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Research article

Effect of rigid boundary on Rayleigh wave in an incompressible heterogeneous medium over an incompressible half-space

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Abstract: In the present problem, an attempt has been made to study the propagation of Rayleigh waves in an incompressible medium with polynomial variation (m) of rigidity over an incompressible half-space under rigid layer. Instead of using the Whittaker function, the expansion formula proposed by Newlands has been used for a better result in shallow depth. The velocity equation has been calculated and the results are shown in figures. The study in the assumed medium, the authors obtained that the phase velocity of Rayleigh waves increases except for the polynomial variation of rigidity m=1, 2 and 3.

Keywords: heterogeneity; incompressible; rigidity; rigid boundary; wave propagation
Mathematics Subject Classification: 86A15

1. Introduction

The study of the propagation of Rayleigh waves in the layered non-homogeneous earth is central interested in theoretical seismologists. As a result of intensive studies of many authors, a good amount of information about the propagation of Rayleigh waves with different variations in density and rigidity is available. A book of Ewing, Jardetzky and Press (1957) [1], Achenbach (1973) [2], Miklowitz (1978) [3], Pilant (1979) [4] and many other different books contains a vast amount of information on the propagation of Rayleigh waves. After these books were published, many publications on these topics are available in different journals. Wang et al. (2008) [5] studied the propagation and localization of Rayleigh waves in disordered piezoelectric phononic-crystals. Liu and Liu (2004) [6] studied the propagation characteristics of Rayleigh waves in orthotropic fluid-saturated porous media. Abd-All et al. (2011) [7] studied the propagation of Rayleigh waves in generalized magneto-thermo-elastic orthotropic material under initial stress and gravity field. Rayleigh wave propagation in layered heterogeneous media has been studied in detail by Wilson (1942) [8], Biot (1965) [9], Newlands (1950) [10] and Stonely (1934) [11]. Dutta (1963) [12] illustrated Rayleigh wave propagation in a two-layer anisotropic media whereas propagation of Rayleigh wave in an elastic half-space of orthotropic and viscoelastic material has been discussed by Abd-Alla (1999) [13] and Abd-Alla (2015) [14]. Sharma and Mohinder (2004) [15] studied Rayleigh-lamb waves in the magneto-thermo-elastic homogeneous isotropic plate. Mitra (1957) [16] studied Rayleigh waves in multi-layered media. In some of the studies reported their solution is expressed in Whittaker function (1990) [17] and the computation is involved with the asymptotic expansion of the function. The effects of tidal waves are studied by different authors such as Z. Hu et al. (2015,2015,2017) [18–20] , Chen et al. (2018) [21] and Suzuki et al. (2019) [22]. The expansion is valid on larger depth but does not give good result in shallow depth. Newlands (1950) [10] found a new approach to the problem and obtained a solution that may be used in shallow depth. Following Newlands, in this paper attempt has been made to study the effect of rigid boundary on the propagation of Rayleigh wave in an incompressible heterogeneous medium over an incompressible half-space. The non-homogeneity has been taken in variation in the rigidity.

In this paper, the author dealt with the effect of the rigid boundary of propagation on Rayleigh waves in an incompressible heterogeneous layer over an initially stressed incompressible half-space. The heterogeneity has been taken in rigidity as and density as and in the upper layer. The equation of the upper layer has been solved by the Newlands method (1950).

2. Formulation of the problem

Suppose an incompressible medium of thickness H under rigid layer with shear modulus rigidity $\mu = \mu_0(1 + bz)^m$ and density $\rho = \rho_0$ lying over another incompressible half-space with constant shear modulus μ_2 and density ρ_2, being taken as the vertical distance from the origin at the interface of an incompressible medium and incompressible half-space whereas the rigid surface exits at $z = -H$. The downward direction of z has been taken as positive in Figure 1.

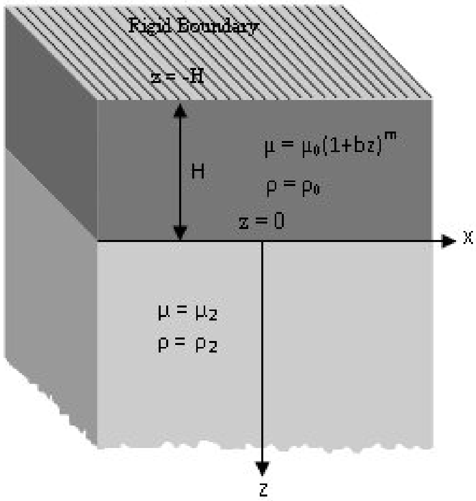

Figure 1. Geometry of the problem.

Consider the wave propagating along x-axis with wave velocity C and wave-length $\frac{2\pi}{k}$. Let u and w are the displacement component in the x- and z- direction respectively at the point (x,y,z) at any time. Suppose that apart from a factor $e^{ik(x-Ct)}$ and u ,w are functions of z only.

3. Equation of motion and solution

3.1. Equation of motion and solution for upper layer

The equation of motion in two diminutions for an elastic solid are

$$\begin{aligned} &\frac{\partial}{\partial x}\left\{\lambda\Delta + 2\mu\frac{\partial u}{\partial x}\right\} + \frac{\partial}{\partial z}\left\{\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right\} = \rho\frac{\partial^2 u}{\partial t^2} \\ &\frac{\partial}{\partial z}\left\{\lambda\Delta + 2\mu\frac{\partial w}{\partial z}\right\} + \frac{\partial}{\partial x}\left\{\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right\} = \rho\frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{1}$$

where, since the medium is incompressible,

$$\begin{aligned} &\Delta = \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) = 0 \\ &\qquad \nabla^2\varphi = 0 \end{aligned} \tag{2}$$

Substituting,

$$\begin{aligned} u &= \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z} \\ w &= \frac{\partial \phi}{\partial z} - \frac{\partial \psi}{\partial x} \end{aligned} \tag{3}$$

where ϕ and ψ being scalar and vector potentials, by $\mu = \mu_0(1 + bz)^m$,incompressibility condition $\Delta = 0$, together with $lim_{\lambda\to\infty,\Delta\to 0}\lambda\Delta \to P_1$,whereP_1 the hydrostatic stress, Eq (1) takes the form as

Again, substituting,

$$\begin{aligned}\frac{\partial}{\partial x}[-P_1 + 2\mu_0 bm(1+bz)^{m-1}w + \rho k^2C^2\phi] + \frac{\partial}{\partial z}[\mu\nabla^2\psi + \rho k^2C^2\psi] = 0\\ \frac{\partial}{\partial z}[-P_1 + 2\mu_0 bm(1+bz)^{m-1}w + \rho k^2C^2\phi] - \frac{\partial}{\partial x}[\mu\nabla^2\psi + \rho k^2C^2\psi] = 0\end{aligned} \tag{4}$$

which are satisfied by

$$\begin{aligned}P_1 = 2\mu_0 bm(1+bz)^{m-1}w + \rho k^2C^2\phi\\ \mu\nabla^2\psi = \rho k^2C^2\psi\end{aligned} \tag{5}$$

From Eq (5)

$$\begin{aligned}\frac{\partial^2\psi}{\partial z^2} + k^2\left[\frac{\rho c^2}{\mu_0(1+bz)^m} - 1\right]\psi(z) = 0\\ \frac{\partial^2\phi}{\partial z^2} - k^2\Phi = 0\end{aligned} \tag{6}$$

Taking $Z = (1+bz)$ in Eq (6), we have

$$\frac{d^2\psi}{dZ^2} - \left(\frac{k}{b}\right)^2\left(1 - \frac{\rho C^2}{\mu_0 Z^m}\right)\psi = 0 \tag{7}$$

In power of $\left(\frac{k}{b}\right)^2$, the series solution of Eq (7), may be written as

$$\psi(Z) = \psi_0(Z) + \left(\frac{k}{b}\right)^2\psi_1(Z) + \ldots\ldots + \left(\frac{k}{b}\right)^{2n}\psi_n(Z) + \ldots \tag{8}$$

Hence,

$$\left[\psi_0''(Z) + \left(\frac{k}{b}\right)^2\psi_1''(Z) + \ldots\ldots\right] - \left(\frac{k}{b}\right)^2\left[1 - \frac{\rho C^2}{\mu_0 Z^m}\right]\left[\psi_0(Z) + \left(\frac{k}{b}\right)^2\psi_1(Z) + \ldots..\right] = 0 \tag{9}$$

Thus,if

$$\begin{aligned}\psi_0''(Z) = 0\\ \psi_1''(Z) = \left[1 - \frac{\rho C^2}{\mu_0 Z^m}\right]\psi_0\\ .\\ .\\ \psi_{n+1}''(Z) = \left[1 - \frac{\rho C^2}{\mu_0 Z^m}\right]\psi_n(Z)\end{aligned} \tag{10}$$

Since the series [Eq (9)] converges and involves with two constants, then it is a valid solution. We have,

$$\psi_n'(1) = \psi_n(1) = 0 \tag{11}$$

$$\psi_n'(Z) = \frac{A_2}{b}, \psi_0(Z) = A_1 + \frac{A_2}{b}(Z-1) = A_1 + A_2 z \tag{12}$$

Due to linearity of Eq (7), we have

$$\psi = A_1\left[\psi_0^{(1)}(Z) + \left(\frac{k}{b}\right)^2 \psi_1^{(1)}(Z) + ...\right] + A_2\left[\psi_0^{(2)}(Z) + \left(\frac{k}{b}\right)^2 \psi_1^{(2)}(Z) + ...\right] = A_1\psi^1 + A_2\psi^2 \tag{13}$$

Where,

$$\begin{aligned} \psi_0^{(1)} &= 1 \\ \psi_0^{(2)} &= \frac{Z-1}{b} = z \end{aligned} \tag{14}$$

Now,

$$\psi_1 = \int_1^Z d\xi \int_1^\xi \left(1 - \frac{\rho C^2}{\mu_0 t^m}\right)\psi_0(t)dt = \int_1^Z d\xi \int_1^\xi \left(1 - \frac{\rho C^2}{\mu_0 t^m}\right)\left[A_1 + \frac{A_2}{b}(t-1)\right]dt$$
$$= A_1\left\{\frac{(Z-1)^2}{2} - \frac{\rho C^2}{\mu_0}[ZlogZ - (Z-1)]\right\} + A_2\left\{\frac{(Z-1)^3}{2} - \frac{\rho C^2}{\mu_0}\left[\frac{Z^2}{2} - Z + \frac{1}{2} - (ZlogZ - (Z-1))\right]\right\} \tag{15}$$

And the solution of $\psi^{(1)}$ and $\psi^{(2)}$ are readily obtained as follows:
Case I : When m=1

$$\psi_1 = A_1\left\{\frac{(Z-1)^2}{2} - \frac{\rho C^2}{\mu_0}[ZlogZ - (Z-1)]\right\} + \frac{A_2}{b}\left\{\frac{(Z-1)^3}{2} - \frac{\rho C^2}{\mu_0}\left[\frac{1}{2} - \frac{Z^2}{2} + ZlogZ\right]\right\} \tag{16}$$

The solutions are

$$\begin{aligned} \psi_Z^{(1)} &= 1 + \left(\frac{k}{b}\right)^2\left[\frac{(Z-1)^2}{2} - \frac{\rho C^2}{\mu_0}\{ZlogZ - (Z-1)\}\right] \\ \psi_Z^2 &= (Z-1) + \left(\frac{k}{b}\right)^2\left[\frac{(Z-1)^3}{6} - \frac{\rho C^2}{\mu_0}\left\{\frac{1}{2} - \frac{Z^2}{2} + ZlogZ\right\}\right] \end{aligned} \tag{17}$$

Case II: when m=2

$$\psi_1 = A_1\left[\frac{(Z-1)^2}{2} - \frac{\rho C^2}{\mu_0}\{logZ - (Z-1)\}\right] + \frac{A_2}{b}\left[\frac{(Z-1)^3}{6} - \frac{\rho C^2}{\mu_0}\{(Z+1)logZ + 2(1-Z)\}\right] \tag{18}$$

The solution are

$$\begin{aligned} \psi_Z^{(1)} &= 1 + \left(\frac{k}{b}\right)^2\left[\frac{(Z-1)^2}{2} - \frac{\rho C^2}{\mu_0}\{logZ - (Z-1)\}\right] \\ \psi_Z^{(2)} &= (Z-1) + \left(\frac{k}{b}\right)^2\left[\frac{(Z-1)^3}{6} - \frac{\rho C^2}{\mu_0}\{(Z+1)logZ + 2(1-Z)\}\right] \end{aligned} \tag{19}$$

Case III: when m=3

$$\psi_1 = A_1\left[\frac{(Z-1)^2}{2}+\frac{\rho C^2}{\mu_0}\left\{1-\frac{Z}{2}-\frac{1}{2Z}\right\}\right]+\frac{A_2}{b}\left[\frac{(Z-1)^3}{6}+\frac{\rho C^2}{\mu_0}\left\{logZ+\frac{1}{2Z}-\frac{Z}{2}\right\}\right] \tag{20}$$

The solution are

$$\begin{aligned}\psi_Z^{(1)} &= 1+\left(\frac{k}{b}\right)^2\left[\frac{(Z-1)^2}{2}+\frac{\rho C^2}{\mu_0}\left\{1-\frac{Z}{2}-\frac{1}{2Z}\right\}\right]\\ \psi_Z^{(2)} &= (Z-1)+\left(\frac{k}{b}\right)^2\left[\frac{(Z-1)^3}{6}+\frac{\rho C^2}{\mu_0}\left\{logZ+\frac{1}{2Z}-\frac{Z}{2}\right\}\right]\end{aligned} \tag{21}$$

Case IV: when any value of m except 1,2 and 3

$$\begin{aligned}\psi_1 = A_1\left[\frac{(Z-1)^2}{2}+\frac{\rho C^2}{\mu_0}\frac{1}{(1-m)}\left\{(Z-1)+\frac{1}{(2-m)}\left(1-\frac{1}{Z^{m-2}}\right)\right\}\right]\\ +\frac{A_2}{b}\left[\frac{(Z-1)^3}{6}+\frac{\rho C^2}{\mu_0}\frac{1}{(2-m)}\left\{(Z-1)+\frac{1}{(3-m)}\left(1-\frac{1}{Z^{m-3}}\right)\right\}+\frac{\rho C^2}{\mu_0}\frac{1}{(1-m)}\left\{(1-Z)+\frac{1}{(2-m)}\left(\frac{1}{Z^{m-2}}-1\right)\right\}\right]\end{aligned} \tag{22}$$

The solutions are

$$\psi_Z^{(1)} = 1+\left(\frac{k}{b}\right)^2\left[\frac{(Z-1)^2}{2}+\frac{\rho C^2}{\mu_0}\frac{1}{(1-m)}\left\{(Z-1)+\frac{1}{(2-m)}\left(1-\frac{1}{Z^{m-2}}\right)\right\}\right] \tag{23}$$

$$\begin{aligned}\psi_Z^{(2)} = (Z-1)+\left(\frac{k}{b}\right)^2\Bigg[\frac{(Z-1)^3}{6}+\frac{\rho C^2}{\mu_0}\frac{1}{(2-m)}\left\{(Z-1)+\frac{1}{(3-m)}\left(1-\frac{1}{Z^{(m-3)}}\right)\right\}\\ +\frac{\rho C^2}{\mu_0}\frac{1}{(1-m)}\left\{(1-Z)+\frac{1}{(2-m)}\left(\frac{1}{Z^{(m-2)}}-1\right)\right\}\Bigg]\end{aligned} \tag{24}$$

Hence the final solutions are

$$\begin{aligned}\phi &= [Q_1cosh(kz)+Q_2sinh(kz)]cosk(x-Ct)\\ \psi &= [A_1\psi^{(1)}(z)+A_2\psi^{(2)}(z)]sink(x-Ct)\end{aligned} \tag{25}$$

Hence the displacement and stress component along x- and z- direction in the incompressible layer sandwiched between the rigid surface and a half-space are given by

$$\begin{aligned}u &= \left[-k(Q_1cosh(kz)+Q_2sinh(kz))+\left(A_1\frac{\partial}{\partial z}\psi^{(1)}(Z)+A_2\frac{\partial}{\partial z}\psi^{(2)}(Z)\right)\right]sink(x-Ct)\\ w &= \left[k(Q_1sinh(kz)+Q_2cosh(kz))+k\left(A_1\psi^{(1)}(Z)+A_2\psi^{(2)}(Z)\right)\right]cosk(x-Ct)\\ \sigma_{xz} &= 2k\left[-\mu\frac{\partial\phi}{\partial z}+k\left(\mu-\frac{\varepsilon\mu_0}{2}\right)\psi\right]\\ \sigma_{zz} &= 2\left[k^2\phi\left(\mu-\frac{\varepsilon\mu_0}{2}\right)\psi-\mu_0bm(1+bz)^{m-1}\frac{\partial\phi}{\partial z}+\mu_0bm(1+bz)^{m-1}k\psi-\mu k\frac{\partial\psi}{\partial z}\right]\end{aligned} \tag{26}$$

where, $\varepsilon = \dfrac{C^2\rho}{\mu_0}$

3.2. Solution for the homogeneous half-space

The values of ϕ and ψ as

$$\begin{aligned} \phi &= Re^{-kz} cosk(x - Ct) \\ \psi &= S e^{-knz} sink(x - Ct) \end{aligned} \tag{27}$$

Where, $n = \left(1 - \frac{\rho_2 C^2}{\mu_2}\right)$

Hence the displacement and stress component along x- and z- direction in the incompressible layer sandwiched between the rigid surface and a half space are given by

$$\begin{aligned} u &= \left[-kRe^{-kz} - S kne^{-knz}\right] sink(x - Ct) \\ w &= \left[-kRe^{-kz} - S ke^{-knz}\right] cosk(x - Ct) \\ \sigma_{xz} &= 2k\left[-\mu\frac{\partial\phi}{\partial z} + k\left(\mu - \frac{\varepsilon\delta\mu_0}{2}\right)\psi\right] \\ \sigma_{zz} &= 2\left[k^2\phi\left(\mu - \frac{\varepsilon\delta\mu_0}{2}\right)\psi - \mu k\frac{\partial\psi}{\partial z}\right] \end{aligned} \tag{28}$$

where, $\delta = \frac{\rho_2}{\rho_0}$

3.3. Boundary conditions and dispersion relation

From the Figure 1 we can use the following boundary conditions:
(i) At $z = 0$, the displacements are continuous
$u_1 = u_2$
$w_1 = w_2$
(ii) At the interface z=0 , the continuity of the stress requires that
$(\sigma_{xz})_1 = (\sigma_{xz})_2$
$(\sigma_{zz})_1 = (\sigma_{zz})_2$
(iii) At z=-H , the displacement vanishes
$u_1 = 0$
$w_1 = 0$

By using the boundary condition (i) and (ii), we have

$$\begin{aligned} -kQ_1 + A_2 b &= -kR - knS \\ Q_2 - A_1 &= -R - S \\ -Q_2 + \left(1 - \frac{\varepsilon}{2}\right)A_1 &= \frac{\mu_2 R}{\mu_0} + \left(\frac{\mu_2}{\mu_0} - \frac{\varepsilon\delta}{2}\right)S \\ kQ_1\left(1 - \frac{\varepsilon}{2}\right) - bmQ_2 + bmA_1 - bA_2 &= kR\left(\frac{\mu_2}{\mu_0} - \frac{\varepsilon\delta}{2}\right) + \frac{\mu_2}{\mu_0}knS \end{aligned} \tag{29}$$

By using the boundary condition (iii), we have

$$\begin{aligned} (Q_1 cosh(kz) + Q_2 sinh(kz)) - \frac{1}{k}\left(A_1 \frac{\partial}{\partial z}\psi^{(1)}(Z) + A_2 \frac{\partial}{\partial z}\psi^{(2)}(Z)\right) = 0 \\ k(Q_1 sinh(kz) + Q_2 cosh(kz)) + k\left(A_1\psi^{(1)}(Z) + A_2\psi^{(2)}(Z)\right) = 0 \end{aligned} \tag{30}$$

From the Eqs (29) and (30), we have the relation between Q_1, Q_2, A_1 and A_2

$$\begin{aligned} \frac{1}{2}\varepsilon Q_1 &= D_1 R + E_1 S \\ \frac{1}{2}\varepsilon Q_2 &= D_2 R + E_2 S \\ \frac{1}{2}\varepsilon A_1 &= D_3 R + E_3 S \\ \frac{1}{2}\varepsilon A_2 &= D_4 R + E_4 S \end{aligned} \tag{31}$$

where

$$\begin{aligned} D_1 &= \frac{1}{k}\left[k - \frac{bmD_2}{\varepsilon/2} + \frac{bmD_3}{\varepsilon/2} - k\left(\frac{\mu_2}{\mu_0} - \frac{\varepsilon\delta}{2}\right)\right] \\ D_2 &= D_3 - \frac{\varepsilon}{2} \\ D_3 &= 1 - \frac{\mu_2}{\mu_0} \\ D_4 &= \frac{1}{b}\left(kD_1 - \frac{\varepsilon k}{2}\right) \end{aligned} \tag{32}$$

$$\begin{aligned} E_1 &= \frac{1}{k}\left[kn - \frac{bmD_2}{\varepsilon/2} + \frac{bmD_3}{\varepsilon/2} - \frac{\mu_2}{\mu_0}kn\right] \\ E_2 &= E_3 - \frac{\varepsilon}{2} \\ E_3 &= 1 - \frac{\mu_2}{\mu_0} + \frac{\varepsilon\delta}{2} \\ E_4 &= \frac{1}{b}\left(kE_1 - \frac{\varepsilon kn}{2}\right) \end{aligned} \tag{33}$$

$$\begin{aligned} M_1 &= (coshkz)_{z=-H} \\ M_2 &= (sinhkz)_{z=-H} \\ M_3 &= -\frac{1}{k}\left[\frac{\partial}{\partial z}\psi^{(1)}(Z)\right]_{z=-H} \\ M_4 &= -\frac{1}{k}\left[\frac{\partial}{\partial z}\psi^{(2)}(Z)\right]_{z=-H} \end{aligned} \tag{34}$$

$$\begin{aligned} N_1 &= (sinhkz)_{z=-H} \\ N_2 &= (coshkz)_{z=-H} \\ N_3 &= (\psi^{(1)}(Z))_{z=-H} \\ N_4 &= (\psi^{(2)}(Z))_{z=-H} \end{aligned} \quad (35)$$

The consistency of Eq (30) for non-trivial solution of R and S implies that,

$$\Sigma_{i=1}^{4} D_i M_i \Sigma_{j=1}^{4} E_j N_j = \Sigma_{k=1}^{4} D_k N_k \Sigma_{l=1}^{4} E_l M_l \quad (36)$$

Equation (36) is the required equation for the dispersion of Rayleigh surface wave in an incompressible heterogeneous layer lying over an incompressible homogeneous elastic half-space under rigid layer.

4. Numerical results and discussion of the result

On the above study the authors are used to parametric relation are taken as $bH = 0.5, \frac{\rho_2}{\rho_0} = 1.2, \frac{\mu_2}{\mu_0} = 1.8.$, respectively as given in Table 1. For equation, the dispersion curve of Rayleigh waves has been calculated in Eq (35) for different values of m starting from 0 to 5 and the authors observed that the nature of phase velocity for different values of $\frac{2\pi}{kH}$ in an incompressible heterogeneous under rigid layer. The results are presented in Figures 2 and 3. In Figure 2 for different values of m=1,2 and 3 respectively being shows that in the assumed medium under a rigid layer, Rayleigh wave velocity decreases with increases of $\frac{2\pi}{kH}$.We also observed that in case of linear, quadratic and cubic variation of rigidity in an incompressible heterogeneous under rigid layer as the phase velocity of Rayleigh waves increases as rigidity increases. On the other hand in case in case of the phase velocity of Rayleigh waves increases as $\frac{2\pi}{kH}$ increases under rigid layer. In case the homogeneous incompressible elastic medium under rigid layer the phase velocity of Rayleigh waves more compares to m=0, 0.5, 4 and 5 .

Table 1. Parameters of Figure 2 and 3.

Figure no.	Curve no.	m
	1	1
2	2	2
	3	3
	1	0
	2	0.5
3	3	4
	4	5

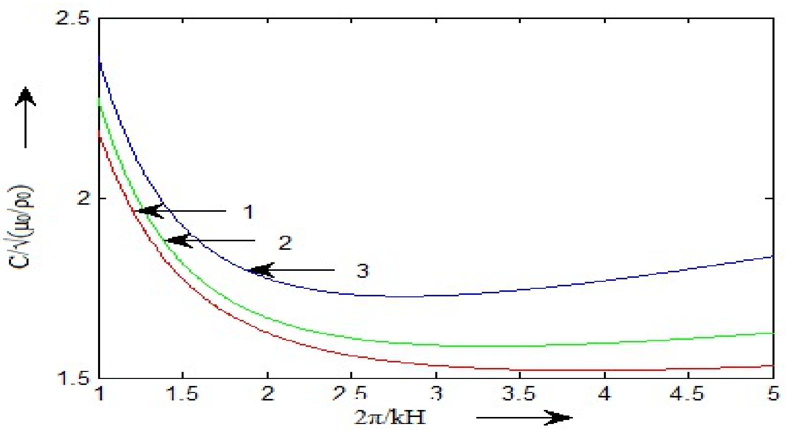

Figure 2. Dispersion curve for Rayleigh wave in an incompressible heterogeneous layer over incompressible homogeneous under rigid layer for m=1, 2 and 3.

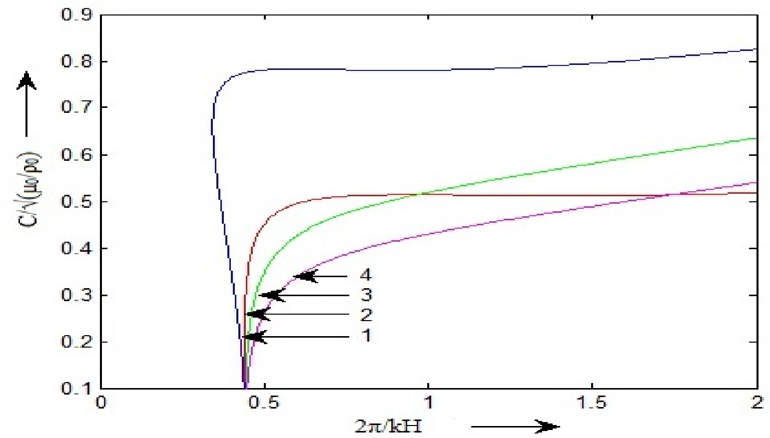

Figure 3. Dispersion curve for Rayleigh wave in an incompressible heterogeneous layer over an incompressible homogeneous under rigid layer for m=0, 0.5, 4 and 5.

5. Conclusions

From the above study, the authors reveals that some of the important facts regarding the propagation of Rayleigh waves in assumed medium and half-space under rigid layer as
(i) The phase velocity of Rayleigh waves in an assumed medium under rigid layer decreases in the linear, quadratic and cubic variation of rigidity only.
(ii)The phase velocity of Rayleigh waves increases for different values of m=0, 0.5, 4 and 5.
(iii)The phase velocity of Rayleigh waves in an assumed medium under a rigid layer decreases only m=1,2 and 3 otherwise increases.

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Conflict of interest

The authors declare no conflict of interest in this paper.

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