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Research article

Optical solitons solutions for perturbed time fractional nonlinear Schrodinger equation via two strategic algorithms

S. Owyed[1], M. A. Abdou[2,3], A. Abdel-Aty[2,4,*] and H. Dutta[5]

[1] Department of Mathematics, College of Sciences, University of Bisha, P. O. Box 344, Bisha, 61922, Saudi Arabia
[2] Department of Physics, College of Sciences, University of Bisha, P. O. Box 344, Bisha, 61922, Saudi Arabia
[3] Physics Department, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt
[4] Physics Department, Faculty of Science, Al-Azhar University, 71524 Assiut, Egypt
[5] Department of Mathematics, Faculty of Science, Gauhati University, Guwahati 781014, India

* **Correspondence:** Email: amabdelaty@ub.edu.sa; Tel: +966533590123.

Abstract: In this work, two algorithms namely, the generalized exp(-w(ξ)) and rational (G'/G^2)-expansion methods are suggested for constructing new optical solitons solutions for the perturbed fractional nonlinear Schrodinger equation. The solutions include hyperbolic, trigonometric or rational function. Our results indicate that, group of new solutions are obtained with much reliability, accuracy and efficiency of the proposed methods. Eventually, our pending may become of wide relevance in addition to realize the main features and even propagation of the nonlinear waves in fractal medium.

Keywords: optical soliton solutions; generalized exp(-w(ξ)) method; extended rational (G'/G^2) expansion method; the perturbed fractional time nonlinear Schrodinger equation; conformable fractional
Mathematics Subject Classification: 35A20, 35A99, 83C15, 65Z05

1. Introduction

Very recently, there are many models that describe the telecommunications industry, namely, nonlinear Schrodinger's equations, Manakov model (GLL) equation, Gerdjikov-Ivanov model (LPD) equation and many other models of special interest in nonlinear optics [1–14]. On the other hand, fractional calculus has a wide array of various applications in nonlinear science [15–45]. Schrodinger equation one of the important equations in the last century and widely used in the area of quantum mechanics and quantum optics. Several methods are used in solving Schrodinger equation including approximate, exact and numerical solutions, starting from the perturbation, variational and WKB methods [46–51]. Recently, there are many methods were introduced to solve this important equation. On the other hand, the Schrodinger equation and its solutions in the fractional from is studied in many papers [28–33]. This paper studies the possibility to find solutions for perturbed fractional time nonlinear Schrodinger equation. In this study, two algorithms schemes are used for constructing the optical solitons solutions to the mentioned model. They are generalized exp(-w(ξ)) method and rational (G'/G^2) expansion method.

The outlines of this work are given as: Section 2 presents the fractional calculus of conformable fractional derivatives and some properties; section 3.A, 3.B introduce the two integrations schemes; sections 4.A, 4.B present the new solitons solutions of Eq (13); and the last section is the conclusion.

2. Preliminaries

The derivative used in this paper is, the conformable derivative of order α and can be defined as:
Defintion: Let g:(0, ∞) → R, flowing are definition, properties and theorem about used conformable derivatives [22]:

$$\text{(a)}\quad U_\alpha (g)(t) = \lim_{\varepsilon \to 0} \left(g(t+ \varepsilon\, t^{1-\alpha}) - g(t)\right) / \varepsilon ,\ t>0, 0< \alpha <1, \tag{1}$$

$$\text{(b)}\ U_\alpha (bg+ch) = B\, U_\alpha (g)+C\, U_\alpha (h),\quad B, C \in R, \tag{2}$$

$$\text{(c)}\ U_\alpha (t^\lambda) = \lambda\, t^{\lambda-\alpha},\ \lambda \in \tag{3}$$

$$\text{(d)}\ U_\alpha (gh)=g\, U_\alpha (h) + h\, U_\alpha (g), \tag{4}$$

$$\text{(e)}\ U_\alpha (g/h)=(h\, U_\alpha (g) - g\, U_\alpha (h))/ h^2 \tag{5}$$

If g is differentiable, then $U_\alpha (g)(t)= t^{1-\alpha} (dg/dh)$.

Theorem: Let, g:(0, ∞) → R be differentiable and α differentiable function, then

$$U_\alpha (g*h) = t^{1-\alpha} h'(t)\, g'(h(t)) \tag{6}$$

The time fractional nonlinear Schrodinger equation with temporal evolution is given in its dimensionless as follows [9].

$$i\frac{\partial^\alpha v}{\partial t^\alpha} + v_{xx} + \gamma v|v|^2 + i[\gamma_1 v_{xxx} + \gamma_2 |v|^2 v_x + \gamma_3 (|v|^2)_x v\,] = 0, t > 0, 0 < \alpha < 1 \tag{7}$$

In Eq (7) γ_1 *represents* dispersion term, γ_2 is nonlinear dispersion and γ_3 is the nonlinear dispersion term.

To gain the optical soliton solutions of the Eq (7). With the aid of the hypothesis as [23–28].

$$v(x,t)=V(\xi),\quad \xi=ax+\frac{bt^{\alpha}}{\Gamma(1+\alpha)} \tag{8}$$

Then Eq (7) reads

$$ibV'+a^2V"+\gamma V|V|^2+i[\gamma_1 a^3V"'+\gamma_2 a|V|^2V'+\gamma_3 a(|V|^2)'V\ ]=0. \tag{9}$$

Where V is a complex function defines as

$$V(\xi)=e^{iL\xi}U(\xi). \tag{10}$$

Where L is a constant, and $U(\xi)$ is a real function. Making use Eq (10), then Eq (9) yields

$$(-bL-a^2L^2+\gamma_1 a^3 l^3)U-(\gamma-\gamma_2 aL)U^3+(a^2-3\gamma_1 a^3 L)U"=0. \tag{11}$$

$$(b+2a^2L-3\gamma_1 a^3L^2)U'+\gamma_1 a^3U"'+(\gamma_2 a+2\gamma_3 a)U^2U'=0. \tag{12}$$

Equation (12), can be rewritten as:

$$(b+2a^2L-3\gamma_1 a^3L^2)U+\gamma_1 a^3U"+\frac{1}{3}(\gamma_2 a+2\gamma_3 a)U^3=0. \tag{13}$$

3. A quick of two algorithms schemes

Given the general fractional nonlinear evolution equation as follows:

$$H\left(v,D_t^{\alpha}v,D_x^{\beta}v,D_{tt}^{2\alpha}v,D_{xx}^{\beta}v,D_t^{\alpha}D_x^{\beta}v,....\right),\ \ 0<\alpha,\ \beta<1, \tag{14}$$

Making use of e fractional complex transformation as:

$$v\,(x,t)=V(\xi),\qquad \xi=\frac{kx^{\beta}}{\Gamma(1+\beta)}+\frac{ct^{\alpha}}{\Gamma(1+\alpha)}, \tag{15}$$

Where k and c are to be determined later, with

$$D_t^{\alpha}v=\sigma_t'\frac{dV}{d\xi}D_t^{\alpha}\xi,\ \ D_x^{\alpha}v=\sigma_x'\frac{dV}{d\xi}D_x^{\alpha}\xi, \tag{16}$$

Where $\sigma_x'=\sigma_t'=l$, where l is a constant. Then Eq (14) becomes

$$Z\left(V,V',V",V"',....\right)=0, \tag{17}$$

3.1. The $exp(-w(\xi))$–expansion method

In view of this method [24], we express the solution of Eq (17) as:

$$U(\xi)=\frac{\sum_{i=0}^{N}\alpha_i(\exp\,(-w(\xi)))^i}{\sum_{j=0}^{M}\beta_j(exp\,(-w(\xi)))^j} \tag{18}$$

Where α_i, β_j are constants to be specified later, and $w(\xi)$ satisfies:

$$w'(\xi) = \exp(-w(\xi)) + \mu \exp(w(\xi)) + \lambda \tag{19}$$

Where λ and μ are constants to be evaluated later. The general solutions of Eq (19) reads

Case I: when $\delta = \lambda^2 - 4\mu > 0$ and $\mu \neq 0$, gain the hyperbolic function solution is

$$w_1(\xi) = \ln\left(\frac{-\sqrt{\delta}\tanh\left(\frac{\sqrt{\delta}}{2}(\xi + c_1)\right) - \lambda}{2\mu}\right) \tag{20}$$

Case II: If $\delta = \lambda^2 - 4\mu < 0$ and, $\mu \neq 0$, then the trigonometric function solution is:

$$w_2(\xi) = \ln\left(\frac{\sqrt{-\delta}\tan\left(\frac{\sqrt{-\delta}}{2}(\xi + c_1)\right) - \lambda}{2\mu}\right) \tag{21}$$

Case III: If $\lambda^2 - 4\mu > 0,\ \mu = 0$ and $\lambda \neq 0$, then the hyperbolic function solution

$$w_3(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + c_1)) - 1}\right) \tag{22}$$

Case IV: If $\lambda^2 - 4\mu = 0,\ \mu \neq 0$ and $\lambda \neq 0,$ gain the rational function solution is

$$w_4(\xi) = \ln\left(-\frac{2(\lambda(\xi + c_1) + 2)}{\lambda^2(\xi + c_1) - 1}\right) \tag{23}$$

Case V: If $\lambda^2 - 4\mu = 0,\ \mu = 0$ and $\lambda = 0$, implies to the rational function solution

$$w_5(\xi) = \ln(\xi + c_1) \tag{24}$$

Where c_1 is constant of integration, $\xi = \frac{kx^{\beta}}{\Gamma(1+\beta)} + \frac{ct^{\alpha}}{\Gamma(1+\alpha)}$.

With the aid of Eq (18) and Eq (19) into Eq (13), yields the coefficients of $\exp(-w(\xi))$ to zero, gives a set of algebraic equations which can be solved to find $\alpha_i, \beta_i\ \mu, \lambda, c, k$. Inserting these values in Eq (18), then the new optical soliton solutions of Eq (14) are obtained.

3.2. The extended rational (G'/G^2) expansion method

In view of the $\left(\frac{G'}{G^2}\right)$-expansion method [36–39]. The quick gain of this method the solution of Eq (17) can be expressed as

$$U(\xi) = \frac{\sum_{i=0}^{N} a_i \psi^i(\xi)}{\sum_{j=0}^{M} b_j \psi^j(\xi)\xi}, \tag{25}$$

Where $\psi(\xi) = \left(\frac{G\prime(\xi)}{G^2(\xi)}\right)$ satisfies

$$\psi'(\xi) = \mu + \lambda\psi^2(\xi), \tag{26}$$

Where $\lambda \neq 0$ and $\mu \neq 1$ and a_0, a_i, b_j to be determined. In terms of the general solutions of Eq (19) which can be classified as:

Case I: Trigonometric function solution for $\lambda\mu > 0$

$$\psi(\xi) = \sqrt{\frac{\mu}{\lambda}} \frac{(k_1 cos\sqrt{\lambda\mu}\xi + k_2 sin\sqrt{\lambda\mu}\xi)}{(k_2 cos\sqrt{\lambda\mu}\xi - k_1 sin\sqrt{\lambda\mu}\xi\xi)}. \tag{27}$$

Case II: Hyperbolic function solution for $\lambda\mu < 0$, then

$$\psi(\xi) = -\sqrt{\frac{|\mu\lambda|}{\lambda}} \frac{(k_1 sinh\sqrt{|\lambda\mu|}\xi + k_2 cosh\sqrt{|\lambda\mu|}\xi)}{(k_1 sinh\sqrt{|\lambda\mu|}\xi + k_1 sinh\sqrt{|\lambda\mu|}\xi\xi)}. \tag{28}$$

Case III: Rational function solution for $\lambda \neq 0$ and $\mu = 0$, then

$$\psi(\xi) = -\frac{k_1}{\lambda(k_1 + k_2\xi)}. \tag{29}$$

Where k_1 and k_2 are constants.

Step 2: The integer N and M in Eq (25) and Eq (18) are determined by balancing between the highest order derivative via

$$D\left(\frac{\partial^q u}{\partial \xi^q}\right) = N - M + q \tag{30}$$

$$D(u^\alpha \left(\frac{\partial^q u}{\partial \xi^q}\right)^\beta) = (N - M)\alpha + \beta(N - M + q) \tag{31}$$

Where α, q, β are real constant.

Step 3: Making use Eq (25) with Eq (26) in Eq (13), collecting the same power $\psi^i(\xi)$, and to zero, we gain a system of algebraic equations, solved it for values of a_i, b_j, c. Then optical soliton solutions of Eq (24) are given.

4. Optical soliton solutions of perturbed time FNSE

4.1. Optical soliton solutions using the generalized exp(-w(ξ)) method

Consequently, to solve Eq (13) using $exp(-w(\xi))$ method [30], considering the balance principle to Eq (13), we obtain $N = M + 1$. By taking M = 1, then N = 2. Then the solution of Eq (13) admits to:

$$U(\xi) = \frac{\alpha_0+\alpha_1\, exp(-w(\xi))+\alpha_2\, exp(-2w(\xi))}{\beta_0+\beta_1\, exp(-w(\xi))} \tag{32}$$

Inserting Eq (32) into Eq (13) with Eq (19), collecting all power of $exp(-\varphi(\xi))$, and using symbolic computation program gives,

Case I:

$$L = L, a = a, b = \frac{1}{2}(-4L - 4\gamma_1 a\mu + 6\gamma_1 aL^2 + \gamma_1 a\lambda^2)a^2,\ \alpha_0 = \alpha_0,\ \alpha_1 = \frac{\lambda\alpha_0}{2\mu},$$

$$\alpha_2 = 0, \beta_0 = 0,\ \beta_1 = \beta_1\ , \gamma_3 = \frac{\alpha_0^2\gamma_1 + 6\gamma_1\mu^2 a^2\beta_1^2}{\alpha_0^2}. \tag{33}$$

Case II:

$$L = L, a = a, b = 3\gamma_1 a^3 L^2 + \frac{1}{2}\gamma_1 a^3\lambda^2 - 2a^3\gamma_1\mu - 2a^2 L,$$

$$\alpha_0 = \alpha_0\,,\ \ \alpha_1 = 0,\ \alpha_2 = 0,\ \beta_1 = \beta_1,$$

$$\beta_0 = \frac{1}{2}\lambda\beta_1, \gamma_3 = -\frac{3\lambda^4\gamma_1 a^2\beta_1^2 - 24\mu\lambda^2\gamma_1 a^2\beta_1^2 + 8\gamma_2\alpha_0^2 + 48\mu^2\gamma_1 a^2\beta_1^2}{16\alpha_0^2} \tag{34}$$

Case III:

$$L = L, a = a, b = \frac{a(-6\beta_1^2 aL - \gamma_2\alpha_1^2 - 2\gamma_3\alpha_1^2 + 9\beta_1^2\gamma_1 a^2 L^2)}{3}, \alpha_0 = \frac{\alpha_1\beta_0}{\beta_1}\ \ \alpha_1 = \alpha_1, \alpha_2 = 0,$$

$$\beta_0 = \beta_0, \beta_1 = \beta_1, \gamma_3 = \gamma_3. \tag{35}$$

In view of Eq (33) with Eqs (20–24) into Eq (32), admits to the following solutions as

$$U_i(\xi) = \frac{\alpha_0 + \dfrac{\lambda\alpha_0}{2\mu}\exp(-w_i(\xi))}{\beta_0 + \beta_1\, exp(-w_i(\xi))}, i = 1, \ldots, 5$$

$$v_i(x,t) = \frac{\alpha_0 + \frac{\lambda\alpha_0}{2\mu}\exp(-w_i(\xi))}{\beta_0+\beta_1\, exp(-w_i(\xi))}\, e^{iL\xi}, i = 1, \ldots, 5 \tag{36}$$

$$\xi = ax + \frac{(-4L - 4\gamma_1 a\mu + 6\gamma_1 aL^2 + \gamma_1 a\lambda^2)a^2 t^\alpha}{2\Gamma(1+\alpha)}$$

By using Eq (34) with Eqs (20–24) into Eq (32), we gain the following exact solutions:

$$U_i(\xi) = \frac{\alpha_0}{\frac{1}{2}\lambda\beta_1 + \beta_1\, exp(-w_i(\xi))}, i = 1, \dots, 5 \tag{37}$$

$$v_i(x,t) = \frac{\alpha_0}{\frac{1}{2}\lambda\beta_1 + \beta_1\, exp(-w_i(\xi))} e^{iL\xi}, i = 1, \dots, 5 \tag{38}$$

$$\xi = ax + \frac{(3\gamma_1 a^3 L^2 + \frac{1}{2}\gamma_1 a^3 \lambda^2 - 2a^3\gamma_1 \mu - 2a^2 L)t^\alpha}{2\Gamma(1+\alpha)}.$$

According to (Case III) with Eqs (20–24) into Eq (32), admits to solutions of Eq (7) as

$$U_i(\xi) = \frac{\frac{\alpha_1\beta_0}{\beta_1} + \alpha_1 \exp(-w_i(\xi))}{\beta_0 + \beta_1\, exp(-w_i(\xi))}, i = 1, \dots, 5 \tag{39}$$

$$v_i(x,t) = \frac{\frac{\alpha_1\beta_0}{\beta_1} + \alpha_1 \exp(-w_i(\xi))}{\beta_0 + \beta_1\, exp(-w_i(\xi))} e^{iL\xi}, i = 1, \dots, 5 \tag{40}$$

$$\xi = ax + \frac{a(-6\beta_1^2 aL - \gamma_2\alpha_1^2 - 2\gamma_3\alpha_1^2 + 9\beta_1^2\gamma_1 a^2 L^2)t^\alpha}{3\Gamma(1+\alpha)}$$

Where $w_i(\xi), i = 1, \dots, 5$ are given in Eqs (20–24),

It is worth noting that all explored solutions Eqs (36–40) presents optical soliton solutions to Eq (7).

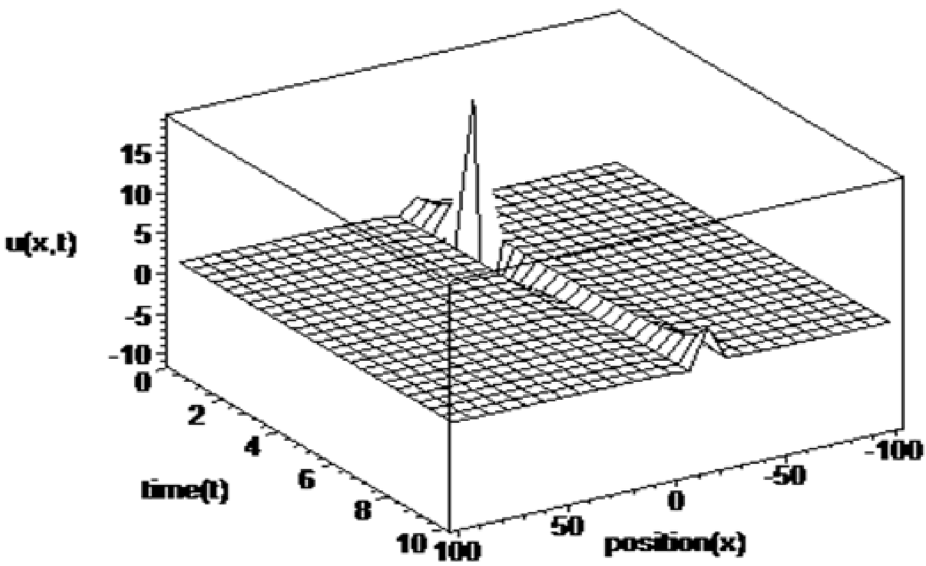

Figure 1. The rational function solution of Eq (36) using Exp method with a fixed parameter via case (I) with fixed parameters and fractal parameter α =1.

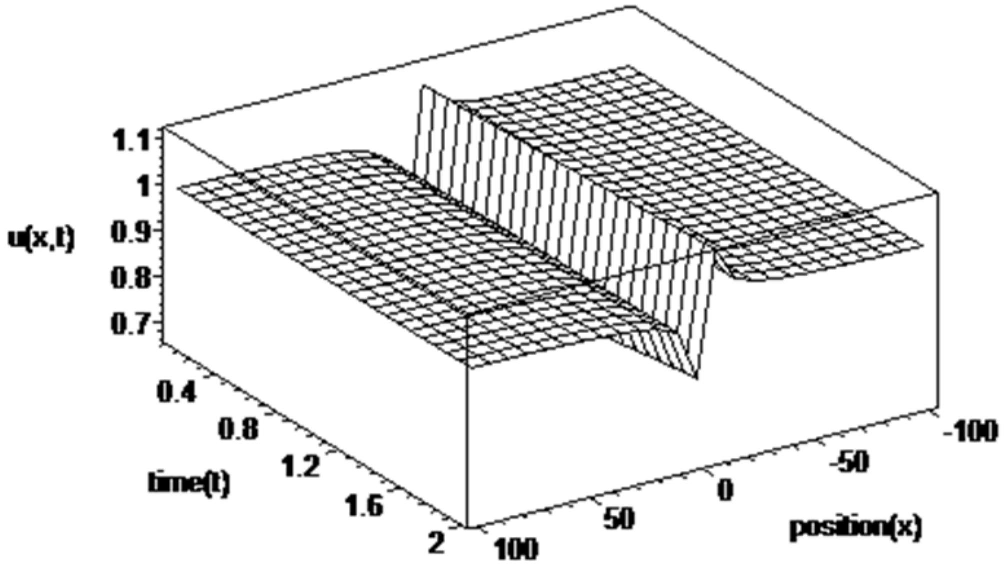

Figure 2. The hyperbolic function solution of Eq (36) using Exp method using case [I]. Eq (33) with fixed parameters and fractal parameter α = 0.75.

4.2. Optical soliton solutions via rational $\left(\frac{G'}{G^2}\right)$ expansion method

This section, is devoted for obtaining the optical soliton solutions of Eq (13) via the rational $\left(\frac{G'}{G^2}\right)$ method [34,35]. Consider the balance between U'', and U^3, we get $N = M + 1$. For $M = 1$, we have $N = 2$. Therefore the solutions of Eq.(25) becomes

$$U(\xi) = \frac{a_0 + a_1\psi(\xi) + a_2\psi^2(\xi)}{b_0 + b_1\psi(\xi)}, \quad \psi(\xi) = \frac{G'(\xi)}{G^2(\xi)} \tag{41}$$

Eqs (41) is employed to Eqs (13) having the same power of $\psi(\xi)$ and equating to zero, we have a set of algebraic equations. By solving it, we obtain

Case I:

$$L = \frac{a + \sqrt{a^2 + 6\gamma_1^2 a^4 \mu\lambda + 3\gamma_1 ab}}{3\gamma_1 a^2}, a_0 = 0, \quad a_1 = -\frac{6\gamma_1 a\mu b_0}{\sqrt{-\frac{6\gamma_1}{\gamma_2 + 2\gamma_3}}(\gamma_2 + 2\gamma_3)},$$

$$a_2 = \sqrt{-\frac{6\gamma_1}{\gamma_2 + 2\gamma_3}}\,\mu b_1 a, \quad b_0 = b_0, \ b_1 = b_1. \tag{42}$$

Subsequently, in view of Eq (42) with Eqs (27–29) into Eq (41), admits to the solutions of Eq. (7) as

$$U_i(\xi) = \frac{-\frac{6\gamma_1 a\mu b_0}{\sqrt{-\frac{6\gamma_1}{\gamma_2 + 2\gamma_3}}(\gamma_2 + 2\gamma_3)}\psi_i(\xi) + \sqrt{-\frac{6\gamma_1}{\gamma_2 + 2\gamma_3}}\,\mu b_1 a\,\psi_i^2(\xi)}{b_0 + b_1\psi_i(\xi)} \tag{43}$$

$$v_i(x,t)=\frac{-\frac{6\gamma_1 a\mu b_0}{\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}(\gamma_2+2\gamma_3)}\psi_i(\xi)+\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}\mu b_1 a\,\psi_i^2(\xi)}{b_0+b_1\psi_i(\xi)}e^{i\left(\frac{a+\sqrt{a^2+6\gamma_1^2a^4\mu\lambda+3\gamma_1 ab}}{3\gamma_1 a^2}\right)\xi} \tag{44}$$

$\psi_i(\xi)=\left(\frac{G_i'(\xi)}{G_i^2(\xi)}\right)$, $\xi=ax+\frac{b\,t^\alpha}{\Gamma(1+\alpha)}$, $(i=1,2,3)$ are given in Eqs (27–29).

Case II:

$$L=\frac{a+\sqrt{a^2-\gamma_1^2a^4\mu\lambda+3\gamma_1 ab}}{3\gamma_1 a^2},\quad a_0=-\frac{6\gamma_1 ab_1\lambda}{\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}(\gamma_2+2\gamma_3)},\ a_2=\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}\ \mu b_1 a,$$

$$a_1=0,\quad b_0=0,\ b_1=b_1 \tag{45}$$

In view of Eq (45) with (27–29) into Eq (41), we gain the new solutions for Eq (7) as

$$U_i(\xi)=\frac{-\frac{6\gamma_1 ab_1\lambda}{\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}(\gamma_2+2\gamma_3)}+\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}\mu b_1 a\psi_i^2(\xi)}{b_1\psi_i(\xi)} \tag{46}$$

$$v_i(x,t)=\frac{-\frac{6\gamma_1 ab_1\lambda}{\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}(\gamma_2+2\gamma_3)}+\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}\mu b_1 a\psi_i^2(\xi)}{b_1\psi_i(\xi)}e^{i\left(\frac{a+\sqrt{a^2-\gamma_1^2a^4\mu\lambda+3\gamma_1 ab}}{3\gamma_1 a^2}\right)\xi} \tag{47}$$

$\psi_i(\xi)=\left(\frac{G_i'(\xi)}{G_i^2(\xi)}\right)$, $\xi=ax+\frac{b\,t^\alpha}{\Gamma(1+\alpha)}$, $(i=1,2,3)$ are given in Eqs (27–29).

Case III:

$$L=\frac{a+\sqrt{a^2+24\gamma_1^2a^4\mu\lambda+3\gamma_1 ab}}{3\gamma_1 a^2},\quad a_0=\frac{6\gamma_1 ab_1\lambda}{\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}(\gamma_2+2\gamma_3)},\ a_2=\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}\ \mu b_1 a,$$

$$a_1=0,\quad b_0=0,\ b_1=b_1. \tag{48}$$

Making use Eq (48) with Eqs (27–29) into Eq (41), we write down explicitly the following solutions of Eq (7) as

$$U_i(\xi)=\frac{\frac{6\gamma_1 ab_1\lambda}{\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}(\gamma_2+2\gamma_3)}+\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}\mu b_1 a\psi_i^2(\xi)}{\beta_1\psi_i(\xi)} \tag{49}$$

$$v_i(x,t)=\frac{\frac{6\gamma_1 ab_1\lambda}{\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}(\gamma_2+2\gamma_3)}+\sqrt{-\frac{6\gamma_1}{\gamma_2+2\gamma_3}}\mu b_1 a\psi_i^2(\xi)}{b_1\psi_i(\xi)}e^{i\left(\frac{a+\sqrt{a^2+24\gamma_1^2a^4\mu\lambda+3\gamma_1 ab}}{3\gamma_1 a^2}\right)\xi} \tag{50}$$

$\psi_i(\xi)=\left(\frac{G_i'(\xi)}{G_i^2(\xi)}\right)$, $\xi=ax+\frac{b\,t^\alpha}{\Gamma(1+\alpha)}$, $(i=1,2,3)$ are given in Eqs (27–29).

Case IV:

$$L = \frac{a+\sqrt{a^2+6\gamma_1^2 a^4 \mu\lambda+3\gamma_1 ab}}{3\gamma_1 a^2}, \quad a_0 = -\frac{b_1\lambda a}{\sqrt{-\frac{\gamma_2+2\gamma_3}{6\gamma_1}}}, \; a_1 = \sqrt{6}\sqrt{\frac{\mu\lambda a^2\gamma_1}{\gamma_2+2\gamma_3}},$$

$$a_2 = 0, \quad b_0 = \frac{\sqrt{-\frac{\gamma_2+2\gamma_3}{6\gamma_1}}\sqrt{6}\sqrt{\frac{\mu\lambda a^2\gamma_1}{\gamma_2+2\gamma_3}}b_1}{\mu a}, \;\; b_1 = b_1. \tag{51}$$

In view of Eq (51) with Eqs (27–29) into Eq (41), we obtain

$$U_i(\xi) = \frac{-\frac{b_1\lambda a}{\sqrt{-\frac{\gamma_2+2\gamma_3}{6\gamma_1}}}+\sqrt{6}\sqrt{\frac{\mu\lambda a^2\gamma_1}{\gamma_2+2\gamma_3}}\psi_i(\xi)}{\frac{\sqrt{-\frac{\gamma_2+2\gamma_3}{6\gamma_1}}\sqrt{6}\sqrt{\frac{\mu\lambda a^2\gamma_1}{\gamma_2+2\gamma_3}}b_1}{\mu a}+b_1\psi_i(\xi)} \tag{52}$$

$$v_i(x,t) = \frac{-\frac{\beta_1\lambda a}{\sqrt{-\frac{\gamma_2+2\gamma_3}{6\gamma_1}}}+\sqrt{6}\sqrt{\frac{\mu\lambda a^2\gamma_1}{\gamma_2+2\gamma_3}}\psi_i(\xi)}{\frac{\sqrt{-\frac{\gamma_2+2\gamma_3}{6\gamma_1}}\sqrt{6}\sqrt{\frac{\mu\lambda a^2\gamma_1}{\gamma_2+2\gamma_3}}\beta_1}{\mu a}+\beta_1\psi_i(\xi)} e^{i\left(\frac{a+\sqrt{a^2+24\gamma_1^2 a^4\mu\lambda+3\gamma_1 ab}}{3\gamma_1 a^2}\right)\xi} \tag{53}$$

$\psi_i(\xi) = \left(\frac{G_i'(\xi)}{G_i^2(\xi)}\right)$ $\xi = ax + \frac{b\,t^\alpha}{\Gamma(1+\alpha)}$, $(i = 1,2,3)$ are given in Eqs (27–29).

Thus, the optical solutions of Eq (7) can be captured by Eq (49–53), under the constraints Eq (10) via $V(\xi) = e^{iL\xi}U(\xi)$.

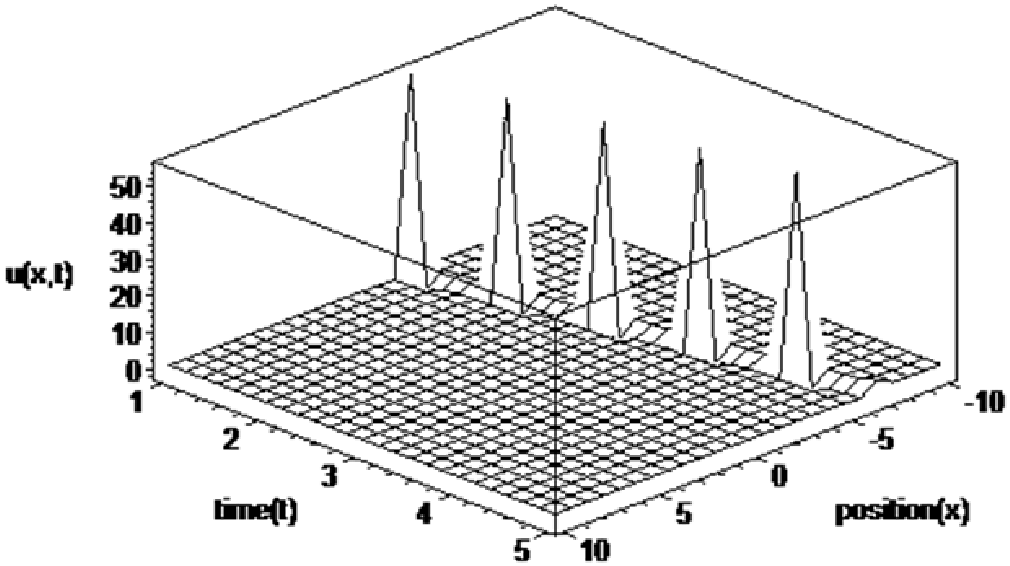

Figure 3. The trigonometric function solution of Eq (52) using Exp method using case [I] Eq (33) with fixed parameters, and fractal parameter α =1.

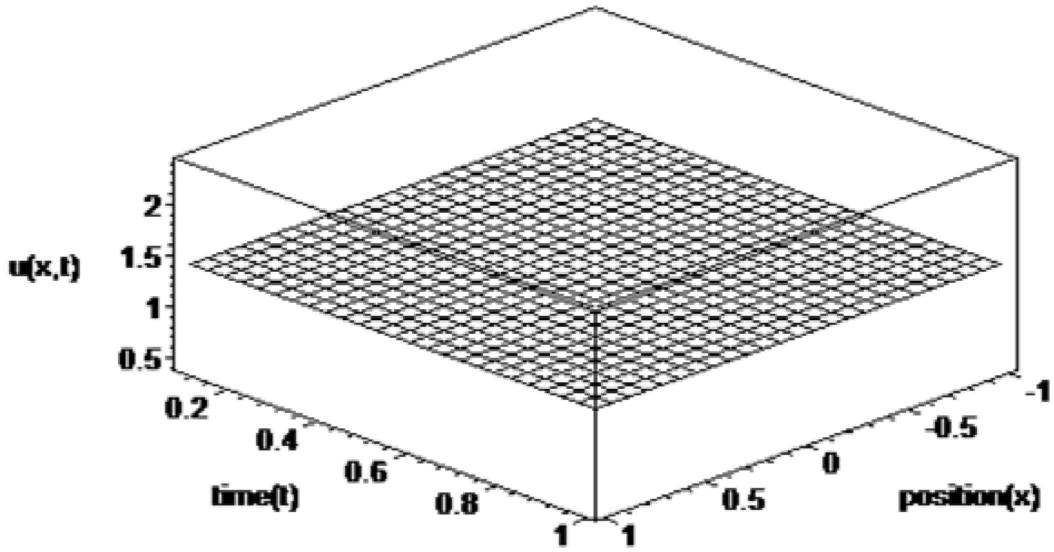

Figure 4. The 3D dimensional of the exact solution (trigonometric function solution) of Eq. (53) using (G/G^2) expansion method with a fixed parameter with α =1.

5. Conclusions

In this work, the generalized $exp(-w(\xi))$ and rational $\left(\frac{G'}{G^2}\right)$ expansion methods have been applied for finding the new optical soliton solutions for the perturbed time fractional nonlinear Schrodinger equation with the help of conformable fractional derivative. The results are group of new solutions of Schrodinger equation, where the proposed methods proved better reliability, accuracy and efficiency. The obtained results and figures (1-4) conclude that, the fractional parameter α plays the main rule in the solutions, for example if we substituted by $\alpha = 1$, the solutions become the same with the obtained by the normal derivative.

Conflict of interest

The authors declare that there is no conflict of interest in this paper.

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