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Research article

Algorithms for single-valued neutrosophic decision making based on TOPSIS and Clustering methods with new distance measure

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Abstract: Single-valued neutrosophic set (SVNS) is an important contrivance for directing the decision-making queries with unknown and indeterminant data by employing a degree of "acceptance", "indeterminacy", and "non-acceptance" in quantitative terms. Under this set, the objective of this paper is to propose some new distance measures to find discrimination between the SVNSs. The basic axioms of the measures have been highlighted and examined their properties. Furthermore, to examine the relevance of proposed measures, an extended TOPSIS ("Technique for Order Preference by Similarity to Ideal Solution") method is introduced to solve the group decision-making problems. Additionally, a new clustering technique is proposed based on the stated measures to classify the objects. The advantages, comparative analysis as well as superiority analysis is given to shows its influence over existing approaches.

Keywords: single-valued neutrosophic set; information measure; TOPSIS; clustering algorithm; decision-making
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1. Introduction

Multi-criteria decision making (MCDM) plays a vital role in our daily lives. In this competitive environment, our goal is to determine the best option that must be inspected toward the numerous criteria. However, in many cases, it is difficult for a person to opt for a suitable one due to the presence of several kinds of uncertainties in the data, which may occur due to a lack of knowledge or human error. Thus, the process of MCDM becomes growing these days and generally involves the following three phases.

i) Choose a proper scale to evaluate the given objects;

ii) Aggregate the information, using the suitable technique, to obtain the tendency value of each objects;

iii) Rank the given objects to select the best one(s).

As the decision process becomes complex day-by-day due to a huge number of uncertainties present in the information. Thus to feel it deeply and concisely, a theory of fuzzy set (FS) gave by Zadeh [1] plays a vital role in the decision-making problems (DMPs) by allowing a membership degree (MDs) to each element. Later on, Atanassov [2] prolongs the FSs to intuitionistic FSs (IFSs) by adding non-membership degrees (NMDs) along with MDs such that their sum can't pass one. In modern life, the complex system requires the uncertainties in views of indeterminacy and hence the present sets, FS or IFS, are incapable to deal with the information correctly. To consider it, Smarandache [3] presented neutrosophic set (NS) by involving the three independent functions namely "truth", "indeterminacy" and "falsity" which are the standard or non-standard real subsets of $]^{-}0, 1^{+}[$. However, for software engineering proposals the classical unit interval $[0, 1]$ is used. Thus, Wang et al. [4] enriches the NS to SVNS in which ranges of the independent degrees are taken as $[0, 1]$ instead of $]^{-}0, 1^{+}[$. Since its appearance and the ability to tackle the indeterminacy at the initial stage of data, SVNS is one of the hot topics to tackle the DMPs. In the literature, such theory is widely to solve the DMPs and are classified into two aspects, namely aggregation operators (AOs) and information measures (IMs), which are presented below:

1) *The basic results of SVNS:* Ye [5] firstly defined the operational laws for SVNSs. However, to order the given SVNSs, Peng et al. [6] defined the score function, while Nancy and Garg [7] presented an improved score function. Rani and Garg [8] presented the subtraction and division operations for interval NSs.

2) *AOs based approaches:* For example, Ye [5] initiated the idea of weighted averaging (WA) and weighted geometric (WG) operators. Peng et al. [6] presented the ordered WA and WG operators. Liu et al. [9] define the operators based on Hamacher norm. Nancy and Garg [10] developed Frank t-norm based AOs for DMPs. Garg and Nancy [11] defined logarithm operational laws based AOs for SVNS. Wei and Zhang [12] defined some Bonferroni mean AOs. Yang and Li [13] presented power AOs for SVNS. Garg and Nancy [14] presented the power aggregation operators for the linguistic SVNSs. Ji et al. [15] developed the frank prioritized BM operators for solving DMPs. Garg [16] defined the concept of neutrality operational laws and its based AOs for solving the decision-making problems. Garg and Nancy [17] developed the hybrid Heronian mean AOs by considering the concept of Choquet and frank norm operational laws for SVNSs.

3) *IMs based approaches:* In the literature, several measures such as distance, similarity, entropy are reviewed by the scholars. For instance, Majumdar [18] defined the distance measures for SVNSs. Huang [19] defined the distance measures between two SVNSs and hence defined the similarity, entropy and index of the distance measures to solve the clustering and the decision-making problems. Liu and Luo [20] defined the weighted distance measures for SVNSs. Garg and Nancy [21] defined the biparametric distance measures for SVNSs to solve the decision-making problems. Wu et al. [22] defined the cross-entropy for SVNSs. Garg and Nancy [23] defined the

entropy measure of order α for SVNSs. Mondal and Pramanik [24], Mondal et al. [25] defined the tangential and logarithm measures to compute the degree of similarity between two or more SVNSs. Liu et al. [26] discussed the multicriteria model for the selection of the transport service using SVNS features. A concept of divergence measure for SVNS is proposed by the authors in [27] and utilized it to solve the decision making problems.

The above-mentioned approaches are widely applicable in different fields. However, the approaches based on IMs are extensively reviewed. Among them, a "Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS)" [28] is a well-known approach that is working on the principle to pick the best one according to its minimum distance from the target set. For it, the two ideals namely PIS("Positive Ideal Set") and NIS ("Negative Ideal Set") are considered and the working of the TOPSIS method depends on it. In the TOPSIS method, both the inclinations such as similarity or dis-similarity are considered together to reach the target set. Based on these features, several researchers have addressed the problem of TOPSIS to solve the MCDM problem under the SVNS environment. For example, Biswas et al. [29] firstly presented the model of TOPSIS for SVNS. Pouresmaeil et al. [30] presented an MCDM method based on TOPSIS and VIKOR ("VIseKriterijumska Optimizacija I Kompromisno Resenje") methods. Selvachandran et al. [31] presented an extended TOPSIS method based on the maximum-deviation method while Peng and Dai [32] presented a modified TOPSIS method to solve the MCDM problems. Nancy and Garg [27] presented divergence measures based TOPSIS method under the SVNS. Mukhametzyanov and Pamucar [33] discuss the TOPSIS method for solving the MCDM problems through statistical analysis. Apart from the above scheme, Ruspini [34] built up a strategy for arranging perceptions into groups with the end goal that each cluster is as homogeneous as conceivable in the FS domain. The strategy is known as clustering which clusters the fuzzy information into the various leveled structure based on the proximity matrix. Inspired by this idea, Ye [35] introduced the clustering method for the SVNS minimum spanning tree. Again, Ye [36] presented another clustering algorithm based on the similarity measure which is obtained from distance measure. Since clustering has applications in various fields like image processing, data mining, medical diagnosis, machine learning, etc. therefore authors extend these applications of clustering analysis in the SVNS environment [37–40]. Thus, from the above studies, we conclude that SVNS is one of the most favorable environment to access the alternatives.

Considering the versatility of SVNS and the quality of the TOPSIS method, the theme of the present study is to examine the new distance measure to compute the degree of discrimination between the given sets. Also, we study their relevant axioms and the properties to show its validity. To reach the target precisely, we extended the given TOPSIS approach for DMPs. Holding all the above tips in mind, the main objective of the present work is listed as

(i) to define some new distance measures for given numbers under SVNS environment.

(ii) to develop an algorithm to determine the MCDM problems based on the extended TOPSIS approach.

(iii) to test the presented approach with a numerical example.

(iv) to impersonate a new clustering algorithm based on the proposed measures.

The major assets of the presented TOPSIS method over the others as the basic TOPSIS method aggregate the decision matrices by the aggregation operator and then decide relative coefficients based on the aggregated matrices. But in our approach, we find the relative coefficient of each decision matrix individually and then give the final results for each alternative. Then we aggregate the results of each decision-maker and get new relative coefficients for final ranking results. That is, this approach gives the individual as well as aggregated ranking results of decision-makers.

The rest of the text is designed as. Section 2 gives brief review on SVNS. Section 3 trades with new distance measures along with their characteristics. In Section 4, we offer an extended group TOPSIS method based on proposed measures to solve the MCDM problem. The applicability of the approach is discussed through a case study. In Section 5, a new clustering algorithm is presented and explained with a numerical example. Section 6 gives the advantages of the study. Finally, a concrete conclusion is given in Section 7.

2. Preliminaries

In it, we discuss some basic terms associated with SVNS in universal set $\mathcal{X}$.

Definition 2.1. [3] A neutrosophic set $\mathcal{N}$ is given as

$$\mathcal{N} = \{(x, \varsigma_{\mathcal{N}}(x), \tau_{\mathcal{N}}(x), \upsilon_{\mathcal{N}}(x)) \mid x \in \mathcal{X}\} \tag{2.1}$$

where $\varsigma_{\mathcal{N}}(x), \tau_{\mathcal{N}}(x), \upsilon_{\mathcal{N}}(x) : \mathcal{X} \to]^{-}0, 1^{+}[$ are the degrees of "acceptance", "indeterminacy" and "non-acceptance" such that $^{-}0 \leq \sup \varsigma_{\mathcal{N}}(x) + \sup \tau_{\mathcal{N}}(x) + \sup \upsilon_{\mathcal{N}}(x) \leq 3^{+}$.

Definition 2.2. [4] A single-valued neutrosophic set $\mathcal{N}$ in $\mathcal{X}$ is stated as

$$\mathcal{N} = \{(x, \varsigma_{\mathcal{N}}(x), \tau_{\mathcal{N}}(x), \upsilon_{\mathcal{N}}(x)) \mid x \in \mathcal{X}\} \tag{2.2}$$

where $\varsigma_{\mathcal{N}}, \tau_{\mathcal{N}}, \upsilon_{\mathcal{N}} \in [0, 1]$ and $0 \leq \varsigma_{\mathcal{N}} + \tau_{\mathcal{N}} + \upsilon_{\mathcal{N}} \leq 3$ for each $x \in \mathcal{X}$. We call a pair $\mathcal{N} = (\varsigma_{\mathcal{N}}, \tau_{\mathcal{N}}, \upsilon_{\mathcal{N}})$, throughout this article, and known as SVN number (SVNN).

Definition 2.3. [4] For two SVNNs $\mathcal{N}_1 = (\varsigma_1, \tau_1, \upsilon_1)$ and $\mathcal{N}_2 = (\varsigma_2, \tau_2, \upsilon_2)$, some basic operations are defined as

(i) $\mathcal{N}_1 \subseteq \mathcal{N}_2$ if $\varsigma_1 \leq \varsigma_2, \tau_1 \geq \tau_2, \upsilon_1 \geq \upsilon_2$.

(ii) $\mathcal{N}_1 \cap \mathcal{N}_2 = (\min(\varsigma_1, \varsigma_2), \max(\tau_1, \tau_2), \max(\upsilon_1, \upsilon_2))$.

(iii) $\mathcal{N}_1 \cup \mathcal{N}_2 = (\max(\varsigma_1, \varsigma_2), \min(\tau_1, \tau_2), \min(\upsilon_1, \upsilon_2))$.

(iv) $\mathcal{N}_1 = \mathcal{N}_2$ if and only if $\mathcal{N}_1 \subseteq \mathcal{N}_2$ and $\mathcal{N}_2 \subseteq \mathcal{N}_1$.

(v) Complement: $\mathcal{N}_1^c = (\upsilon_1, \tau_1, \varsigma_1)$.

Definition 2.4. Let $\Psi(\mathcal{X})$ be the collections of all SVNSs over $\mathcal{X}$. A real-valued function $\mathcal{D} : \Psi(\mathcal{X}) \to \Psi(\mathcal{X})$ is termed as distance measures, if for $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3 \in \Psi(\mathcal{X})$, $\mathcal{D}$ satisfies the following axioms.

(P1) $0 \leq \mathcal{D}(\mathcal{N}_1, \mathcal{N}_2) \leq 1$;

(P2) $\mathcal{D}(\mathcal{N}_1, \mathcal{N}_2) = 0 \Leftrightarrow \mathcal{N}_1 = \mathcal{N}_2$;

(P3) $\mathcal{D}(\mathcal{N}_1, \mathcal{N}_2) = \mathcal{D}(\mathcal{N}_2, \mathcal{N}_1)$;

(P4) If $\mathcal{N}_1 \subseteq \mathcal{N}_2 \subseteq \mathcal{N}_3$ then $\mathcal{D}(\mathcal{N}_1, \mathcal{N}_2) \leq \mathcal{D}(\mathcal{N}_1, \mathcal{N}_3)$ and $\mathcal{D}(\mathcal{N}_2, \mathcal{N}_3) \leq \mathcal{D}(\mathcal{N}_1, \mathcal{N}_3)$.

3. Proposed distance measure

This section presents new distance measure for SVNSs and investigating their properties.

Definition 3.1. For two SVNSs $\mathcal{N}_1 = \{(\varsigma_{\mathcal{N}_1}(x_j), \tau_{\mathcal{N}_1}(x_j), \upsilon_{\mathcal{N}_1}(x_j)) \mid x_j \in \mathcal{X}\}$ and $\mathcal{N}_2 = \{(\varsigma_{\mathcal{N}_2}(x_j), \tau_{\mathcal{N}_2}(x_j), \upsilon_{\mathcal{N}_2}(x_j)) \mid x_j \in \mathcal{X}\}$, a proposed distance measure $\mathcal{D}$ between them is stated as.

$$\mathcal{D}_\lambda(\mathcal{N}_1, \mathcal{N}_2) = \left[\sum_{j=1}^{n} w_j \left(\sum_{r=1}^{3} \beta_r \eta_r(x_j)\right)^{\lambda}\right]^{1/\lambda} \tag{3.1}$$

where $\lambda > 0, \beta_r \in [0,1]$, $\sum\limits_{r=1}^{3} \beta_r = 1$ and $w_j \in [0,1]$, $\sum\limits_{j=1}^{n} w_j = 1$,

$$\eta_1(x_j) = \frac{\psi_1(x_j) + \psi_2(x_j) + \psi_3(x_j)}{6};$$
$$\psi_1(x_j) = \left| \begin{array}{l} \left|(1 - \varsigma_{\mathcal{N}_1}(x_j)) + \dfrac{\mid 1 - \varsigma_{\mathcal{N}_1}(x_j) + \tau_{\mathcal{N}_1}(x_j) + \upsilon_{\mathcal{N}_1}(x_j) \mid}{3}\right| \\ - \left|(1 - \varsigma_{\mathcal{N}_2}(x_j)) + \dfrac{\mid 1 - \varsigma_{\mathcal{N}_2}(x_j) + \tau_{\mathcal{N}_2}(x_j) + \upsilon_{\mathcal{N}_2}(x_j) \mid}{3}\right| \end{array} \right|;$$
$$\psi_2(x_j) = \left| \begin{array}{l} \left|\tau_{\mathcal{N}_1}(x_j) + \dfrac{\mid 1 - \varsigma_{\mathcal{N}_1}(x_j) + \tau_{\mathcal{N}_1}(x_j) + \upsilon_{\mathcal{N}_1}(x_j) \mid}{3}\right| \\ - \left|\tau_{\mathcal{N}_2}(x_j) + \dfrac{\mid 1 - \varsigma_{\mathcal{N}_2}(x_j) + \tau_{\mathcal{N}_2}(x_j) + \upsilon_{\mathcal{N}_2}(x_j) \mid}{3}\right| \end{array} \right|;$$
$$\psi_3(x_j) = \left| \begin{array}{l} \left|\upsilon_{\mathcal{N}_1}(x_j) + \dfrac{\mid 1 - \varsigma_{\mathcal{N}_1}(x_j) + \tau_{\mathcal{N}_1}(x_j) + \upsilon_{\mathcal{N}_1}(x_j) \mid}{3}\right| \\ - \left|\upsilon_{\mathcal{N}_2}(x_j) + \dfrac{\mid 1 - \varsigma_{\mathcal{N}_2}(x_j) + \tau_{\mathcal{N}_2}(x_j) + \upsilon_{\mathcal{N}_2}(x_j) \mid}{3}\right| \end{array} \right|;$$
$$\eta_2(x_j) = \max\left(\mid \varsigma_{\mathcal{N}_1}(x_j) - \varsigma_{\mathcal{N}_2}(x_j) \mid, \mid \tau_{\mathcal{N}_1}(x_j) - \tau_{\mathcal{N}_2}(x_j) \mid, \mid \upsilon_{\mathcal{N}_1}(x_j) - \upsilon_{\mathcal{N}_2}(x_j) \mid\right);$$
$$\eta_3(x_j) = \left|\frac{2 + \varsigma_{\mathcal{N}_1}(x_j) - \tau_{\mathcal{N}_1}(x_j) - \upsilon_{\mathcal{N}_1}(x_j)}{3} - \frac{2 + \varsigma_{\mathcal{N}_2}(x_j) - \tau_{\mathcal{N}_2}(x_j) - \upsilon_{\mathcal{N}_2}(x_j)}{3}\right|.$$

Theorem 3.1. The above defined measure $\mathcal{D}$ satisfies the properties of Definition 2.4.

Proof. Consider two SVNSs $\mathcal{N}_1$ and $\mathcal{N}_2$ defined over $\mathcal{X}$.

(P1) Since for each j, $\varsigma_{\mathcal{N}_1}(x_j), \tau_{\mathcal{N}_1}(x_j), \upsilon_{\mathcal{N}_1}(x_j) \in [0,1]$, therefore, $(1-\varsigma_{\mathcal{N}_1}(x_j)) \in [0,1]$ and $\mid (1-\varsigma_{\mathcal{N}_1}(x_j)) + \tau_{\mathcal{N}_1}(x_j) + \upsilon_{\mathcal{N}_1}(x_j) \mid /3 \in [0,1]$, hence

$$0 \leq \left|(1 - \varsigma_{\mathcal{N}_1}(x_j)) + \frac{\mid (1 - \varsigma_{\mathcal{N}_1}(x_j)) + \tau_{\mathcal{N}_1}(x_j) + \upsilon_{\mathcal{N}_1}(x_j) \mid}{3}\right| \leq 2,$$
$$\text{and} \qquad 0 \leq \left|(1 - \varsigma_{\mathcal{N}_2}(x_j)) + \frac{\mid (1 - \varsigma_{\mathcal{N}_2}(x_j)) + \tau_{\mathcal{N}_2}(x_j) + \upsilon_{\mathcal{N}_2}(x_j) \mid}{3}\right| \leq 2$$

Hence,

$$\begin{aligned}\psi_1(x_j) &= \left| \left|(1-\varsigma_{\mathcal{N}_1}(x_j)) + \frac{|(1-\varsigma_{\mathcal{N}_1}(x_j)) + \tau_{\mathcal{N}_1}(x_j) + \upsilon_{\mathcal{N}_1}(x_j)|}{3}\right| \right.\\ &\quad \left. - \left|(1-\varsigma_{\mathcal{N}_2}(x_j)) + \frac{|(1-\varsigma_{\mathcal{N}_2}(x_j)) + \tau_{\mathcal{N}_2}(x_j) + \upsilon_{\mathcal{N}_2}(x_j)|}{3}\right| \right| \\ &\in [0,2]\end{aligned}$$

Similarly, we get $\psi_2(x_j) \in [0,2], \psi_3(x_j) \in [0,2]$. From this, we have

$$\eta_1(x_j) = \frac{\psi_1(x_j) + \psi_2(x_j) + \psi_3(x_j)}{6} \in [0,1].$$

Further, we have

$$|\varsigma_{\mathcal{N}_1}(x_j) - \varsigma_{\mathcal{N}_2}(x_j)|, |\tau_{\mathcal{N}_1}(x_j) - \tau_{\mathcal{N}_2}(x_j)|, |\upsilon_{\mathcal{N}_1}(x_j) - \upsilon_{\mathcal{N}_2}(x_j)| \in [0,1].$$

Thus,

$$\eta_2(x_j) = \max\begin{pmatrix} |\varsigma_{\mathcal{N}_1}(x_j) - \varsigma_{\mathcal{N}_2}(x_j)|, \\ |\tau_{\mathcal{N}_1}(x_j) - \tau_{\mathcal{N}_2}(x_j)|, \\ |\upsilon_{\mathcal{N}_1}(x_j) - \upsilon_{\mathcal{N}_2}(x_j)| \end{pmatrix} \in [0,1]$$

Also, by definition of SVNS,

$$0 \leq \frac{2 + \varsigma_{\mathcal{N}_1}(x_j) - \tau_{\mathcal{N}_1}(x_j) - \upsilon_{\mathcal{N}_1}(x_j)}{3} \leq 1$$
$$\text{and} \qquad 0 \leq \frac{2 + \varsigma_{\mathcal{N}_2}(x_j) - \tau_{\mathcal{N}_2}(x_j) - \upsilon_{\mathcal{N}_2}(x_j)}{3} \leq 1$$

Therefore,

$$\left|\frac{2 + \varsigma_{\mathcal{N}_1}(x_j) - \tau_{\mathcal{N}_1}(x_j) - \upsilon_{\mathcal{N}_1}(x_j)}{3} - \frac{2 + \varsigma_{\mathcal{N}_2}(x_j) - \tau_{\mathcal{N}_2}(x_j) - \upsilon_{\mathcal{N}_2}(x_j)}{3}\right| \in [0,1],$$

which means $\eta_3(x_j) \in [0,1]$. Since, $\eta_1(x_j), \eta_2(x_j), \eta_3(x_j), \beta_r(x_j) \in [0,1]; (r = 1,2,3)$, therefore, $\left(\sum_{r=1}^{3} \beta_r \eta_r(x_j)\right)^{\lambda} \in [0,1]$, for any real $\lambda > 0$. Also, for $w_j \in [0,1]$, we get

$$\mathcal{D}_\lambda(\mathcal{N}_1, \mathcal{N}_2) = \sum_{j=1}^{n} w_j \left(\sum_{r=1}^{3} \beta_r \eta_r(x_j)\right)^{\lambda} \in [0,1].$$

(P2) For $\mathcal{N}_1 = \mathcal{N}_2$, we have $\varsigma_{\mathcal{N}_1}(x_j) = \varsigma_{\mathcal{N}_2}(x_j)$, $\tau_{\mathcal{N}_1}(x_j) = \tau_{\mathcal{N}_2}(x_j)$, $\upsilon_{\mathcal{N}_1}(x_j) = \upsilon_{\mathcal{N}_2}(x_j)$ for $j = 1,2,..,n$, which implies $\psi_1(x_j) = 0$, $\psi_2(x_j) = 0$, $\psi_3(x_j) = 0$, i.e., $\eta_1(x_j) = 0$. Also, we get, $\eta_2(x_j) = 0$ and $\eta_3(x_j) = 0$. Hence, $\mathcal{D}_\lambda(\mathcal{N}_1, \mathcal{N}_2) = 0$.

On the other hand, we assume $\mathcal{D}_\lambda(\mathcal{N}_1, \mathcal{N}_2) = 0$, which implies $\left(\sum_{r=1}^{3} \beta_r \eta_r(x_j)\right)^{\lambda} = 0$. Since $\beta_r \eta_r(x_j) \geq 0$ therefore $\sum_{r=1}^{3} \beta_r \eta_r(x_j) = 0$, implies $\beta_r \eta_r(x_j) = 0, \forall r, j$. Thus, we have $\eta_r(x_j) = 0$, $\forall r$. From this, we get $\varsigma_{\mathcal{N}_1}(x_j) = \varsigma_{\mathcal{N}_2}(x_j)$, $\tau_{\mathcal{N}_1}(x_j) = \tau_{\mathcal{N}_2}(x_j)$, $\upsilon_{\mathcal{N}_1}(x_j) = \upsilon_{\mathcal{N}_2}(x_j)$. Hence we get, $\mathcal{N}_1 = \mathcal{N}_2$.

(P3) Since each $\eta_r(x_j)$; $(r = 1,2,3)$, is symmetric therefore $\mathcal{D}_\lambda(\mathcal{N}_1,\mathcal{N}_2) = \mathcal{D}_\lambda(\mathcal{N}_2,\mathcal{N}_1)$.

(P4) If $\mathcal{N}_1 \subseteq \mathcal{N}_2 \subseteq \mathcal{N}_3$, then $\varsigma_{\mathcal{N}_1}(x_j) \le \varsigma_{\mathcal{N}_2}(x_j) \le \varsigma_{\mathcal{N}_3}(x_j)$; $\tau_{\mathcal{N}_1}(x_j) \ge \tau_{\mathcal{N}_2}(x_j) \ge \tau_{\mathcal{N}_3}(x_j)$; $\upsilon_{\mathcal{N}_1}(x_j) \ge \upsilon_{\mathcal{N}_2}(x_j) \ge \upsilon_{\mathcal{N}_3}(x_j)$ which implies

$$\left|\left|(1-\varsigma_{\mathcal{N}_1}(x_j)) + \frac{|1-\varsigma_{\mathcal{N}_1}(x_j)+\tau_{\mathcal{N}_1}(x_j)+\upsilon_{\mathcal{N}_1}(x_j)|}{3}\right| - \left|(1-\varsigma_{\mathcal{N}_2}(x_j)) + \frac{|1-\varsigma_{\mathcal{N}_2}(x_j)+\tau_{\mathcal{N}_2}(x_j)+\upsilon_{\mathcal{N}_2}(x_j)|}{3}\right|\right|$$
$$\le \left|\left|(1-\varsigma_{\mathcal{N}_1}(x_j)) + \frac{|1-\varsigma_{\mathcal{N}_1}(x_j)+\tau_{\mathcal{N}_1}(x_j)+\upsilon_{\mathcal{N}_1}(x_j)|}{3}\right| - \left|(1-\varsigma_{\mathcal{N}_3}(x_j)) + \frac{|1-\varsigma_{\mathcal{N}_3}(x_j)+\tau_{\mathcal{N}_3}(x_j)+\upsilon_{\mathcal{N}_3}(x_j)|}{3}\right|\right|,$$

that is, $\psi_1(\mathcal{N}_1,\mathcal{N}_2)(x_j) \le \psi_1(\mathcal{N}_1,\mathcal{N}_3)(x_j)$. Similarly, we get $\psi_2(\mathcal{N}_1,\mathcal{N}_2)(x_j) \le \psi_2(\mathcal{N}_1,\mathcal{N}_3)(x_j)$ and $\psi_3(\mathcal{N}_1,\mathcal{N}_2)(x_j) \le \psi_3(\mathcal{N}_1,\mathcal{N}_3)(x_j)$, therefore, we get $\eta_1(\mathcal{N}_1,\mathcal{N}_2)(x_j) \le \eta_1(\mathcal{N}_1,\mathcal{N}_3)(x_j)$.

Further, we get $|\ \varsigma_{\mathcal{N}_1}(x_j) - \varsigma_{\mathcal{N}_2}(x_j)\ |\le|\ \varsigma_{\mathcal{N}_1}(x_j) - \varsigma_{\mathcal{N}_3}(x_j)\ |$, $|\ \tau_{\mathcal{N}_1}(x_j) - \tau_{\mathcal{N}_2}(x_j)\ |\le|\ \tau_{\mathcal{N}_1}(x_j) - \tau_{\mathcal{N}_3}(x_j)\ |$ and $|\ \upsilon_{\mathcal{N}_1}(x_j) - \upsilon_{\mathcal{N}_2}(x_j)\ |\le|\ \upsilon_{\mathcal{N}_1}(x_j) - \upsilon_{\mathcal{N}_3}(x_j)\ |$. Thus, we get $\eta_2(\mathcal{N}_1,\mathcal{N}_2)(x_j) \le \eta_2(\mathcal{N}_1,\mathcal{N}_3)(x_j)$.

Also,

$$\left|\frac{2+\varsigma_{\mathcal{N}_1}(x_j)-\tau_{\mathcal{N}_1}(x_j)-\upsilon_{\mathcal{N}_1}(x_j)}{3} - \frac{2+\varsigma_{\mathcal{N}_2}(x_j)-\tau_{\mathcal{N}_2}(x_j)-\upsilon_{\mathcal{N}_2}(x_j)}{3}\right|$$
$$\le \left|\frac{2+\varsigma_{\mathcal{N}_1}(x_j)-\tau_{\mathcal{N}_1}(x_j)-\upsilon_{\mathcal{N}_1}(x_j)}{3} - \frac{2+\varsigma_{\mathcal{N}_3}(x_j)-\tau_{\mathcal{N}_3}(x_j)-\upsilon_{\mathcal{N}_3}(x_j)}{3}\right|$$

Thus, we get, $\eta_3(\mathcal{N}_1,\mathcal{N}_2)(x_j) \le \eta_3(\mathcal{N}_1,\mathcal{N}_3)(x_j)$. Hence, $\mathcal{D}_\lambda(\mathcal{N}_1,\mathcal{N}_2) \le \mathcal{D}_\lambda(\mathcal{N}_1,\mathcal{N}_3)$.

□

The above defined measure is illustrated with a numerical example as follow.

Example 3.1. Consider two SVNSs $\mathcal{N}_1 = \{(x_1, 0.7, 0.4, 0.4), (x_2, 0.5, 0.2, 0.1)\}$ and $\mathcal{N}_2 = \{(x_1, 0.2, 0.5, 0.4), (x_2, 0.6, 0.2, 0.2)\}$, with weight vector $(0.7, 0.3)^T$. By considering $\beta_1 = \beta_2 = \beta_3 = 1/3$ and $\lambda = 2$, we have

$$\psi_1(x_1) = \left|0.3 + \frac{0.3+0.4+0.4}{3}\right| - \left|0.8 + \frac{0.8+0.5+0.4}{3}\right| = 0.7$$
$$\psi_2(x_1) = \left|0.4 + \frac{0.3+0.4+0.4}{3}\right| - \left|0.5 + \frac{0.8+0.5+0.4}{3}\right| = 0.3$$
$$\psi_3(x_1) = \left|0.4 + \frac{0.3+0.4+0.4}{3}\right| - \left|0.4 + \frac{0.8+0.5+0.4}{3}\right| = 0.2$$
$$\eta_1(x_1) = \frac{0.7+0.3+0.2}{6} = 0.2000$$

Similarly, we get $\eta_1(x_2) = 0.0333$.

$$\eta_2(x_1) = \max(0.5, 0.1, 0) = 0.5; \quad \eta_2(x_2) = \max(0.1, 0, 0.1) = 0.1$$

and

$$\eta_3(x_1) = \left|\frac{2+0.7-0.4-0.4}{3} - \frac{2+0.2-0.5-0.4}{3}\right| = 0.1999$$

$$\eta_3(x_2) = \left|\frac{2+0.5-0.2-0.1}{3} - \frac{2+0.6-0.2-0.2}{3}\right| = 0$$

and hence, by Eq. (3.1)

$$\begin{aligned}\mathcal{D}(\mathcal{N}_1, \mathcal{N}_2) &= \left(0.7\left(\frac{1}{3}(0.2+0.5+0.1999)\right)^2 + 0.3\left(\frac{1}{3}(0.0333+0.1+0)\right)^2\right)^{1/2} \\ &= 0.2522\end{aligned}$$

Next, we define the degree of similarity based on proposed measure as follows.

Definition 3.2. A real-valued function $\mathcal{S}$ is termed as similarity measure between SVNSs $\mathcal{N}_1$ and $\mathcal{N}_2$ and defined as

$$\mathcal{S}_\lambda(\mathcal{N}_1, \mathcal{N}_2) = 1 - \mathcal{D}_\lambda(\mathcal{N}_1, \mathcal{N}_2) \tag{3.2}$$

Theorem 3.2. The measure defined in Definition 3.2 have the following features:

(S1) $0 \leq \mathcal{S}_\lambda(\mathcal{N}_1, \mathcal{N}_2) \leq 1$.

(S2) $\mathcal{S}(\mathcal{N}_1, \mathcal{N}_2) = 1$ if $\mathcal{N}_1 = \mathcal{N}_2$.

(S3) $\mathcal{S}(\mathcal{N}_1, \mathcal{N}_2) = \mathcal{S}(\mathcal{N}_2, \mathcal{N}_1)$.

(S4) If $\mathcal{N}_1 \subseteq \mathcal{N}_2 \subseteq \mathcal{N}_3$ then $\mathcal{S}(\mathcal{N}_1, \mathcal{N}_3) \leq \mathcal{S}(\mathcal{N}_1, \mathcal{N}_2)$ and $\mathcal{S}(\mathcal{N}_1, \mathcal{N}_3) \leq \mathcal{S}(\mathcal{N}_2, \mathcal{N}_3)$.

Proof. For two SVNSs $\mathcal{N}_1$ and $\mathcal{N}_2$, we have

(S1) Since, $0 \leq \mathcal{D}_\lambda(\mathcal{N}_1, \mathcal{N}_2) \leq 1$, therefore $0 \leq 1 - \mathcal{D}_\lambda(\mathcal{N}_1, \mathcal{N}_2) \leq 1$, i.e. $0 \leq \mathcal{S}_\lambda(\mathcal{N}_1, \mathcal{N}_2) \leq 1$.

(S2) $\mathcal{S}_\lambda(\mathcal{N}_1, \mathcal{N}_2) = 1 \Leftrightarrow \mathcal{D}_\lambda(\mathcal{N}_1, \mathcal{N}_2) = 0$ if $\mathcal{N}_1 = \mathcal{N}_2$.

(S3) It follows from definition.

(S4) If $\mathcal{N}_1 \subseteq \mathcal{N}_2 \subseteq \mathcal{N}_3$, then $\mathcal{D}(\mathcal{N}_1, \mathcal{N}_2) \leq \mathcal{D}(\mathcal{N}_1, \mathcal{N}_3)$ and $\mathcal{D}(\mathcal{N}_2, \mathcal{N}_3) \leq \mathcal{D}(\mathcal{N}_1, \mathcal{N}_3)$, which implies $1 - \mathcal{D}(\mathcal{N}_1, \mathcal{N}_2) \geq 1 - \mathcal{D}(\mathcal{N}_1, \mathcal{N}_3)$ and $1 - \mathcal{D}(\mathcal{N}_2, \mathcal{N}_3) \geq 1 - \mathcal{D}(\mathcal{N}_1, \mathcal{N}_3)$, that is, $\mathcal{S}(\mathcal{N}_1, \mathcal{N}_3) \leq \mathcal{S}(\mathcal{N}_1, \mathcal{N}_2)$ and $\mathcal{S}(\mathcal{N}_1, \mathcal{N}_3) \leq \mathcal{S}(\mathcal{N}_2, \mathcal{N}_3)$.

□

4. MCDM method based on extended TOPSIS method

In this section, we offer a novel TOPSIS method based on proposed measures to handle the group DMPs. Further, a real-life example is given to demonstrate it and the validity test is conducted to justify it.

4.1. Proposed approach

Consider a MCGDM ("Multi-Criteria Group Decision Making") problem with "m" alternatives $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_m\}$ assessed under "n" criteria $\mathfrak{B} = \{\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n\}$ by "l" distinct experts $\mathfrak{R}^{(1)}, \mathfrak{R}^{(2)}, \ldots, \mathfrak{R}^{(l)}$. Each expert $\mathfrak{R}^{(z)}$ $(z = 1, 2, \ldots, l)$ evaluate $\mathcal{V}_i$ $(i = 1, 2, \ldots, m)$ under $\mathfrak{B}_j$ $(j = 1, 2, \ldots, n)$ under SVNS environment and recorded their rating in terms of SVNNs as $\alpha_{ij}^{(z)} = \left(\varsigma_{ij}^{(z)}, \tau_{ij}^{(z)}, \upsilon_{ij}^{(z)}\right)$. Assume that $w^{(z)} = \left(w_1^{(z)}, w_2^{(z)}, \ldots, w_n^{(z)}\right)^T$ with $w_j^{(z)} > 0$ and $\sum\limits_{j=1}^{n} w_j^{(z)} = 1$ be weight vector of the criteria and $\xi = (\xi_1, \xi_2, \ldots, \xi_l); \xi_z > 0; \sum\limits_{z=1}^{l} \xi_z = 1$ be for experts. The collective values of all expert $\mathfrak{R}^{(z)}$ for m alternatives are represented in decision matrix $\mathcal{R}^{(z)} = \left(\alpha_{ij}^{(z)}\right)_{m\times n}$ given as

$$\mathcal{R}^{(z)} = \begin{array}{c} \\ \mathcal{V}_1 \\ \mathcal{V}_2 \\ \vdots \\ \mathcal{V}_m \end{array} \begin{array}{c} \begin{array}{cccc} \mathfrak{B}_1 & \mathfrak{B}_2 & \ldots & \mathfrak{B}_n \end{array} \\ \begin{pmatrix} \alpha_{11}^{(z)} & \alpha_{12}^{(z)} & \ldots & \alpha_{1n}^{(z)} \\ \alpha_{21}^{(z)} & \alpha_{22}^{(z)} & \ldots & \alpha_{2n}^{(z)} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1}^{(z)} & \alpha_{m2}^{(z)} & \ldots & \alpha_{mn}^{(z)} \end{pmatrix} \end{array}$$

To select the finest alternative(s), the procedure steps (whose flowchart is presented in Fig. 1) are summarized as follows:

Step 1: Arrange the SVN decision matrix $\mathcal{R}\left(\alpha_{ij}^{(z)}\right)_{m\times n}$ for each decision maker $\mathfrak{R}$.

Step 2: Normalize the information if required, by converting the cost type criteria into the benefit type.

Step 3: Compute PIA ("Positive Ideal Alternative"), $\mathcal{V}^{(z)+}$, and NIA ("Negative Ideal Alternative") $\mathcal{V}^{(z)-}$, for each expert $\mathfrak{R}^{(z)}$, $(z = 1, 2, \ldots, l)$, as

$$\mathcal{V}^{(z)+} = \left(\max_j \left(\varsigma_{ij}^{(z)}\right), \min_j \left(\tau_{ij}^{(z)}\right), \min_j \left(\upsilon_{ij}^{(z)}\right)\right)_{1\times n} \tag{4.1}$$

and

$$\mathcal{V}^{(z)-} = \left(\min_j \left(\varsigma_{ij}^{(z)}\right), \max_j \left(\tau_{ij}^{(z)}\right), \max_j (\upsilon_{ij}^{(z)})\right)_{1\times n} \tag{4.2}$$

Step 4: Calculate $(\mathcal{D}_\lambda)_i^{(z)+}$ and $(\mathcal{D}_\lambda)_i^{(z)-}$ from the PIA and NIA, respectively, corresponding to each decision maker.

Step 5: Compute the closeness degree for experts as:

$$\mathfrak{R}_i^{(z)} = \frac{(\mathcal{D}_\lambda)_i^{(z)-}}{(\mathcal{D}_\lambda)_i^{(z)-} + (\mathcal{D}_\lambda)_i^{(z)+}} \tag{4.3}$$

provided $(\mathcal{D}_\lambda)_i^{(z)+} \neq 0$, $\mathfrak{R}_i^{(z)} \in [0, 1]$.

Step 6: From Eq. (4.3), we may obtain the different ordering based on the each expert opinion and hence it is difficult to compromise on a single task. To overcome it, we aggregate the expert preferences by using weight $\xi_z > 0$, $\sum_{z=1}^{l} \xi_z = 1$ to each expert as

$$(\mathcal{D}_\lambda)_i^+ = \sum_{z=1}^{l} \xi_z \, (\mathcal{D}_\lambda)_i^{(z)+} \tag{4.4}$$

and

$$(\mathcal{D}_\lambda)_i^- = \sum_{z=1}^{l} \xi_z \, (\mathcal{D}_\lambda)_i^{(z)-} \tag{4.5}$$

Step 7: The overall closeness degree $\mathfrak{I}_i$ of each alternative $\mathcal{V}_i$, $(i = 1, 2, \ldots, m)$ is computed as

$$\mathfrak{I}_i = \frac{(\mathcal{D}_\lambda)_i^-}{(\mathcal{D}_\lambda)_i^- + (\mathcal{D}_\lambda)_i^+} \tag{4.6}$$

provided $(\mathcal{D}_\lambda)_i^+ \neq 0$ and rank them accordingly.

4.2. Illustrative example

To illustrate the approach, we consider the following example, which can be read as

A travel agency naming, Marricot Trip mate , has excelled in providing travel related services to domestic and Inbound tourists . Agency wants to provide more facilities like detailed information, online booking capabilities, allow to book and sell airline tickets, car rentals, hotels, and other travel related services etc. to their customers. For this purpose, agency intends to find an appropriate information technology (IT) software development company that delivers affordable solutions through software development. To complete this motive, agency forms a set of five companies (alternatives), namely, Zensar Tech ($\mathcal{V}_1$), NIIT Tech ($\mathcal{V}_2$), HCL Tech($\mathcal{V}_3$) and Hexaware Tech($\mathcal{V}_4$) and the selection is held on the basis of the different criteria, namely, Technology Expertise ($\mathfrak{B}_1$), Service quality ($\mathfrak{B}_2$), Project Management ($\mathfrak{B}_3$), Industry Experience ($\mathfrak{B}_4$). The agency hires the three experts $\mathfrak{R}^{(1)}, \mathfrak{R}^{(2)}$ and $\mathfrak{R}^{(3)}$ for evaluation of the considered $\mathcal{V}_i (i = 1, 2, \ldots, 5)$ under $\mathfrak{B}_j (j = 1, 2, 3, 4)$. For computation, we take $\lambda = 2$ and $\beta_1 = \beta_2 = \beta_3 = 1/3$. Then, the following steps of the stated method are executed to find the best one(s).

Step 1: The rating information of each expert is summarized in Table 1.

Step 2: As $\mathfrak{B}_j$'s are of benefit type, so no need of normalization.

Step 3: The PIA and NIA are computed by Eqs. (4.1) and (4.2), and summarized in Table 2.

Step 4: By applying Eq. (3.1), the positive and negative degrees of the measurement values for each expert are represented in Table 3. For instance, for expert $\mathfrak{R}^{(1)}$, the values of $(\mathcal{D}_\lambda)_1^{(1)+} = 0.1484$, $(\mathcal{D}_\lambda)_2^{(1)+} = 0.1427$, $(\mathcal{D}_\lambda)_3^{(1)+} = 0.1713$ and $(\mathcal{D}_\lambda)_4^{(1)+} = 0.2194$. Similarly, from NIA, we get $(\mathcal{D}_\lambda)_1^{(1)-} = 0.1844$, $(\mathcal{D}_\lambda)_2^{(1)-} = 0.2172$, $(\mathcal{D}_\lambda)_3^{(1)-} = 0.1870$ and $(\mathcal{D}_\lambda)_4^{(1)-} = 0.1637$. The others values are tabulated in Table 3.

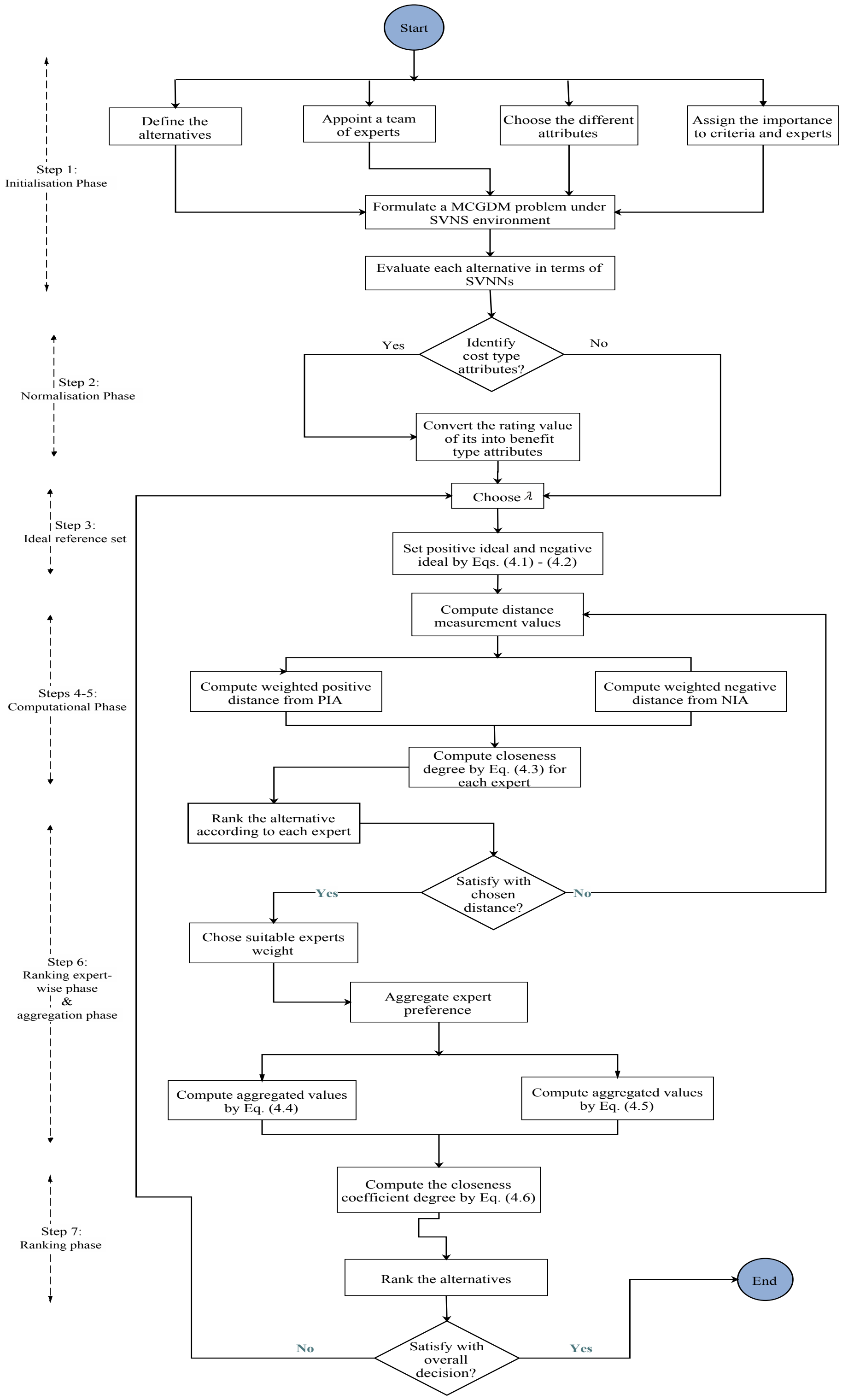

Figure 1. Flowchart of the proposed approach

Step 5: By Eq. (4.3), closeness degrees $\mathfrak{K}_i^{(z)}$ for each expert $\Re^{(z)}$ are computed and summarized in the third column of the each expert in Table 3. It is seen that for $\Re^{(1)}$ expert, the best one is $\mathcal{V}_2$

Table 1. Decision matrix in terms of SVNN

Expert		$\mathfrak{B}_1$	$\mathfrak{B}_2$	$\mathfrak{B}_3$	$\mathfrak{B}_4$
$\mathfrak{R}^{(1)}$	$\mathcal{V}_1$	$(0.5, 0.1, 0.3)$	$(0.5, 0.1, 0.4)$	$(0.7, 0.1, 0.2)$	$(0.3, 0.2, 0.1)$
	$\mathcal{V}_2$	$(0.4, 0.2, 0.3)$	$(0.3, 0.2, 0.4)$	$(0.9, 0.0, 0.1)$	$(0.5, 0.3, 0.2)$
	$\mathcal{V}_3$	$(0.4, 0.3, 0.1)$	$(0.5, 0.1, 0.3)$	$(0.5, 0.0, 0.4)$	$(0.6, 0.2, 0.2)$
	$\mathcal{V}_4$	$(0.6, 0.1, 0.2)$	$(0.2, 0.2, 0.5)$	$(0.4, 0.3, 0.2)$	$(0.7, 0.2, 0.1)$
	$w^{(1)}$	0.30	0.25	0.25	0.20
$\mathfrak{R}^{(2)}$	$\mathcal{V}_1$	$(0.6, 0.1, 0.2)$	$(0.5, 0.3, 0.1)$	$(0.5, 0.1, 0.3)$	$(0.2, 0.3, 0.4)$
	$\mathcal{V}_2$	$(0.4, 0.4, 0.1)$	$(0.6, 0.3, 0.1)$	$(0.5, 0.2, 0.2)$	$(0.7, 0.1, 0.2)$
	$\mathcal{V}_3$	$(0.2, 0.2, 0.3)$	$(0.6, 0.2, 0.1)$	$(0.4, 0.1, 0.3)$	$(0.4, 0.3, 0.3)$
	$\mathcal{V}_4$	$(0.6, 0.1, 0.3)$	$(0.1, 0.2, 0.6)$	$(0.1, 0.3, 0.5)$	$(0.2, 0.3, 0.2)$
	$w^{(2)}$	0.40	0.30	0.20	0.10
$\mathfrak{R}^{(3)}$	$\mathcal{V}_1$	$(0.2, 0.1, 0.7)$	$(0.4, 0.1, 0.6)$	$(0.5, 0.2, 0.5)$	$(0.2, 0.1, 0.6)$
	$\mathcal{V}_2$	$(0.4, 0.3, 0.6)$	$(0.4, 0.2, 0.5)$	$(0.1, 0.2, 0.8)$	$(0.5, 0.3, 0.5)$
	$\mathcal{V}_3$	$(0.2, 0.2, 0.7)$	$(0.2, 0.3, 0.7)$	$(0.3, 0.3, 0.7)$	$(0.2, 0.1, 0.7)$
	$\mathcal{V}_4$	$(0.5, 0.5, 0.4)$	$(0.2, 0.3, 0.8)$	$(0.2, 0.1, 0.6)$	$(0.3, 0.3, 0.6)$
	$w^{(3)}$	0.25	0.30	0.35	0.10

Table 2. PIA & NIA for each expert

Expert		$\mathfrak{B}_1$	$\mathfrak{B}_2$	$\mathfrak{B}_3$	$\mathfrak{B}_4$
$\mathfrak{R}^{(1)}$	PIA	$(0.6, 0.1, 0.1)$	$(0.5, 0.1, 0.3)$	$(0.9, 0.0, 0.1)$	$(0.7, 0.2, 0.1)$
	NIA	$(0.4, 0.3, 0.3)$	$(0.2, 0.2, 0.5)$	$(0.4, 0.3, 0.4)$	$(0.3, 0.3, 0.2)$
$\mathfrak{R}^{(2)}$	PIA	$(0.6, 0.1, 0.1)$	$(0.6, 0.2, 0.1)$	$(0.5, 0.1, 0.2)$	$(0.7, 0.1, 0.2)$
	NIA	$(0.2, 0.4, 0.3)$	$(0.1, 0.3, 0.6)$	$(0.1, 0.3, 0.5)$	$(0.2, 0.3, 0.4)$
$\mathfrak{R}^{(3)}$	PIA	$(0.4, 0.1, 0.4)$	$(0.4, 0.1, 0.5)$	$(0.5, 0.1, 0.5)$	$(0.5, 0.1, 0.5)$
	NIA	$(0.2, 0.5, 0.7)$	$(0.2, 0.3, 0.8)$	$(0.1, 0.3, 0.8)$	$(0.2, 0.3, 0.7)$

Table 3. Measurement values from ideal alternatives corresponding to each expert

	$\mathfrak{R}^{(1)}$			$\mathfrak{R}^{(2)}$			$\mathfrak{R}^{(3)}$		
	$(\mathcal{D}_\lambda)_i^{(1)+}$	$(\mathcal{D}_\lambda)_i^{(1)-}$	$\mathfrak{K}_i^{(1)}$	$(\mathcal{D}_\lambda)_i^{(2)+}$	$(\mathcal{D}_\lambda)_i^{(2)-}$	$\mathfrak{K}_i^{(2)}$	$(\mathcal{D}_\lambda)_i^{(3)+}$	$(\mathcal{D}_\lambda)_i^{(3)-}$	$\mathfrak{K}_i^{(3)}$
$\mathcal{V}_1$	0.1484	0.1844	0.5541	0.1308	0.3137	0.7057	0.1385	0.2449	0.6388
$\mathcal{V}_2$	0.1427	0.2172	0.6034	0.1392	0.2964	0.6805	0.2095	0.1725	0.4515
$\mathcal{V}_3$	0.1713	0.1870	0.5161	0.2001	0.2655	0.5702	0.2163	0.1238	0.3640
$\mathcal{V}_4$	0.2194	0.1637	0.4272	0.2879	0.1885	0.3957	0.2181	0.1590	0.4216

while $\mathcal{V}_1$ for the other experts. As the best and worst alternatives change accordingly to the experts and hence it is hard to grasp and select the optimal one. To defeat the ambiguity, we consider the expert importance and aggregate their values.

Step 6: Use $\xi = (0.25, 0.40, 0.35)$ of the experts and Eqs. (4.4) and (4.5), we compute the aggregated values are $(\mathcal{D}_2)_1^+ = 0.1379$, $(\mathcal{D}_2)_1^- = 0.2573$, $(\mathcal{D}_2)_2^+ = 0.1647$, $(\mathcal{D}_2)_2^- = 0.2332$, $(\mathcal{D}_2)_3^+ =$

0.1986, $(\mathcal{D}_2)_3^- = 0.1952$, $(\mathcal{D}_2)_4^+ = 0.2464$ and $(\mathcal{D}_2)_4^- = 0.1720$.

Step 7: Compute $\Im_i$ by Eq. (4.6) and get $\Im_1 = 0.6511$, $\Im_2 = 0.5861$, $\Im_3 = 0.4957$ and $\Im_4 = 0.4111$. Since $\Im_1 > \Im_2 > \Im_3 > \Im_4$ and hence ordering of the alternatives is $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_3 \succ \mathcal{V}_4$. Therefore, $\mathcal{V}_1$ is the best choice.

Further, to reach the power of the parameter λ on to the process, we modify the parameter λ and achieve the proposed TOPSIS method on the estimated data. The final optimal ranking of the given numbers is taken and results are recorded in Table 4. It is obviously seen that the ranking order for all values of λ is not alike. For instance, when $\lambda = 1, 2$ then the optimal alternative has been received as $\mathcal{V}_1$ while the worst one is $\mathcal{V}_4$. On the other hand, for other values of λ's such as $\lambda = 5, 10, \ldots$, we get $\mathcal{V}_2$ as the best one. Hence, a person can examine the impact of λ during the process and pick the best one accordingly. This analysis will help the decision-maker to follow the given DMP more profoundly and encourage him/her to select the parameter according to the requirement of the process. Also, this parameter makes the approach more manageable as compared to others regarding the selection of the final decision.

Table 4. Effect of λ on the ranking process

λ	Values of $\Im$'s for				Ordering
	$\mathcal{V}_1$	$\mathcal{V}_2$	$\mathcal{V}_3$	$\mathcal{V}_4$	
1	0.6876	0.5965	0.5101	0.3804	$\mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_3 \succ \mathcal{V}_4$
2	0.6511	0.5861	0.4957	0.4111	$\mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_3 \succ \mathcal{V}_4$
5	0.5803	0.5842	0.4972	0.4305	$\mathcal{V}_2 \succ \mathcal{V}_1 \succ \mathcal{V}_3 \succ \mathcal{V}_4$
10	0.5512	0.5849	0.5007	0.4371	$\mathcal{V}_2 \succ \mathcal{V}_1 \succ \mathcal{V}_3 \succ \mathcal{V}_4$
50	0.5330	0.5874	0.5030	0.4395	$\mathcal{V}_2 \succ \mathcal{V}_1 \succ \mathcal{V}_3 \succ \mathcal{V}_4$
100	0.5312	0.5879	0.5033	0.4392	$\mathcal{V}_2 \succ \mathcal{V}_1 \succ \mathcal{V}_3 \succ \mathcal{V}_4$

4.3. Validity test

To verify the completion of the stated method, we inquire about their validity through the following three testing criteria, as established by Wang and Triantaphyllou [41].

Test criteria 1: An effective decision-making method should not change the indication of the best alternative on replacing a non-optimal alternative by another worse alternative without changing the relative importance of each decision criteria.

Test criteria 2: An effective decision-making method should follow transitive property.

Test criteria 3: When an decision-making problem is decomposed into smaller problems and the same decision-making method is applied to smaller problems to rank the alternatives, a combined ranking of the alternatives should be identical to the original ranking of un-decomposed problem.

4.3.1. Under criterion 1

For the given problem, the best alternative obtained as $\mathcal{V}_1$ and $\mathcal{V}_4$ as non-optimal. So, to test under "*criteria 1*", we update the rating values of $\mathcal{V}_3$ with the arbitrary new values for each expert, and tabulated in Table 5. Then, by implementing the proposed TOPSIS method on it, we compute the final closeness degree $\Im_i$ for each alternative and get $\Im_1 = 0.6653$, $\Im_2 = 0.5938$, $\Im_3 = 0.4022$ and $\Im_4 = 0.4503$. Based on it, we easily obtain that $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_4 \succ \mathcal{V}_3$ and suggest us that $\mathcal{V}_1$ is still best alternative. Therefore, the stated algorithm is valid "*under test criterion 1*".

Table 5. Updated rating values of $\mathcal{V}_3$ to each expert

	$\mathfrak{B}_1$	$\mathfrak{B}_2$	$\mathfrak{B}_3$	$\mathfrak{B}_4$
$\mathfrak{R}^{(1)}$	$(0.2, 0.1, 0.2)$	$(0.4, 0.2, 0.1)$	$(0.3, 0.0, 0.1)$	$(0.5, 0.2, 0.2)$
$\mathfrak{R}^{(2)}$	$(0.1, 0.2, 0.4)$	$(0.5, 0.3, 0.1)$	$(0.4, 0.2, 0.4)$	$(0.4, 0.3, 0.3)$
$\mathfrak{R}^{(3)}$	$(0.2, 0.4, 0.7)$	$(0.4, 0.3, 0.8)$	$(0.2, 0.4, 0.7)$	$(0.2, 0.3, 0.7)$

4.3.2. Testing for criteria 2 and 3

Under it, we split the given problem into three subproblems with consists $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3\}$, $\{\mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\}$, and $\{\mathcal{V}_3, \mathcal{V}_4, \mathcal{V}_1\}$ as an alternative. Now, applying stated algorithm on individual subproblem and hence obtain their respective ranking as $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_3$, $\mathcal{V}_2 \succ \mathcal{V}_4 \succ \mathcal{V}_3$, and $\mathcal{V}_1 \succ \mathcal{V}_4 \succ \mathcal{V}_3$. By merging all, we get $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_3 \succ \mathcal{V}_4$, and it states the validity of suggested method "*under test criteria 2 and 3*".

4.4. Comparative study

For the comparison view, an examination has been arranged with the existing studies [25, 29] under SVNS and interpreted as follows.

4.4.1. Comparison with approach given by Mondal et al. [25]

Mondal et al. [25] performed the logarithm similarity-based MCGDM approach to solve the DMPs. We implement their approach to the considered data and their procedure steps are organized as follows.

Step 1: The information about the alternatives are listed in Table 1.

Step 2: Aggregated the experts preferences by taking average of their numbers and the resultant values (called as "central decision matrix") are listed in Table 6.

Table 6. Aggregated values by weighted average

	$\mathfrak{B}_1$	$\mathfrak{B}_2$	$\mathfrak{B}_3$	$\mathfrak{B}_4$
$\mathcal{V}_1$	$(0.4333, 0.1000, 0.4000)$	$(0.4667, 0.1667, 0.3667)$	$(0.5667, 0.1333, 0.3333)$	$(0.2000, 0.2000, 0.3667)$
$\mathcal{V}_2$	$(0.4000, 0.3000, 0.3333)$	$(0.4333, 0.2333, 0.3333)$	$(0.5000, 0.1333, 0.3667)$	$(0.5667, 0.2333, 0.3000)$
$\mathcal{V}_3$	$(0.2667, 0.2333, 0.3667)$	$(0.4333, 0.2000, 0.3667)$	$(0.4000, 0.1333, 0.4667)$	$(0.4000, 0.2000, 0.4000)$
$\mathcal{V}_4$	$(0.5667, 0.2333, 0.3000)$	$(0.1667, 0.2333, 0.6333)$	$(0.2333, 0.2333, 0.4333)$	$(0.4000, 0.2337, 0.3000)$

Step 3: From the values of Table 6, we compute the ideal alternative $\mathcal{V}^*$ as

$$\mathcal{V}^* = \left\{ \begin{array}{c} (0.5667, 0.1000, 0.3000), (0.4667, 0.1667, 0.3000), \\ (0.5667, 0.1333, 0.3000), (0.5667, 0.2000, 0.3000) \end{array} \right\}$$

Step 4: Compute the attribute weights ω_i by

$$\omega_i = \frac{1 - E_j}{n - \sum_{j=1}^{n} E_j}$$

where $E_j = 1 - \frac{1}{n}\sum_{i=1}^{m}\left[\left(\varsigma_{ij} + \upsilon_{ij}\right)\log_2\left(2 - \tau_{ij}^2\right)\right]$ and hence we get $\omega = (0.2283, 0.2406, 0.3311, 0.2000)^T$.

Step 5: With information ω, we compute the logarithm similarity (LS) values

$$LS(\mathcal{V}_i, \mathcal{V}^*) = \frac{1}{n}\left[\begin{array}{l} \frac{1}{2}\sum\limits_{i=1}^{m}\omega_i \log_4 (4- \mid \varsigma_{\mathcal{V}_i} - \varsigma_{\mathcal{V}^*} \mid - \mid \tau_{\mathcal{V}_i} - \tau_{\mathcal{V}^*} \mid - \mid \upsilon_{\mathcal{V}_i} - \upsilon_{\mathcal{V}^*} \mid) \\ + \frac{1}{2}\sum\limits_{i=1}^{2}\omega_i \log_2 (2 - \max (\mid \varsigma_{\mathcal{V}_i} - \varsigma_{\mathcal{V}^*} \mid, \mid \tau_{\mathcal{V}_i} - \tau_{\mathcal{V}^*} \mid, \mid \upsilon_{\mathcal{V}_i} - \upsilon_{\mathcal{V}^*} \mid)) \end{array}\right]$$

of each alternative from $\mathcal{V}^*$ and get $LS(\mathcal{V}_1, \mathcal{V}^*) = 0.9461$, $LS(\mathcal{V}_2, \mathcal{V}^*) = 0.9517$, $LS(\mathcal{V}_3, \mathcal{V}^*) = 0.9095$ and $LS(\mathcal{V}_4, \mathcal{V}^*) = 0.8643$ respectively.

Step 6: Based on these values, we obtain $\mathcal{V}_2 \succ \mathcal{V}_1 \succ \mathcal{V}_3 \succ \mathcal{V}_4$ as ranking and hence $\mathcal{V}_2$ is best choice.

4.4.2. Comparison with approach given by Biswas et al. [29]

By implementing the TOPSIS approach as given by Biswas et al. [29] on to the considered data, we initially take all the experts and criteria at the same level. Then, to execute their approach, we aggregate the different expert preferences by using WA operator as suggested by [5]. Based on their obtained values, PIA ($\mathcal{V}^+$) and NIA ($\mathcal{V}^-$) are obtained as $\mathcal{V}^+ = \{(\mathfrak{B}_1, 0.5691, 0.1000, 0.2621), (\mathfrak{B}_2, 0.4687, 0.1442, 0.2714), (\mathfrak{B}_3, 0.6443, 0.0000, 0.2520), (\mathfrak{B}_4, 0.5783, 0.1817, 0.2289)\}$ and $\mathcal{V}^- = \{(\mathfrak{B}_1, 0.2732, 0.2289, 0.3476), (\mathfrak{B}_2, 0.1680, 0.2289, 0.6214), (\mathfrak{B}_3, 0.2440, 0.2080, 0.4380), (\mathfrak{B}_4, 0.2348, 0.2621, 0.3476)\}$, respectively. Now, by utilizing Euclidean distance between $\mathcal{V}_i$ and PIA/NIA, we compute the closeness degrees $\mathfrak{C}_i$'s as $\mathfrak{C}_1 = 0.6152$, $\mathfrak{C}_2 = 0.7381$, $\mathfrak{C}_3 = 0.5402$ and $\mathfrak{C}_4 = 0.3727$. Thus, ordering are $\mathcal{V}_2 \succ \mathcal{V}_1 \succ \mathcal{V}_3 \succ \mathcal{V}_4$ and the best alternative is $\mathcal{V}_2$.

From the above-computed decisions, it is analyzed that, the best alternative, as well as the ordering position of other alternatives obtained by using current approaches, is not alike to the proposed approach. However, these changes are evident as in both existing approaches all decision matrices are collaborated into a single matrix by some idea and then final results are decided. But the proposed approach offers the decision based on each decision-maker and then search the final decision by considering the decisions of all the experts. Moreover, in our approach, each decision-maker has his/her weight vector for criteria but in the existing approaches, this can never be accessible. Thus, we can say that the proposed approach is somehow superior to the existing approaches.

5. Proposed clustering method

In this section, we present a novel SVN cluster method based on the proposed similarity measure $\mathcal{S}$ to cluster the heterogenous object in the homogenous way. The description of the analysis is given hereafter.

Definition 5.1. [36] For a collection of SVNNs $\mathcal{V}_i$, a matrix $\bar{N} = (s_{ik})_{m\times m}$, where $s_{ik} = \mathcal{S}_\lambda(\mathcal{V}_i, \mathcal{V}_k)$, $(i, k = 1, 2, \ldots, m)$ is called as Similarity matrix between the SVNNs. Also, the matrix $\bar{N}$ has the following properties:

(i) $0 \leq s_{ik} \leq 1$.

(ii) $s_{ii} = 1$.

(iii) $s_{ik} = s_{ki}$; where $i, k = 1, 2, \ldots, m$.

Definition 5.2. [42] A matrix $\bar{N}^2 = \bar{N} \circ \bar{N} = (\tilde{s}_{ij})_{m\times m}$ where $\tilde{s}_{ik} = \max\limits_{v}\{\min(s_{iv}, s_{vk})\}_{m\times m}$ is called similarity composition matrix.

Definition 5.3. [42] If $\bar{N}^2 \subseteq \bar{N}$ i.e. $\max\limits_{u}\big(\min(s_{iu}, s_{uk})\big) \leq s_{ik}\ \forall\ i, k$, then $\bar{N}^2$ is termed as "equivalent similarity matrix (ESM)".

Definition 5.4. [42] For similarity matrix $\bar{N} = (s_{ik})_{m\times m}$, and in the compositions $\bar{N} \to \bar{N}^2 \to \bar{N}^4 \to \ldots \to \bar{N}^{2^z} \to \ldots$, $\exists\, z \in \mathbf{Z}^+$ such that $\bar{N}^{2^z} = \bar{N}^{2^{z+1}}$ and then $\bar{N}^{2^z}$ is also an ESM.

Definition 5.5. [42] For an ESM $\bar{N} = (s_{ik})_{m\times m}$, the matrix $\bar{N}_\alpha = (\tilde{s}^\alpha_{ik})_{m\times m}$ is termed $\alpha-$cutting matrix of $\bar{N}$, where

$$\tilde{s}^\alpha_{ij} = \begin{cases} 0 & ; \quad \tilde{s}_{ik} \leq \alpha \\ 1 & ; \quad \tilde{s}_{ik} \geq \alpha \end{cases}$$

where $\alpha \in [0, 1]$ is the confidence level.

Next, we present a clustering algorithm based on proposed measure $\mathcal{S}_\lambda$ whose description are as follows.

Assume m alternatives $\{Q_1, Q_2, \ldots, Q_m\}$ which are described by n criteria $\{\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_n\}$. These choices are assessed by an expert in terms of SVNNs. The target of this task is to classify the given Q_i's into their equivalence classes. For it, a method has been suggested which are summarized in the following steps:

Step 1: Construct the similarity matrix $\bar{N} = (s_{ik})_{m\times m}$, $s_{ik} = \mathcal{S}_\lambda(\mathcal{V}_i, \mathcal{V}_k), (i, k = 1, 2, \ldots, m)$. Here $\mathcal{S}_\lambda$ is computed by Eq. (3.2).

Step 2: Obtain the ESM $\bar{N}^{2^p} \triangleq \bar{N} = (\tilde{s}_{ik})_{m\times m}$ by making use of composition of matrices as given in Definition 5.2.

Step 3: Construct the $\alpha-$ cut matrix $\bar{N}_\alpha = (\tilde{s}^\alpha_{ik})_{m\times m}$ by Definition 5.5.

Step 4: Classify the identical Q_i and Q_k into the same class. .

The above mentioned algorithm is demonstrated through an example as

Consider five brands of mobile phones, say, Q_1, Q_2, Q_3, Q_4, Q_5, which are selected under the six criteria, namely price of mobile phone ($\mathfrak{B}_1$), appearance ($\mathfrak{B}_2$), memory ($\mathfrak{B}_3$), operating system ($\mathfrak{B}_4$), performance ($\mathfrak{B}_5$) and processor ($\mathfrak{B}_6$). The aim is to classify the phones with these criteria. An expert

Table 7. Rating values of each object

	$\mathfrak{B}_1$	$\mathfrak{B}_2$	$\mathfrak{B}_3$	$\mathfrak{B}_4$	$\mathfrak{B}_5$	$\mathfrak{B}_6$
Q_1	$(0.3, 0.2, 0.5)$	$(0.6, 0.3, 0.1)$	$(0.4, 0.3, 0.3)$	$(0.8, 0.1, 0.1)$	$(0.1, 0.3, 0.6)$	$(0.5, 0.2, 0.4)$
Q_2	$(0.6, 0.3, 0.3)$	$(0.5, 0.4, 0.2)$	$(0.6, 0.2, 0.1)$	$(0.7, 0.2, 0.1)$	$(0.3, 0.1, 0.6)$	$(0.4, 0.3, 0.3)$
Q_3	$(0.4, 0.2, 0.4)$	$(0.8, 0.2, 0.1)$	$(0.5, 0.3, 0.1)$	$(0.6, 0.1, 0.2)$	$(0.4, 0.1, 0.5)$	$(0.3, 0.2, 0.2)$
Q_4	$(0.2, 0.4, 0.4)$	$(0.4, 0.5, 0.1)$	$(0.9, 0.2, 0.0)$	$(0.8, 0.2, 0.1)$	$(0.2, 0.3, 0.5)$	$(0.7, 0.3, 0.1)$
Q_5	$(0.5, 0.3, 0.2)$	$(0.3, 0.2, 0.6)$	$(0.6, 0.1, 0.3)$	$(0.7, 0.1, 0.1)$	$(0.6, 0.2, 0.2)$	$(0.5, 0.2, 0.3)$

gives the rating of each phone over $\mathfrak{B}_j$'s in terms of SVNNs. The complete summary of their ratings are listed in Table 7. To implemented the stated algorithm, we choose $\lambda = 2$ and $\beta_1 = \beta_2 = \beta_3 = 1/3$.

Now, we utilize the proposed measure $\mathcal{S}$ to assemble the phones Q_i, which involves the subsequent steps:

Step 1: By using Eq. (3.2), calculate the degrees of similarity between the phones cars i.e., $\mathcal{S}(Q_i, Q_k)$ (i,k=1,2, …, 5). Thus, a similarity matrix $\bar{N}$ is obtained as:

$$\bar{N} = \begin{pmatrix} 1.0000 & 0.8637 & 0.8576 & 0.8100 & 0.7687 \\ 0.8637 & 1.0000 & 0.8791 & 0.8309 & 0.8450 \\ 0.8576 & 0.8791 & 1.0000 & 0.7896 & 0.8055 \\ 0.8100 & 0.8309 & 0.7896 & 1.0000 & 0.7836 \\ 0.7687 & 0.8450 & 0.8055 & 0.7836 & 1.0000 \end{pmatrix}$$

Step 2: Compute the matrix $\bar{N}^2$, using Definition 5.2, given as:

$$\bar{N}^2 = \bar{N} \circ \bar{N} = \begin{pmatrix} 1.0000 & 0.8637 & 0.8637 & 0.8309 & 0.8450 \\ 0.8637 & 1.0000 & 0.8791 & 0.8309 & 0.8450 \\ 0.8637 & 0.8791 & 1.0000 & 0.8309 & 0.8450 \\ 0.8309 & 0.8309 & 0.8309 & 1.0000 & 0.8309 \\ 0.8450 & 0.8450 & 0.8450 & 0.8309 & 1.0000 \end{pmatrix}$$

Since $\bar{N}^2 \neq \bar{N}$, therefore we compute $\bar{N}^4$.

$$\bar{N}^4 = \bar{N}^2 \circ \bar{N}^2 = \begin{pmatrix} 1.0000 & 0.8637 & 0.8637 & 0.8309 & 0.8450 \\ 0.8637 & 1.0000 & 0.8791 & 0.8309 & 0.8450 \\ 0.8637 & 0.8791 & 1.0000 & 0.8309 & 0.8450 \\ 0.8309 & 0.8309 & 0.8309 & 1.0000 & 0.8309 \\ 0.8450 & 0.8450 & 0.8450 & 0.8309 & 1.0000 \end{pmatrix}$$

Since, $\bar{N}^4 = \bar{N}^2$, therefore, $\bar{N}^2$ is an ESM.

Step 3: Assume $\alpha = 0.8637$ and by Definition 5.5, $\bar{N}_\alpha$ becomes

$$\bar{N}_\alpha = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{5.1}$$

Step 4: From Eq. (5.1), we divide Q_i into three classes as

$$\{Q_1, Q_2, Q_3\}, \quad \{Q_4\} \quad \text{and} \quad \{Q_5\}$$

This means the phones Q_1, Q_2 and Q_3 are more similar to each other than that of the alternative in other clusters.

Further, by examining the various $\alpha-$ cuts, we get the different classes. Thus, a comprehensive analysis based on the $\alpha-$ cut is placed in Table 8. From this Table 8, we recognize that the decision-maker has only one way to partition the set of alternatives in a particular number of classes. The above review unfolds the importance of different values of confidence level α on the clustering process and also investigates the role of α in the flexibility of the algorithm.

Table 8. Different clustering classes for different confidence levels

Class	Confidence level	Clustering results
1	$0.0000 \leq \alpha \leq 0.8309$	$\{Q_1, Q_2, Q_3, Q_4, Q_5\}$
2	$0.8309 < \alpha \leq 0.8450$	$\{Q_1, Q_2, Q_3, Q_5\}, \{Q_4\}$
3	$0.8450 \leq \alpha \leq 0.8637$	$\{Q_1, Q_2, Q_3\}$, $\{Q_4\}$, $\{Q_5\}$
4	$0.8637 \leq \alpha \leq 0.8791$	$\{Q_1\}$,$\{Q_2, Q_3\}$, $\{Q_4\}$, $\{Q_5\}$
5	$0.8791 \leq \alpha \leq 1.0000$	$\{Q_1\}$,$\{Q_2\}$,$\{Q_3\}$, $\{Q_4\}$, $\{Q_5\}$

However, the value of confidence level α is chosen by a decision-maker from 0(smallest) to 1(biggest). Based on their values, we summarize the clustering results and their corresponding α-level matrices whose description are given as

1) If $0 \leq \alpha \leq 0.8309$,

$$\bar{N}_\alpha = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

2) If $0.8390 < \alpha \leq 0.8450$,

$$\bar{N}_\alpha = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

3) If $0.8450 < \alpha \leq 0.8637$,

$$\bar{N}_\alpha = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

4) If $0.8637 < \alpha \leq 0.8791$,

$$\bar{N}_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

5) If $0.8791 < \alpha \leq 1$,

$$\bar{N}_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

From above $\alpha-$ cutting matrices, it can easily noticed that when $0 \leq \alpha \leq 0.8309$, $0.8309 < \alpha < 0.8450$, $0.8450 < \alpha \leq 0.8637$, $0.8637 < \alpha \leq 0.8791$ and $0.8791 < \alpha \leq 1$, the alternatives are classified into 1,2,3,4 and 5 classes respectively. This reflects that as given alternatives are more differentiated with increase of α value.

6. Advantages of the proposed work

The major benefits from the proposed approach over the existing approaches are listed as below.

1) This paper highlights the significance of taking the idea of agreeness, falsity, and disagrees in one envelop in form of SVNSs. As in real DM problems, the degree of membership and nonmembership may work independently, so this generalization of IFSs is more superior and innovative in the evaluation of the information.

2) The proposed methodology is extremely flexible due to the presence of the parameter λ in the formulation of both proposed measures. Due to the presence of the parameter, the decision-maker has the opportunity to give his/her decision by taking different semantics in his mind. This makes the proposed work more friendly and impressionable for the decision-maker who is working on different kinds of DM processes.

3) The proposed method practices the notion of TOPSIS for making the final decision. In this approach, the raking results are finding not by aggregating all the decision matrices but evaluated firstly based on the individual decision-maker and then get a final decision by considering the results given by each decision-maker. In this manner, this approach displays the individual as well as the aggregated decision on the final choice by considering the reference points.

4) Moreover, the proposed similarity measure can be used to cluster the heterogeneous data which is used in various concepts like data mining, image processing, DM problems, medical imaging and so on.

7. Conclusion

The key contribution of the work can be summarized below.

1) The examined study employs the three independent degrees namely MD, NMD and degree of indeterminacy to check the vagueness in the data.

2) This paper offers new distance measures for estimating the degree of discrimination between the two or more SVNSs. Traditionally, all the measurements are computed by using either Hamming or Euclidean distance measures [18–20], which may not furnish the proper choice to the expert. To succeed it, revised distance measures are injected in this work which supplies an alternative way to trade with the SVNN information.

3) An extended TOPSIS method has been introduced with the stated distance measures and by consideration the multi-experts. The advantages of the stated method are that it not only taken into the account the degree of discrimination but also takes the degree of similarity between the observation, to avoid the decision only based on the small distances. Also, the ideal alternatives i.e., PIA ($\mathcal{V}^+$) and NIA ($\mathcal{V}^-$) are considered as constant rather it is dependent on the given observation. Finally, the presented TOPSIS method is based on the additional parameter λ which will make a decision maker flexible to choose their alternatives based on their preferences or goals.

4) The MCGDM algorithm based on the recommended TOPSIS is explained, which is more generalized and flexible with the parameter λ to the decision-maker. The significance of the parameter λ is shown in detail (Table 4). To sustain their performance, a validity test is examined which ensures their reliableness and preciseness.

5) A new clustering algorithm is presented based on the proposed similarity measures under the different confidence levels of the expert. The main objective of this algorithm is to classify the heterogenous objects into the homogenous classes. The applicability of this algorithm is explained with a numerical example and classify the objects with different levels of preferences of the expert.

In the future, we shall lengthen the application of the proposed measures to the diverse fuzzy environment as well as different fields of application such as supply chain management, emerging decision problems, brain hemorrhage, risk evaluation, etc [43–47].

Conflict of interest

The authors declare no conflicts of interest.

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