Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 5(3): 2143–2152.
DOI:10.3934/math.2020142
Received: 10 December 2019
Accepted: 21 February 2020
Published: 27 February 2020

Research article

On deferred statistical convergence of sequences of sets

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Abstract: The main purpose of this paper is to introduce the concepts of Wijsman deferred statistical convergence and Wijsman strong deferred Cesàro summability for sequences of sets.

Keywords: deferred density; deferred Cesàro mean; statistical convergence; Wijsman convergence
Mathematics Subject Classification: Primary: 40A05, 40C05; Secondary: 46A45

1. Introduction

The idea of statistical convergence was introduced by Fast [1] and Steinhaus [2] independently in the same year 1951 and since then several generalizations and applications of this concept have been investigated by various authors namely Bhardwaj and Dhawan [3, 4], Cakalli [5], Cinar et al. [6], Caserta et al. [7], Colak [8], Connor [9], Et et al. [10–12], Esi et al. [13], Fridy [14], Hazarika et al. [15], Isik et al. [16, 17], Mursaleen [18], Nuray and Rhoades [19], Salat [20], Savas and Et [21], Srivastava and Et [22], Sengul et al. [23, 24], Yilmazturk and Kucukaslan [25] and many others.

The idea of statistical convergence depends upon the density of subsets of the set $\mathbb{N}$ of natural numbers. The density of a subset $\mathbb{E}$ of $\mathbb{N}$ is defined by

$$\delta(\mathbb{E}) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_{\mathbb{E}}(k),$$

provided that the limit exists, where $\chi_{\mathbb{E}}$ is the characteristic function of the set $\mathbb{E}$. It is clear that any finite subset of $\mathbb{N}$ has zero natural density and that

$$\delta\left(\mathbb{E}^c\right) = 1 - \delta\left(\mathbb{E}\right).$$

A sequence $x = (x_k)_{k\in\mathbb{N}}$ is said to be statistically convergent to L if, for every $\varepsilon > 0$, we have

$$\delta\left(\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}\right) = 0.$$

In this case, we write

$$x_k \xrightarrow{\text{stat}} L \quad \text{as} \quad k \to \infty \qquad \text{or} \qquad \text{S-lim}_{k\to\infty} x_k = L.$$

Agnew [26] introduced the concept of deferred Cesàro mean of real (or complex) valued sequences $x = (x_k)$ defined by

$$\left(D_a^b(x)\right)_n = \frac{1}{b_n - a_n} \sum_{k=a_n+1}^{b_n} x_k,\ n = 1, 2, 3, \ldots, \tag{1}$$

where $a = (a_n)$ and $b = (b_n)$ are two sequences of non-negative integers satisfying

$$a_n < b_n \text{ and } \lim_{n\to\infty} b_n = \infty. \tag{2}$$

Deferred density of $\mathbb{K} \subset \mathbb{N}$ defined by

$$\delta_a^b(\mathbb{K}) = \lim_{n\to\infty} \frac{|\{k : a_n < k \le b_n, k \in \mathbb{K}\}|}{b_n - a_n}.$$

A sequence $x = (x_k)$ is said to be deferred statistically convergent to L provided that

$$\lim_{n\to\infty} \frac{|\{a_n < k \le b_n : |x_k - L| \ge \varepsilon\}|}{b_n - a_n} = 0$$

for each $\varepsilon > 0$ and it is written by $S_a^b - \lim x_k = L$ [27].

Let (X, ρ) be a metric space. The distance $d(x, A)$ from a point x to a non-empty subset A of (X, ρ) is defined to be

$$d(x, A) = \inf_{y\in A} \rho(x, y).$$

If $\sup_k d(x, A_k) < \infty$ (for each $x \in X$), then we say that the sequence $\{A_k\}$ is bounded.

2. Main results

In the present section we shall give the definitions of Wijsman deferred statistical convergence and Wijsman strong deferred Cesàro summability and examine some inclusion properties regarding these concepts.

Definition 1. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2), A and A_k be non-empty closed subsets of X for each k. A sequence $\{A_k\}$ is said to be Wijsman deferred statistically convergent to A (or WS_d−convergent) provided that

$$\lim_{n\to\infty} \frac{1}{b_n - a_n} |\{a_n < k \le b_n : |d(x, A_k) - d(x, A)| \ge \varepsilon\}| = 0$$

for each $\varepsilon > 0$ and for each $x \in X$, and it is written by $A_k \longrightarrow A\,(WS_d)$ or $WS_d - \lim A_k = A$. The set of all WS_d−convergent sequences will be denoted by WS_d. If $b_n = n$, $a_n = 0$, then we write WS instead of WS_d.

If we take $b_n = k_n$, $a_n = k_{n-1}$, where (k_n) is a lacunary sequence, then WS_d−convergence is the same as Wijsman lacunary statistical convergence given by Bhardwaj and Dhawan [4].

As an example, consider the following sequence:

Let $b_n = k_n$, $a_n = k_{n-1}$, where (k_n) is a lacunary sequence and consider a sequence of sets defined by

$$A_k = \begin{cases} \{3x\}, & k_{n-1} < k < k_{n-1} + \sqrt{h_n} \\ \{0\}, & otherwise \end{cases}$$

For $X = \mathbb{R}$, $\rho(x, y) = |x - y|$, $A = \{1\}$ and $x > 1$, we have

$$\frac{1}{h_n} |\{k \in I_n : |d(x, A_k) - d(x, \{1\})| \geq \varepsilon\}| \to 0,$$

so $WS_d - \lim A_k = \{1\}$.

Definition 2. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2), A and A_k non-empty closed subsets of X for each k. We say that the sequence $\{A_k\}$ is Wijsman strong deferred Cesàro convergent to A (or WN_d−convergent) if for each $x \in X$,

$$\lim_{n\to\infty} \frac{1}{b_n - a_n} \sum_{a_n+1}^{b_n} |d(x, A_k) - d(x, A)| = 0,$$

and we write $A_k \longrightarrow A(WN_d)$ or $WN_d - \lim A_k = A$. The set of all WN_d−convergent sequences is denoted by WN_d. If $b_n = n$, $a_n = 0$, then we write WN instead of WN_d.

If we take $b_n = k_n$, $a_n = k_{n-1}$, where (k_n) is a lacunary sequence, then WN_d−convergence coincides with Wijsman lacunary strong Cesàro convergence given by Bhardwaj and Dhawan [4].

As an example, consider the following sequence:

Let $b_n = k_n$, $a_n = k_{n-1}$, where (k_n) is a lacunary sequence and consider a sequence defined by

$$A_k = \begin{cases} \left\{\frac{xk}{2}\right\}, & k_{n-1} < k < k_{n-1} + \sqrt{h_n} \\ \{0\}, & otherwise \end{cases}$$

Let $X = \mathbb{R}$, $x, y \in X$, $\rho(x, y) = |x - y|$, $A = \{1\}$ and $x > 1$. Since

$$\frac{1}{h_n} \sum_{k\in I_n} |d(x, A_k) - d(x, \{1\})| \to 0$$

the sequence $\{A_k\}$ is WN_d−convergent to $\{1\}$.

Here and in what follows we suppose that the sets A and A_k (for each $k \in \mathbb{N}$) are non-empty closed subsets of X for each k.

Theorem 1. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2) and $A, A_k \subset X$, then every Wijsman strong deferred Cesàro convergent sequence is Wijsman deferred statistically convergent, but the converse is not true.

Proof. First part of the proof is easy. For the converse, take $b_n = k_n$, $a_n = k_{n-1}$, where (k_n) is a lacunary sequence, $\rho(x, y) = |x - y|$, $X = \mathbb{R}$ and define $\{A_k\}$ by

$$A_k = \begin{cases} \{h_n\} & \text{if } k \in I_n \text{ such that } k = k_{n-1} + 1 \\ \{0\}, & \text{otherwise} \end{cases}.$$

Note that $\{A_k\}$ is not a bounded sequence and for each $x \in X$, we have

$$\begin{aligned}
\frac{1}{h_n}\sum_{k\in I_n}|d(x,A_k)-d(x,0)| &= \frac{1}{h_n}|d(x,A_{k_{n-1}+1})-d(x,0)| \\
&= \frac{1}{h_n}|d(x,h_n)-d(x,0)| \\
&\leq \frac{1}{h_n}|(x-h_n)-(x-0)| \\
&= \frac{1}{h_n}h_n \to 1\,(n\to\infty)
\end{aligned}$$

so $\{A_k\} \notin WN_d$, but

$$\frac{1}{h_n}|\{k\in I_n : |d(x,A_k)-d(x,\{0\})|>\varepsilon\}| = \frac{1}{h_n} \to 0\,(n\to\infty)$$

and so $\{A_k\} \in WS_d$.

From Theorem 1 we have the following:

Corollary 1. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2) and $A, A_k \subset X$, then every Wijsman strong Cesàro convergent sequence is Wijsman statistically convergent, but the converse is not true.

Theorem 2. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2) and $A, A_k \subset X$. If the sets A and A_k (for each $k \in \mathbb{N}$) are bounded, then every Wijsman deferred statistically convergent sequence is Wijsman strong deferred Cesàro convergent.

Proof. Let $\{A_k\}$ be Wijsman deferred statistically convergent to A and $\varepsilon > 0$ be given. Then there exists $x \in X$ such that

$$\lim_{n\to\infty}\frac{1}{b_n-a_n}|\{a_n<k\leq b_n : |d(x,A_k)-d(x,A)|\geq\varepsilon\}| = 0.$$

Since A, A_k (for each $k \in \mathbb{N}$) are bounded, we can write (for each $x \in X$ and $\varepsilon > 0$)

$$\begin{aligned}
\frac{1}{b_n-a_n}\sum_{a_n+1}^{b_n}|d(x,A_k)-d(x,A)| &= \frac{1}{b_n-a_n}\sum_{\substack{a_n+1\\|d(x,A_k)-d(x,A)|\geq\varepsilon}}^{b_n}|d(x,A_k)-d(x,A)| \\
&\quad+\frac{1}{b_n-a_n}\sum_{\substack{a_n+1\\|d(x,A_k)-d(x,A)|<\varepsilon}}^{b_n}|d(x,A_k)-d(x,A)| \\
&\leq \frac{M}{b_n-a_n}|\{a_n<k\leq b_n : |d(x,A_k)-d(x,A)|\geq\varepsilon\}|+\varepsilon
\end{aligned}$$

Taking limit $n \to \infty$, we get $WN_d - \lim A_k = A$.

From Theorem 2 we have the following:

Corollary 2. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2) and $A, A_k \subset X$. If the sets A and A_k (for each $k \in \mathbb{N}$) are bounded, then every Wijsman statistically convergent sequence is Wijsman strong Cesàro convergent.

Theorem 3. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2) and $A, A_k \subset X$, If $\lim_n \frac{b_n - a_n}{n} = a > 0, (a \in \mathbb{R})$ and $b_n < n$, then every $WS-$convergent sequence to A is WS_d-convergent to A.

Proof. Let $\{A_k\}$ be a $WS-$convergent sequence to A, $\lim_n \frac{b_n - a_n}{n} = a > 0$ and $b_n < n$. For a given $\varepsilon > 0$, we have

$$\{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\} \supseteq \{a_n < k \leq b_n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}$$

therefore

$$\begin{aligned} & \frac{1}{n} |\{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}| \\ \geq \; & \frac{1}{n} |\{a_n < k \leq b_n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}| \\ = \; & \frac{b_n - a_n}{n} \frac{1}{b_n - a_n} |\{a_n < k \leq b_n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}| . \end{aligned}$$

So $\{A_k\}$ is WS_d-convergent to A.

Theorem 4. Let $(a_n), \left(a_n^{'}\right), \left(b_n^{'}\right)$ and (b_n) be sequences of non-negative integers satisfying the following condition and (2)

$$a_n < a_n^{'} < b_n^{'} < b_n \text{ for all } n \in \mathbb{N}, \tag{3}$$

$A, A_k \subset X$ and suppose that the sets $\left\{k : a_n < k \leq a_n^{'}\right\}$ and $\left\{k : b_n^{'} < k \leq b_n\right\}$ are finite for all $n \in \mathbb{N}$, then every $WS_{d'}-$convergent sequence is WS_d-convergent, where

$$WS_{d'} = \left\{\mathbf{A} = (A_k) : \lim_{n \to \infty} \frac{1}{b_n^{'} - a_n^{'}} \left|\left\{a_n^{'} < k \leq b_n^{'} : |d(x, A_k) - d(x, A)| \geq \varepsilon\right\}\right| = 0\right\}.$$

Proof. Let us assume that the sequence $\{A_k\}$ is $WS_{d'}-$convergent. Then for any $\varepsilon > 0$ we have

$$\begin{aligned} & \frac{1}{b_n - a_n} |\{k : a_n < k \leq b_n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}| \\ \leq \; & \frac{1}{b_n^{'} - a_n^{'}} \left|\left\{k : a_n < k \leq a_n^{'} : |d(x, A_k) - d(x, A)| \geq \varepsilon\right\}\right| \\ & + \frac{1}{b_n^{'} - a_n^{'}} \left|\left\{k : a_n^{'} < k \leq b_n^{'} : |d(x, A_k) - d(x, A)| \geq \varepsilon\right\}\right| \\ & + \frac{1}{b_n^{'} - a_n^{'}} \left\{k : b_n^{'} < k \leq b_n : |d(x, A_k) - d(x, A)| \geq \varepsilon\right\}. \end{aligned}$$

Taking limit when $n \to \infty$, we get

$$\lim \frac{1}{b_n - a_n} |\{k : a_n < k \leq b_n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}| = 0.$$

Theorem 5. Let $(a_n), \left(a_n^{'}\right), \left(b_n^{'}\right)$ and (b_n) be sequences of non-negative integers satisfying (2) and (3) such that

$$\lim \frac{b_n - a_n}{b_n^{'} - a_n^{'}} = a > 0, \;\; (a \in \mathbb{R}) \tag{4}$$

and $A, A_k \subset X$, then every WS_d−convergent sequence is $WS_{d'}$−convergent.
Proof. It is easy to see that the inclusion

$$\begin{aligned} & \left\{k : a_n^{'} < k \le b_n^{'} : |d(x, A_k) - d(x, A)| \ge \varepsilon\right\} \\ \subset \ & \{k : a_n < k \le b_n : |d(x, A_k) - d(x, A)| \ge \varepsilon\} \end{aligned}$$

holds and so the following inequality too

$$\begin{aligned} & \left|\left\{k : a_n^{'} < k \le b_n^{'} : |d(x, A_k) - d(x, A)| \ge \varepsilon\right\}\right| \\ \le \ & |\{k : a_n < k \le b_n : |d(x, A_k) - d(x, A)| \ge \varepsilon\}| . \end{aligned}$$

Therefore we have

$$\begin{aligned} & \frac{1}{b_n^{'} - a_n^{'}} \left|\left\{k : a_n^{'} < k \le b_n^{'} : |d(x, A_k) - d(x, A)| \ge \varepsilon\right\}\right| \\ \le \ & \frac{b_n - a_n}{b_n^{'} - a_n^{'}} \frac{1}{b_n - a_n} |\{k : a_n < k \le b_n : |d(x, A_k) - d(x, A)| \ge \varepsilon\}| . \end{aligned}$$

Taking limits when $n \to \infty$, we get

$$\lim \frac{1}{b_n^{'} - a_n^{'}} \left|\left\{k : a_n^{'} < k \le b_n^{'} : |d(x, A_k) - d(x, A)| \ge \varepsilon\right\}\right| = 0.$$

Theorem 6. Let $(a_n), \left(a_n^{'}\right), \left(b_n^{'}\right)$ and (b_n) be sequences of non-negative integers satisfying $(2), (3), (4)$ and $A, A_k \subset X$, then every WN_d−convergent sequence is $WN_{d'}$−convergent.
Proof. Proof follows from the inequality

$$\begin{aligned} \frac{1}{b_n - a_n} \sum_{a_n+1}^{b_n} |d(x, A_k) - d(x, A)| & \ge \frac{1}{b_n - a_n} \sum_{a_n^{'}+1}^{b_n^{'}} |d(x, A_k) - d(x, A)| \\ & \ge \frac{b_n^{'} - a_n^{'}}{b_n - a_n} \frac{1}{b_n^{'} - a_n^{'}} \sum_{a_n^{'}+1}^{b_n^{'}} |d(x, A_k) - d(x, A)| . \end{aligned}$$

Theorem 7. Let $(a_n), \left(a_n^{'}\right), \left(b_n^{'}\right)$ and (b_n) be sequences of non-negative integers satisfying (2) and (3), A_k (for each $k \in \mathbb{N}$) and A are bounded ($A, A_k \subset X$) and suppose that the sets $\left\{k : a_n < k \le a_n^{'}\right\}$ and $\left\{k : b_n^{'} < k \le b_n\right\}$ are finite for all $n \in \mathbb{N}$, then every $WN_{d'}$−convergent sequence is WN_d−convergent.
Proof. Since A_k (for each $k \in \mathbb{N}$) and A are bounded, we have $|d(x, A_k) - d(x, A)| \le M$ for some $M > 0$. So we have

$$\begin{aligned} \frac{1}{b_n - a_n} \sum_{a_n+1}^{b_n} |d(x, A_k) - d(x, A)| \ = \ & \frac{1}{b_n - a_n} \sum_{a_n+1}^{a_n^{'}} |d(x, A_k) - d(x, A)| \\ & + \frac{1}{b_n - a_n} \sum_{a_n^{'}+1}^{b_n^{'}} |d(x, A_k) - d(x, A)| \end{aligned}$$

$$+\frac{1}{b_n - a_n}\sum_{b_n'+1}^{b_n} |d(x, A_k) - d(x, A)|$$

$$\leq \frac{2}{b_n' - a_n'} MO(1) + \frac{1}{b_n' - a_n'}\sum_{a_n'+1}^{b_n'} |d(x, A_k) - d(x, A)|.$$

Taking limit when $n \to \infty$, we get

$$\lim \frac{1}{b_n - a_n}\sum_{a_n+1}^{b_n} |d(x, A_k) - d(x, A)| = 0.$$

In the following theorem, by changing the conditions on the sequences (a_n) and (b_n) we give the same relation with Theorem 3.

Theorem 8. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2) and $A, A_k \subset X$, and let $\liminf_n \frac{b_n}{a_n} > 1$. If the sequece $\{A_k\}$ is Wijsman statistically convergent to A, then it is Wijsman deferred statistically convergent to A.

Proof. Let $\liminf_n \frac{b_n}{a_n} > 1$, then we can find a number $r > 0$ such that $\frac{b_n}{a_n} > 1 + r$ for sufficiently large n, which implies that

$$\frac{b_n - a_n}{b_n} \geq \frac{r}{1+r} \Longrightarrow \frac{1}{b_n} \geq \frac{r}{(1+r)}\frac{1}{(b_n - a_n)}.$$

If $WS - \lim A_k = A$, then we have

$$\begin{aligned}\frac{1}{b_n}\left|\{k \leq b_n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}\right| &\geq \frac{1}{b_n}\left|\{a_n < k \leq b_n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}\right| \\ &\geq \frac{r}{(1+r)}\frac{1}{(b_n - a_n)}\left|\{a_n < k \leq b_n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}\right|.\end{aligned}$$

So we have $WS_d - \lim A_k = A$.

Theorem 9. Let $A, A_k \subset X$ and $sup_n\left(\frac{b_n}{b_n - a_n}\right) < \infty$. Let (a_n) and (b_n) be sequences of non-negative integers satisfying (2) such that

$i)$ $\lim\limits_{n\to\infty} a_n = \infty$,

$ii)$ $\lim\limits_{n\to\infty} b_n - a_n = \infty$.

If $\{A_k\}$ is Wijsman strong Cesàro convergent to A, then it is Wijsman strong deferred Cesàro convergent to A.

Proof. Suppose that $sup_n \frac{b_n}{b_n - a_n} < \infty$, then $sup_n\left(\frac{a_n}{b_n - a_n}\right) < \infty$. In this case we can find positive numbers M and K such that $\frac{b_n}{b_n - a_n} \leq M$ and $\frac{a_n}{b_n - a_n} \leq K$. Then we have

$$\frac{1}{(b_n - a_n)}\sum_{a_n+1}^{b_n} |d(x, A_n) - d(x, A)| = \frac{1}{(b_n - a_n)}\sum_{k=1}^{b_n} |d(x, A_k) - d(x, A)|$$

$$
\begin{aligned}
&-\frac{1}{(b_n-a_n)}\sum_{k=1}^{a_n}|d(x,A_k)-d(x,A)| \\
=\;& \frac{b_n}{(b_n-a_n)}\frac{1}{b_n}\sum_{k=1}^{b_n}|d(x,A_k)-d(x,A)| \\
&-\frac{a_n}{(b_n-a_n)}\frac{1}{a_n}\sum_{k=1}^{a_n}|d(x,A_k)-d(x,A)| \\
<\;& \frac{M}{b_n}\sum_{k=1}^{b_n}|d(x,A_k)-d(x,A)|+\frac{K}{a_n}\sum_{k=1}^{a_n}|d(x,A_k)-d(x,A)|.
\end{aligned}
$$

So $\{A_k\}$ is WN_d-convergent to A.

In the following theorems, by changing the conditions on the sequences (a_n) and (b_n) we give the same relation with Theorem 9.

Theorem 10. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2) and $A, A_k \subset X$, If $\liminf_n \dfrac{b_n}{a_n} > 1$, then $WN \subset WN_d$.

Theorem 11. Let (a_n) and (b_n) be sequences of non-negative integers satisfying the conditions (2) and $A, A_k \subset X$, if $\liminf_n \dfrac{(b_n - a_n)}{n} > 0$ and $b_n < n$ then $WN \subset WN_d$.

3. Conclusion

The concepts of Wijsman statistical convergence and Wijsman strong Cesàro summability for sequences of sets were introduced and studied by Nuray and Rhoades [19] in 2012 and then the concepts were improved by Bhardwaj et al. [4], Esi et al. [13], Hazarika et al. [15] and Sengul [24]. In this paper we study the concepts of Wijsman deferred statistical convergence and Wijsman strong deferred Cesàro summability for sequences of sets. The results which we obtained in this study are much more general than those obtained by others. To get these general results, we introduce some of fairly wide classes of sequences of sets using two sequences of non-negative integers satisfying the conditions $a_n < b_n$ and $\lim_{n\to\infty} b_n = \infty$. Researchers who are working in this area can study the concepts of Wijsman deferred statistical convergence of order α and Wijsman strong deferred Cesàro summability of order α for sequences of sets, where $0 < \alpha \leq 1$.

Acknowledgments

This research was supported by FUBAP (The Management Union of the Scientific Research Projects of Firat University) under the Project Number: FUBAB FF.19.05.

Conflict of interest

The authors declare that they have no conflict of interests.

References

1. H. Fast, *Sur la convergence statistique*, Colloq. Math., **2** (1951), 241–244.
2. H. Steinhaus, *Sur la convergence ordinaire et la convergence asymptotique*, Colloq. Math., **2** (1951), 73–74.
3. V. K. Bhardwaj, S. Dhawan, *Density by moduli and lacunary statistical convergence*, Abstr. Appl. Anal., **2016** (2016), 9365037.
4. V. K. Bhardwaj, S. Dhawan, *Density by moduli and Wijsman lacunary statistical convergence of sequences of sets*, J. Inequal. Appl., **2017** (2017), 25.
5. H. Cakalli, *Lacunary statistical convergence in topological groups*, Indian J. Pure Appl. Math., **26** (1995), 113–119.
6. M. Cinar, M. Karakas, M. Et, *On pointwise and uniform statistical convergence of order α for sequences of functions*, Fixed Point Theory Appl., **2013** (2013), 33.
7. A. Caserta, G. Di Maio, L. D. R. Kočinac, *Statistical convergence in function spaces*, Abstr. Appl. Anal., **2011** (2011), 420419.
8. R. Colak, *Statistical convergence of order α*, In: *Modern Methods in Analysis and Its Applications*, Anamaya Pub, New Delhi, 2010, 121–129.
9. J. S. Connor, *The statistical and strong $p-$Cesàro convergence of sequences*, Analysis, **8** (1988), 47–63.
10. M. Et, B. C. Tripathy, A. J. Dutta, *On pointwise statistical convergence of order α of sequences of fuzzy mappings*, Kuwait J. Sci., **41** (2014), 17–30.
11. M. Et, R. Colak, Y. Altin, *Strongly almost summable sequences of order α*, Kuwait J. Sci., **41** (2014), 35–47.
12. M. Et, S. A. Mohiuddine, A. Alotaibi, *On $\lambda-$statistical convergence and strongly $\lambda-$summable functions of order α*, J. Inequal. Appl., **2013** (2013), 469.
13. A. Esi, N. L. Braha, A. Rushiti, *Wijsman $\lambda-$statistical convergence of interval numbers*, Bol. Soc. Parana. Mat., **35** (2017), 9–18.
14. J. Fridy, *On statistical convergence*, Analysis, **5** (1985), 301–313.
15. B. Hazarika, A. Esi, N. L. Braha, *On asymptotically Wijsman lacunary $\sigma-$statistical convergence of set sequences*, J. Math. Anal., **4** (2013), 33–46
16. M. Isik, K. E. Akbas, *On $\lambda-$statistical convergence of order α in probability*, J. Inequal. Spec. Funct., **8** (2017), 57–64.
17. M. Isik, K. E. Akbas, *On asymptotically lacunary statistical equivalent sequences of order α in probability*, ITM Web of Conferences, **13** (2017), 01024.
18. M. Mursaleen, *$\lambda-$ statistical convergence*, Math. Slovaca, **50** (2000), 111–115.
19. F. Nuray, B. E. Rhoades, *Statistical convergence of sequences of sets*, Fasc. Math., **49** (2012), 87–99.
20. T. Salat, *On statistically convergent sequences of real numbers*, Math. Slovaca, **30** (1980), 139–150.

21. E. Savas, M. Et, *On* $(\Delta_\lambda^m, I)-$*statistical convergence of order* α, Period. Math. Hungar., **71** (2015), 135–145.

22. H. M. Srivastava, M. Et, *Lacunary statistical convergence and strongly lacunary summable functions of order* α, Filomat, **31** (2017), 1573–1582.

23. H. Sengul, M. Et, *On* $I-$*lacunary statistical convergence of order* α *of sequences of sets*, Filomat, **31** (2017), 2403–2412.

24. H. Sengul, *On Wijsman* $I-$*lacunary statistical equivalence of order* (η, μ), J. Inequal. Spec. Funct., **9** (2018), 92–101.

25. M. Yilmazturk, M. Kucukaslan, *On strongly deferred Cesàro summability and deferred statistical convergence of the sequences*, Bitlis Eren Univ. J. Sci. Technol., **3** (2011), 22–25.

26. R. P. Agnew, *On deferred Cesàro means*, Ann. Math., **33** (1932), 413–421.

27. M. Kucukaslan, M. Yilmazturk, *On deferred statistical convergence of sequences*, Kyungpook Math. J., **56** (2016), 357–366.

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