AIMS Mathematics

AIMS Mathematics, 5(3): 2346–2368.
DOI: 10.3934/math.2020155
Received: 30 September 2019
Accepted: 07 February 2020
Published: 03 March 2020

http://www.aimspress.com/journal/Math

Research article

Numerical computation of fractional Kersten-Krasil'shchik coupled KdV-mKdV system occurring in multi-component plasmas

Amit Goswami[1], Sushila[2,*], Jagdev Singh[3] and Devendra Kumar[4]

[1] Department of Physics, Jagan Nath University, Jaipur-303901, Rajasthan, India
[2] Department of Physics, Vivekananda Global University, Jaipur-303012, Rajasthan, India
[3] Department of Mathematics, JECRC University, Jaipur-303905, Rajasthan, India
[4] Department of Mathematics, University of Rajasthan, Jaipur-302004, Rajasthan, India

*** Correspondence:** Email: sushila.jag@gmail.com.

Abstract: In this paper, we study the nonlinear behaviour of multi-component plasma. For this an efficient technique, called Homotopy perturbation Sumudu transform method (HPSTM) is introduced. The power of method is represented by solving the time fractional Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system. This coupled nonlinear system usually arises as a description of waves in multi-component plasmas, traffic flow, electric circuits, electrodynamics and elastic media, shallow water waves etc. The prime purpose of this study is to provide a new class of technique, which need not to use small parameters for finding approximate solution of fractional coupled systems and eliminate linearization and unrealistic factors. Numerical solutions represent that proposed technique is efficient, reliable, and easy to use to large variety of physical systems. This study shows that numerical solutions gained by HPSTM are very accurate and effective for analysis the nonlinear behaviour of system. This study also states that HPSTM is much easier, more convenient and efficient than other available analytical methods.

Keywords: Kersten-Krasil'shchik coupled KdV-mKdV system; Sumudu transform; Homotopy perturbation method; He's polynomials; multi-component plasmas
Mathematics Subject Classification: 26A33, 35A22, 35R11, 76X05

1. Introduction

Plasma is also known as ionized state of the matter. In this state of matter, Plasma contains multi-component such as free electrons, ions neutrals, dust etc. The multi-component plasmas deal with the partially or fully ionized state of plasma. It also fulfils the condition of quasi neutrality. The multi-component plasmas play a crucial role in plasma discharge and other processing industry. The multi-component plasma having more than two components while general plasma having ions and electrons. The wide range of plasmas itself give an opportunity to analysis such plasmas in distinct scales. There are two main areas of multi-component plasma such as dusty plasma and the negative ions plasma. Both areas of multi-component plasma have its wide range of emerging applications in science and engineering. The dusty plasmas are the important from the perspective of space application to fusion possibility. On the other hand, negative ions plasmas open up the door to understand microelectronics fabrication technology to large scale fusion exploration [1–8].

In the past decades, researchers paid attention to study the multi-component plasma mathematically and analysis the obtained results. The fractional nonlinear partial differential equations are widely used to model the complex phenomenon in many fields of physics and engineering such as hydrodynamics, lattice vibration, acoustics plasma physics, optical fibre, wave propagation, fluid dynamics, etc. The fractional differential equations have great interest by the many researchers due to its applicability in many branches of science and technology. The main benefit of taking fractional order coupled system in this application is its nonlocal property. An integer type differential operator is a class of local operator while fractional type differential operator is a class of nonlocal operator. The main benefit of taking fractional type operator is carrying historical information for determining future state of a system. Its means all future state of a system carries historical information of their previous states and uses this information for determining next future state of a system [9–14].

In the past decades, many researchers are used to various techniques for solving fractional nonlinear partial differential equation and find approximate and exact solutions of the fractional evolution equations. Some of them are homotopy perturbation method, Backlund transformation method, homotopy analysis technique, Tanh method, Adomian decomposition algorithm, variational iteration algorithm, Laplace decomposition method and many more [15–20]. In the past time, Singh et al. [21,22] give an excellent technique for solving fractional nonlinear systems. This scheme is a unique combine form of Sumudu transform with HPM known as HPSTM. The nonlinear term can be decomposed by using He's polynomials which is proposed by Ghorbani and Saberi-Nadjafi [23] and Ghorbani [24].

Time fractional Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system and homogeneous two component time fractional coupled third order KdV systems are very important fractional nonlinear systems for describing the behaviour of waves in multi-component plasma and elaborate various nonlinear phenomena in plasma physics. In present paper, we show applicability of HPSTM for study the time fractional Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system and homogeneous two component time fractional coupled third order KdV system. HPSTM gives exceptional precision in analogous to numerical results and having high precisions with the numerical results. The HPSTM find solutions in convergent series with computable element in straightforward approaches, which need not to apply perturbation, linearization or contrary assumptions. In this work, we apply HPSTM on time fractional Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system and homogeneous two component time fractional coupled third order KdV system and find approximate solution, exact solution and absolute error. Variations in results show by the graphically as well as numerically.

This work is arranged as: in section 2, we describe mathematical model of problem, in section 3, analysis of HPSTM method presented, in section 4, we apply HPSTM on fractional Kersten-Krasil'shchik coupled KdV-mKdV system and homogeneous two component time fractional coupled third order KdV system, in section 5, results and discussions of physical systems are given, in section 6, conclusions are presented. At the last, references are given.

2. Mathematical model of the problem

The fractional coupled systems are widely applied to study complex behaviour of plasma contains multi components such as ions, free electrons, atoms etc. Many researchers made efforts to study this behaviour numerically. In this direction, recently Paul Kersten and Joseph Krasil'shchik studied KdV equation and modified KdV equation and proposed absolute complexity between coupled KdV–mKdV nonlinear systems for study the behaviour of nonlinear systems. Numerous variations of this Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system has been introduced by many researchers [27–34].

Among these variations, the mathematical model for describing behaviour of multi-component plasma for waves propagating in positive ξ axis, known as nonlinear fractional Kersten-Krasil'shchik coupled KdV-mKdV system is given by:

$$D_\eta^\alpha \rho + \rho_{3\xi} - 6\rho\rho_\xi + 3\omega\omega_{3\xi} + 3\omega_\xi\omega_{2\xi} - 3\rho_\xi\omega^2 + 6\rho\omega\omega_\xi = 0,\ \eta > 0,\ \xi \in R,\ 0 < \alpha \le 1 \quad (2.1)$$

$$D_\eta^\alpha \omega + \omega_{3\xi} - 3\omega^2\omega_\xi - 3\rho\omega_\xi + 3\rho_\xi\omega = 0,\ \ \eta > 0,\ \xi \in R,\ 0 < \alpha \le 1, \quad (2.2)$$

where η is temporal coordinate and ξ is spatial coordinate. The factor α is represents order of the fractional operator. This operator is studied in the Caputo form.
When $\alpha = 1$, fractional coupled system converts to classical system as:

$$\rho_\eta + \rho_{3\xi} - 6\rho\rho_\xi + 3\omega\omega_{3\xi} + 3\omega_\xi\omega_{2\xi} - 3\rho_\xi\omega^2 + 6\rho\omega\omega_\xi = 0,\ \eta > 0,\ \xi \in R, \quad (2.3)$$

$$\omega_\eta + \omega_{3\xi} - 3\omega^2\omega_\xi - 3\rho\omega_\xi + 3\rho_\xi\omega = 0,\ \ \eta > 0,\ \xi \in R, \quad (2.4)$$

If we put $\omega = 0$ then Kersten-Krasil'shchik coupled KdV-mKdV system convert into well-known KdV system as:

$$\rho_\eta + \rho_{3\xi} - 6\rho\rho_\xi = 0,\ \eta > 0,\ \xi \in R, \quad (2.5)$$

If we put $\rho = 0$ then Kersten-Krasil'shchik coupled KdV-mKdV system convert into well-known modified KdV system as:

$$\omega_\eta + \omega_{3\xi} - 3\omega^2\omega_\xi = 0,\ \ \eta > 0,\ \xi \in R, \quad (2.6)$$

In view of that, Kersten-Krasil'shchik coupled KdV-mKdV system can be assumed combination between KdV system and mKdV system represented by (2.3) to (2.6).

In this study, we also consider fractional nonlinear two component homogeneous time fractional coupled third order KdV system as:

$$D_\eta^\alpha \rho - \rho_{3\xi} - \rho\rho_\xi - \omega\omega_\xi = 0,\ \eta > 0,\ \xi \in R,\ 0 < \alpha \le 1, \tag{2.7}$$

$$D_\eta^\alpha \omega + 2\omega_{3\xi} - \rho\omega_\xi = 0,\ \ \eta > 0,\ \xi \in R,\ 0 < \alpha \le 1, \tag{2.8}$$

where η is temporal coordinate and ξ is spatial coordinate, α is a factor represents order of the fractional operator. This operator is studied in the Caputo form.
When $\alpha = 1$, fractional coupled system converts to classical system as:

$$\rho_\eta - \rho_{3\xi} - \rho\rho_\xi - \omega\omega_\xi = 0,\ \eta > 0,\ \xi \in R, \tag{2.9}$$

$$\omega_\eta + 2\omega_{3\xi} - \rho\omega_\xi = 0,\ \ \eta > 0,\ \xi \in R. \tag{2.10}$$

3. Basic plan of HPSTM

Assume a nonhomogeneous, nonlinear time fractional coupled system as [21,22]:

$$D_\eta^\alpha \rho(\xi,\eta) + R\rho(\xi,\eta) + N\rho(\xi,\eta) = g(\xi,\eta), \tag{3.1}$$

with the initial condition (IC)

$$\rho(\xi,0) = f(\xi), \tag{3.2}$$

where $D_\eta^\alpha \rho(\xi,\eta)$ is fractional order Caputo derivative of the function $\rho(\xi,\eta), g(\xi,\eta)$ represent source term, R shows linear differential derivative and N shows nonlinear differential derivative.
Using Sumudu transform [25,26] on Eq. (3.1), we get

$$S\left[D_\eta^\alpha \rho(\xi,\eta)\right] + S[R\rho(\xi,\eta)] + S[N\rho(\xi,\eta)] = S[g(\xi,\eta)]. \tag{3.3}$$

By using identity of Sumudu transform, we have

$$S[\rho(\xi,\eta)] = f(\xi) + u^\alpha S[g(\xi,\eta)] - u^\alpha S[R\rho(\xi,\eta) + N\rho(\xi,\eta)]. \tag{3.4}$$

Using inverse Sumudu transform on Eq. (3.4), we have

$$\rho(\xi,\eta) = G(\xi,\eta) - S^{-1}\left[u^\alpha S[R\rho(\xi,\eta) + N\rho(\xi,\eta)]\right]. \tag{3.5}$$

Here $G(\xi,\eta)$ is an expression due to source term and IC.
Use of HPM on Eq. (3.5) gives

$$\rho(\xi,\eta) = \sum_{n=0}^{\infty} p^n \rho_n(\xi,\eta). \tag{3.6}$$

Nonlinear term can be disintegrated as:

$$N\rho(\xi,\eta) = \sum_{n=0}^{\infty} p^n H_n(\rho), \tag{3.7}$$

here $H_n(\rho)$ are He's polynomials [23,24] which are expressed as:

$$H_n(\rho_0,\rho_1,\rho_2,\dots,\rho_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{i=0}^{\infty} p^i \rho_i\right)\right]_{p=0}, n = 0,1,2,3 \dots. \tag{3.8}$$

Incorporating Eq. (3.6) and Eq. (3.7) in Eq. (3.5), we have

$$\sum_{n=0}^{\infty} p^n \rho_n(\xi,\eta) = G(\xi,\eta) - p\left(S^{-1}\left[u^\alpha S[R\sum_{n=0}^{\infty} p^n \rho_n(\xi,\eta) + \sum_{n=0}^{\infty} p^n H_n(\rho)]\right]\right). \tag{3.9}$$

Equation (3.9) represents combination of He's polynomials, Sumudu transform and HPM.
Comparing coefficients of the same powers of p, we have

$$p^0: \rho_0(\xi,\eta) = G(\xi,\eta),$$

$$p^1: \rho_1(\xi,\eta) = -S^{-1}\left[u^\alpha S[R\rho_0(\xi,\eta) + H_0(\rho)]\right],$$

$$p^2: \rho_2(\xi,\eta) = -S^{-1}\left[u^\alpha S[R\rho_1(\xi,\eta) + H_1(\rho)]\right], \tag{3.10}$$

$$p^3: \rho_3(\xi,\eta) = -S^{-1}\left[u^\alpha S[R\rho_2(\xi,\eta) + H_2(\rho)]\right],$$

$$\vdots$$

$$p^n: \rho_n(\xi,\eta) = -S^{-1}\left[u^\alpha S[R\rho_{n-1}(\xi,\eta) + H_{n-1}(\rho)]\right].$$

By the same manner, one can evaluate other elements of $\rho_n(\xi,\eta)$ and then find series solution. We approximate numerical solution of $\rho(\xi,\eta)$ as:

$$\rho(\xi,\eta) = \lim_{N\to\infty} \sum_{n=0}^{N} \rho_n(\xi,\eta). \tag{3.11}$$

Equation (3.11) represents series solution, which converges very fast.

4. Numerical experiments

Example 4.1. Assume time fractional Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system as:

$$D_\eta^\alpha \rho + \rho_{3\xi} - 6\rho\rho_\xi + 3\omega\omega_{3\xi} + 3\omega_\xi\omega_{2\xi} - 3\rho_\xi\omega^2 + 6\rho\omega\omega_\xi = 0,\ \eta > 0,\ \xi \in R,\ 0 < \alpha \le 1,$$

$$D_\eta^\alpha \omega + \omega_{3\xi} - 3\omega^2\omega_\xi - 3\rho\omega_\xi + 3\rho_\xi\omega = 0,\ \ \eta > 0,\ \xi \in R,\ 0 < \alpha \le 1, \tag{4.1}$$

with the initial condition

$$\rho(\xi,\ 0) = c - 2c\, sech^2(\sqrt{c}\xi),$$

$$\omega(\xi,\ 0) = 2\sqrt{c}\, sech(\sqrt{c}\xi). \tag{4.2}$$

Using Sumudu transform on Eq. (4.1) by the application of initial condition given by Eq. (4.2), we get

$$S[\rho(\xi,\ \eta)] = c - 2c\, sech^2(\sqrt{c}\xi) - u^\alpha S\left[\rho_{3\xi} - 6\rho\rho_\xi + 3\omega\omega_{3\xi} + 3\omega_\xi\omega_{2\xi} - 3\rho_\xi\omega^2 + 6\rho\omega\omega_\xi\right],$$

$$S[\omega(\xi,\ \eta)] = 2\sqrt{c}\, sech(\sqrt{c}\xi) - u^\alpha S\left[\omega_{3\xi} - 3\omega^2\omega_\xi - 3\rho\omega_\xi + 3\rho_\xi\omega\right]. \tag{4.3}$$

Apply inverse Sumudu transform, we get

$$\rho(\xi,\ \eta) = c - 2c\, sech^2(\sqrt{c}\xi)$$

$$-S^{-1}\left[u^\alpha S\left[\rho_{3\xi} - 6\rho\rho_\xi + 3\omega\omega_{3\xi} + 3\omega_\xi\omega_{2\xi} - 3\rho_\xi\omega^2 + 6\rho\omega\omega_\xi\right]\right],$$

$$\omega(\xi,\ \eta) = 2\sqrt{c}\, sech(\sqrt{c}\xi) - S^{-1}\left[u^\alpha S\left[\omega_{3\xi} - 3\omega^2\omega_\xi - 3\rho\omega_\xi + 3\rho_\xi\omega\right]\right]. \tag{4.4}$$

Use HPM on Eq. (4.4), we get

$$\sum_{n=0}^{\infty} p^n \rho_n(\xi,\eta) = c - 2c\,sech^2(\sqrt{c}\xi)$$
$$-p\left(S^{-1}\left[u^{\alpha} S\left[\left(\sum_{n=0}^{\infty} p^n \rho_n(\xi,\eta)\right)_{3\xi} + \left(\sum_{n=0}^{\infty} p^n H_n(\rho)\right)\right]\right]\right),$$
$$\sum_{n=0}^{\infty} p^n \omega_n(\xi,\eta) = 2\sqrt{c}\,sech(\sqrt{c}\xi)$$
$$-p\left(S^{-1}\left[u^{\alpha} S\left[\left(\sum_{n=0}^{\infty} p^n \omega_n(\xi,\eta)\right)_{3\xi} + \left(\sum_{n=0}^{\infty} p^n H_n(\omega)\right)\right]\right]\right). \quad (4.5)$$

Nonlinear steps given by He's polynomials $H_n(\rho)$ and $H_n(\omega)$, which are given as [23,24]:

$$\sum_{n=0}^{\infty} p^n H_n(\rho) = -6\rho\rho_{\xi} + 3\omega\omega_{3\xi} + 3\omega_{\xi}\omega_{2\xi} - 3\rho_{\xi}\omega^2 + 6\rho\omega\omega_{\xi},$$

$$\sum_{n=0}^{\infty} p^n H_n(\omega) = -3\omega^2\omega_{\xi} - 3\rho\omega_{\xi} + 3\rho_{\xi}\omega. \quad (4.6)$$

Values of components of He's polynomials are given by

$$H_0(\rho) = -6\rho_0(\rho_0)_{\xi} + 3\omega_0(\omega_0)_{3\xi} + 3(\omega_0)_{\xi}(\omega_0)_{2\xi} - 3(\rho_0)_{\xi}\omega_0^2 + 6\rho_0\omega_0(\omega_0)_{\xi}$$

$$H_1(\rho) = -6\rho_1(\rho_0)_{\xi} - 6\rho_0(\rho_1)_{\xi} + 3\omega_1(\omega_0)_{3\xi} + 3\omega_0(\omega_1)_{3\xi} + 3(\omega_0)_{\xi}(\omega_1)_{2\xi}$$
$$+3(\omega_0)_{2\xi}(\omega_1)_{\xi} - 3(\rho_1)_{\xi}\omega_0^2 - 6(\rho_0)_{\xi}\omega_0\omega_1 + 6\rho_0\omega_1(\omega_0)_{\xi} + 6\rho_0\omega_0(\omega_1)_{\xi}$$
$$+ 6\rho_1\omega_0(\omega_0)_{\xi} \quad (4.7)$$

$$H_2(\rho) = -6\rho_2(\rho_0)_{\xi} - 6\rho_1(\rho_1)_{\xi} - 6\rho_0(\rho_2)_{\xi} + 3\omega_2(\omega_0)_{3\xi} + 3\omega_1(\omega_1)_{3\xi} + 3\omega_0(\omega_2)_{3\xi}$$
$$+3(\omega_0)_{\xi}(\omega_2)_{2\xi} + 3(\omega_1)_{\xi}(\omega_1)_{2\xi} + 3(\omega_0)_{2\xi}(\omega_2)_{\xi} - 3(\rho_2)_{\xi}\omega_0^2 - 6(\rho_1)_{\xi}\omega_0\omega_1$$
$$-6(\rho_0)_{\xi}\omega_0\omega_2 + 6\rho_0\omega_2(\omega_0)_{\xi} + 6\rho_1\omega_1(\omega_0)_{\xi} + 6\rho_2\omega_0(\omega_0)_{\xi} + 6\rho_0\omega_1(\omega_1)_{\xi}$$
$$+6\rho_1\omega_0(\omega_1)_{\xi} + 6\rho_0\omega_0(\omega_2)_{\xi}$$

$$H_3(\rho) = -6\rho_3(\rho_0)_{\xi} - 6\rho_2(\rho_1)_{\xi} - 6\rho_1(\rho_2)_{\xi} - 6\rho_0(\rho_3)_{\xi} + 3\omega_3(\omega_0)_{3\xi} + 3\omega_2(\omega_1)_{3\xi}$$
$$+3\omega_1(\omega_2)_{3\xi} + 3\omega_0(\omega_3)_{3\xi} + 3(\omega_0)_{\xi}(\omega_3)_{2\xi} + 3(\omega_1)_{\xi}(\omega_2)_{2\xi} + 3(\omega_1)_{2\xi}(\omega_2)_{\xi}$$
$$+3(\omega_0)_{2\xi}(\omega_3)_{\xi} - 3(\rho_3)_{\xi}\omega_0^2 - 6(\rho_2)_{\xi}\omega_0\omega_1 - 6(\rho_1)_{\xi}\omega_0\omega_2 - 3(\rho_1)_{\xi}\omega_1^2$$
$$+\rho_0\omega_3(\omega_0)_{\xi} + 6\rho_1\omega_2(\omega_0)_{\xi} + 6\rho_2\omega_1(\omega_0)_{\xi} + 6\rho_3\omega_0(\omega_0)_{\xi} + 6\rho_0\omega_2(\omega_1)_{\xi}$$
$$+66\rho_1\omega_1(\omega_1)_{\xi} + 6\rho_2\omega_0(\omega_1)_{\xi} + 6\rho_0\omega_1(\omega_2)_{\xi} + 6\rho_1\omega_0(\omega_2)_{\xi} + 6\rho_0\omega_0(\omega_3)_{\xi}$$
$$\vdots$$

and

$$H_0(\omega) = -3\omega_0^2(\omega_0)_{\xi} - 3\rho_0(\omega_0)_{\xi} + 3\omega_0(\rho_0)_{\xi}$$

$$H_1(\omega) = -3\omega_0^2(\omega_1)_{\xi} - 6\omega_0\omega_1(\omega_0)_{\xi} - 3\rho_1(\omega_0)_{\xi} - 3\rho_0(\omega_1)_{\xi} + 3\omega_1(\rho_0)_{\xi} + 3\omega_0(\rho_1)_{\xi}$$

$$H_2(\omega) = -3\omega_0^2(\omega_2)_\xi - 6\omega_0\omega_1(\omega_1)_\xi - 6\omega_0\omega_2(\omega_0)_\xi - 3\omega_2^2(\omega_0)_\xi - 3\rho_2(\omega_0)_\xi$$

$$-3\rho_1(\omega_1)_\xi - 3\rho_0(\omega_2)_\xi + 3\omega_2(\rho_0)_\xi - 3\omega_1(\rho_1)_\xi - 3\omega_0(\rho_2)_\xi \tag{4.8}$$

$$H_3(\omega) = -3\omega_0^2(\omega_3)_\xi - 6\omega_0\omega_1(\omega_2)_\xi - 6\omega_0\omega_2(\omega_1)_\xi - 6\omega_0\omega_3(\omega_0)_\xi - 6\omega_1\omega_2(\omega_0)_\xi$$

$$-3\omega_3^2(\omega_0)_\xi - 3\rho_3(\omega_0)_\xi - 3\rho_2(\omega_1)_\xi - 3\rho_1(\omega_2)_\xi - 3\rho_0(\omega_3)_\xi + 3\omega_3(\rho_0)_\xi$$

$$-3\omega_2(\rho_1)_\xi - 3\omega_1(\rho_2)_\xi - 3\omega_0(\rho_3)_\xi$$

$$\vdots$$

Comparing the coefficients of same powers of p, we have

$$p^0:\ \rho_0(\xi, \eta) = c - 2csech^2(\sqrt{c}\xi),$$

$$p^1:\ \rho_1(\xi, \eta) = -S^{-1}\left[u^\alpha S[(\rho_0)_{3\xi} + H_0(\rho)]\right]$$

$$= 8c^{5/2}sinh(\sqrt{c}\xi)sech^3(\sqrt{c}\xi)\frac{\eta^\alpha}{\Gamma(\alpha+1)}, \tag{4.9}$$

$$p^2:\ \rho_2(\xi, \eta) = -S^{-1}\left[u^\alpha S[(\rho_1)_{3\xi} + H_1(\rho)]\right]$$

$$= -16c^4[2cosh^2(\sqrt{c}\xi) - 3]sech^4(\sqrt{c}\xi)\frac{\eta^{2\alpha}}{\Gamma(2\alpha+1)},$$

$$p^3:\ \rho_3(\xi, \eta) = -S^{-1}\left[u^\alpha S[(\rho_2)_{3\xi} + H_2(\rho)]\right]$$

$$= 128\, c^{11/2}[cosh^2(\sqrt{c}\xi) - 3]sinh(\sqrt{c}\xi)sech^5(\sqrt{c}\xi)\frac{\eta^{3\alpha}}{\Gamma(3\alpha+1)},$$

$$p^4:\ \rho_4(\xi, \eta) = -S^{-1}\left[u^\alpha S[(\rho_3)_{3\xi} + H_3(\rho)]\right]$$

$$= -256\, c^7[2cosh^4(\sqrt{c}\xi) - 15cosh^2(\sqrt{c}\xi) + 15]sech^6(\sqrt{c}\xi)\frac{\eta^{4\alpha}}{\Gamma(4\alpha+1)},$$

$$\vdots$$

and

$$p^0: \omega_0(\xi, \eta) = 2\sqrt{c}sech(\sqrt{c}\xi),$$

$$p^1:\ \omega_1(\xi, \eta) = -S^{-1}\left[u^\alpha S[(\omega_0)_{3\xi} + H_0(\omega)]\right]$$

$$= -4c^2sinh(\sqrt{c}\xi)sech^2(\sqrt{c}\xi)\frac{\eta^\alpha}{\Gamma(\alpha+1)}, \tag{4.10}$$

$$p^2:\ \omega_2(\xi, \eta) = -S^{-1}\left[u^\alpha S[(\omega_1)_{3\xi} + H_1(\omega)]\right]$$

$$= 8c^{7/2}[cosh^2(\sqrt{c}\xi) - 2]sech^3(\sqrt{c}\xi)\frac{\eta^{2\alpha}}{\Gamma(2\alpha+1)},$$

$$p^3:\ \omega_3(\xi, \eta) = -S^{-1}\left[u^\alpha S[(\omega_2)_{3\xi} + H_2(\omega)]\right]$$

$$= -16c^5\left[cosh^2(\sqrt{c}\xi) - 6\right]sinh(\sqrt{c}\xi)sech^4(\sqrt{c}\xi)\frac{\eta^{3\alpha}}{\Gamma(3\alpha+1)},$$

$$p^4:\ \omega_4(\xi,\ \eta) = -S^{-1}\left[u^\alpha S\left[(\omega_3)_{3\xi} + H_3(\omega)\right]\right]$$

$$= 32c^{13/2}\left[cosh^4(\sqrt{c}\xi) - 20cosh^2(\sqrt{c}\xi) + 24\right]sech^5(\sqrt{c}\xi)\frac{\eta^{4\alpha}}{\Gamma(4\alpha+1)},$$

$\vdots$

Hence series solution is given by

$$\rho(\xi,\ \eta) = \sum_{i=0}^{\infty}\rho_i(\xi,\ \eta)$$

$$= c - 2csech^2(\sqrt{c}\xi) + 8c^{5/2}sinh(\sqrt{c}\xi)sech^3(\sqrt{c}\xi)\frac{\eta^\alpha}{\Gamma(\alpha+1)}$$

$$-16c^4\left[2cosh^2(\sqrt{c}\xi) - 3\right]sech^4(\sqrt{c}\xi)\frac{\eta^{2\alpha}}{\Gamma(2\alpha+1)} + 128c^{11/2}\left[cosh^2(\sqrt{c}\xi) - 3\right]$$

$$sinh(\sqrt{c}\xi)sech^5(\sqrt{c}\xi)\frac{\eta^{3\alpha}}{\Gamma(3\alpha+1)} - 256c^7\left[2cosh^4(\sqrt{c}\xi) - 15cosh^2(\sqrt{c}\xi) + 15\right]$$

$$sech^6(\sqrt{c}\xi)\frac{\eta^{4\alpha}}{\Gamma(4\alpha+1)} + \cdots, \tag{4.11}$$

and

$$\omega(\xi,\ \eta) = \sum_{i=0}^{\infty}\omega_i(\xi,\ \eta)$$

$$= 2\sqrt{c}sech(\sqrt{c}\xi) - 4c^2sinh(\sqrt{c}\xi)sech^2(\sqrt{c}\xi)\frac{\eta^\alpha}{\Gamma(\alpha+1)} + 8c^{7/2}\left[cosh^2(\sqrt{c}\xi) - 2\right]$$

$$sech^3(\sqrt{c}\xi)\frac{\eta^{2\alpha}}{\Gamma(2\alpha+1)} - 16c^5\left[cosh^2(\sqrt{c}\xi) - 6\right]sinh(\sqrt{c}\xi)sech^4(\sqrt{c}\xi)\frac{\eta^{3\alpha}}{\Gamma(3\alpha+1)}$$

$$+32c^{13/2}\left[cosh^4(\sqrt{c}\xi) - 20cosh^2(\sqrt{c}\xi) + 24\right]sech^5(\sqrt{c}\xi)\frac{\eta^{4\alpha}}{\Gamma(4\alpha+1)} - \cdots. \tag{4.12}$$

Putting $\alpha = 1$ in (4.11) and (4.12), we get solution of the problem as:

$$\rho(\xi,\ \eta) = c - 2csech^2(\sqrt{c}\xi) + 8\eta c^{5/2}sinh(\sqrt{c}\xi)sech^3(\sqrt{c}\xi) - 8\eta^2c^4$$

$$\left[2cosh^2(\sqrt{c}\xi) - 3\right]sech^4(\sqrt{c}\xi) + \frac{64}{3}\eta^3c^{11/2}\left[cosh^2(\sqrt{c}\xi) - 3\right]sinh(\sqrt{c}\xi)$$

$$sech^5(\sqrt{c}\xi)\frac{\eta^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{32}{3}\eta^4c^7\left[2cosh^4(\sqrt{c}\xi) - 15cosh^2(\sqrt{c}\xi) + 15\right]$$

$$sech^6(\sqrt{c}\xi) + \cdots, \tag{4.13}$$

and

$$\omega(\xi,\ \eta) = 2\sqrt{c}sech(\sqrt{c}\xi) - 4\eta c^2sinh(\sqrt{c}\xi)sech^2(\sqrt{c}\xi) + 4\eta^2c^{7/2}\left[cosh^2(\sqrt{c}\xi) - 2\right]$$

$$sech^3(\sqrt{c}\xi) - \frac{8}{3}\eta^3c^5\left[cosh^2(\sqrt{c}\xi) - 6\right]sinh(\sqrt{c}\xi)sech^4(\sqrt{c}\xi) + \frac{4}{3}\eta^4c^{13/2}$$

$$\left[cosh^4(\sqrt{c}\xi) - 20cosh^2(\sqrt{c}\xi) + 24\right]sech^5(\sqrt{c}\xi) - \cdots. \tag{4.14}$$

The solution represents by Eq. (4.14) is similar to exact solution in closed form as:

$$\rho(\xi,\eta) = c - 2c sech^2\left[\sqrt{c}(\xi + 2c\eta)\right],$$

$$\omega(\xi,\eta) = 2\sqrt{c} sech\left[\sqrt{c}(\xi + 2c\eta)\right]. \quad (4.15)$$

Example 4.2. Assume homogeneous two component time fractional coupled third order KdV system as:

$$D_\eta^\alpha \rho - \rho_{3\xi} - \rho\rho_\xi - \omega\omega_\xi = 0,\ \eta > 0,\ \xi \in R,\ 0 < \alpha \le 1,$$
$$D_\eta^\alpha \omega + 2\omega_{3\xi} - \rho\omega_\xi = 0,\ \ \eta > 0,\ \xi \in R,\ 0 < \alpha \le 1, \quad (4.16)$$

with the initial condition

$$\rho(\xi,\ 0) = 3 - 6\, tanh^2\left(\frac{\xi}{2}\right),$$

$$\omega(\xi,\ 0) = -3c\sqrt{2}\, tanh^2\left(\frac{\xi}{2}\right). \quad (4.17)$$

Using Sumudu transform on Eq. (4.16) by the application of initial condition given by Eq. (4.17), we get

$$S[\rho(\xi,\ \eta)] = 3 - 6\, tanh^2\left(\frac{\xi}{2}\right) + u^\alpha S[\rho_{3\xi} + \rho\rho_\xi + \omega\omega_\xi],$$

$$S[\omega(\xi,\ \eta)] = -3c\sqrt{2}\, tanh^2\left(\frac{\xi}{2}\right) - u^\alpha S[2\omega_{3\xi} - \rho\omega_\xi]. \quad (4.18)$$

Apply inverse Sumudu transform, we get

$$\rho(\xi,\ \eta) = 3 - 6\, tanh^2\left(\frac{\xi}{2}\right) + S^{-1}\left[u^\alpha S[\rho_{3\xi} + \rho\rho_\xi + \omega\omega_\xi]\right],$$

$$\omega(\xi,\ \eta) = -3c\sqrt{2}\, tanh^2\left(\frac{\xi}{2}\right) - S^{-1}\left[u^\alpha S[2\omega_{3\xi} - \rho\omega_\xi]\right]. \quad (4.19)$$

Use HPM on Eq. (4.19), we get

$$\sum_{n=0}^{\infty} p^n \rho_n(\xi,\ \eta) = 3 - 6\, tanh^2\left(\frac{\xi}{2}\right)$$

$$+p\left(S^{-1}\left[u^\alpha S\left[\left(\sum_{n=0}^{\infty} p^n \rho_n(\xi,\ \eta)\right)_{3\xi} + \left(\sum_{n=0}^{\infty} p^n H_n(\rho)\right)\right]\right]\right),$$

$$\sum_{n=0}^{\infty} p^n \omega_n(\xi,\ \eta) = -3c\sqrt{2}\, tanh^2\left(\frac{\xi}{2}\right)$$

$$-p\left(S^{-1}\left[u^\alpha S\left[2\left(\sum_{n=0}^{\infty} p^n \omega_n(\xi,\ \eta)\right)_{3\xi} - \left(\sum_{n=0}^{\infty} p^n H_n(\omega)\right)\right]\right]\right). \quad (4.20)$$

Nonlinear steps given by He's polynomials $H_n(\rho)$ and $H_n(\omega)$, which are given as:

$$\sum_{n=0}^{\infty} p^n H_n(\rho) = \rho\rho_\xi + \omega\omega_\xi,$$

$$\sum_{n=0}^{\infty} p^n H_n(\omega) = -\rho\omega_\xi. \quad (4.21)$$

Values of factors of He's polynomials are given as [23,24]:

$$H_0(\rho) = \rho_0(\rho_0)_\xi + \omega_0(\omega_0)_\xi$$
$$H_1(\rho) = \rho_1(\rho_0)_\xi + \rho_0(\rho_1)_\xi + \omega_1(\omega_0)_\xi + \omega_0(\omega_1)_\xi \quad (4.22)$$

$$H_2(\rho) = \rho_2(\rho_0)_\xi + \rho_1(\rho_1)_\xi + \rho_0(\rho_2)_\xi + \omega_2(\omega_0)_\xi + \omega_1(\omega_1)_\xi + \omega_0(\omega_2)_\xi$$
$$H_3(\rho) = \rho_3(\rho_0)_\xi + \rho_2(\rho_1)_\xi + \rho_1(\rho_2)_\xi + \rho_0(\rho_3)_\xi + \omega_3(\omega_0)_\xi + \omega_2(\omega_1)_\xi + \omega_1(\omega_2)_\xi + \omega_0(\omega_3)_\xi$$
$$\vdots$$

and

$$H_0(\omega) = -\rho_0(\omega_0)_\xi \tag{4.23}$$
$$H_1(\omega) = -\rho_1(\omega_0)_\xi - \rho_0(\omega_1)_\xi$$
$$H_2(\omega) = -\rho_2(\omega_0)_\xi - \rho_1(\omega_1)_\xi - \rho_0(\omega_2)_\xi$$
$$H_3(\omega) = -\rho_3(\omega_0)_\xi - \rho_2(\omega_1)_\xi - \rho_1(\omega_2)_\xi - \rho_0(\omega_3)_\xi$$
$$\vdots$$

Comparing coefficients of same powers of *p*, we have

$$p^0:\ \rho_0(\xi,\eta) = 3 - 6\,tanh^2\left(\frac{\xi}{2}\right),$$
$$p^1:\ \rho_1(\xi,\eta) = S^{-1}\left[u^\alpha S\left[(\rho_0)_{3\xi} + H_0(\rho)\right]\right]$$
$$= -6\,sech^2\left(\frac{\xi}{2}\right) tanh\left(\frac{\xi}{2}\right)\frac{\eta^\alpha}{\Gamma(\alpha+1)}, \tag{4.24}$$
$$p^2:\ \rho_2(\xi,\eta) = S^{-1}\left[u^\alpha S\left[(\rho_1)_{3\xi} + H_1(\rho)\right]\right]$$
$$= 3\left[2 + 7\,sech^2\left(\frac{\xi}{2}\right) - 15\,sech^4\left(\frac{\xi}{2}\right)\right] sech^2\left(\frac{\xi}{2}\right)\frac{\eta^{2\alpha}}{\Gamma(2\alpha+1)},$$
$$p^3:\ \rho_3(\xi,\eta) = -S^{-1}\left[u^\alpha S\left[(\rho_2)_{3\xi} + H_2(\rho)\right]\right]$$
$$\vdots$$

and

$$p^0: \omega_0(\xi,\eta) = -3c\sqrt{2}\,tanh^2\left(\frac{\xi}{2}\right),$$
$$p^1:\ \omega_1(\xi,\eta) = -S^{-1}\left[u^\alpha S\left[2(\omega_0)_{3\xi} - H_0(\omega)\right]\right]$$
$$= 3c\sqrt{2}\,sech^2\left(\frac{\xi}{2}\right) tanh\left(\frac{\xi}{2}\right)\frac{\eta^\alpha}{\Gamma(\alpha+1)}, \tag{4.25}$$
$$p^2:\ \omega_2(\xi,\eta) = -S^{-1}\left[u^\alpha S\left[(\omega_1)_{3\xi} - H_1(\omega)\right]\right]$$
$$= \frac{3c\sqrt{2}}{2}\left[2 + 21\,sech^2\left(\frac{\xi}{2}\right) - 24\,sech^4\left(\frac{\xi}{2}\right)\right] sech^2\left(\frac{\xi}{2}\right)\frac{\eta^{2\alpha}}{\Gamma(2\alpha+1)},$$
$$p^3:\ \omega_3(\xi,\eta) = -S^{-1}\left[u^\alpha S\left[(\omega_2)_{3\xi} - H_2(\omega)\right]\right]$$
$$\vdots$$

Hence series solution is given by

$$\rho(\xi,\eta) = \sum_{i=0}^{\infty}\rho_i(\xi,\eta)$$
$$= 3 - 6\,tanh^2\left(\frac{\xi}{2}\right) - 6\,sech^2\left(\frac{\xi}{2}\right) tanh\left(\frac{\xi}{2}\right)\frac{\eta^\alpha}{\Gamma(\alpha+1)}$$

$$+3\left[2+7\,sech^2\left(\frac{\xi}{2}\right)-15\,sech^4\left(\frac{\xi}{2}\right)\right]sech^2\left(\frac{\xi}{2}\right)\frac{\eta^{2\alpha}}{\Gamma(2\alpha+1)}-\cdots, \quad (4.26)$$

and

$$\omega(\xi,\,\eta)=\sum_{i=0}^{\infty}\omega_i(\xi,\,\eta)$$

$$=-3c\sqrt{2}\,tanh^2\left(\frac{\xi}{2}\right)+3c\sqrt{2}\,sech^2\left(\frac{\xi}{2}\right)tanh\left(\frac{\xi}{2}\right)\frac{\eta^{\alpha}}{\Gamma(\alpha+1)}$$

$$+\frac{3c\sqrt{2}}{2}\left[2+21\,sech^2\left(\frac{\xi}{2}\right)-24\,sech^4\left(\frac{\xi}{2}\right)\right]sech^2\left(\frac{\xi}{2}\right)\frac{\eta^{2\alpha}}{\Gamma(2\alpha+1)}+\cdots. \quad (4.27)$$

Putting $\alpha = 1$ in (4.26) and (4.27), we get solution of the problem as:

$$\rho(\xi,\,\eta)=3-6\,tanh^2\left(\frac{\xi}{2}\right)-6\eta\,sech^2\left(\frac{\xi}{2}\right)tanh\left(\frac{\xi}{2}\right)+$$

$$\frac{3}{2}\eta^2\left[2+7\,sech^2\left(\frac{\xi}{2}\right)-15\,sech^4\left(\frac{\xi}{2}\right)\right]sech^2\left(\frac{\xi}{2}\right)-\cdots, \quad (4.28)$$

and

$$\omega(\xi,\,\eta)=-3c\sqrt{2}\,tanh^2\left(\frac{\xi}{2}\right)+3\,\eta c\sqrt{2}\,sech^2\left(\frac{\xi}{2}\right)tanh\left(\frac{\xi}{2}\right)$$

$$+\frac{3c\sqrt{2}}{4}\eta^2\left[2+21\,sech^2\left(\frac{\xi}{2}\right)-24\,sech^4\left(\frac{\xi}{2}\right)\right]sech^2\left(\frac{\xi}{2}\right)+\cdots. \quad (4.29)$$

The solution given by Eq. (4.29) is similar to closed form solution as:

$$\rho(\xi,\eta)=3-6\,tanh^2\left(\frac{\xi+\eta}{2}\right),$$

$$\omega(\xi,\eta)=-3c\sqrt{2}\,tanh^2\left(\frac{\xi+\eta}{2}\right). \quad (4.30)$$

5. Results and discussions

Figure 1 (a)–(b) shows profile of the closed form solution and HPSTM solution at $\alpha = 1$. The profile of error between both results $|\rho_{Exact} - \rho_{HPSTM}|$ for nonlinear time fractional Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system when $c = 1$ and $\alpha = 1$ for $0 \le \eta \le 0.01$ and $-3 \le \xi \le 3$ for Eq. (4.1) by the application of initial condition represented by the Eq. (4.2) of $\rho(\xi,\eta)$ is shown by Figure 1(c). Figure 1 (a)–(c) shows that profile of the HPSTM solution is analogous to exact solution of $\rho(\xi,\eta)$. The analytical solutions gained by HPSTM have high precisions at sixth term approximation. The numerical solutions represent high level of precisions between exact solution and HPSTM solution for $\rho(\xi,\eta)$. It can be analyzed by the Figure 2 that as spatial coordinate vary with time it gives to first decrease and then increase in the $\rho(\xi,\eta)$ for wave propagation in nonlinear plasma medium at $\eta = 0.01$. Figure 3 shows the comparison of profile of $\rho(\xi,\eta)$ at different values of α for $-3 \le \xi \le 3$ and $\eta \le 0.01$.

Table 1 represents that exact solution is analogous to HPSTM solution and have large level accuracy between the numerical results. Table 1 indicates that absolute error is 10^{-9}order between numerical results for $\rho(\xi,\eta)$.

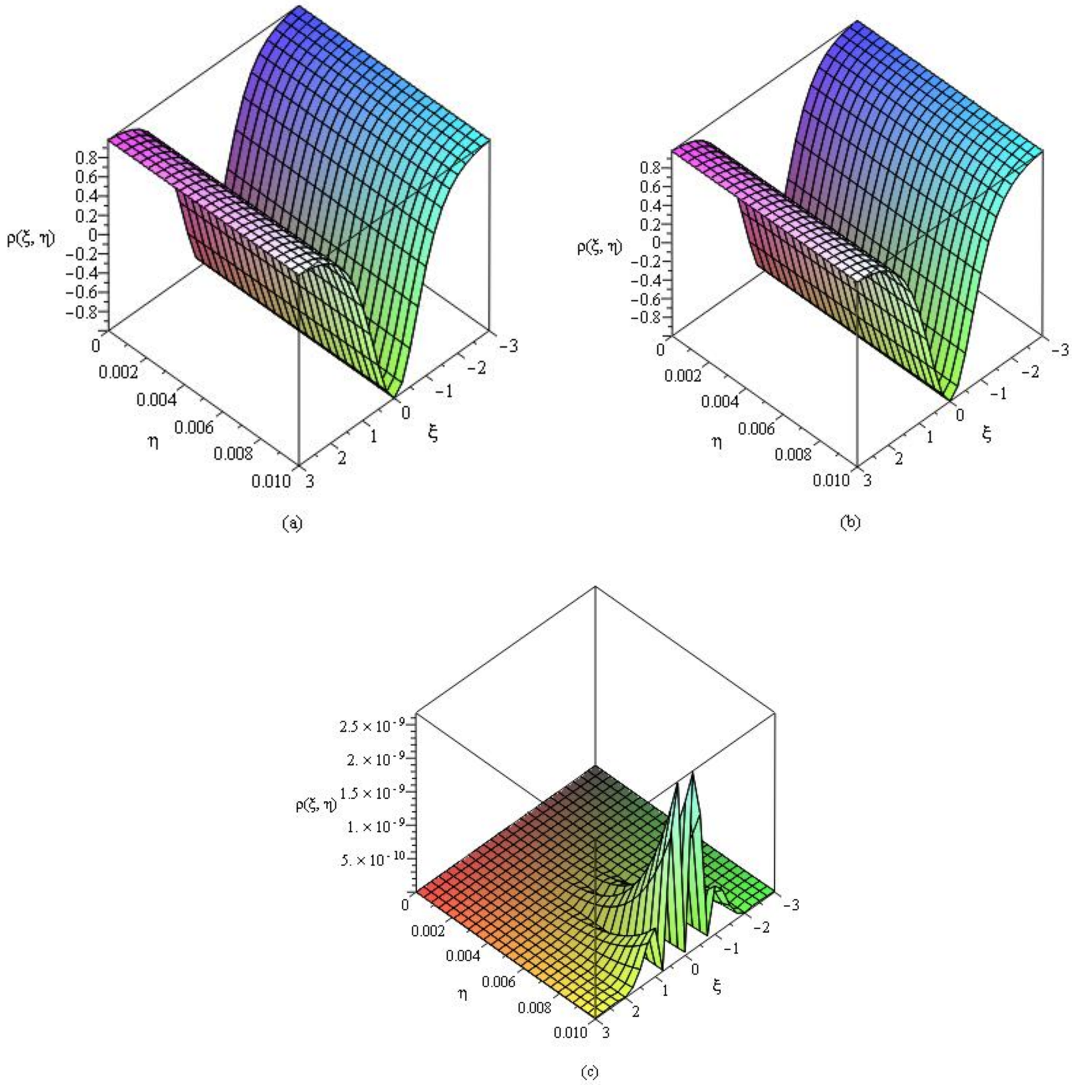

Figure 1. Profile of solution of $\rho(\xi, \eta)$ for Eq. (4.1) when $c = 1$: (a) Exact solution, (b) HPSTM solution at $\alpha = 1$, (c) Absolute Error $|\rho_{Exact} - \rho_{HPSTM}|$ for $0 \leq \eta \leq 0.01$, $-3 \leq \xi \leq 3$ and $\alpha = 1$.

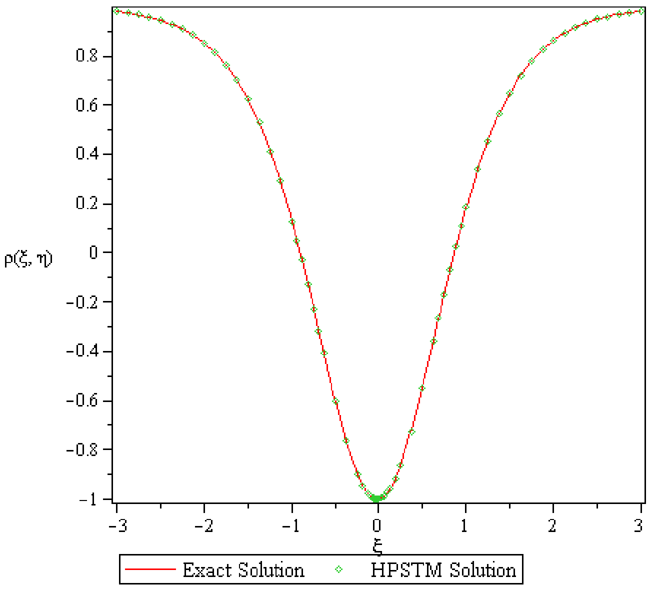

Figure 2. Profile of exact solution and HPSTM solution of $\rho(\xi,\eta)$ for $\eta = 0.01$ and $-3 \leq \xi \leq 3$.

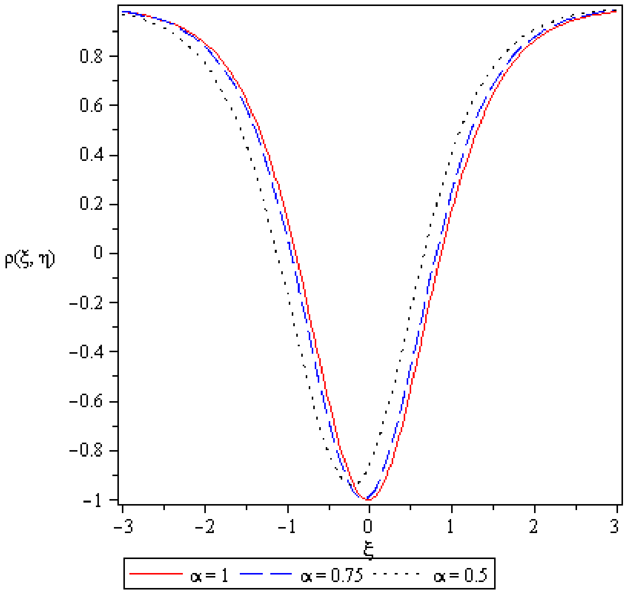

Figure 3. Comparison of profile of $\rho(\xi,\eta)$ at different values of α for$-3 \leq \xi \leq 3$ and $\eta = 0.01$.

Table 1. Variation of exact solution with HPSTM solution of $\rho(\xi,\eta)$ at $\eta = 0.01$.

ξ	Exact Solution	HPSTM Solution	Absolute Error
3.0	0.9810379547	0.9810379579	1.957810944E−09
2.5	0.9488741084	0.9488741131	4.588520200E−09
2.0	0.8640471736	0.8640471812	7.500972059E−09
1.5	0.6514628688	0.6514628649	3.864259000E−09
1.0	0.1853889664	0.1853888931	7.330837940E−08
0.5	−0.5436053970	−0.5436054423	4.527637000E−08
0.0	−0.9992002130	−0.9992000000	2.130000000E−07
−0.5	−0.6017332810	−0.6017333223	4.129203000E−08
−1.0	0.1342165142	0.1342164396	7.470826874E−08
−1.5	0.6252890662	0.6252890617	4.510941000E−09
−2.0	0.8531473885	0.8531473961	7.543603848E−09
−2.5	0.9446752749	0.9446752795	4.629439800E−09
−3.0	0.9794667886	0.9794667902	1.994434944E−09

Figure 4 (a)–(b) shows profile of the closed form solution and HPSTM solution at $\alpha = 1$.The profile of error between both solutions $|\omega_{Exact} - \omega_{HPSTM}|$ for nonlinear time fractional Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system when $c = 1$ and $\alpha = 1$ for $0 \le \eta \le 0.01$ and $-3 \le \xi \le 3$ for Eq. (4.1) by the application of initial condition represented by the Eq. (4.2) of $\omega(\xi,\eta)$ is given by Figure 4(c). Figure 4 (a)–(c) shows that profile of the HPSTM solution is analogous to exact solution of $\omega(\xi,\eta)$. The analytical solutions gained by HPSTM have high precisions at sixth term approximation. The numerical solutions represent large level of precisions between exact solution and HPSTM solution for $\omega(\xi,\eta)$. It can be analyzed by the Figure 5 that as spatial coordinate vary with time it gives to first increase and then decrease in the $\omega(\xi,\eta)$ for wave propagation in nonlinear plasma medium at $\eta = 0.01$. Figure 6 shows the comparison of profile of $\omega(\xi,\eta)$ at different values of α for $-3 \le \xi \le 3$ and $\eta \le 0.01$.

Table 2 represents that exact solution is analogous to HPSTM solution and have large level accuracy between the numerical results. Table 2 indicates that absolute error is 10^{-12}order between numerical results for $\omega(\xi,\eta)$.

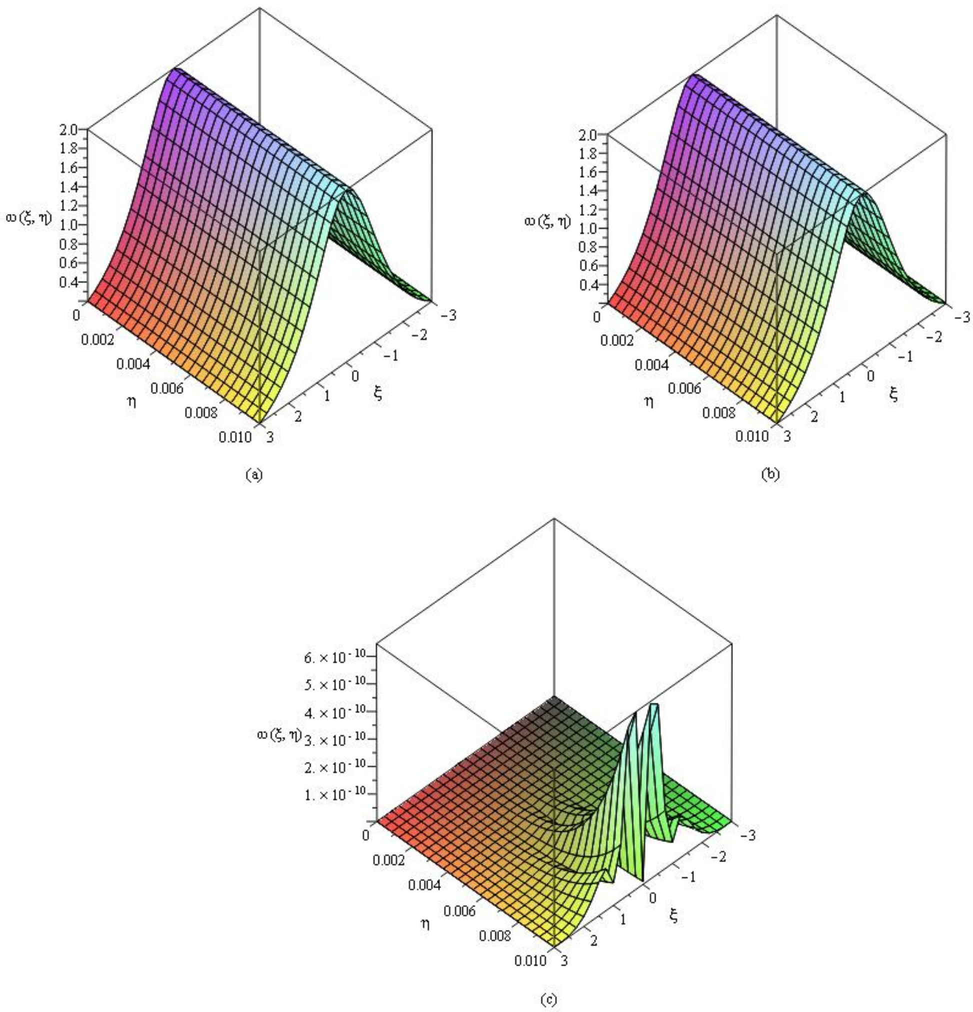

Figure 4. Profile of $\omega(\xi,\eta)$ for Eq. (4.1) when $c = 1$: (a) Exact solution, (b) HPSTM solution at $\alpha = 1$, (c) Absolute Error $|\omega_{Exact} - \omega_{HPSTM}|$ for $0 \le \eta \le 0.01$, $-3 \le \xi \le 3$ and $\alpha = 1$.

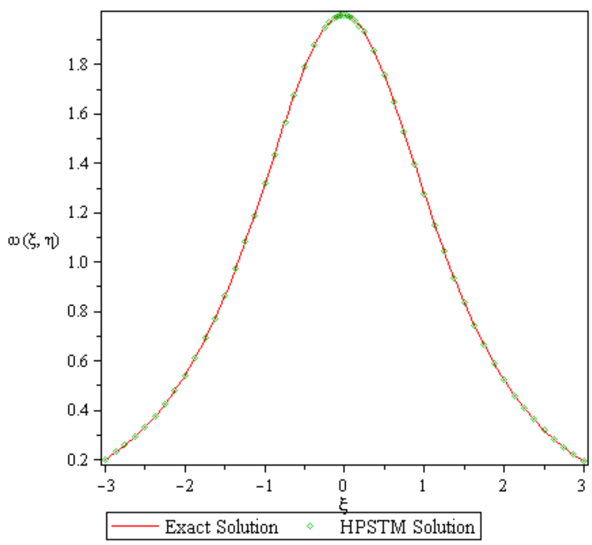

Figure 5. Profile of exact solution and HPSTM solution of $\omega(\xi,\eta)$ for $\eta = 0.01$ and $-3 \le \xi \le 3$.

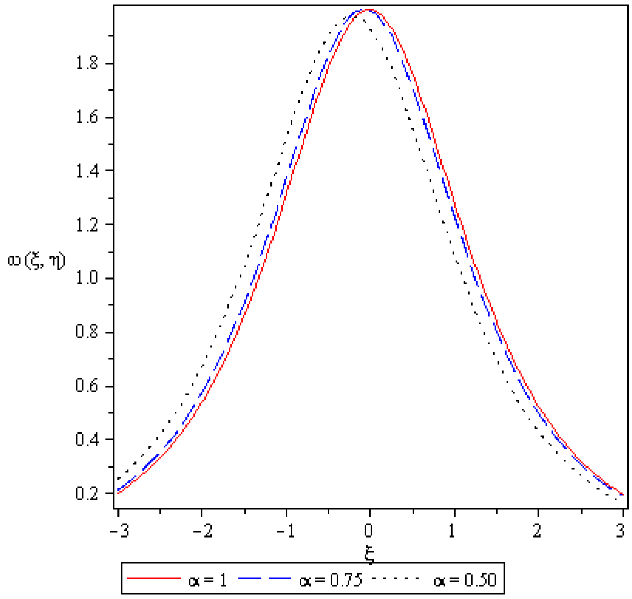

Figure 6. Comparison of profile of $\omega(\xi,\eta)$ at different values of α for $-3 \le \xi \le 3$ and $\eta = 0.01$.

Table 2. Variation of exact solution with HPSTM solution of $\omega(\xi,\eta)$ at $\eta = 0.01$.

ξ	Exact Solution	HPSTM Solution	Absolute Error
3.0	0.1947410861	0.1947410860	1.001455080E−10
2.5	0.3197683274	0.3197683273	1.358172040E−10
2.0	0.5214457334	0.5214457333	6.868329502E−11
1.5	0.8349097330	0.8349097328	2.056274600E−10
1.0	1.2764098350	1.2764098350	3.489344755E−10
0.5	1.7570460420	1.7570460420	6.046615540E−10
0.0	1.9996000670	1.9996000670	3.333333500E−10
−0.5	1.7898230530	1.7898230520	4.983904460E−10
−1.0	1.3158901820	1.3158901820	3.908333123E−10
−1.5	0.8656915546	0.8656915546	3.031660000E−12
−2.0	0.5419457750	0.5419457750	7.044098959E−11
−2.5	0.3326401212	0.3326401213	5.249600400E−11
−3.0	0.2026485202	0.2026485201	2.472076832E−10

Figure 7 (a)–(b) shows profile of the closed form solution and HPSTM solution at $\alpha = 1$. The error between both solutions $|\rho_{Exact} - \rho_{HPSTM}|$ for nonlinear two component homogeneous time fractional coupled third order KdV system when $\alpha = 1$ for $0 \le \eta \le 0.0001$ and $-5 \le \xi \le 5$ for Eq. (4.16) by the application of initial condition represented by the Eq. (4.17) of $\rho(\xi,\eta)$ is given by

Figure 7(c). Figure 7 (a)–(c) shows that profile of the HPSTM solution is analogous to exact solution of $\rho(\xi,\eta)$. The analytical solutions gained by HPSTM have high accuracy at 4th term approximation. The numerical solutions represent very high level of agreement between exact solution and HPSTM solution for $\rho(\xi,\eta)$. It can be analyzed by the Figure 8 that as spatial coordinate vary with time it gives to first increase and then decrease in the $\rho(\xi,\eta)$ for wave propagation in nonlinear plasma medium at $\eta = 0.01$. Figure 9 shows the comparison of profile of $\rho(\xi,\eta)$ at different values of α for $-5 \le \xi \le 5$ and $\eta \le 0.1$.

Table 3 represents that exact solution is analogous to HPSTM solution and have large level accuracy between the numerical results. Table 3 indicates that absolute error is 10^{-10}order between numerical results for $\rho(\xi,\eta)$.

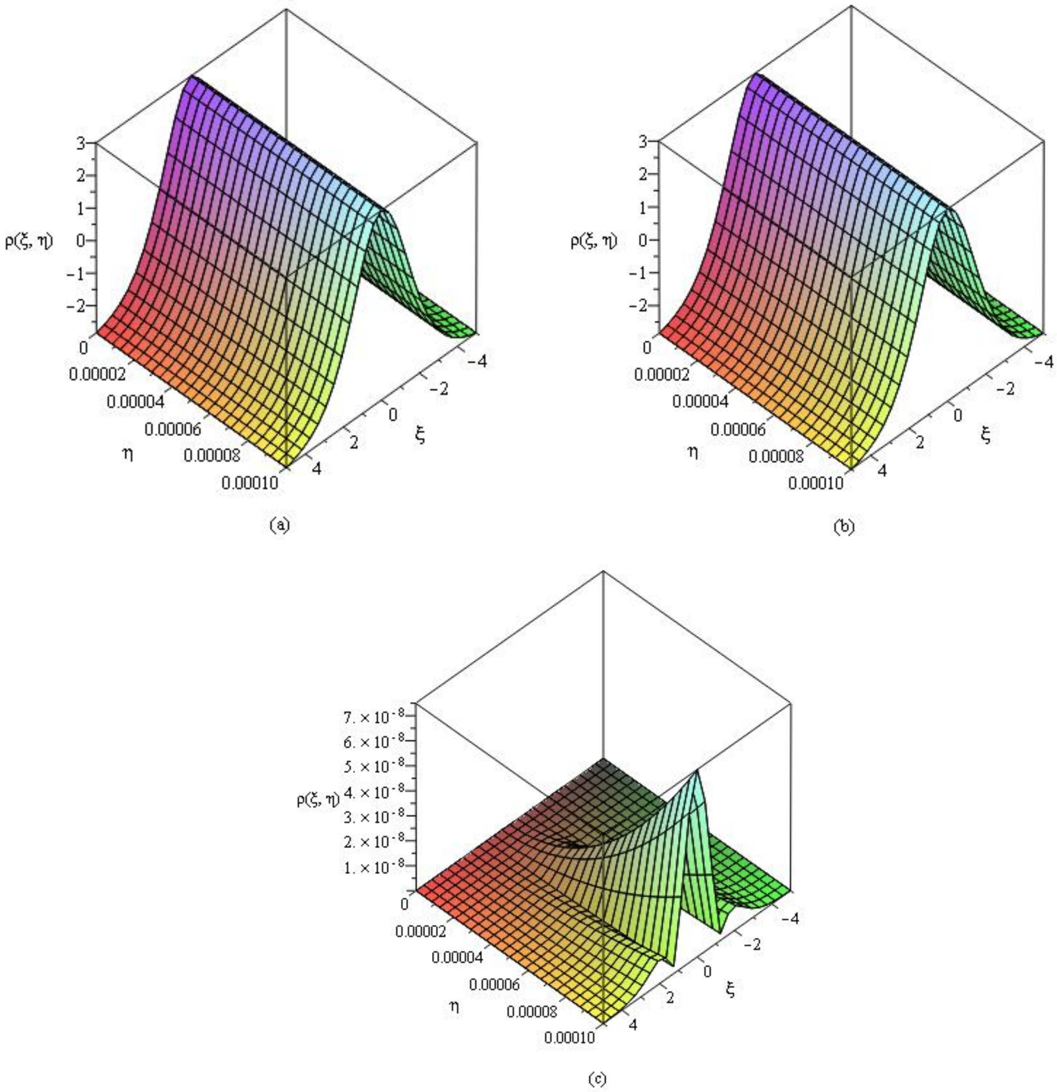

Figure 7. Profile of $\rho(\xi,\eta)$ for Eq. (4.16): (a) Exact solution, (b) HPSTM solution at $\alpha = 1$, (c) Absolute Error $|\rho_{Exact} - \rho_{HPSTM}|$ for $0 \le \eta \le 0.0001, -5 \le \xi \le 5$ and $\alpha = 1$.

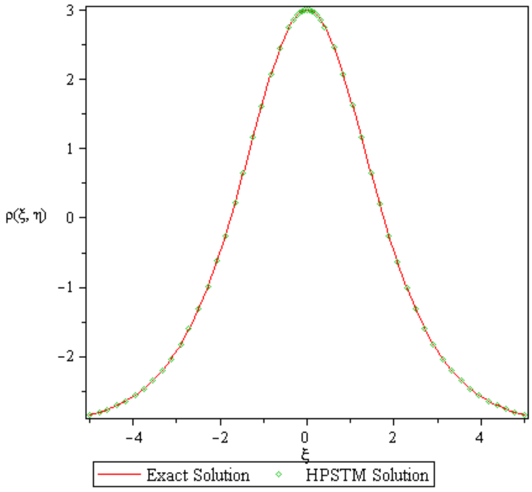

Figure 8. Profile of exact solution and HPSTM solution of $\rho(\xi, \eta)$ for $\eta = 0.01$ and $-5 \leq \xi \leq 5$.

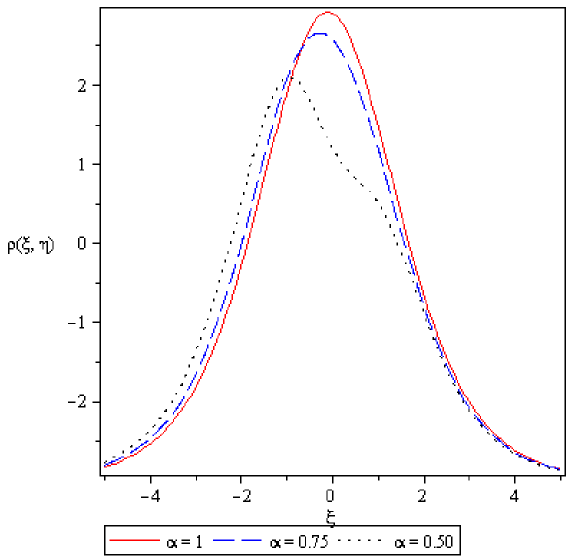

Figure 9. Comparison of profile of $\rho(\xi, \eta)$ at different values of α for $-5 \leq \xi \leq 5$ and $\eta = 0.1$.

Table 3. Variation of exact solution with HPSTM solution at $\eta = 0.0001$.

ξ	Exact Solution	HPSTM Solution	Absolute Error
5	−2.840462381	−2.840462380	1.444186835E−09
4	−2.576135914	−2.576135916	7.505047070E−10
3	−1.915858303	−1.915858299	4.760423501E−09
2	−0.480345856	−0.480345845	1.127798505E−08
1	1.718468335	1.718468318	1.736182461E−08
0	2.999999985	2.999999910	7.500000002E−08
−1	1.718904452	1.718904435	1.715811279E−08
−2	−0.479962035	−0.479962025	1.021909119E−08
−3	−1.915662023	−1.915662020	4.383116417E−09
−4	−2.576054184	−2.576054186	1.160759310E−10
−5	−2.840430898	−2.840430897	5.811609752E−10

Figure 10 (a)–(b) shows profile of the closed form solution and HPSTM solution at $\alpha = 1$. The error between both solutions $|\omega_{Exact} - \omega_{HPSTM}|$ for nonlinear two component homogeneous time fractional coupled third order KdV System when $c = 0.00001$ and $\alpha = 1$ for $0 \leq \eta \leq 0.001$ and $-5 \leq \xi \leq 5$ for Eq. (4.16) by the application of initial condition represented by the Eq. (4.17) of $\omega(\xi, \eta)$ is given by Figure 10(c). Figure 10 (a)–(c) shows that profile of the HPSTM solution is analogous to exact solution of $\omega(\xi, \eta)$. The analytical solutions gained by HPSTM have high accuracy at 4^{th} term approximation. The numerical solutions represent high level of agreement between exact solution and HPSTM solution for $\omega(\xi, \eta)$. It can be analyzed by the Figure 11 that as spatial coordinate vary with time it gives to first increase and then decrease in the $\omega(\xi, \eta)$ for wave propagation in nonlinear plasma medium at $\eta = 0.001$. Figure 12 shows the comparison of profile of $\omega(\xi, \eta)$ at different values of α for $-5 \leq \xi \leq 5$ and $\eta \leq 0.1$.

Table 4 represents that exact solution is analogous to HPSTM solution and have large level accuracy between the numerical results. Table 4 indicates that absolute error is 10^{-9} order between numerical results for $\omega(\xi, \eta)$.

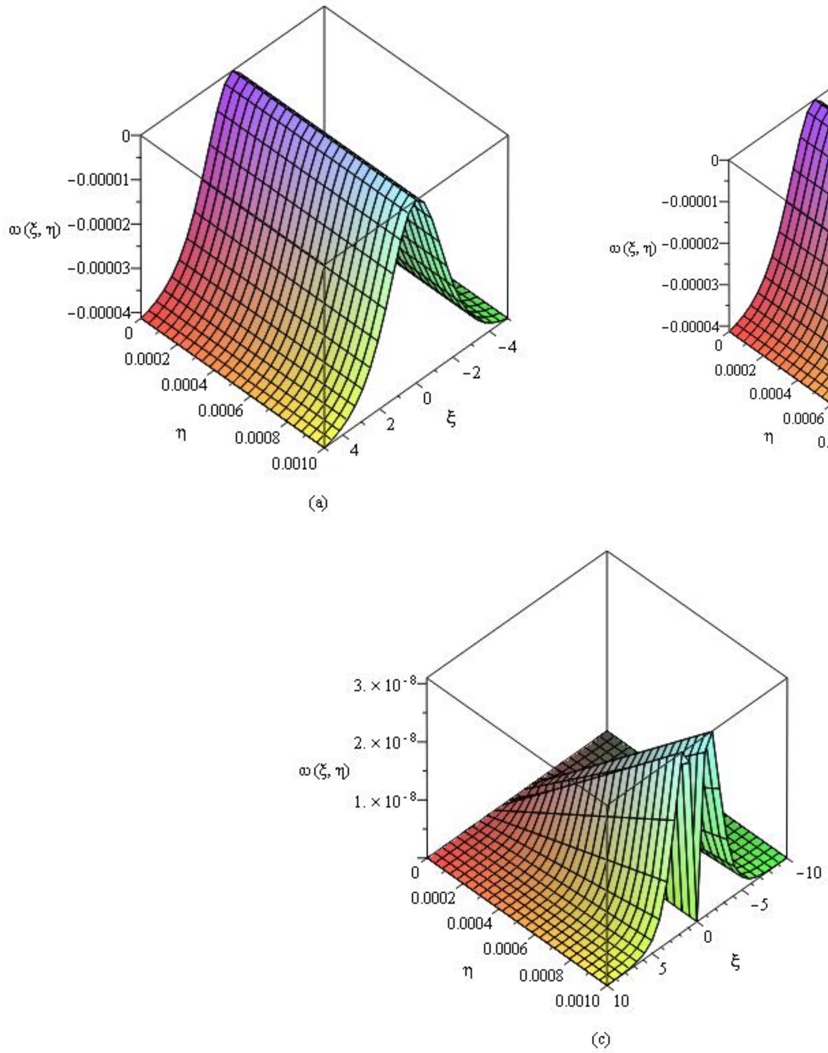

Figure 10. Profile of $\omega(\xi, \eta)$ for Eq. (4.16) when $c = 0.00001$ (a) Exact solution, (b) HPSTM solution at $\alpha = 1$, (c) Absolute Error $|\omega_{Exact} - \omega_{HPSTM}|$ for $0 \leq \eta \leq 0.001$, $-5 \leq \xi \leq 5$ and $\alpha = 1$.

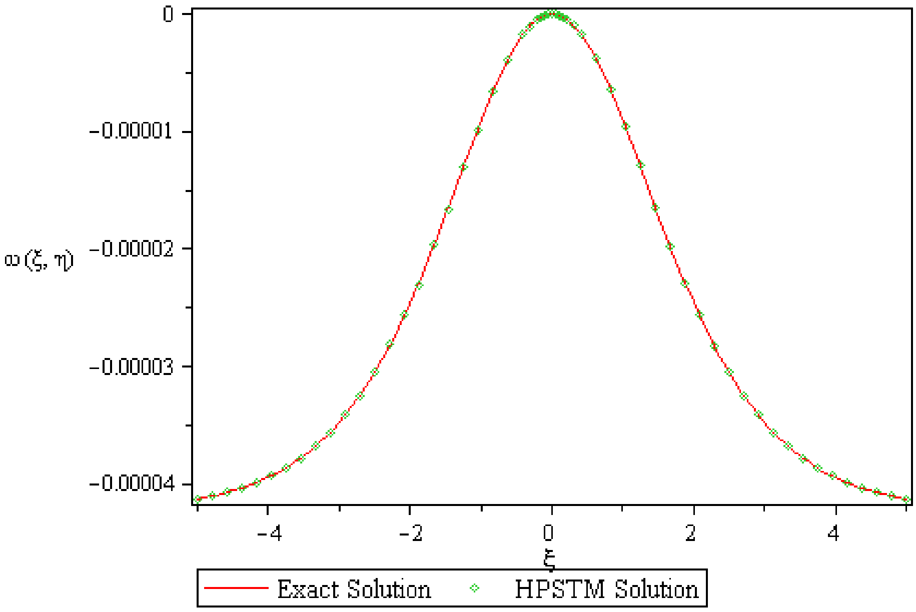

Figure 11. Profile of exact solution and HPSTM solution of $\omega(\xi,\eta)$ for $\eta = 0.001$ and $-5 \le \xi \le 5$.

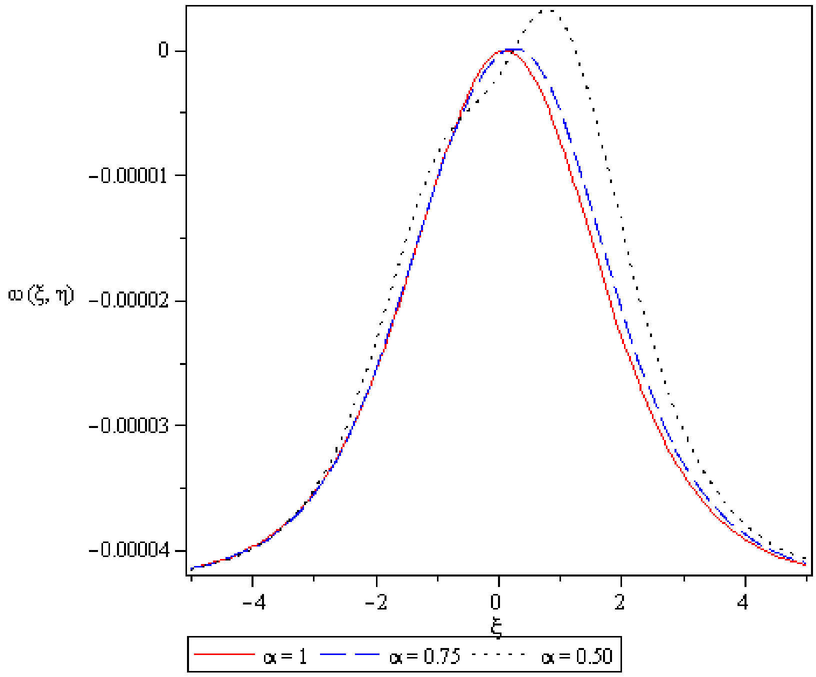

Figure 12. Comparison of profile of $\omega(\xi,\eta)$ at different values of α for $-5 \le \xi \le 5$ and $\eta = 0.1$.

Table 4. Variation of exact solution with HPSTM solution of $\omega(\xi,\eta)$ at $\eta = 0.001$.

ξ	Exact Solution	HPSTM Solution	Absolute Error
5	−0.00004129930679	−0.00004129708040	2.226375861E−09
4	−0.00003943183452	−0.00003942605404	5.780438670E−09
3	−0.00003476661022	−0.00003475272434	1.388587893E−08
2	−0.00002462197136	−0.00002459480514	2.716621800E−08
1	−0.000009075677432	−0.000009044805675	3.087175865E−08
0	−1.060659995000000	−1.060660172000000	1.769181166E−07
−1	−0.000009044839294	−0.000009075643815	3.080452365E−08
−2	−0.00002459483119	−0.00002462194532	2.711411868E−08
−3	−0.00003475273115	−0.00003476660340	1.387225658E−08
−4	−0.00003942605525	−0.00003943183334	5.778090290E−09
−5	−0.00004129708058	−0.00004129930661	2.226059143E−09

6. Conclusions

The HPSTM is applied precisely and accurately for study time fractional Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system and time fractional two component homogeneous time fractional coupled third order KdV System. The obtained results compared with the exact solution and absolute errors between the HPSTM solution and exact solution find graphically as well as numerically. We find classical solution of these systems by putting $\alpha = 1$. The results reveals that the suggested algorithm is very efficient and powerful method for solving various types nonlinear fractional coupled systems occurring in various areas of physics and technology. At the end, the proposed technique may be taken as a new tool for over other available analytical methods and used to study many fractional nonlinear coupled systems occurring in plasma.

Conflict of interest

The authors declare no conflict of interest.

References

1. A. R. Seadawy, *Stability analysis for Zakharov-Kuznetsov equation of weakly nonlinear ionacoustic waves in a plasma,* Computers and Mathematics with Applications, **67** (2014), 172–180.
2. R. Zhang, L. Yang, Q. Liu, et al. *Dynamics of nonlinear Rossby waves in zonally varying flow with spatial-temporal varying topography,* Appl. Math.Comput., **346** (2019), 666–679.
3. A. R. Seadawy, *Stability analysis for two-dimensional ion-acoustic waves in quantum plasmas,* Physics of Plasmas, **21** (2014), 052107.

4. A. Goswami, J. Singh, D. Kumar, *A reliable algorithm for KdV equations arising in warm plasma*, Nonlinear Engineering, **5** (2016), 7–16.
5. A. R. Seadawy, *Nonlinear wave solutions of the three-dimensional Zakharov–Kuznetsov–Burgers equation in dusty plasma*, Physica A, **439** (2015), 124–131.
6. R. Zhang, Q. Liu, L. Yang, et al. *Nonlinear planetary-synoptic wave interaction under generalized beta effect and its solutions*, Chaos, Solitons and Fractals, **122** (2019), 270–280.
7. A. R. Seadawy, *Three-dimensional nonlinear modified Zakharov–Kuznetsov equation of ion-acoustic waves in a magnetized plasma*, Comput. Math. Appl., **71** (2016), 201–212.
8. T. Kakutani, H. Ono, *Weak non-linear hydromagnetic waves in a cold collision free plasma*, J. Phys. Soc. JPN, **26** (1969), 1305–1318.
9. A. R. Seadawy, *Stability analysis solutions for nonlinear three-dimensional modified Korteweg–de Vries–Zakharov–Kuznetsov equation in a magnetized electron–positron plasma*, Physica A, **455** (2016), 44–51.
10. A. R. Seadawy, *Ion acoustic solitary wave solutions of two-dimensional nonlinear Kadomtsev–Petviashvili–Burgers equation in quantum plasma*, Mathematical methods and applied Sciences, **40** (2017), 1598–1607.
11. A. R. Seadawy, *Solitary wave solutions of two-dimensional nonlinear Kadomtsev–Petviashvili dynamic equation in dust-acoustic plasmas*, Pramana, **89** (2017), 49.
12. R. Zhang, L. Yang, *Nonlinear Rossby waves in zonally varying flow under generalized beta approximation*, Dynam. Atmos. Oceans, **85** (2019), 16–27.
13. R. Zhang, L. Yang, J. Song, et al. *(2 + 1)-Dimensional nonlinear Rossby solitary waves under the effects of generalized beta and slowly varying topography*, Nonlinear Dynam., **90** (2017), 815–822.
14. Q. Liu, R. Zhang, L. Yang, et al. *A new model equation for nonlinear Ross by waves and some of its solutions*, Phys. Lett. A, **383** (2019), 514–525.
15. J. Singh, D. Kumar, S. Kumar, *A new fractional model of nonlinear shock wave equation arising in flow of gases*, Nonlinear Engineering, **3** (2014), 43–50.
16. J. Singh, D. Kumar, S. Kumar, *A reliable algorithm for solving discontinued problems arising in nanotechnology*, Scientia Iranica, **20** (2013), 1059–1062.
17. A. Goswami, J. Singh, D. Kumar, et al. *An efficient analytical technique for fractional partial differential equations occurring in ion acoustic waves in plasma*, Journal of Ocean Engineering and Science, **4** (2019), 85–99.
18. J. H. He, *Homotopy perturbation method: a new nonlinear analytical technique*, Appl. Math. Comput., **135** (2003), 73–79.
19. A. Goswami, J. Singh, D. Kumar, et al. *An analytical approach to the fractional Equal Width equations describing hydro-magnetic waves in cold plasma*, Physica A, **524** (2019), 563–575.
20. A. Goswami, J. Singh, D. Kumar, *Numerical simulation of fifth order KdV equations occurring in magneto-acoustic waves*, Ain Shams Eng. J., **9** (2018), 2265–2273.
21. J. Singh, D. Kumar, D. Sushila, *Homotopy perturbation Sumudu transform method for nonlinear equations*, Adv. Theor. Appl. Math. Mech., **4** (2011), 165–175.
22. D. Kumar, J. Singh, D. Baleanu, *A new analysis for fractional model of regularized long wave equation arising in ion acoustic plasma waves*, Math. Method. Appl. Sci., **40** (2017), 5642–5653.

23. A. Ghorbani, J. Saberi-Nadjafi, *He's homotopy perturbation method for calculating Adomian polynomials*, Int. J. Nonlin. Sci. Num., **8** (2007), 229–232.
24. A. Ghorbani, *Beyond Adomian polynomials: He polynomials*, Chaos, Solitons and Fractals, **39** (2009), 1486–1492.
25. G. K. Watugala, *Sumudu transform- a new integral transform to solve differential equations and control engineering problems*, Integrated Education, **24** (1993), 35–43.
26. F. B. M. Belgacem, A. A. Karaballi, S. L. Kalla, *Analytical investigations of the Sumudu transform and applications to integral production equations*, Math. Probl. Eng., **3** (2003), 103–118.
27. Y. Qin, Y. T. Gao, X. Yu, et al. *Bell polynomial approach and N-soliton solutions for a coupled KdV-mKdV system*, Commun. Theor. Phys., **58** (2012), 73–78.
28. W. Rui, X. Qi, *Bilinear approach to quasi-periodic wave solutions of the Kersten-Krasil'shchik coupled KdV-mKdV system*, Bound. Value Probl., **2016** (2016), 130.
29. P. Kersten, J. Krasil'shchik, *Complete integrability of the coupled KdV-mKdV system*, Adv. Stud. Pure Math., **89** (2000), 151–171.
30. Y. Keskin, G. Oturanc, *Reduced differential transform method for partial differential equations*, Int. J. Nonlinear Sci. Numer. Simul., **10** (2009), 741–749.
31. Y. Keskin, G. Oturanc, *Reduced differential transform method for generalized KdV equations*, Math. Comput. Appl., **15** (2010), 382–393.
32. Y. C. Hon, E. G. Fan, *Solitary wave and doubly periodic wave solutions for the Kersten-Krasil'shchik coupled KdV-mKdV system*, Chaos, Solitons and Fractals, **19** (2004), 1141–1146.
33. A. K. Kalkanli, S. Y. Sakovich, I. Yurdusen, *Integrability of Kersten-Krasil'shchik coupled KdV-mKdV equations: singularity analysis and Lax pair*, J. Math. Phys., **44** (2003), 1703–1708.
34. A. F. Qasim, M. O. Al-Amr, *Approximate Solution of the Kersten-Krasil'shchik Coupled KdV-mKdV System via Reduced Differential Transform Method*, Eurasian J. Sci. Eng., **4** (2018), 1–9.