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Research article

Computing the conjugacy classes and character table of a non-split extension $2^{6\cdot}(2^5{:}S_6)$ from a split extension $2^6{:}(2^5{:}S_6)$

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Abstract: In this paper, we will demonstrate how the character table of a sub-maximal subgroup $2^6{:}(2^5{:}S_6)$ of the sporadic simple group Fi_{22} can be used to obtain the conjugacy classes and character table of a non-split extension of the form $2^{6\cdot}(2^5{:}S_6)$, which sits maximal in the unique non-split extension $2^{6\cdot}Sp_6(2)$.

Keywords: coset analysis; Fischer-Clifford matrices; non-split extension; inertia factor; projective character table; fusion map; restriction of characters
Mathematics Subject Classification: 20C15, 20C40

1. Introduction

In the papers [2] and [16], the character tables of the non-split extension $\overline{G_3} = 2^{6\cdot}Sp_6(2)$ and split extension $2^6{:}Sp_6(2)$ were successfully computed by the method of Fischer-Clifford matrices [8]. In the ATLAS [6] we found that $N_{Fi_{22}}(2^6) \cong 2^6{:}Sp_6(2)$ is a maximal subgroup of the smallest Fischer sporadic simple group Fi_{22} of index 694980. Here the elementary abelian group 2^6 is a pure $2B$-group, where $2B$ denotes a class of involutions in Fi_{22}. Dempwolff in [7] proved that a unique non-split extension (up to isomorphism) of the form $\overline{G_n} = 2^{2n\cdot}Sp_{2n}(2)$ does exist for all $n \geq 2$, where $\overline{G_n}/2^{2n} \cong Sp_{2n}(2)$ acts faithfully on 2^{2n}. In [2] it was noted that $2^6{:}Sp_6(2)$ and $\overline{G_3}$ give rise to the same character table. The groups $2^6{:}Sp_6(2)$ and $\overline{G_3}$ have subgroups of types $M_1 = 2^6{:}(2^5{:}S_6)$ and $\overline{G} = 2^{6\cdot}(2^5{:}S_6)$, where M_1 and $\overline{G}$ are pre-images of a maximal subgroup $2^5{:}S_6$ of index 63 in $Sp_6(2)$, under the natural epimorphism modulo 2^6.

In this paper, it will be shown that the character tables of M_1 and $\overline{G}$ coincide and how the conjugacy classes of $\overline{G}$ can be obtained from the classes of M_1 by "restricting" characters of $\overline{G_3}$ (see Section 5 of this paper) to characters of M_1. In this regard, the format of the character tables of M_1 and $\overline{G_3}$ (see [2] and [13]) which were obtained by the method of Fischer-Clifford matrices, plays an important role.

The power maps of $\overline{G}$ and the fusion map of $\overline{G}$ into $\overline{G_3}$ are also computed. Most of our computations are done in the computer algebra systems MAGMA [5] and GAP [21]. For concepts and definitions used in this paper, the readers are referred to the review paper on Fischer-Clifford theory [3] and [1, 11, 12, 17–19].

2. Theory of Fischer-Clifford matrices

Since the ordinary character tables of the groups $2^6{:}Sp_6(2)$, $\overline{G_3}$, M_1 and $\overline{G}$ have been computed by the technique of Fischer-Clifford matrices, a brief theoretical background of this technique will be given in this section. In Section 4, it will be shown that only the ordinary irreducible characters of the inertia factors will be used in the construction of the character table of $\overline{G}$. Therefore, only the case where every irreducible character of N can be extended to its inertia group in the extension group $N.G$ will be discussed. Here the author will follow closely the work of the authors in [16].

Let $\overline{G} = N.G$ be an extension of N by G and $\theta \in Irr(N)$, where $Irr(N)$ denotes the irreducible characters of N. Define θ^g by $\theta^g(n) = \theta(gng^{-1})$ for $g \in \overline{G}$, $n \in N$ and $\theta^g \in Irr(N)$. Let $\overline{H} = \left\{x \in \overline{G} | \theta^x = \theta\right\} = I_{\overline{G}}(\theta)$ be the inertia group of θ in $\overline{G}$. We say that θ is extendible to $\overline{H}$ if there exists $\phi \in Irr(\overline{H})$ such that $\phi\downarrow_N = \theta$. If θ is extendible to $\overline{H}$, then by Gallagher [11], we have

$$\left\{\gamma | \gamma \in Irr(\overline{H}), <\gamma\downarrow_N, \theta> \neq 0\right\} = \left\{\beta\phi | \beta \in Irr(\overline{H}/N)\right\}.$$

Let $\overline{G}$ have the property that every irreducible character of N can be extended to its inertia group. Now let $\theta_1 = 1_N, \theta_2, \cdots, \theta_t$ be representatives of the orbits of $\overline{G}$ on $Irr(N)$, $\overline{H_i} = I_{\overline{G}}(\phi_i)$, $1 \leq i \leq t$, $\phi_i \in Irr(\overline{H_i})$ be an extension of θ_i to $\overline{H_i}$ and $\beta \in Irr(\overline{H_i})$ such that $N \subseteq ker(\beta)$. Then

$$Irr(\overline{G}) = \bigcup_{i=1}^{t} \{(\beta\phi_i)^{\overline{G}} \,|\, \beta \in Irr(\overline{H_i}), N \subseteq ker(\beta)\} = \bigcup_{i=1}^{t} \{(\beta\phi_i)^{\overline{G}} \,|\, \beta \in Irr(\overline{H_i}/N)\}$$

Hence the irreducible characters of $\overline{G}$ will be divided into blocks, where each block corresponds to an inertia group $\overline{H_i}$.

Let H_i be the inertia factor group and ϕ_i be an extension of θ_i to $\overline{H_i}$. Take $\theta_1 = 1_N$ as the identity character of N, then $\overline{H_1} = \overline{G}$ and $H_1 \cong G$. Let $X(g) = \{x_1, x_2, \cdots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $N\overline{g}$ whose images under the natural homomorphism $\overline{G} \longrightarrow G$ are in the class $[g]$ of G and we take $x_1 = \bar{g}$. We define

$$R(g) = \{(i, y_k) \,|\, 1 \leq i \leq t, H_i \cap [g] \neq \emptyset, 1 \leq k \leq r\},$$

where y_k runs over representatives of the conjugacy classes of elements of H_i which fuse into $[g]$. Let $\{y_{l_k}\}$ be the representatives of conjugacy classes of $\overline{H_i}$ which contain liftings of y_k under the natural homomorphism $\overline{H_i} \longrightarrow H_i$. Then we define the Fischer-Clifford matrix $M(g)$ by $M(g) = (a^j_{(i,y_k)})$, where

$$a^j_{(i,y_k)} = \sum_l{}' \frac{|C_{\overline{G}}(x_j)|}{|C_{\overline{H_i}}(y_{l_k})|}\phi_i(y_{l_k}),$$

with columns indexed by $X(g)$ and rows indexed by $R(g)$ and where $\sum_l'$ is the summation over all l for which $y_{l_k} \sim x_j$ in $\overline{G}$. Then the partial character table of $\overline{G}$ on the classes $\{x_1, x_2, \cdots, x_{c(g)}\}$ is given by

$$\begin{bmatrix} C_1(g)\,M_1(g) \\ C_2(g)\,M_2(g) \\ \vdots \\ C_t(g)\,M_t(g) \end{bmatrix}$$ where the Fischer-Clifford matrix $M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix}$ is divided into blocks $M_i(g)$ with each block corresponding to an inertia group $\overline{H}_i$ and $C_i(g)$ is the partial character table of H_i consisting of the columns corresponding to the classes that fuse into $[g]$. Hence the full character table of $\overline{G}$ will be $\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_t \end{bmatrix}$, where $\Delta_i = [C_i(1)M_i(1)|C_i(g_2)M_i(g_2)|...|C_i(g_k)M_i(g_k)]$ with $\{1, g_1, g_2, ..., g_k\}$ the representatives of conjugacy classes of G. We can also observe that $|Irr(\overline{G})| = |Irr(H_1)| + |Irr(H_2)| + ... + |Irr(H_t)|$.

3. The action of G on N and $Irr(N)$

The group $\overline{G_3} = 2^{6\cdot}Sp_6(2)$ was constructed in [2] as a permutation group on 128 points and it was shown that $\overline{G_3}$ has an inertia group $\overline{G} = 2^{6\cdot}(2^5{:}S_6)$ which belongs to a non-split extension of a reducible module of dimension 6 over $GF(2)$ for the maximal subgroup $2^5{:}S_6$ of $Sp_6(2)$. Using the generators of $\overline{G_3}$ given in [2], the group $\overline{G}$ is constructed as the centralizer $C_{\overline{G}_3}(2A)$ of the class of involutions $2A$ within $\overline{G_3}$.

Let $\overline{G} = 2^{6\cdot}(2^5{:}S_6)$ be the non-split extension of $N = 2^6$ by $G = 2^5{:}S_6$. The group $2^5{:}S_6$ is the stabilizer of a vector in the action of $Sp_6(2)$ on its natural 6-dimensional module 2^6. This action is the same in both the split and non-split extensions $2^6{:}Sp_6(2)$ and $\overline{G_3}$. This immediately defines the action of $2^5{:}S_6$ on the module 2^6. Note that the action of the split extension $M_1 = 2^6{:}(2^5{:}S_6)$ on 2^6 is the same as the action of $2^5{:}S_6$ on 2^6. The group G can be constructed as a matrix group of dimension 6 over the finite field $GF(2)$ within $Sp_6(2)$. Now with the action of G on $N = 2^6$, where we view N as the vector space of dimension six over $GF(2)$, we will obtain four orbits of lengths 1, 1, 30 and 32 with corresponding point stabilizers G, G, $2^4{:}S_5$ and S_6, respectively. By Brauer's Theorem [10] the action of G on $Irr(N)$ will also produce 4 orbits and since the action is self-dual, the orbit lengths will be $1, 1, 30$ and 32 with corresponding inertia factor groups $H_1 = H_2 = G$, $H_3 = 2^4{:}S_5$ and $H_4 = S_6$.

4. The ordinary character table of $\overline{G} = 2^{6\cdot}(2^5{:}S_6)$

Having obtained the inertia factors $H_1 = H_2 = G$, $H_3 = 2^4{:}S_5$ and $H_4 = S_6$ for the action of G on $Irr(N)$, we can formed the Fischer-Clifford matrix $M(1A)$ corresponding to the identity coset $N1_{\overline{G}} = N$ as follows:

$$M(1A) = \begin{array}{r} \\ 23040 \\ 23040 \\ 768 \\ 720 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 1474560 & 1474560 & 49152 & 46080 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 30 & 30 & -2 & 0 \\ 32 & -32 & 0 & 0 \end{pmatrix} \\ \begin{array}{cccc} 1 & 1 & 30 & 32 \end{array} \end{array}$$

The column weights above the matrix $M(1A)$ are the centralizer orders $|C_{\overline{G}}(\overline{g})|$ of the classes $1A, 2A, 2B$ and $2C$ of $\overline{G}$ (see Table 5) coming from the identity coset $N(\overline{1A}) = N$ by means of the technique of coset-analysis (see [14], [15] and [16]). Whereas, the row weights to the left of the matrix $M(1A)$ represent the centralizer orders $|C_{H_i}(1A)|$ of the inertia factors H_i on the identity element $1A$.

Table 1. The partial character table of $\overline{G}$ for coset N.

$[g]_{\overline{G}}$	1A	2A	2B	2C
χ_1	1	1	1	1
χ_{38}	a	a	a	-a
χ_{38+t_2}	30b	30b	-2b	0
$\chi_{38+t_2+t_3}$	32c	-32c	0	0

Table 1 is the partial ordinary character table of $\overline{G}$ on the classes $1A, 2A, 2B$ and $2C$ of $\overline{G}$, where each of the 4 lines of Table 1 corresponds to the first row of entries of the sub-matrices $C_i(1A)M_i(1A), i = 1,2,3,4$. $M_i(1A)$ and $C_i(1A)$ correspond to the rows of the Fischer-Clifford matrix $M(1A)$ and columns of the projective character tables of the inertia factors H_i, respectively, which are associated with the classes $[1A]_{H_i}$ of the inertia factors H_i which fuse into the class $[1A]_G$ of G. Also, note that the character values in the 1st column of Table 1 are the degrees of the ordinary irreducible characters $\chi_1, \chi_{38}, \chi_{38+t_2}$ and $\chi_{38+t_2+t_3}$ of $\overline{G}$. The characters $\chi_1, \chi_{38}, \chi_{38+t_2}$ and $\chi_{38+t_2+t_3}$ occupy the first position for each block of characters coming from an inertia subgroup $\overline{H_i}$ of $\overline{G}$, where $37, t_2$ and t_3 represent the number $|IrrProj(H_i, \alpha_i)|$ of irreducible projective characters with associated factor set α_i for the inertia factors H_1, H_2 and H_3, respectively. Now $deg(\eta_1) = 1, deg(\phi_1) = a, deg(\psi_1) = b$ and $deg(\gamma_1) = c$ are the degrees of the irreducible projective characters $\mathbf{1}_G$, ϕ_1, ψ_1 and γ_1 which occupy the first position in each set $IrrProj(H_i, \alpha_i)$, $i = 1, 2, ..., 4$, respectively.

We copy a small part of the ordinary character table of $2^{6\cdot}Sp_6(2)$ (see Table 11.12 in [4]), containing the values of the character $63a$ on the classes $1A, 2A$:

$[g]_{\overline{G_3}}$	$1A$	$2A$
$63a$	63	-1

Now the classes $1A, 2A, 2B$ and $2C$ of $\overline{G}$ consist of the elements of N (see Table 5). If we decompose $(63a)_N$ into the the set $Irr(N)$ and also notice that $< (63a)_N, 1_N > = 0$, then $(63a)_N = a(\chi_{38})_N + b(\chi_{38+t_2})_N + c(\chi_{38+t_2+t_3})_N$, where a, b and c are defined as above. If we take into account the fusion of the classes $1A, 2A, 2B$ and $2C$ of $\overline{G}$ into the classes of $1A$ and $2A$ of $\overline{G_3}$, and the decomposition of $(63a)_N$ into the set $Irr(N)$, then the following set of equations (by restricting the character values of $63a$ to Table 1) is obtained:

1. $(63a)_N(1A) = a + 30b + 32c = 63$
2. $(63a)_N(2A) = a + 30b - 32c = -1$
3. $(63a)_N(2B) = a - 2b = -1$
4. $(63a)_N(2C) = -a = -1$

Solving the above equations simultaneously, we obtain that $a = b = 1 = c = 1$ and hence $deg(\phi_1) = deg(\psi_1) = deg(\gamma_1) = 1$. We can conclude that only the ordinary irreducible characters tables of the inertia factors H_i will be involved in the construction of the ordinary character table of $2^{6\cdot}(2^5{:}S_6)$. This means that the ordinary irreducible characters of the split extension $M_1 = 2^6{:}(2^5{:}S_6)$ (Table 9.7 in [13]) are the same as the ones for the non-split extension $\overline{G}$, but the class orders of the two groups will differ as it will be shown in Section 5.

5. The conjugacy classes of $\overline{G}$

In this section, we will compute the order of an element $\overline{g}$ in a conjugacy class $[\overline{g}]_{\overline{G}}$ of $\overline{G}$ from the conjugacy classes and ordinary irreducible characters of both M_1 and $\overline{G}_3$. For both $\overline{G}$ and M_1, the centralizer orders for each class of elements coming from a corresponding coset $N\overline{g}$, $g \in 2^5{:}S_6$, will be the same, but their class orders may be different. The method of coset-analysis was used to compute the conjugacy classes of elements of $2^6{:}(2^5{:}S_6)$ (see Table 5) and $\overline{G}_3$ (Table 1 in [2]). Let $\overline{G} = N.G$ be an extension of N by G, where N is abelian. Then for $g \in G$, we write $\overline{g}$ for a lifting of g in $\overline{G}$ under the natural homomorphism $\overline{G} \longrightarrow G$. We consider a coset $N\overline{g}$ for each class representative g of G, writing k for number of orbits of N acting by conjugation on the coset $N\overline{g}$, and f_j for the numbers of these fused by the action of $\left\{\overline{h} : h \in C_G(g)\right\}$. Note if $\overline{G}$ is a split extension then $\overline{g}$ becomes g. The order of the centralizer $C_{\overline{G}}(x)$ for each element $x \in \overline{G}$ in a conjugacy class $[x]_{\overline{G}}$ is given by $|C_{\overline{G}}(x)| = \frac{k|C_G(g)|}{f_j}$.

For example, let consider the classes of $M_1 = 2^6{:}(2^5{:}S_6)$ obtained from the cosets $N(2A)$ and $N(2E)$ (see Table 3), where $2A$ and $2E$ are classes of involutions in $2^5{:}S_6$. In addition, we consider also the partial character table of M_1 corresponding to the cosets $N(2A)$ and $N(2E)$ (see [13]), which was computed by the technique of Fischer-Clifford matrices. We obtained also from [13] that $(\phi_1 = 63a)_{2^6:(2^5:S_6)} = \chi_{38} + \chi_{75} + \chi_{127}$, $(\phi_2 = 63b)_{2^6:(2^5:S_6)} = \chi_{39} + \chi_{76} + \chi_{128}$, $(\phi_3 = 315a)_{2^6:(2^5:S_6)} = \chi_{43} + \chi_{86} + \chi_{94} + \chi_{132}$ and $(\phi_4 = 315b)_{2^6:(2^5:S_6)} = \chi_{40} + \chi_{75} + \chi_{80} + \chi_{94} + \chi_{130}$, where ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are ordinary irreducible characters of $2^6{:}Sp_6(2)$ of degrees 63 and 315 which are restricted to irreducible characters of M_1 by the technique of set intersection (see [9, 14, 16]).

From Table 4 we notice that classes $2A$ and $2E$ of $2^5{:}S_6$ are fusing into the class $2A$ of $Sp_6(2)$. Hence the classes of $\overline{G}$, which will be obtained from the cosets $N(\overline{2A})$ and $N(\overline{2E})$ using coset analysis, will fuse into the classes of $\overline{G}_3$ lying above the class $2A$ of $Sp_6(2)$. Since the character tables of $\overline{G}$ and M_1 coincide, the corresponding cosets $N(\overline{2A})$ and $N(\overline{2E})$ for both of the groups will produce the same number of classes and share the same class centralizer orders and partial character tables. Also $\overline{G}_3$ and $2^6{:}Sp_6(2)$ share the same character table and therefore we can expect that the irreducible character χ_{32} of degree 63 of $\overline{G}_3$ (see Table 2) will restrict to the same irreducible characters as above-mentioned character $\phi_2 = 63b$.

Suppose that Table 3 is the partial character table of $\overline{G}$ corresponding to the cosets N, $N(\overline{2A})$ and $N(\overline{2E})$ and irreducible characters of degrees 1, 30 and 32. Now the ordinary character χ_{32} of $\overline{G}_3$ in Table 2 will restrict to the sum of the irreducible characters χ_{39}, χ_{76} and χ_{128} of $\overline{G}$ in Table 3. If the character values of χ_{32} on the classes $4A$, $4B$ and $2B$ coming from the coset $N(\overline{2A})$ in Table 2 and the character values of the restricted character $(\chi_{32})_{\overline{G}} = \chi_{39} + \chi_{76} + \chi_{128}$ on the classes $2D, 2E, 4A, 2L, 2M, 2N, 2O, 4I$ and $4J$ coming from the cosets $N(\overline{2A})$ and $N(\overline{2E})$ in Table 3 are taking into consideration, then the class orders of $2D, 2E, 2L, 2M, 2N$ and $2O$ are forced to change from order 2 to order 4 whereas the class orders of $4A, 4I$ and $4J$ are forced to change from order 4 to order 2. Hence we obtained the classes of $\overline{G}$, with their respective class orders and centralizer orders (see Table 5), associated with the cosets $N(\overline{2A})$ and $N(\overline{2E})$. In a similar fashion, we obtained all the classes of $\overline{G}$, with their class and centralizer orders, using the above restricted characters ϕ_1, ϕ_2, ϕ_3 and ϕ_4 together with the ordinary character tables of $\overline{G}_3$ and M_1. See Table 5 where all the information concerning the conjugacy classes of M_1 and $\overline{G}$ are listed. For the explanation of the parameters used in Table 5 the readers are referred to [16] and [20].

Table 2. The partial character table of $\overline{G}_3 = 2^{6\cdot}Sp_6(2)$.

$[g]_{Sp_6(2)}$	1A		2A		
$[g]_{2^{6\cdot}Sp_6(2)}$	1A	2A	4A	4B	2B
χ_{32}	63	-1	-29	3	-1

Table 3. The partial character table of $M_1 = 2^6{:}(2^5{:}S_6)$.

$[g]_{2^5:S_6}$	1A				2A			2E					
$[g]_{2^6:(2^5:S_6)}$	1A	2A	2B	2C	2D	2E	4A	2L	2M	2N	2O	4I	4J
χ_{39}	1	1	1	-1	1	1	-1	-1	-1	-1	1	1	-1
χ_{76}	30	30	-2	0	-30	2	0	-12	-12	4	2	-2	0
χ_{128}	32	-32	0	0	0	0	0	-16	16	0	0	0	0
$\chi_{39}+\chi_{76}+\chi_{128}$	63	-1	-1	-1	-29	3	-1	-29	3	3	3	-1	-1

Table 4. The fusion of $2^5{:}S_6$ into $Sp(6,2)$.

$[h]_{2^5:S_6}$ $\longrightarrow$	$[g]_{Sp(6,2)}$	$[h]_{2^5:S_6}$ $\longrightarrow$	$[g]_{Sp(6,2)}$	$[h]_{2^5:S_6}$ $\longrightarrow$	$[g]_{Sp(6,2)}$	$[h]_{2^5:S_6}$ $\longrightarrow$	$[g]_{Sp(6,2)}$
$1A$	$1A$	$2J$	$2D$	$4G$	$4E$	$6E$	$6B$
$2A$	$2A$	$3A$	$3A$	$4H$	$4E$	$6F$	$6D$
$2B$	$2B$	$3B$	$3C$	$4I$	$4B$	$6G$	$6G$
$2C$	$2C$	$4A$	$4B$	$4J$	$4C$	$6H$	$6F$
$2D$	$2C$	$4B$	$4C$	$5A$	$5A$	$8A$	$8A$
$2E$	$2A$	$4C$	$4D$	$6A$	$6B$	$8B$	$8B$
$2F$	$2B$	$4D$	$4A$	$6B$	$6A$	$10A$	$10A$
$2G$	$2D$	$4E$	$4D$	$6C$	$6D$	$12A$	$12A$
$2H$	$2D$	$4F$	$4E$	$6D$	$6E$	$12B$	$12B$
$2I$	$2C$						

Table 5. The conjugacy classes of elements of the groups M_1 and $\overline{G}$.

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[\overline{g}]_{M_1}$	$[\overline{g}]_{\overline{G}}$	$\lvert C_{M_1}(\overline{g})\rvert$ and $\lvert C_{\overline{G}}(\overline{g})\rvert$
$1A$	64	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$1A$	$1A$	1474560
		$f_2 = 1$	$(1,0,1,0,0,1)$	$(1,0,1,0,0,1)$	$2A$	$2A$	1474560
		$f_3 = 30$	$(0,1,0,0,0,0)$	$(0,1,0,0,0,0)$	$2B$	$2B$	49152
		$f_4 = 32$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$2C$	$2C$	46080
$2A$	32	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2D$	$4A$	737280
		$f_2 = 15$	$(0,1,0,0,0,0)$	$(0,0,0,0,0,0)$	$2E$	$4B$	49152
		$f_3 = 16$	$(1,0,0,0,0,0)$	$(1,0,1,0,0,0)$	$4A$	$2D$	46080
$2B$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2F$	$2E$	24576
		$f_2 = 3$	$(1,1,1,1,1,0)$	$(0,0,0,0,0,0)$	$2G$	$2F$	8192
		$f_3 = 4$	$(0,0,0,1,1,1)$	$(1,0,1,0,0,1)$	$4B$	$4C$	6144
		$f_4 = 8$	$(1,0,0,0,0,0)$	$(0,1,0,0,1,0)$	$4C$	$4D$	3072
$2C$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2H$	$4E$	24576
		$f_2 = 3$	$(0,1,0,0,0,0)$	$(0,0,0,0,0,0)$	$2I$	$4F$	8192
		$f_3 = 4$	$(1,1,0,0,1,1)$	$(1,0,1,0,0,1)$	$4D$	$2G$	6144
		$f_4 = 8$	$(1,0,0,0,0,0)$	$(0,1,1,0,0,0)$	$4E$	$4G$	3072
$2D$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2J$	$4H$	12288
		$f_2 = 3$	$(1,0,0,0,0,1)$	$(0,0,0,0,0,0)$	$2K$	$4I$	4096
		$f_3 = 4$	$(1,0,0,0,0,0)$	$(0,1,1,0,0,0)$	$4F$	$4J$	3072
		$f_4 = 4$	$(1,1,1,1,0,0)$	$(1,1,0,0,0,1)$	$4G$	$4K$	3072
		$f_5 = 4$	$(1,0,1,0,1,0)$	$(1,0,1,0,0,1)$	$4H$	$2H$	3072
$2E$	32	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2L$	$4L$	24576
		$f_2 = 1$	$(1,1,0,0,0,1)$	$(0,0,0,0,0,0)$	$2M$	$4M$	24576
		$f_3 = 6$	$(1,0,0,0,0,1)$	$(0,0,0,0,0,0)$	$2N$	$4N$	4096
		$f_4 = 8$	$(1,1,0,0,1,1)$	$(0,1,1,0,0,0)$	$2O$	$4O$	3072
		$f_5 = 8$	$(0,1,0,0,1,1)$	$(0,1,1,0,0,0)$	$4I$	$2I$	3072
		$f_6 = 8$	$(0,1,0,0,0,1)$	$(0,0,0,0,0,0)$	$4J$	$2J$	3072
$2F$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2P$	$2K$	6144
		$f_2 = 1$	$(1,1,1,1,1,0)$	$(0,0,0,0,0,0)$	$2Q$	$2L$	6144
		$f_3 = 2$	$(1,1,1,1,0,1)$	$(0,0,0,0,0,0)$	$2R$	$2M$	3072
		$f_4 = 6$	$(1,1,1,1,1,1)$	$(0,0,0,1,0,1)$	$4K$	$4P$	1024
		$f_5 = 6$	$(0,1,1,1,1,1)$	$(1,1,1,0,1,1)$	$4L$	$4Q$	1024
$2G$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2S$	$4R$	3072
		$f_2 = 1$	$(1,0,0,1,1,1)$	$(1,0,1,0,0,1)$	$4M$	$2N$	3072
		$f_3 = 3$	$(1,1,1,1,1,1)$	$(1,1,1,1,1,0)$	$4N$	$4S$	1024
		$f_4 = 3$	$(1,1,1,1,0,0)$	$(1,0,1,1,0,0)$	$4O$	$4T$	1024
$2H$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2T$	$4U$	1024
		$f_2 = 1$	$(1,1,1,0,1,1)$	$(1,0,0,0,0,1)$	$4P$	$4V$	1024
		$f_3 = 1$	$(1,1,1,1,0,0)$	$(1,1,0,0,0,1)$	$4Q$	$4W$	1024
		$f_4 = 1$	$(1,1,1,1,1,0)$	$(1,0,1,0,0,1)$	$4R$	$2O$	1024
		$f_5 = 2$	$(1,1,1,1,1,1)$	$(1,0,0,0,0,1)$	$4S$	$4X$	512
		$f_6 = 2$	$(1,1,1,1,0,1)$	$(1,1,1,0,0,1)$	$4T$	$4Y$	512
$2I$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2U$	$4Z$	2048
		$f_2 = 1$	$(0,1,1,1,0,1)$	$(0,0,0,0,0,0)$	$2V$	$4AA$	2048
		$f_3 = 2$	$(0,1,1,1,1,1)$	$(1,0,1,1,1,0)$	$4U$	$2P$	1024
		$f_4 = 2$	$(1,1,1,1,1,1)$	$(1,0,1,1,1,0)$	$2W$	$4AB$	1024
		$f_5 = 2$	$(1,1,1,1,0,1)$	$(0,0,0,0,0,0)$	$4V$	$2Q$	1024
		$f_6 = 4$	$(1,1,1,1,0,0)$	$(1,1,0,1,0,0)$	$4W$	$4AC$	512
		$f_7 = 4$	$(1,0,1,1,1,1)$	$(1,1,0,1,0,0)$	$4X$	$4AD$	512
$2J$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2X$	$4AE$	1024
		$f_2 = 1$	$(1,1,1,1,0,1)$	$(1,0,1,0,0,1)$	$4Y$	$4AF$	1024
		$f_3 = 1$	$(1,1,1,0,0,1)$	$(1,0,1,1,1,0)$	$4Z$	$2R$	1024
		$f_4 = 1$	$(1,1,1,1,1,1)$	$(0,0,0,1,1,1)$	$4AA$	$4AG$	1024
		$f_5 = 2$	$(1,1,1,1,1,0)$	$(0,1,1,0,1,0)$	$4AB$	$4AH$	512
		$f_6 = 2$	$(1,1,0,1,1,1)$	$(1,1,0,0,1,1)$	$4AC$	$4AI$	512

Table 5 (continued)

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[\overline{g}]_{M_1}$	$[\overline{g}]_{\overline{G}}$	$\lvert C_{M_1}(\overline{g})\rvert$ and $\lvert C_{\overline{G}}(\overline{g})\rvert$
3A	16	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	3A	3A	2304
		$f_2 = 1$	(1, 0, 1, 0, 0, 1)	(1, 0, 1, 0, 0, 1)	6A	6A	2304
		$f_3 = 6$	(0, 0, 0, 1, 0, 0)	(0, 0, 0, 0, 1, 1)	6B	6B	384
		$f_4 = 8$	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	6C	6C	288
3B	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	3B	3B	144
		$f_2 = 1$	(1, 0, 1, 0, 0, 1)	(1, 0, 1, 0, 0, 1)	6D	6D	144
		$f_3 = 2$	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	6E	6E	72
4A	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AD	8A	1536
		$f_2 = 3$	(1, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 0)	4AE	8B	512
		$f_3 = 4$	(1, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	8A	4AJ	384
4B	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AF	8C	1536
		$f_2 = 3$	(0, 1, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AG	8D	512
		$f_3 = 4$	(1, 0, 0, 0, 0, 0)	(1, 0, 1, 0, 0, 1)	8B	4AK	384
4C	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AH	4AL	512
		$f_2 = 1$	(1, 0, 0, 1, 0, 0)	(0, 0, 0, 0, 0, 0)	4AI	4AM	512
		$f_3 = 2$	(1, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AJ	4AN	256
4D	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AK	4AO	512
		$f_2 = 1$	(1, 0, 0, 1, 0, 0)	(0, 0, 0, 0, 0, 0)	4AL	4AP	512
		$f_3 = 2$	(0, 0, 1, 0, 0, 1)	(0, 0, 0, 0, 0, 0)	4AM	4AQ	256
4E	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4A	4AR	256
		$f_2 = 1$	(1, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AO	4AS	256
		$f_3 = 1$	(1, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 0)	4AP	4AT	256
		$f_4 = 1$	(1, 0, 0, 0, 1, 0)	(0, 0, 0, 0, 0, 0)	4AQ	4AU	256
4F	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AR	8E	128
		$f_2 = 1$	(1, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	8C	4AV	128
		$f_3 = 1$	(0, 1, 0, 0, 0, 0)	(1, 0, 1, 1, 1, 0)	8D	4AW	128
		$f_4 = 1$	(1, 1, 0, 0, 0, 0)	(1, 0, 1, 1, 1, 0)	4AS	8F	128
4G	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AT	8G	128
		$f_2 = 1$	(0, 0, 0, 1, 1, 0)	(1, 0, 1, 0, 0, 1)	8E	4AX	128
		$f_3 = 1$	(1, 1, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 0)	4AU	8H	128
		$f_4 = 1$	(1, 1, 0, 1, 1, 1)	(1, 0, 1, 0, 0, 1)	8F	4AY	128
4H	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AV	8I	128
		$f_2 = 1$	(1, 0, 0, 0, 1, 0)	(1, 0, 1, 0, 1, 1)	4AW	8J	128
		$f_3 = 1$	(0, 1, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 0)	8G	4AZ	128
		$f_4 = 1$	(1, 1, 0, 0, 1, 1)	(1, 0, 1, 0, 1, 1)	8H	4BA	128
4I	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4AX	8K	256
		$f_2 = 1$	(1, 0, 1, 0, 0, 1)	(0, 0, 0, 0, 0, 0)	4AY	8L	256
		$f_3 = 2$	(1, 0, 0, 0, 1, 0)	(1, 0, 1, 0, 1, 1)	4AZ	8M	128
		$f_4 = 2$	(1, 1, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	8I	4BB	128
		$f_5 = 2$	(1, 1, 0, 0, 1, 1)	(0, 0, 1, 0, 0, 0)	8J	4BC	128
4J	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	4BA	8N	256
		$f_2 = 1$	(1, 0, 1, 1, 1, 0)	(0, 0, 0, 0, 0, 0)	4BB	8O	256
		$f_3 = 2$	(1, 0, 0, 0, 0, 0)	(0, 0, 0, 1, 1, 1)	4BC	8P	128
		$f_4 = 2$	(1, 1, 1, 1, 0, 0, 1)	(0, 0, 0, 1, 1, 1)	8K	4BD	128
		$f_5 = 2$	(1, 1, 0, 0, 1, 1)	(0, 0, 0, 0, 0, 0)	8L	4BE	128
5A	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	5A	5A	40
		$f_2 = 1$	(1, 0, 1, 0, 0, 1)	(1, 0, 1, 0, 0, 1)	10A	10A	40
		$f_3 = 2$	(1, 0, 0, 0, 0, 0)	(1, 1, 0, 0, 0, 0)	10B	10B	20

Table 5 (continued)

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[\overline{g}]_{M_1}$	$[\overline{g}]_{\overline{G}}$	$\lvert C_{M_1}(\overline{g})\rvert$ and $\lvert C_{\overline{G}}(\overline{g})\rvert$
6A	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6F	12A	1152
		$f_2 = 3$	(1, 1, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 0)	6G	12B	384
		$f_3 = 4$	(1, 0, 0, 0, 0, 0)	(1, 0, 1, 0, 0, 1)	12A	6F	288
6B	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6H	6G	192
		$f_2 = 1$	(1, 0, 1, 0, 1, 1)	(1, 0, 1, 0, 0, 1)	12B	12C	192
		$f_3 = 2$	(1, 0, 1, 1, 0, 1)	(0, 1, 1, 0, 0, 0)	12C	12D	96
6C	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6I	12E	192
		$f_2 = 1$	(1, 0, 1, 0, 1, 1)	(1, 0, 1, 0, 0, 1)	12D	6H	192
		$f_3 = 2$	(1, 0, 1, 1, 1, 1)	(0, 1, 1, 0, 1, 0)	12E	12F	96
6D	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6J	12G	72
		$f_2 = 1$	(0, 0, 1, 0, 0, 1)	(0, 0, 1, 0, 0, 1)	12F	6I	72
6E	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6K	12H	192
		$f_2 = 1$	(1, 0, 1, 0, 0, 1)	(0, 0, 0, 0, 0, 0)	6L	12I	192
		$f_3 = 2$	(1, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6M	12J	96
		$f_4 = 2$	(1, 0, 1, 0, 1, 1)	(0, 1, 1, 0, 0, 0)	12G	6J	96
		$f_5 = 2$	(1, 0, 0, 0, 1, 0)	(0, 1, 1, 0, 0, 0)	12H	6K	96
6F	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6N	12K	96
		$f_2 = 1$	(1, 0, 0, 0, 0, 0)	(0, 1, 1, 0, 0, 0)	12I	6L	96
		$f_3 = 1$	(1, 0, 1, 0, 1, 1)	(1, 1, 0, 0, 0, 1)	12J	12L	96
		$f_4 = 1$	(1, 0, 0, 0, 1, 0)	(1, 0, 1, 0, 0, 1)	12K	12M	96
6G	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6O	12N	24
		$f_2 = 1$	(1, 0, 0, 0, 0, 0)	(1, 0, 1, 0, 0, 1)	12L	6M	24
6H	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6P	6N	48
		$f_2 = 1$	(1, 1, 1, 0, 1, 1)	(0, 0, 0, 0, 0, 0)	6Q	6O	48
		$f_3 = 2$	(1, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	6R	6P	24
8A	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	8M	8Q	32
		$f_2 = 1$	(1, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	8N	8R	32
8B	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	8O	8S	32
		$f_2 = 1$	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	8P	8T	32
10A	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	10C	20A	20
		$f_2 = 1$	(1, 0, 0, 0, 0, 0)	(1, 0, 1, 0, 0, 1)	20A	10C	20
12A	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	12M	24A	48
		$f_2 = 1$	(1, 0, 0, 0, 0, 0)	(1, 0, 1, 0, 0, 1)	24A	12O	48
12B	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0)	12N	24B	48
		$f_2 = 1$	(1, 0, 0, 0, 0, 0)	(1, 0, 1, 0, 0, 1)	24B	12P	48

6. The power maps of the elements of $2^6{:}(2^5{:}S_6)$

By restricting some ordinary characters of $\overline{G}_3$ to $\overline{G}$ and also computing the structure constants (using GAP) for the set $Irr(\overline{G})$, we ensure that the consistency checks of Programme E [22] for the set $Irr(\overline{G})$ are satisfied. The information about the conjugacy classes found in Table 5 can be used to compute the power maps for the elements of $\overline{G}$ and Programme E is used to confirm that the character table of $\overline{G}$ produces the unique p-powers listed in Table 6.

Table 6. The power maps of the elements of $2^{6\cdot}(2^5{:}S_6)$.

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	$[g]_G$	$[x]_{\overline{G}}$	2	3	5	$[g]_G$	$[x]_{\overline{G}}$	2	3	5	$[g]_G$	$[x]_{\overline{G}}$	2	3	5
1A	1A				2A	4A	2A			4F	8E	4F			4G	8G	4AA		
	2A	1A				4B	2A				4AV	2G				4AX	2Q		
	2B	1A				2D	1A				4AW	2G				8H	4Z		
	2C	1A									8F	4E				4AY	2Q		
2B	2E	1A			2C	4E	2A			4H	8I	4AA			4I	8K	4Z		
	2F	1A				4F	2A				8J	4Z				8L	4Z		
	4C	2A				2G	1A				4AZ	2Q				8M	4Z		
	4D	2B				4G	2B				4BA	2Q				4BB	2Q		
																4BC	2Q		
2D	4H	2B			2E	4L	2B			4J	8N	4Z			5A	5A			1A
	4I	2B				4M	2B				8O	4Z				10A	5A		2A
	4J	2B				4N	2B				8P	4Z				10B	5A		2C
	4K	2A				4O	2B				4BD	2Q							
	2H	1A				2I	1A				4BE	2Q							
						2J	1A												
2F	2K	1A			2G	4R	2A			6A	12A	6A	4A		6B	6G	3A	2E	
	2L	1A				2N	1A				12B	6A	4B			12C	6A	4C	
	2M	1A				4S	2B				6F	3A	2D			12D	6B	4D	
	4P	2B				4T	2B												
	4Q	2B																	
2H	4U	2B			2I	4Z	2B			6C	12E	6A	4E		6D	12G	6D	4A	
	4V	2B				4AA	2B				6H	3A	2G			6I	3B	2D	
	4W	2A				2P	1A				12F	6B	4G						
	2O	1A				4AB	2B												
	4X	2B				2Q	1A												
	4Y	2B				4AC	2B												
						4AD	2B												
2J	4AE	2B			3A	3A		1A		6E	12H	6B	4L		6F	12K	6B	4H	
	4AF	2A				6A	3A	2A			12I	6B	4M			6L	3A	2H	
	2R	1A				6B	3A	2B			12J	6B	4O			12L	6B	4J	
	4AG	2B				6C	3A	2C			6J	3A	2I			12M	6A	4K	
	4AH	2B									6K	3A	2J						
	4AI	2B																	
3B	3B		1A		4A	8A	4E			6G	12N	6D	4R		6H	6N	3B	2K	
	6D	3B	2A			8B	4E				6M	3B	2N			6O	3B	2L	
	6E	3B	2C			4AJ	2G									6P	3B	2M	
4B	8C	4E			4C	4AL	2E			8A	8Q	4AL			8B	8S	4AO		
	8D	4E				4AM	2F				8R	4AM				8T	4AP		
	4AK	2G				4AN	2F												
4D	4AO	2E			4E	4AR	2E			10A	20A	10A		4A	12A	24A	12E	8A	
	4AP	2F				4AS	2F				10C	5A		2D		12O	6H	4AJ	
	4AQ	2F				4AT	2F												
						4AU	2F												
										12B	24B	12E	8C						
											12P	6H	4AK						

7. The fusion of $2^{6\cdot}(2^5{:}S_6)$ into $2^{6\cdot}SP_6(2)$

By making use of the values of ϕ_1, ϕ_2, ϕ_3 and ϕ_4 on the classes of $2^{6\cdot}Sp_6(2)$, the values of $(\phi_1)_{2^{6\cdot}(2^5\cdot S_6)}$, $(\phi_2)_{2^{6\cdot}(2^5:S_6))}$, $(\phi_3)_{2^{6\cdot}(2^5:S_6)}$ and $(\phi_4)_{2^{6\cdot}(2^5:S_6)}$ on the classes of $2^{6\cdot}(2^5{:}S_6)$, Table 4 and the permutation character $\chi(2^{6\cdot}Sp_6(2)|2^{6\cdot}(2^5{:}S_6)) = 1a + 27a + 35a$ of degree 63 of $2^{6\cdot}Sp_6(2)$ acting on $2^{6\cdot}(2^5{:}S_6)$, the complete fusion map of $2^{6\cdot}(2^5{:}S_6)$ into $2^{6\cdot}Sp_6(2)$ is computed and is given in Table 7.

Table 7. The fusion of $2^{6\cdot}(2^5{:}S_6)$ into $2^{6\cdot}Sp_6(2)$.

$[g]_{(2^5{:}S_6)}$	$[x]_{2^{6\cdot}(2^5{:}S_6)}$ $\longrightarrow$	$[y]_{2^{6\cdot}Sp_6(2)}$	$[g]_{(2^5{:}S_6)}$	$[x]_{2^{6\cdot}(2^5{:}S_6)}$ $\longrightarrow$	$[y]_{2^{6\cdot}Sp_6(2)}$
1A	1A	1A	2A	4A	4A
	2A	2A		4B	4B
	2B	2A		2D	2B
	2C	2A			
2B	2E	2C	2C	4E	4D
	2F	2D		4F	4E
	4C	4C		2G	2E
	4D	4C		4G	4F
2D	4H	4D	2E	4L	4A
	4I	4E		4M	4B
	4J	4E		4N	4B
	4K	4F		4O	4B
	2H	2E		2I	2B
				2J	2B
2F	2K	2C	2G	4R	4G
	2L	2D		2N	2F
	2M	2D		4S	4H
	4P	4C		4T	4I
	4Q	4C			
2H	4U	4G	2I	4Z	4D
	4V	4H		4AA	4E
	4W	4I		2P	2E
	2O	2F		4AB	4E
	4X	4H		2Q	2E
	4Y	4I		4AC	4F
				4AD	4F
2J	4AE	4G	3A	3A	3A
	4AF	4H		6A	6A
	2R	2F		6B	6A
	4AG	4I		6C	6A
	4AH	4I			
	4AI	4H			
3B	3B	3C	4A	8A	8A
	6D	6B		8B	8B
	6E	6B		4AJ	4L
4B	8C	8C	4C	4AL	4N
	8D	8D		4AM	4O
	4AK	4M		4AN	4P
4D	4AO	4J	4E	4AR	4N
	4AP	4K		4AS	4O
	4AQ	4K		4AT	4P
				4AU	4P
4F	8E	8E	4G	8G	8E
	4AV	4R		4AX	4R
	4AW	4Q		8H	8F
	8F	8F		4AY	4Q
4H	8I	8E	4I	8K	8A
	8J	8F		8L	8B
	4AZ	4Q		8M	8B
	4BA	4R		4BB	4L
				4BC	4L
4J	8N	8C	5A	5A	5A
	8O	8D		10A	10A
	8P	8D		10B	10A
	4BD	4M			
	4BE	4M			
6A	12A	12A	6B	6G	6D
	12B	12B		12C	12C
	6F	6C		12D	12C
6C	12E	12D	6D	12G	12F
	6H	6F		6I	6G
	12F	12R			
6E	12H	12A	6F	12K	12D
	12I	12B		6L	6F
	12J	12B		12L	12E
	6J	6C		12M	12E
	6K	6C			
6G	12N	12G	6H	6N	6H
	6M	6J		6O	6I
				6P	6I
8A	8Q	8G	8B	8S	8I
	8R	8H		8T	8J
10A	20A	20A	12A	24A	24A
	10C	10B		12O	12H
12B	24B	24B			
	12P	12I			

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Conflict of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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