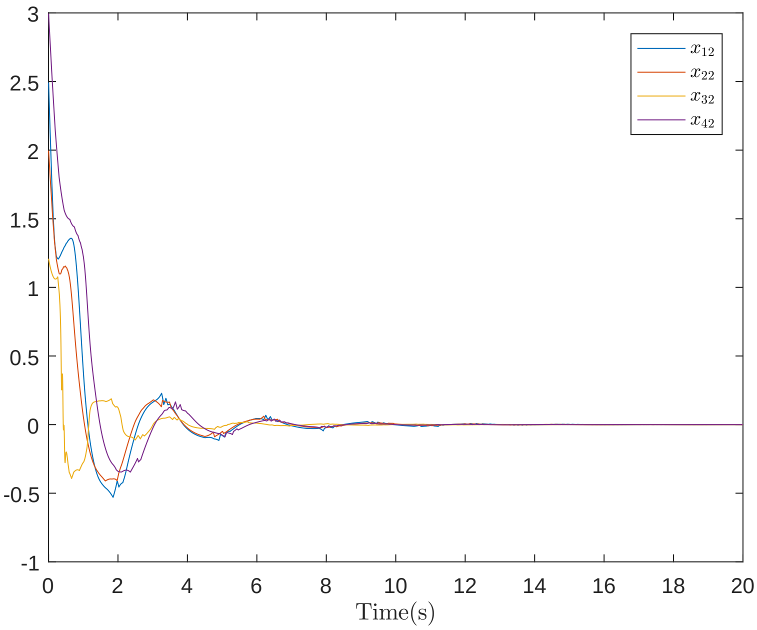

Figure 3. Responses of the state x_{12}, x_{22}, x_{32} and x_{42}.

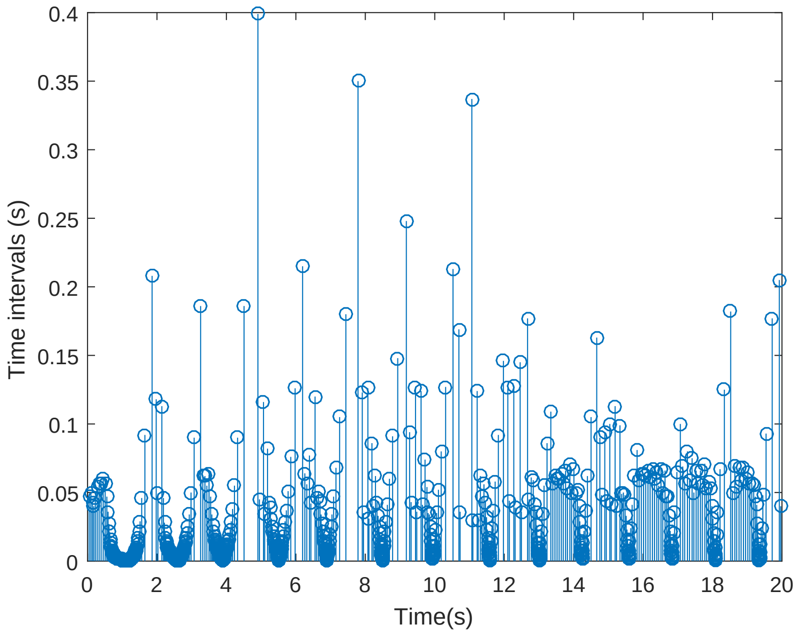

Figure 4. Event-triggered time instants of agent 1.

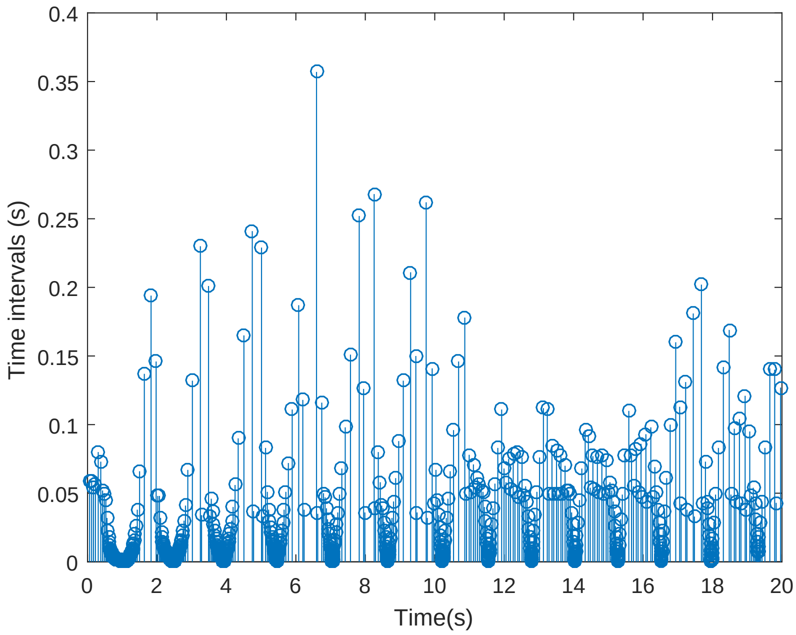

Figure 5. Event-triggered time instants of agent 2.

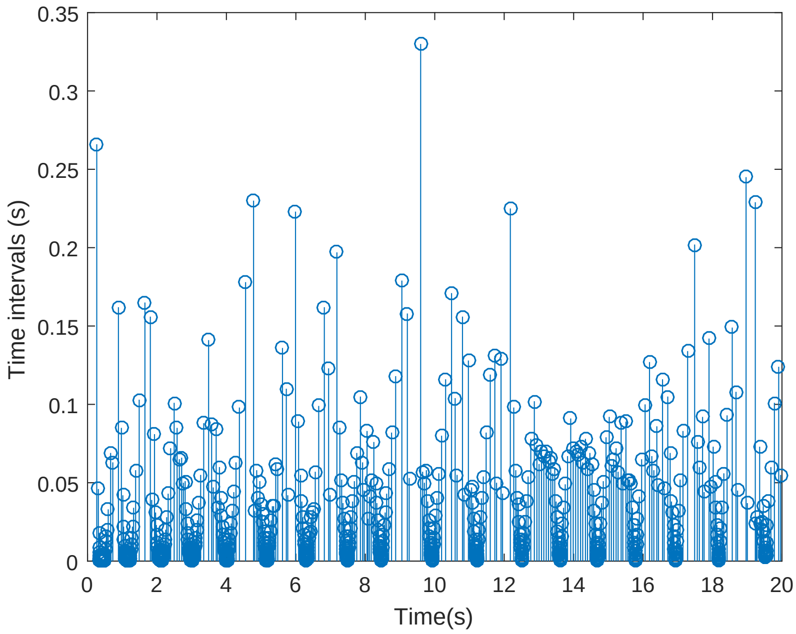

Figure 6. Event-triggered time instants of agent 3.

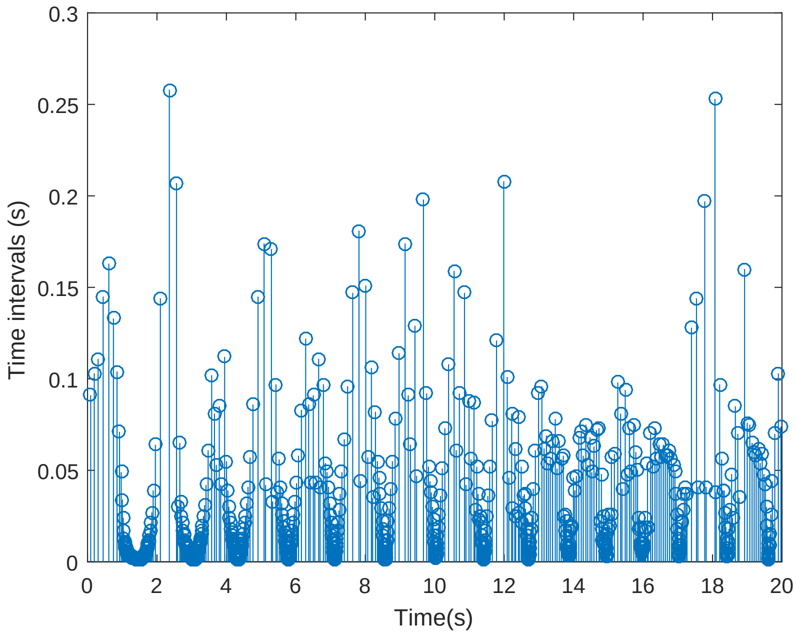

Figure 7. Event-triggered time instants of agent 4.

7. Conclusions

In this paper, an event-triggered fault-tolerant consensus control strategy for nonlinear multi-agent systems has been proposed. Based on the approximate property of neural networks and the model reference control method, the fault-tolerant method has been designed to ensure security for leaderless multi-agent systems, which reduces the exact requirements of control parameters. The ETC scheme based on a relative threshold method has been proposed to reduce communications. The event-triggered scheme has been proved that there is no Zeno behavior. By the Lyapunov stability theory, we have obtained that all signals are bounded. The simulation result has confirmed the validity of proposed approach. In the future, we will continue to study the issue of event-triggered and fault-tolerant control. These problems are of great significance in various practical systems.

Acknowledgments

This work was partially supported by the National Natural Science Foundation of China (61703051), and the Project of Liaoning Province Science and Technology Program under Grant (2019-KF-03-13)

Conflict of interest

The authors declare no conflict of interest in this paper.

References

1. J. Qin, Q. Ma, Y. Shi, et al. *Recent advances in consensus of multi-agent systems: A brief survey*, IEEE Trans. Ind. Electron., **64** (2016), 4972–4983.
2. J. Zhang, X. Chen, G. Gu, *State Consensus for Discrete-time Multi-agent Systems over Time-varying Graphs*, arXiv preprint arXiv:1710.03398, 2017.
3. X. Liang, C. Xu, D. Wang, *Adaptive neural network control for marine surface vehicles platoon with input saturation and output constraints*, AIMS Math., **5** (2020), 587–602.
4. P. Colli, G. Gilardi, J. Sprekels, *Distributed optimal control of a nonstandard nonlocal phase field system*, AIMS Math., **1** (2016), 225–260.
5. H. Liang, L. Zhang, Y. Sun, et al. *Containment control of semi-markovian multi-agent systems with switching topologies*, IEEE Trans. Syst. Man Cybern. Syst., 2019.
6. W. Wang, H. Liang, Y. Pan, et al. *Prescribed performance adaptive fuzzy containment control for nonlinear multi-agent systems using disturbance observer*, IEEE Trans. Cybern., 2020.
7. Y. Zhang, H. Liang, H. Ma, et al. *Distributed adaptive consensus tracking control for nonlinear multi-agent systems with state constraints*, Appl. Math. Comput., **326** (2018), 16–32.
8. W. Liu, S. Zhou, Y. Qi, et al. *Leaderless consensus of multi-agent systems with lipschitz nonlinear dynamics and switching topologies*, Neurocomputing, **173** (2016), 1322–1329.
9. D. Yang, X. Li, J. Qiu, *Output tracking control of delayed switched systems via state-dependent switching and dynamic output feedback*, Nonlinear Anal. Hybrid. Syst., **32** (2019), 294–305.

10. W. Sun, S. Su, J. Xia, et al. *Adaptive fuzzy tracking control of flexible-joint robots with full-state constraints*, IEEE Trans. Syst. Man Cybern. Syst., **49** (2018), 2201–2209.

11. M. Tong, W. Lin, X. Huo, et al. *A model-free fuzzy adaptive trajectory tracking control algorithm based on dynamic surface control*, Int. J. Adv. Robot. Syst., **16** (2019), 1729881419894417.

12. Y. Han, C. Li, Z. Zeng, et al. *Exponential consensus of discrete-time nonlinear multi-agent systems via relative state-dependent impulsive protocols*, Neural Networks, **108** (2018), 192–201.

13. K. Liu, H. Zhu, J. Lu, *Bridging the gap between transmission noise and sampled data for robust consensus of multi-agent systems*, IEEE Trans. Circuits Syst. I, **62** (2015), 1836–1844.

14. W. Wang, H. Liang, Y. Zhang, et al. *Adaptive cooperative control for a class of nonlinear multi-agent systems with dead zone and input delay*, Nonlinear Dyn., **96** (2019), 2707–2719.

15. Q. Zhou, W. Wang, H. Liang, et al. *Observer-based event-triggered fuzzy adaptive bipartite containment control of multi-agent systems with input quantization*, IEEE Trans. Fuzzy Syst., 2019.

16. X. Li, Q. Zhou, P. Li, et al. *Event-triggered consensus control for multi-agent systems against false data injection attacks*, IEEE Trans. Cybern., 2019.

17. Q. Zhou, W. Wang, H. Ma, et al. *Event-triggered fuzzy adaptive containment control for nonlinear multi-agent systems with unknown Bouc-Wen hysteresis input*, IEEE Trans. Fuzzy Syst., 2019.

18. Z. Gu, P. Shi, D. Yue, et al. *Decentralized adaptive event-triggered H_∞ filtering for a class of networked nonlinear interconnected systems*, IEEE Trans. Cybern., **49** (2018), 1570–1579.

19. Z. Chen, Q. Han, Y. Yan, et al. *How often should one update control and estimation: review of networked triggering techniques*, Sci. China Inf. Sci., **63** (2020).

20. L. Xing, C. Wen, Z. Liu, et al. *Event-triggered adaptive control for a class of uncertain nonlinear systems*, IEEE Trans. Autom. Control, **62** (2016), 2071–2076.

21. P. Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, IEEE Trans. Autom. Control, **52** (2007), 1680–1685.

22. D. V. Dimarogonas, E. Frazzoli, K. H. Johansson, *Distributed eventtriggered control for multi-agent systems*, IEEE Trans. Autom. Control, **57** (2011), 1291–1297.

23. Y. Zhang, J. Sun, H. Liang, et al. *Event-triggered adaptive tracking control for multi-agent systems with unknown disturbances*, IEEE Trans. Cybern., 2018.

24. Y. Li, G. Yang, S. Tong, *Fuzzy adaptive distributed event-triggered consensus control of uncertain nonlinear multiagent systems*, IEEE Trans. Syst. Man Cybern. Syst., 2018.

25. C. Nowzari, E. Garcia, J. Cortés, *Event-triggered communication and control of networked systems for multi-agent consensus*, Automatica, **105** (2019), 1–27.

26. M. Ajina, D. Tabatabai, C. Nowzari, *Asynchronous distributed event-triggered coordination for multiagent coverage control*, IEEE Trans. Cybern., 2019.

27. S. Zhu, Y. Liu, Y. Lou, et al. *Stabilization of logical control networks: An event-triggered control approach*, Sci. China Inf. Sci., **63** (2020), 1–11.

28. L. Cao, H. Li, G. Dong, et al. *Event-triggered control for multiagent systems with sensor faults and input saturation*, IEEE Trans. Syst. Man Cybern. Syst., 2019.

29. L. Zhang, H. Liang, Y. Sun, et al. *Adaptive event-triggered fault detection scheme for semi-markovian jump systems with output quantization*, IEEE Trans. Syst. Man Cybern. Syst., 2019.

30. Z. Gu, T. Zhang, Z. Huan, et al. *A novel event-triggered mechanism for networked cascade control system with stochastic nonlinearities and actuator failures*, J. Franklin Inst., **356** (2019), 1955–1974.

31. M. Liu, X. Cao, P. Shi, *Fault estimation and tolerant control for fuzzy stochastic systems*, IEEE Trans. Fuzzy Syst., **21** (2012), 221–229.

32. X. Liu, D. Zhai, T. Li, et al. *Fuzzy-approximation adaptive faulttolerant control for nonlinear pure-feedback systems with unknown control directions and sensor failures*, Fuzzy Sets Syst., **356** (2019), 28–43.

33. C. Deng, G. Yang, *Cooperative adaptive output feedback control for nonlinear multi-agent systems with actuator failures*, Neurocomputing, **199** (2016), 50–57.

34. Q. Fan, G. Yang, *Event-based fuzzy adaptive fault-tolerant control for a class of nonlinear systems*, IEEE Trans. Fuzzy Syst., **26** (2018), 2686–2698.

35. X. Li, G. Yang, *Neural-network-based adaptive decentralized fault-tolerant control for a class of interconnected nonlinear systems*, IEEE Trans. Neural Networks Learn. Syst., **29** (2016), 144–155.

36. H. Yang, P. Shi, X. Li, et al. *Fault-tolerant control for a class of t-s fuzzy systems via delta operator approach*, Signal Proc., **98** (2014), 166–173.

37. P. Du, H. Liang, S. Zhao, et al. *Neural-based decentralized adaptive finite-time control for nonlinear large-scale systems with time-varying output constraints*, IEEE Trans. Syst. Man Cybern. Syst., 2019.

38. X. Zhang, X. Li, J. Cao, et al. *Design of memory controllers for finite-time stabilization of delayed neural networks with uncertainty*, J. Franklin Inst., **355** (2018), 5394–5413.

39. X. Li, B. Zhang, P. Li, et al. *Finite-horizon H-infinity state estimation for periodic neural networks over fading channels*, IEEE Trans. Neural. Networks. Learn. Syst., 2019.

40. W. Sun, S. Su, J. Xia, et al. *Adaptive fuzzy control with high-order barrier Lyapunov functions for high-order uncertain nonlinear systems with full-state constraints*, IEEE Trans. Cybern., 2018.

41. Y. Sheng, F. Lewis, Z. Zeng, et al. *Lagrange stability and finite-time stabilization of fuzzy memristive neural networks with hybrid time-varying delays*, IEEE Trans. Cybern., 2019.

42. H. Liang, Z. Zhang, C. K. Ahn, *Event-triggered fault detection and isolation of discrete-time systems based on geometric technique*, IEEE Trans. Circuits Syst. II, 2019.

43. X. Li, P. Li, Q. Wang, *Input/output-to-state stability of impulsive switched systems*, Syst. Control Lett., **116** (2018), 1–7.

44. L. Zhang, H. K. Lam, Y. Sun, et al. *Fault detection for fuzzy semi-markov jump systems based on interval type-2 fuzzy approach*, IEEE Trans. Fuzzy Syst., 2019.

45. Q. Zhou, P. Du, H. Li, et al. *Adaptive fixed-time control of error-constrained pure-feedback interconnected nonlinear systems*, IEEE Trans. Syst. Man Cybern. Syst., 2019.

46. L. Liu, Y. Liu, S. Tong, *Neural networks-based adaptive nite-time faulttolerant control for a class of strict-feedback switched nonlinear systems*, IEEE Trans. Cybern., **49** (2018), 2536–2545.

47. A. Sahoo, H. Xu, S. Jagannathan, *Neural network-based event-triggered state feedback control of nonlinear continuous-time systems*, IEEE Trans. Neural Networks Learn. Syst., **27** (2015), 497–509.

48. X. Li, X. Yang, T. Huang, *Persistence of delayed cooperative models: Impulsive control method*, J. Franklin Inst., **342** (2019), 130–146.

49. W. Haddad, V. Chellaboina, S. Nersesov, *Impulsive and hybrid dynamical systems: Stability, dissipativity and control*, Princeton University Press, 2006.

50. H. Khalil, *Nonlinear systems*, Upper Saddle River, 2002.