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Research article

Multiplicative topological properties of graphs derived from honeycomb structure

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Abstract: Topological indices are numerical parameters of a molecular graph, which characterize its topology and are usually graph invariant. In quantitative structure-activity relationship/quantitative structure-property relationship study, physico-chemical properties and topological indices such as Randić, atom-bond connectivity (ABC), and geometric-arithmetic (GA) index are used to predict the bioactivity of chemical compounds. Graph theory has found a considerable use in this area of research. In this paper, we are taking Dominating David Derived networks, produced by honeycomb structure of dimension t and obtain analytical closed results of Multiplicative topological indices and acquire exact results of degree based indices.

Keywords: first and second multiplicative Zagreb index; first and second Hyper-Zagreb index; first and second universal Zagreb index; sum and product connectivity of multiplicative indices; multiple atom-bond connectivity and Geometric-Arithmetic index
Mathematics Subject Classification: 05C12, 05C90

1. Introduction

Chemical graph theory is a branch of graph theory in which a chemical compound is represented by simple graph called molecular graph in which vertices are atoms of compound and edges are the atomic bounds. A graph is connected if there is at least one connection between its vertices. Throughout this paper we take Υ a connected graph. Now a day another emerging field is Cheminformatics, which helps to predict biological activities with the relationship of Structure-property and quantitative structure-activity. Topological indices are valuable parameters that are given by graph theory. A number that describe the topology of a graph is called topological index.

A representation of numbers, polynomials and matrices are representations of a graph. Graph has its own characteristics which can be determined by topological indices and the topology of graph remains unchanged under automorphism of graph. In the different classes of indices, degree based topological indices are of extraordinary significance and assume an essential job in substance graph hypothesis and especially in science. In increasingly exact manner, a topological index $Top(H)$ of a graph, is a number with the property that for each graph G isomorphic to H, $Top(H) = Top(G)$. The idea of topological index originated from Wiener [26] while he was dealing with boiling point of paraffin, named this record as *path number*. Later on, renamed as *Wiener index* [7].

A lot of people have been worked in the Chemical Graph Theory. The importance of honeycomb network can not be ignored in all the work of graph theory. The Honeycomb shape is present everywhere in nature, in plants, animal and human cells. No other shape provides such an optimal cover and strength. It is one of the most stable structures. Honeycomb structures are natural or man-made structures that have the geometry of a honeycomb to allow the minimization of the material used to reach minimal weight and maximum strength. A honeycomb structure provides a material with least density and relative high compression properties and shear properties.

1.1. Honeycomb network

Built recursively using the hexagon tessellation [24], honeycomb networks are widely used in computer graphics [24], cellular phone base stations, image processing, and in chemistry as the representation of benzenoid hydrocarbons. Honeycomb network $HC(t)$ is obtained from $HC(t-1)$ by adding a layer of hexagons around the boundary of $HC(t-1)$.

In this paper, we are going to find the topological indices of graphs derived from the honeycomb structure. Dominating David Derived networks are the graphs derived from honeycomb structure.

The method of drawing Dominating David Derived networks (dimension t) is as follows.

STEP 1:-Consider a Honeycomb network $HC(t)$ dimension t, as shown in Figure 1.

STEP 2:-Split each edge into two by embedding another vertex.

STEP 3:-In each hexagon cell, connect the new vertices by an edge if they are at a distance of 4 units within the cell.

STEP 4:-Place vertices at new edge crossings.

STEP 5:-Remove initial vertices and edges of Honeycomb network.

STEP 6:-Split each horizontal edge into two edges by inserting a new vertex. The resulting Graph is called Dominating David Derived system $DDD(t)$ of measurement t [22], as shown in Figure 2.

The First type of Dominating David Derived network $D_1(t)$ can be obtained by connecting vertices of degree two by an edge, which are not in the boundary, as shown in Figure 3.

The second type of Dominating David Derived network $D_2(t)$ can be obtained by sub dividing once the new edge introduced in $D_1(t)$, as shown in Figure 4.

The Third type of Dominating David Derived network $D_3(t)$ can be obtained from $D_1(t)$ by introducing parallel path of length 2 between the vertices of degree two which are not in the boundary. See the Figure 5 for third type of Dominating David derived network of dimension 2, $D_3(2)$.

In this article, Υ is considered a network with a $V(\Upsilon)$ vertex set and an edge set of $E(\Upsilon)$, d_r is the degree of vertex $r \in V(\Upsilon)$.

Some indices associated to Wiener's and Gutman. They derived new topological indices which are named as the first Multiplicative Zagreb index and the second Multiplicative Zagreb index [11] and they are described as:

Let Υ be a graph. Then

$$II_1(\Upsilon) = \prod_{r \in V(\Upsilon)} (d_r)^2, \tag{1.1}$$

$$II_2(\Upsilon) = \prod_{rs \in E(\Upsilon)} (d_r \times d_s). \tag{1.2}$$

V. R. Kulli [15] further described some new and advanced topological indices and he named them as the first Hyper-Zagreb index and the second Hyper-Zagreb index of a graph Υ. They are defined as:

$$HII_1(\Upsilon) = \prod_{rs \in E(\Upsilon)} (d_r + d_s)^2, \tag{1.3}$$

$$HII_2(\Upsilon) = \prod_{rs \in E(\Upsilon)} (d_r \times d_s)^2. \tag{1.4}$$

The first Universal Zagreb index and the second Universal Zagreb index introduced by V. R. Kulli [15]. These indices are defined as:

$$MZ_1^a(\Upsilon) = \prod_{rs \in E(\Upsilon)} (d_r + d_s)^a, \tag{1.5}$$

$$MZ_2^a(\Upsilon) = \prod_{rs \in E(\Upsilon)} (d_r \times d_s)^a. \tag{1.6}$$

The sum and product connectivity of Multiplicative indices [15] described as:

$$SCII(\Upsilon) = \prod_{rs \in E(\Upsilon)} \frac{1}{\sqrt{d_r + d_s}}, \tag{1.7}$$

$$PCII(\Upsilon) = \prod_{rs \in E(\Upsilon)} \frac{1}{\sqrt{d_r \times d_s}}. \tag{1.8}$$

Wei Gao et al. [9] define new topological indices which are named as Multiple atom-bond Connectivity index and Multiple Geometric-Arithmetic index and these indices are defined as follow:

$$ABC_M(\Upsilon) = \sum_{rs \in E(\Upsilon)} \sqrt{\frac{M_r + M_s - 2}{M_r \times M_s}}, \tag{1.9}$$

$$GA_M(\Upsilon) = \sum_{rs \in E(\Upsilon)} \frac{2\sqrt{M_r \times M_s}}{(M_r + M_s)}, \tag{1.10}$$

where

$$M_r = \prod_{rs \in E(\Upsilon)} d_s, M_s = \prod_{rs \in E(\Upsilon)} d_r.$$

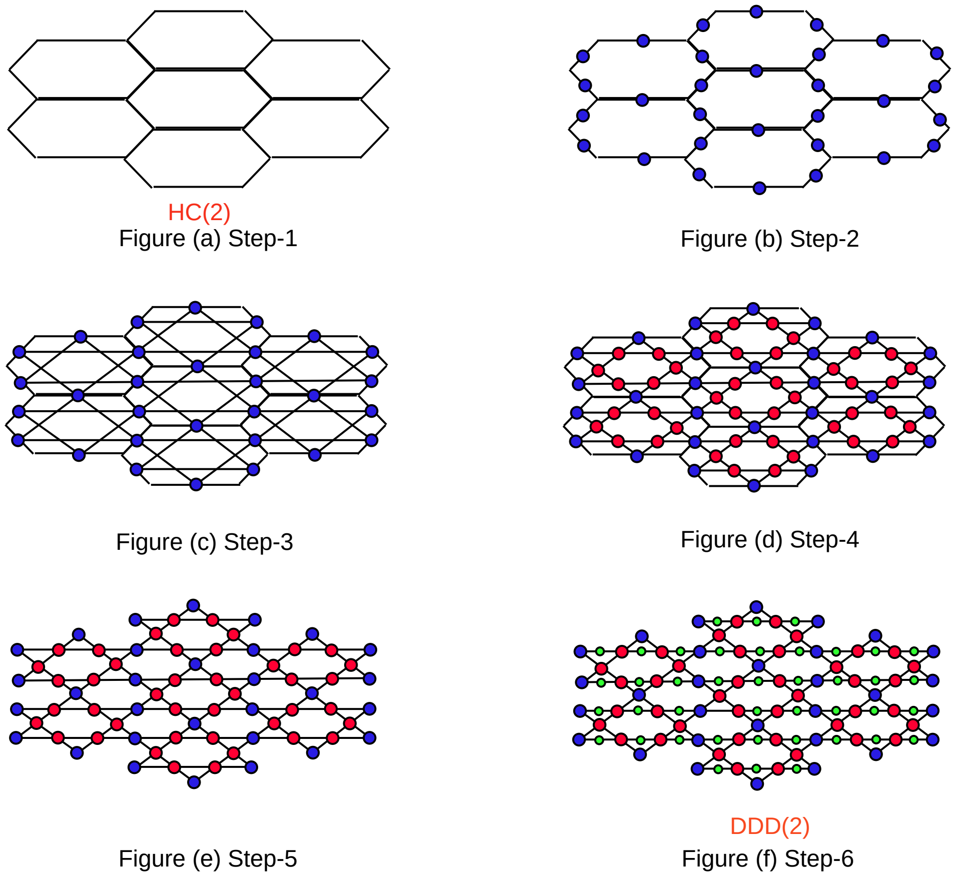

Figure 1. Construction Algorithm for Dominating David Derived network $DDD(2)$.

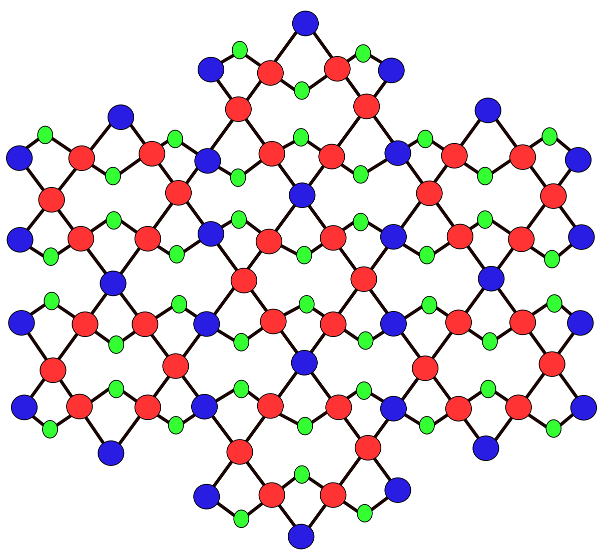

Figure 2. Isomorphic graph of DDD(2).

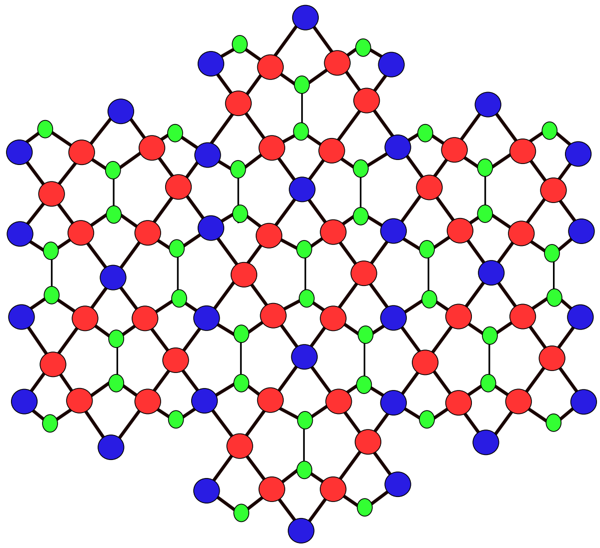

Figure 3. First type of Dominating David Derived network $D_1(2)$.

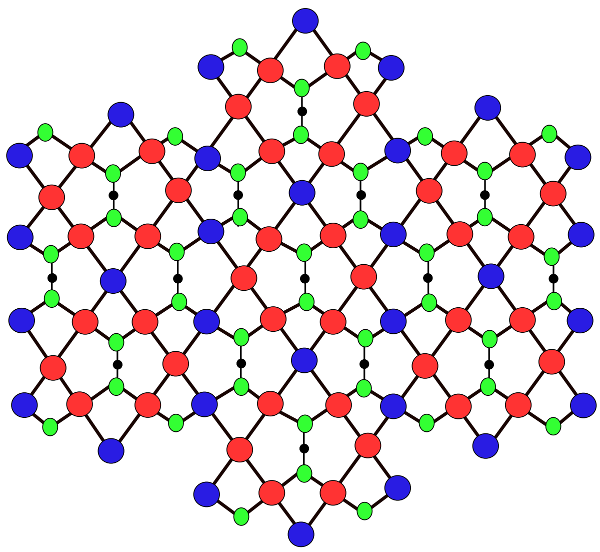

Figure 4. Second type of Dominating David Derived network $D_2(2)$.

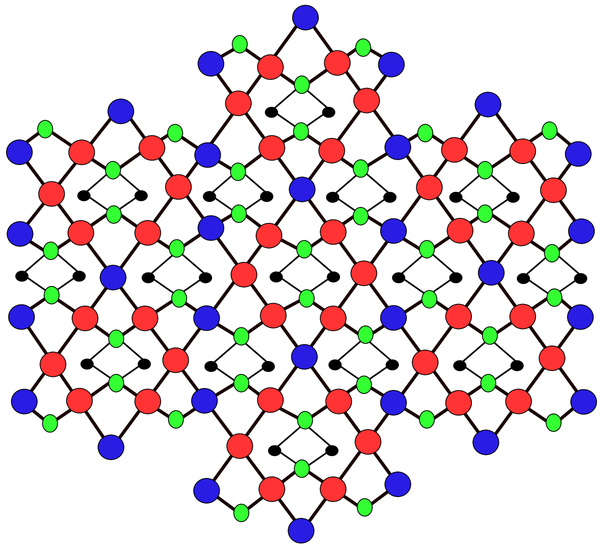

Figure 5. Third type of Dominating David Derived network $D_3(2)$.

2. Results

We have study the multiplicative indices such as first and second multiplicative Zagreb Index, first and second hyper-Zagreb index, first and second Universal Zagreb index, sum and product connectivity of multiplicative indices, Multiple atom-bond connectivity index, Multiple Geometric-Arithmetic index and give closed formulae of these indices for Dominating David Derived networks. Haidar *et al.* studied degree based topological indices for various networks [2]. Nowadays, there is an extensive research activity on *ABC* and *GA* indices and their invariants, for further study of topological indices of various graph families see, [1–5, 8, 12–14, 16–20, 23, 27]. For the basic notations and definitions, see [6, 21, 26].

2.1. Results for first type of Dominating David Derived networks

In this section, we calculate degree-based topological indices of the dimension t for first type of Dominating David Derived networks. In the coming theorems, we compute some important multiplicative indices.

Theorem 2.1.1. Consider the first type of Dominating David Derived network $\Upsilon_1 \cong D_1(t)$ for $t \in \mathbb{N}$. The first and second multiplicative indices of Zagreb indices are equal to

$$II_1(\Upsilon_1) = 34560(-1 + 2t)(5 + t(-13 + 9t))(4 + t(-11 + 9t)),$$

$$II_2(\Upsilon_1) = 339738624t(t - 1)(7t - 4)(6 + t(9t - 14))(5 + t(9t - 13))^2.$$

Proof. Let Υ_1 be the first type of Dominating David Derived network. The Υ_1 has $20t - 10$ vertices of degree 2, $18t^2 - 26t + 10$ vertices of degree 3 and $27t^2 - 33t + 12$ vertices of degree 4. The edge set of $D_1(t)$ is divided into three partitions based on the degree of end vertices. Table 1, shows such an edge partition of $D_1(t)$. Thus from (1.1) is follows that,

$$II_1(\Upsilon_1) = \prod_{r \in V(\Upsilon_1)} (d_r)^2.$$

Table 1. Edge partition of first type of Dominating David Derived network ($D_1(t)$) based on degrees of end vertices of each edge.

(d_r, d_s) where $rs \in E(\Upsilon_1)$	Number of edges
$(2,2)$	$4t$
$(2,3)$	$4t-4$
$(2,4)$	$28t-16$
$(3,3)$	$9t^2-13t+5$
$(3,4)$	$36t^2-56t+24$
$(4,4)$	$36t^2-52t+20$

By using vertex partitions, we get

$$\begin{aligned} II_1(\Upsilon_1) &= (2)^2(20t-10)\times(3)^2(18t^2-26t+10)\times(4)^2(27t^2-33t+12), \\ &= 4(20t-10)\times 9(18t^2-26t+10)\times 16(27t^2-33t+12), \end{aligned}$$

By doing some calculations, we have

$$\Longrightarrow II_1(\Upsilon_1) = 34560(-1+2t)(5+t(-13+9t))(4+t(-11+9t)).$$

From (1.2), we have

$$II_2(\Upsilon_1) = \prod_{rs\in E(\Upsilon_1)} (d_r \times d_s).$$

By using Table 1 edge partitions, we get

$$\begin{aligned} II_2(\Upsilon_1) &= 4|E_1(\Upsilon_1(t))|\times 6|E_2\Upsilon 1(t)|\times 8|E_3(\Upsilon 1(t))|\times 9|E_4(\Upsilon 1(t))|\times 12|E_5(\Upsilon 1(t))|\times 16|E_6(\Upsilon 1(t))|, \\ &= 4(4t)\times 6(4t-4)\times 8(28t-16)\times 9(9t^2-13t+5)\times 12(36t^2-56t+24)\times \\ &\quad 16(36t^2-52t+20), \end{aligned}$$

By doing some calculations, we have

$$\Longrightarrow II_2(\Upsilon_1) = 339738624t(t-1)(7t-4)(6+t(9t-14))(5+t(9t-13))^2.$$

□

Now, we compute advance topological indices and name them as the first Hyper-Zegreb index and second Hyper-Zegreb index for first type of Dominating David Derived network $D_1(t)$.

Theorem 2.1.2. Let $\Upsilon_1 \cong D_1(t)$ be the first type of Dominating David Derived network, then

$$HII_1(\Upsilon_1) = 1664719257600t(t-1)(7t-4)(6+t(9t-14))(5+t(9t-13))^2,$$

$$HII_2(\Upsilon_1) = 112717121716224t(t-1)(7t-4)(6+t(9t-14))(5+t(9t-13))^2.$$

Proof. The outcome can be obtained by using the edge partition in Table 1. By using equation (1.3),

$$HII_1(\Upsilon_1) = \prod_{rs\in E(\Upsilon_1)} (d_r + d_s)^2.$$

$$\begin{aligned} HII_1(\Upsilon_1) &= 16|E_1(\Upsilon 1(t))| \times 25|E_2(\Upsilon 1(t))| \times 36|E_3(\Upsilon 1(t))| \times 36|E_4(\Upsilon 1(t))| \times 49|E_5(\Upsilon 1(t))| \times \\ & \quad 64|E_6(\Upsilon 1(t))|, \\ &= 16(4t) \times 25(4t-4) \times 36(28t-16) \times 36(9t^2-13t+5) \times 49(36t^2-56t+24) \times \\ & \quad 64(36t^2-52t+20), \end{aligned}$$

By doing some calculations, we get

$$\Longrightarrow HII_1(\Upsilon_1) = 1664719257600t(t-1)(7t-4)(6+t(9t-14))(5+t(9t-13))^2.$$

Thus from (1.4),

$$HII_2(\Upsilon_1) = \prod_{rs \in E(\Upsilon_1)} (d_r \times d_s)^2.$$

$$\begin{aligned} HII_2(\Upsilon_1) &= 16|E_1(\Upsilon 1(t))| \times 36|E_2(\Upsilon 1(t))| \times 64|E_3(\Upsilon 1(t))| \times 81|E_4(\Upsilon 1(t))| \times 144|E_5(\Upsilon 1(t))| \times \\ & \quad 256|E_6(\Upsilon 1(t))|, \\ &= 16(4t) \times 36(4t-4) \times 64(28t-16) \times 81(9t^2-13t+5) \times 144(36t^2-56t+24) \times \\ & \quad 256(36t^2-52t+20), \end{aligned}$$

$$\Longrightarrow HII_2(\Upsilon_1) = 112717121716224t(t-1)(7t-4)(6+t(9t-14))(5+t(9t-13))^2.$$

□

Now, we compute the first and second Universal Zagrab indices.

Theorem 2.1.3. Let $\Upsilon_1 \cong D_1(t)$ be the first type of Dominating David Derived network, then

$$MZ_1^a(\Upsilon_1) = 2^{10+7a} \times 315^a t(t-1)(7t-4)(6+t(9t-14))(5+t(9t-13))^2,$$

$$MZ_2^a(\Upsilon_1) = 4^{5+6a} \times 81^a t(t-1)(7t-4)(6+t(9t-14))(5+t(9t-13))^2.$$

Proof. We get the outcome with the edge partition in Table 1. It follows from (1.5),

$$MZ_1^a(\Upsilon_1) = \prod_{rs \in E(\Upsilon_1)} (d_r + d_s)^a.$$

$$\begin{aligned} MZ_1^a(\Upsilon_1) &= (4)^a|E_1(\Upsilon 1(t))| \times (5)^a|E_2(\Upsilon 1(t))| \times (6)^a|E_3(\Upsilon 1(t))| \times (6)^a|E_4(\Upsilon_1(t))| \times (7)^a|E_5(\Upsilon 1(t))| \times \\ & \quad (8)^a|E_6(\Upsilon 1(t))|, \\ &= 4^a(4t) \times 5^a(4t-4) \times 6^a(28t-16) \times 6^a(9t^2-13t+5) \times 7^a(36t^2-56t+24) \times \\ & \quad 8^a(36t^2-52t+20), \end{aligned}$$

By doing some calculations, we get

$$\Longrightarrow MZ_1^a(\Upsilon_1) = 2^{10+7a} \times 315^a t(t-1)(7t-4)(6+t(9t-14))(5+t(9t-13))^2.$$

Also from (1.6),

$$MZ_2^a(\Upsilon_1) = \prod_{rs \in E(\Upsilon_1)} (d_r \times d_s)^a.$$

$$\begin{aligned} MZ_2^a(\Upsilon_1) &= (4)^a|E_1(\Upsilon 1(t))| \times (6)^a|E_2(\Upsilon 1(t))| \times (8)^a|E_3(\Upsilon 1(t))| \times (9)^a|E_4(\Upsilon 1(t))| \times (12)^a|E_5(\Upsilon 1(t))| \times \\ &\quad (16)^a|E_6(\Upsilon 1(t))|, \\ &= 4^a(4t) \times 6^a(4t-4) \times 8^a(28t-16) \times 9^a(9t^2-13t+5) \times 12^a(36t^2-56t+24) \times \\ &\quad 16^a(36t^2-52t+20), \end{aligned}$$

By making some calculations, we get

$$\Longrightarrow MZ_2^a(\Upsilon_1) = 4^{5+6a} \times 81^a t(t-1)(7t-4)(6+t(9t-14))(5+t(9t-13))^2.$$

□

The sum and product connectivity of multiplicative indices are computed as follows.

Theorem 2.1.4. Let $\Upsilon_1 \cong D_1(t)$ be the first type of Dominating David Derived network, then

$$SCII(\Upsilon_1) = \frac{64}{3}\sqrt{\frac{2}{35}}t(t-1)(7t-4)(9t^2-14t+6)(9t^2-13t+5)^2,$$

$$PCII(\Upsilon_1) = \frac{16}{9}t(t-1)(7t-4)(9t^2-14t+6)(9t^2-13t+5)^2.$$

Proof. We get the outcome with the edge partition in Table 1. It follows from (1.7),

$$SCII(\Upsilon_1) = \prod_{rs\in E(\Upsilon_1)} \frac{1}{\sqrt{d_r+d_s}}.$$

$$\begin{aligned} SCII(\Upsilon_1) &= \frac{1}{2}|E_1(\Upsilon 1(t))| \times \frac{1}{\sqrt{5}}|E_2(\Upsilon 1(t))| \times \frac{1}{\sqrt{6}}|E_3(\Upsilon 1(t))|\frac{1}{\sqrt{6}}|E_4(\Upsilon 1(t))| \times \frac{1}{\sqrt{7}}|E_5(\Upsilon 1(t))| \times \\ &\quad \frac{1}{\sqrt{8}}|E_6(\Upsilon 1(t))|, \\ &= \frac{1}{2}(4t) \times \frac{1}{\sqrt{5}}(4t-4) \times \frac{1}{\sqrt{6}}(28t-16) \times \frac{1}{\sqrt{6}}(9t^2-13t+5) \times \frac{1}{\sqrt{7}}(36t^2-56t+24) \times \\ &\quad \frac{1}{\sqrt{8}}(36t^2-52t+20), \end{aligned}$$

By doing some calculations, we get

$$\Longrightarrow SCII(\Upsilon_1) = \frac{64}{3}\sqrt{\frac{2}{35}}t(t-1)(7t-4)(9t^2-14t+6)(9t^2-13t+5)^2.$$

Thus from (1.8),

$$PCII(\Upsilon_1) = \prod_{rs\in E(\Upsilon_1)} \frac{1}{\sqrt{d_r \times d_s}}.$$

$$PCII(\Upsilon_1) = \frac{1}{2}|E_1(\Upsilon 1(t))| \times \frac{1}{\sqrt{6}}|E_2(\Upsilon 1(t))| \times \frac{1}{\sqrt{8}}|E_3(\Upsilon 1(t))| \times \frac{1}{3}|E_4(\Upsilon 1(t))| \times \frac{1}{\sqrt{12}}|E_5(\Upsilon 1(t))| \times$$

$$\frac{1}{4}|E_6(\Upsilon 1(t))|,$$

$$= \frac{1}{2}(4t) \times \frac{1}{\sqrt{6}}(4t-4) \times \frac{1}{\sqrt{8}}(28t-16) + \times \frac{1}{\sqrt{9}}(9t^2-13t+5) \times \frac{1}{\sqrt{12}}(36t^2-56t+24) \times \frac{1}{\sqrt{16}}(36t^2-52t+20),$$

By making some calculations, we get

$$\Longrightarrow PCII(\Upsilon_1) = \frac{16}{9}t(t-1)(7t-4)(9t^2-14t+6)(9t^2-13t+5)^2.$$

□

Wei Gao et al. defines topological indices which are named as Multiple atom-bond connectivity index and Multiple Geometric-Arithmetic index and these indices are computed as follows.

Table 2. Edge partition of first type of Dominating David Derived network ($D_1(t)$) based on degrees product of end vertices of each edge.

(M_r, M_s) where $rs \in E(\Upsilon_1)$	Number of edges	(M_r, M_s) where $rs \in E(\Upsilon_1)$	Number of edges
(8,8)	4t	(48,64)	4
(8,48)	4t	(48,96)	4t-4
(8,64)	4	(48,128)	4t-4
(8,128)	4t-4	(48,144)	$36t^2-72t+36$
(12,24)	4t-4	(48,256)	4t-4
(12,64)	4t-4	(64,144)	4t-4
(16,48)	12t-8	(96,128)	4t-4
(16,96)	4t-4	(96,256)	4t-4
(24,24)	2t-2	(128,144)	4t-4
(24,144)	4t-4	(144,256)	$36t^2-76t+40$
(48,48)	$9t^2-7t+3$		

Theorem 2.1.5. Let $\Upsilon_1 \cong D_1(t)$ be the first type of Dominating David Derived network, then

$$ABC_M(\Upsilon_1) = \frac{1}{144}(-18\sqrt{15} - 48\sqrt{17} - 15\sqrt{21} - 18\sqrt{29} - 12\sqrt{46} - 12\sqrt{71} - 9\sqrt{74} + 9\sqrt{94} - 18\sqrt{134} - 12\sqrt{165} - 24\sqrt{186} - 6\sqrt{206} - 12\sqrt{222} - 8\sqrt{249} + 6\sqrt{330} + 30\sqrt{396} + 36\sqrt{570} - 3\sqrt{906}) + \frac{1}{144}t(216 + 72\sqrt{14} + 18\sqrt{15} + 48\sqrt{17} + 15\sqrt{21} + 18\sqrt{29} + 36\sqrt{35} + 12\sqrt{46} + 12\sqrt{71} + 9\sqrt{74} - 21\sqrt{94} + 18\sqrt{134} + 12\sqrt{165} + 36\sqrt{186} + 6\sqrt{206} + 12\sqrt{222} + 8\sqrt{249} - 57\sqrt{398} - 72\sqrt{570} + 3\sqrt{906}) + \frac{1}{144}t^2(27\sqrt{94} + 2\sqrt{398} + 36\sqrt{570}),$$

$$GA_M(\Upsilon_1) = \frac{37377}{1105} - \frac{976\sqrt{2}}{153} + \frac{202\sqrt{3}}{19} - \frac{400\sqrt{6}}{77} + \left(\frac{488\sqrt{6}}{77} + \frac{416\sqrt{2}}{51} - \frac{3238}{133}\sqrt{3} - \frac{377829}{5525}\right)t +$$

$\frac{9}{25}(121 + 50\sqrt{3})t^2.$

Proof. We get the outcome with the edge partition in Table 2. It follows from (1.9),

$$ABC_M(\Upsilon_1) = \sum_{rs\in E(\Upsilon_1)} \sqrt{\frac{M_r + M_s - 2}{M_r \times M_s}}.$$

$$\begin{aligned}
ABC_M(\Upsilon_1) &= \sqrt{\frac{7}{32}}|E_1(\Upsilon 1(t))| + \sqrt{\frac{9}{64}}|E_2(\Upsilon 1(t))| + \sqrt{\frac{35}{256}}|E_3(\Upsilon 1(t))| + \sqrt{\frac{67}{512}}|E_4(\Upsilon 1(t))| + \\
&\sqrt{\frac{17}{144}}|E_5(\Upsilon 1(t))| + \sqrt{\frac{37}{384}}|E_6(\Upsilon 1(t))| + \sqrt{\frac{31}{384}}|E_7(\Upsilon 1(t))| + \sqrt{\frac{55}{768}}|E_8(\Upsilon 1(t))| + \\
&\sqrt{\frac{23}{288}}|E_9(\Upsilon 1(t))| + \sqrt{\frac{83}{1728}}|E_{10}(\Upsilon 1(t))| + \sqrt{\frac{47}{1152}}|E_{11}(\Upsilon 1(t))| + \\
&\sqrt{\frac{55}{1536}}|E_{12}(\Upsilon 1(t))| + \sqrt{\frac{71}{2304}}|E_{13}(\Upsilon 1(t))| + \sqrt{\frac{29}{1024}}|E_{14}(\Upsilon 1(t))| + \\
&\sqrt{\frac{95}{3456}}|E_{15}(\Upsilon 1(t))| + \sqrt{\frac{151}{6144}}|E_{16}(\Upsilon 1(t))| + \sqrt{\frac{103}{4608}}|E_{17}(\Upsilon 1(t))| + \\
&\sqrt{\frac{37}{2048}}|E_{18}(\Upsilon 1(t))| + \sqrt{\frac{175}{12288}}|E_{19}(\Upsilon 1(t))| + \sqrt{\frac{15}{1024}}|E_{20}(\Upsilon 1(t))| + \\
&\sqrt{\frac{199}{18432}}|E_{21}(\Upsilon 1(t))|, \\
&= \sqrt{\frac{7}{32}}(4t) + \sqrt{\frac{9}{64}}(4t) + \sqrt{\frac{35}{256}}(4) + \sqrt{\frac{67}{512}}(4t-4) + \sqrt{\frac{17}{144}}(4t-4) + \sqrt{\frac{37}{384}}(4t- \\
&4) + \sqrt{\frac{31}{384}}(12t-8) + \sqrt{\frac{55}{768}}(4t-4) + \sqrt{\frac{23}{288}}(2t-2) + \sqrt{\frac{83}{1728}}(4t-4) + \\
&\sqrt{\frac{47}{1152}}(9t^2-7t+3) + \sqrt{\frac{55}{1536}}(4) + \sqrt{\frac{71}{2304}}(4t-4) + \sqrt{\frac{29}{1024}}(4t-4) + \\
&\sqrt{\frac{95}{3456}}(36t^2-72t+36) + \sqrt{\frac{151}{6144}}(4t-4) + \sqrt{\frac{103}{4608}}(4t-4) + \sqrt{\frac{37}{2048}}(4t-4) + \\
&\sqrt{\frac{175}{12288}}(4t-4) + \sqrt{\frac{15}{1024}}(4t-4) + \sqrt{\frac{199}{18432}}(36t^2-76t+40),
\end{aligned}$$

By doing some calculations, we get

$$\begin{aligned}
\Longrightarrow ABC_M(\Upsilon_1) &= \frac{1}{144}(-18\sqrt{15} - 48\sqrt{17} - 15\sqrt{21} - 18\sqrt{29} - 12\sqrt{46} - 12\sqrt{71} - 9\sqrt{74} + \\
&9\sqrt{94} - 18\sqrt{134} - 12\sqrt{165} - 24\sqrt{186} - 6\sqrt{206} - 12\sqrt{222} - 8\sqrt{249} + \\
&6\sqrt{330} + 30\sqrt{396} + 36\sqrt{570} - 3\sqrt{906}) + \frac{1}{144}t(216 + 72\sqrt{14} + 18\sqrt{15} + \\
&48\sqrt{17} + 15\sqrt{21} + 18\sqrt{29} + 36\sqrt{35} + 12\sqrt{46} + 12\sqrt{71} + 9\sqrt{74} - 21\sqrt{94} +
\end{aligned}$$

$$18\sqrt{134}+12\sqrt{165}+36\sqrt{186}+6\sqrt{206}+12\sqrt{222}+8\sqrt{249}-57\sqrt{398}-72\sqrt{570}+3\sqrt{906})+\frac{1}{144}t^2(27\sqrt{94}+2\sqrt{398}+36\sqrt{570}).$$

Thus from (1.10),

$$GA_M(\Upsilon_1)=\sum_{rs\in E(\Upsilon_1)}\frac{2\sqrt{M_r\times M_s}}{(M_r+M_s)}.$$

$$\begin{aligned}
GA_M(\Upsilon_1) &= 1|E_1(\Upsilon1(t))|+\frac{2\sqrt{6}}{7}|E_2(\Upsilon1(t))|+\frac{4\sqrt{2}}{9}|E_3(\Upsilon1(t))|+\frac{8}{17}|E_4(\Upsilon1(t))|+\frac{2\sqrt{2}}{3}|E_5(\Upsilon1(t))|+\\
&\frac{8\sqrt{3}}{19}|E_6(\Upsilon1(t))|+\frac{\sqrt{3}}{2}|E_7(\Upsilon1(t))|+\frac{2\sqrt{6}}{7}|E_8(\Upsilon1(t))|+1|E_9(\Upsilon1(t))|+\frac{2\sqrt{6}}{7}|E_{10}(\Upsilon1(t))|+\\
&1|E_{11}(\Upsilon1(t))|+\frac{4\sqrt{3}}{7}|E_{12}(\Upsilon1(t))|+\frac{2\sqrt{2}}{3}|E_{13}(\Upsilon1(t))|+\frac{4\sqrt{6}}{11}|E_{14}(\Upsilon1(t))|+\\
&\frac{\sqrt{3}}{2}|E_{15}(\Upsilon1(t))|+\frac{\sqrt{12288}}{152}|E_{16}(\Upsilon1(t))|+\frac{12}{13}|E_{17}(\Upsilon1(t))|+\frac{\sqrt{12288}}{112}|E_{18}(D_1(t))|+\\
&\frac{\sqrt{24576}}{176}|E_{19}(\Upsilon1(t))|+\frac{\sqrt{18432}}{136}|E_{20}(\Upsilon1(t))|+\frac{24}{25}|E_{21}(\Upsilon1(t))|,\\
&= 1(4t)+\frac{2\sqrt{6}}{7}(4t)+\frac{4\sqrt{2}}{9}(4)+\frac{8}{17}(4t-4)+\frac{2\sqrt{2}}{3}(4t-4)+\frac{8\sqrt{3}}{19}(4t-4)+\\
&\frac{\sqrt{3}}{2}(12t-8)+\frac{2\sqrt{6}}{7}(4t-4)+1(2t-2)+\frac{2\sqrt{6}}{7}(4t-4)+1(9t^2-7t+3)+\frac{4\sqrt{3}}{7}(4)+\\
&\frac{2\sqrt{2}}{3}(4t-4)+\frac{4\sqrt{6}}{11}(4t-4)+\frac{\sqrt{3}}{2}(36t^2-72t+36)+\frac{\sqrt{12288}}{152}(4t-4)+\frac{12}{13}(4t-4)+\\
&\frac{\sqrt{12288}}{112}(4t-4)+\frac{\sqrt{24576}}{176}(4t-4)+\frac{\sqrt{18432}}{136}(4t-4)+\frac{24}{25}(36t^2-76t+40),
\end{aligned}$$

$$\begin{aligned}
\Longrightarrow GA_M(\Upsilon_1) &= \frac{37377}{1105}-\frac{976\sqrt{2}}{153}+\frac{202\sqrt{3}}{19}-\frac{400\sqrt{6}}{77}+\Big(\frac{488\sqrt{6}}{77}+\frac{416\sqrt{2}}{51}-\sqrt{3}\frac{3238}{133}-\\
&\frac{377829}{5525}\Big)t+\frac{9}{25}(121+50\sqrt{3})t^2.
\end{aligned}$$

□

2.2. *Results for second type of Dominating David Derived network*

Now, we are calculating certain degree-based multiplicative topological indices of the $\Upsilon_2 \cong D_2(t)$, where $t \in \mathbb{N}$ for second type of Dominating David Derived network.

Theorem 2.2.1. Consider the second type of Dominating David Derived network $\Upsilon_2 \cong D_2(t)$ for $t \in \mathbb{N}$. The first and second multiplicative Zagreb indices are equal to

$$II_1(\Upsilon_2) = 3456(5+t(-13+9t))(4+t(-11+9t))(-5+t(7+9t)),$$

$$II_2(\Upsilon_2) = 18874368t(7t-4)(6+t(9t-14))(5+t(9t-13))(3+t(9t-11)).$$

Proof. Let Υ_2 be the second type of Dominating David Derived network. The $D_2(t)$ has $9t^2 + 7t - 5$ vertices of degree 2, $18t^2 - 26t + 10$ vertices of degree 3 and $27t^2 - 33t + 12$ vertices of degree 4. The edge set of $D_2(t)$ is divided into five partitions based on the degree of end vertices. Table 3, shows such an edge partition of $D_2(t)$. Thus from (1.1) is follows that

$$II_1(\Upsilon_2) = \prod_{r \in V(\Upsilon_2)} (d_r)^2.$$

By using vertex partitions, we get

$$\begin{aligned} II_1(\Upsilon_2) &= (2)^2(9t^2 + 7t - 5) \times (3)^2(18t^2 - 26t + 10) \times (4)^2(27t^2 - 33t + 12), \\ &= 4(9t^2 + 7t - 5) \times 9(18t^2 - 26t + 10) \times 16(27t^2 - 33t + 12), \end{aligned}$$

By doing some calculations, we have

Table 3. Edge partition of second type of Dominating David Derived network ($D_2(t)$) based on degrees of end vertices of each edge.

(d_r, d_s) where $rs \in E(\Upsilon_2)$	Number of edges
$(2, 2)$	$4t$
$(2, 3)$	$18t^2 - 22t + 6$
$(2, 4)$	$28t - 16$
$(3, 4)$	$36t^2 - 56t + 24$
$(4, 4)$	$36t^2 - 52t + 20$

$$\Longrightarrow II_1(\Upsilon_2) = 3456(5 + t(-13 + 9t))(4 + t(-11 + 9t))(-5 + t(7 + 9t)).$$

Thus from (1.2), we have

$$II_2(\Upsilon_2) = \prod_{rs \in E(\Upsilon_2)} (d_r \times d_s).$$

By using Table 3 edge partitions, we get

$$\begin{aligned} II_2(\Upsilon_2) &= 4|E_1(\Upsilon 2(t))| \times 6|E_2D(\Upsilon 2(t))| \times 8|E_3(\Upsilon 2(t))| \times 12|E_4(\Upsilon 2(t))| \times \\ &\quad 16|E_5(\Upsilon 2(t))|, \\ &= 4(4t) \times 6(18t^2 - 22t + 6) \times 8(28t - 16) \times 12(36t^2 - 56t + 24) \times \\ &\quad 16(36t^2 - 52t + 20), \end{aligned}$$

By doing some calculations, we have

$$\Longrightarrow II_2(\Upsilon_2) = 18874368t(7t - 4)(6 + t(9t - 14))(5 + t(9t - 13))(3 + t(9t - 11)).$$

□

Now, we compute the topological indices named as first Hyper-Zagreb index and second Hyper-Zagreb index of a graph Υ_2.

Theorem 2.2.2. Let $\Upsilon_2 \cong D_2(t)$ be the second type of Dominating David Derived network, then

$$HII_1(\Upsilon_2) = 23121100800t(7t - 4)(6 + t(9t - 14))(5 + t(9t - 13))(3 + t(9t - 11)),$$

$$HII_2(\Upsilon_2) = 695784701952t(7t - 4)(6 + t(9t - 14))(5 + t(9t - 13))(3 + t(9t - 11)).$$

Proof. The outcome is obtained by using the edge partition provided in Table 3. The result is from (1.3),

$$HII_1(\Upsilon_2) = \prod_{rs\in E(\Upsilon_2)} (d_r + d_s)^2.$$

$$\begin{aligned} HII_1(\Upsilon_2) &= 16|E_1(\Upsilon 2(t))| \times 25|E_2(\Upsilon 2(t))| \times 36|E_3(\Upsilon 2(t))| \times 49|E_4(\Upsilon 2(t))| \times 64|E_5(\Upsilon 2(t))|, \\ &= 16(4t) \times 25(18t^2 - 22t + 6) \times 36(28t - 16) \times 49(36t^2 - 56t + 24) \times \\ &\quad 64(36t^2 - 52t + 20), \end{aligned}$$

By doing some calculations, we get

$$\Longrightarrow HII_1(\Upsilon_2) = 23121100800t(7t - 4)(6 + t(9t - 14))(5 + t(9t - 13))(3 + t(9t - 11)).$$

Also from (1.4),

$$HII_2(\Upsilon_2) = \prod_{rs\in E(\Upsilon_2)} (d_r \times d_s)^2.$$

$$\begin{aligned} HII_2(\Upsilon_2) &= 16|E_1(\Upsilon 2(t))| \times 36|E_2(\Upsilon 2(t))| \times 64|E_3(\Upsilon 2(t))| \times 144|E_4(\Upsilon 2(t))| \times 256|E_5(\Upsilon 2(t))|, \\ &= 16(4t) + \times 36(18t^2 - 22t + 6) \times 64(28t - 16) \times 144(36t^2 - 56t + 24) \times \\ &\quad 256(36t^2 - 52t + 20), \end{aligned}$$

$$\begin{aligned} \Longrightarrow HII_2(\Upsilon_2) &= 695784701952t(7t - 4)(6 + t(9t - 14))(5 + t(9t - 13)) \\ &\quad (3 + t(9t - 11)). \end{aligned}$$

□

Now, we calculate first and second Universal-Zagreb indices.

Theorem 2.2.3. Let $\Upsilon_2 \cong D_2(t)$ be the second type of Dominating David Derived network, then

$$MZ_1^a(\Upsilon_2) = 8^{3+2a}105^a t(7t - 4)(6 + t(9t - 14))(5 + t(9t - 13))(3 + t(9t - 11)),$$

$$MZ_2^a(\Upsilon_2) = 8^{3+4a} \times 9^a t(7t - 4)(6 + t(9t - 14))(5 + t(9t - 13))(3 + t(9t - 11)).$$

Proof. The outcome is obtained by using the edge partition provided in Table 3. The result is from (1.5),

$$MZ_1^a(\Upsilon_2) = \prod_{rs\in E(\Upsilon_2)} (d_r + d_s)^a.$$

$$\begin{aligned} MZ_1^a(\Upsilon_2) &= (4)^a|E_1(\Upsilon 2(t))| \times (5)^a|E_2(\Upsilon 2(t))| \times (6)^a|E_3(\Upsilon 2(t))| \times (7)^a|E_4(\Upsilon 2(t))| \times (8)^a|E_5(\Upsilon 2(t))|, \\ &= 4^a(4t) \times 5^a(18t^2 - 22t + 6) \times 6^a(28t - 16) \times 7^a(36t^2 - 56t + 24) \times 8^a(36t^2 - 52t + 20), \end{aligned}$$

By doing some calculations, we get

$$\Longrightarrow MZ_1^a(\Upsilon_2) = 8^{3+2a}105^a t(7t - 4)(6 + t(9t - 14))(5 + t(9t - 13))(3 + t(9t - 11)).$$

Also from (1.6),

$$MZ_2^a(\Upsilon_2) = \prod_{rs\in E(\Upsilon_2)} (d_r \times d_s)^a.$$

$$\begin{aligned} MZ_2^a(\Upsilon_2) &= (4)^a|E_1(\Upsilon 2(t))| \times (6)^a|E_2(\Upsilon 2(t))| \times (8)^a|E_3(\Upsilon 2(t))| \times (12)^a|E_4(\Upsilon 2(t))| \times \\ &\quad (16)^a|E_5(\Upsilon 2(t))|, \\ &= 4^a(4t) \times 6^a(18t^2 - 22t + 6) \times 8^a(28t - 16) \times 12^a(36t^2 - 56t + 24) \times \\ &\quad 16^a(36t^2 - 52t + 20), \end{aligned}$$

$$\Longrightarrow MZ_2^a(\Upsilon_2) = 8^{3+4a} \times 9^a t(7t-4)(6+t(9t-14))(5+t(9t-13))(3+t(9t-11)).$$

□

The sum and product connectivity of multiplicative indices are as follows.

Theorem 2.2.4. Let $\Upsilon_2 \cong D_2(t)$ be the second type of Dominating David Derived network, then

$$SCII(\Upsilon_2) = \frac{64}{\sqrt{105}} t(7t-4)(9t^2-14t+6)(9t^2-13t+5)(9t^2-11t+3),$$

$$PCII(\Upsilon_2) = \frac{8}{3} t(7t-4)(9t^2-14t+6)(9t^2-13t+5)(9t^2-11t+3).$$

Proof. The outcome is obtained by using the edge partition provided in Table 3. The result is from (1.7),

$$SCII(\Upsilon_2) = \prod_{rs\in E(\Upsilon_2)} \frac{1}{\sqrt{d_r + d_s}}.$$

$$\begin{aligned} SCII(\Upsilon_2) &= \frac{1}{2}|E_1(\Upsilon 2(t))| \times \frac{1}{\sqrt{5}}|E_2(\Upsilon 2(t))| \times \frac{1}{\sqrt{6}}|E_3(\Upsilon 2(t))| \times \frac{1}{\sqrt{7}}|E_4(\Upsilon 2(t))| \times \\ &\quad \frac{1}{\sqrt{8}}|E_5(\Upsilon 2(t))|, \\ &= \frac{1}{2}(4t) \times \frac{1}{\sqrt{5}}(18t^2 - 22t + 6) \times \frac{1}{\sqrt{6}}(28t - 16) \times \frac{1}{\sqrt{7}}(36t^2 - 56t + 24) \times \\ &\quad \frac{1}{\sqrt{8}}(36t^2 - 52t + 20), \end{aligned}$$

By doing some calculations, we get

$$\Longrightarrow SCII(\Upsilon_2) = \frac{64}{\sqrt{105}} t(7t-4)(9t^2-14t+6)(9t^2-13t+5)(9t^2-11t+3).$$

Also from (1.8),

$$PCII(\Upsilon_2) = \prod_{rs\in E(\Upsilon_2)} \frac{1}{\sqrt{d_r \times d_s}}.$$

$$
\begin{aligned}
PCII(\Upsilon_2) &= \frac{1}{2}|E_1(\Upsilon 2(t))| \times \frac{1}{\sqrt{6}}|E_2(\Upsilon 2(t))| \times \frac{1}{\sqrt{8}}|E_3(\Upsilon 2(t))| \times \frac{1}{\sqrt{12}}|E_4(\Upsilon 2(t))| \times \frac{1}{4}|E_5(\Upsilon 2(t))|, \\
&= \frac{1}{2}(4t) \times \frac{1}{\sqrt{6}}(18t^2 - 22t + 6) \times \frac{1}{\sqrt{8}}(28t - 16) \times \frac{1}{\sqrt{12}}(36t^2 - 56t + 24) \times \\
&\quad \frac{1}{\sqrt{16}}(36t^2 - 52t + 20),
\end{aligned}
$$

By doing some calculations, we get

$$
\Longrightarrow PCII(\Upsilon_2) = \frac{8}{3}t(7t - 4)(9t^2 - 14t + 6)(9t^2 - 13t + 5)(9t^2 - 11t + 3).
$$

□

Now we calculate Multiple atom-bond connectivity index and Multiple Geometric-Arithmetic index.

Table 4. Edge partition of second type of Dominating David Derived network ($D_2(t)$) based on degrees product of end vertices of each edge.

$(M_r, M_s) where uv \in E(\Upsilon_2)$	Number of edges	$(M_r, M_s) where uv \in E(\Upsilon_2)$	Number of edges
(8,8)	4t	(32,48)	8t-4
(8,48)	4t	(32,96)	4t-4
(8,64)	4	(32,144)	$36t^2 - 72t + 36$
(8,128)	4t-4	(48,64)	4
(9,16)	4t-4	(48,128)	4t-4
(9,32)	$18t^2 - 30t + 14$	(48,256)	4t-4
(12,16)	4t-4	(64,144)	4t-4
(12,64)	4t-4	(96,128)	4t-4
(16,48)	12t-8	(96,256)	4t-4
(16,96)	4t-4	(128,144)	4t-4
(16,144)	4t-4	(144,256)	$36t^2 - 76t + 40$

Theorem 2.2.5. Let $\Upsilon_2 \cong D_2(t)$ be the second type of Dominating David Derived network, then

$$
\begin{aligned}
ABC_M(\Upsilon_2) &= \frac{1}{4080}(-1020\sqrt{13} - 425\sqrt{21} - 1360\sqrt{23} - 510\sqrt{29} + 1020\sqrt{35} - 510\sqrt{42} - \\
&\quad 255\sqrt{74} + 1700\sqrt{78} + 3060\sqrt{87} - 510\sqrt{134} - 340\sqrt{165} - 680\sqrt{186} - 170\sqrt{206} - \\
&\quad 340\sqrt{222} + 170\sqrt{330} - 816\sqrt{395} + 850\sqrt{398} - 720\sqrt{510} - 85\sqrt{906}) + \\
&\quad \frac{1}{4080}t(6120 + 2040\sqrt{13} + 2040\sqrt{14} + 425\sqrt{21} + 1360\sqrt{23} + 510\sqrt{29} + 510\sqrt{42} + \\
&\quad 255\sqrt{74} - 4420\sqrt{78} - 6120\sqrt{87} + 510\sqrt{134} + 340\sqrt{165} + 1020\sqrt{186} + 170\sqrt{206} + \\
&\quad 340\sqrt{222} + 816\sqrt{395} - 1615\sqrt{398} + 720\sqrt{510} + 85\sqrt{906}) + \frac{1}{4080}t^2(3060\sqrt{78} + \\
&\quad 3060\sqrt{87} + 765\sqrt{398}),
\end{aligned}
$$

$$GA_M(\Upsilon_2) = \Big(\frac{146884}{5525} + \frac{1848296\sqrt{2}}{69003} - \frac{1550\sqrt{3}}{133} - \frac{2176\sqrt{6}}{385}\Big) + t\Big(\frac{-315728}{5525} - \frac{414096}{7667}\sqrt{2} + \frac{2120\sqrt{3}}{133} + \frac{3232\sqrt{6}}{385}\Big) + t^2\Big(\frac{864}{25} + \frac{13608\sqrt{2}}{451}\Big).$$

Proof. The outcome is obtained by using the edge partition provided in Table 4. The result is from (1.9),

$$ABC_M(\Upsilon_2) = \sum_{rs\in E(\Upsilon_2)} \sqrt{\frac{M_r + M_s - 2}{M_r \times M_s}}.$$

$$\begin{aligned} ABC_M(\Upsilon_2) &= \sqrt{\frac{7}{32}}|E_1(\Upsilon 2(t))| + \sqrt{\frac{9}{64}}|E_2(\Upsilon 2(t))| + \sqrt{\frac{35}{256}}|E_3(\Upsilon 2(t))| + \sqrt{\frac{67}{512}}|E_4(\Upsilon 2(t))| + \\ &\sqrt{\frac{23}{144}}|E_5(\Upsilon 2(t))| + \sqrt{\frac{13}{96}}|E_6(\Upsilon 2(t))| + \sqrt{\frac{13}{96}}|E_7(\Upsilon_2(t))| + \sqrt{\frac{37}{384}}|E_8(\Upsilon 2(t))| + \\ &\sqrt{\frac{31}{384}}|E_9(\Upsilon 2(t))| + \sqrt{\frac{55}{768}}|E_{10}(\Upsilon 2(t))| + \sqrt{\frac{79}{1152}}|E_{11}(\Upsilon 2(t))| + \sqrt{\frac{13}{256}}|E_{12}(\Upsilon 2(t))| + \\ &\sqrt{\frac{21}{512}}|E_{13}(\Upsilon 2(t))| + \sqrt{\frac{29}{768}}|E_{14}(\Upsilon 2(t))| + \sqrt{\frac{55}{1536}}|E_{15}(\Upsilon 2(t))| + \\ &\sqrt{\frac{29}{1024}}|E_{16}(D_2(t))| + \sqrt{\frac{151}{6144}}|E_{17}(D_2(t))| + \sqrt{\frac{103}{4608}}|E_{18}(\Upsilon 2(t))| + \\ &\sqrt{\frac{37}{2048}}|E_{19}(\Upsilon 2(t))| + \sqrt{\frac{175}{12288}}|E_{20}(\Upsilon 2(t))| + \sqrt{\frac{15}{1024}}|E_{21}(\Upsilon 2(t))| + \\ &\sqrt{\frac{199}{18432}}|E_{22}(\Upsilon 2(t))|, \\ &= \sqrt{\frac{7}{32}}(4t) + \sqrt{\frac{9}{64}}(4t) + \sqrt{\frac{35}{256}}(4) + \sqrt{\frac{67}{512}}(4t-4) + \sqrt{\frac{23}{144}}(4t-4) + \sqrt{\frac{13}{96}}(18t^2 - \\ &30t + 14) + \sqrt{\frac{13}{96}}(4t-4) + \sqrt{\frac{37}{384}}(4t-4) + \sqrt{\frac{31}{384}}(12t-8) + \sqrt{\frac{55}{768}}(4t-4) + \\ &\sqrt{\frac{79}{1152}}(4t-4) + \sqrt{\frac{13}{256}}(8t-4) + \sqrt{\frac{21}{512}}(4t-4) + \sqrt{\frac{29}{768}}(36t^2 - 72t + 36) + \\ &\sqrt{\frac{55}{1536}}(4) + \sqrt{\frac{29}{1024}}(4t-4) + \sqrt{\frac{151}{6144}}(4t-4) + \sqrt{\frac{103}{4608}}(4t-4) + \sqrt{\frac{37}{2048}}(4t - \\ &4) + \sqrt{\frac{175}{12288}}(4t-4) + \sqrt{\frac{15}{1024}}(4t-4) + \sqrt{\frac{199}{18432}}(36t^2 - 76t + 40), \end{aligned}$$

By making some calculations, we are getting

$$\begin{aligned} \Longrightarrow ABC_M(\Upsilon_2) &= \frac{1}{4080}(- 1020\sqrt{13} - 425\sqrt{21} - 1360\sqrt{23} - 510\sqrt{29} + 1020\sqrt{35} - 510\sqrt{42} - \\ &255\sqrt{74} + 1700\sqrt{78} + 3060\sqrt{87} - 510\sqrt{134} - 340\sqrt{165} - 680\sqrt{186} - \end{aligned}$$

$$170\sqrt{206} - 340\sqrt{222} + 170\sqrt{330} - 816\sqrt{395} + 850\sqrt{398} - 720\sqrt{510} - 85\sqrt{906}) + \frac{1}{4080}t(6120 + 2040\sqrt{13} + 2040\sqrt{14} + 425\sqrt{21} + 1360\sqrt{23} + 510\sqrt{29} + 510\sqrt{42} + 255\sqrt{74} - 4420\sqrt{78} - 6120\sqrt{87} + 510\sqrt{134} + 340\sqrt{165} + 1020\sqrt{186} + 170\sqrt{206} + 340\sqrt{222} + 816\sqrt{395} - 1615\sqrt{398} + 720\sqrt{510} + 85\sqrt{906}) + \frac{1}{4080}t^2(3060\sqrt{78} + 3060\sqrt{87} + 765\sqrt{398}).$$

Thus from (1.10),

$$GA_M(\Upsilon_2) = \sum_{rs\in E(\Upsilon_2)} \frac{2\sqrt{M_r \times M_s}}{(M_r + M_s)}.$$

$$\begin{aligned} GA_M(\Upsilon_2) &= 1|E_1(\Upsilon2(t))| + \frac{2\sqrt{6}}{7}|E_2(\Upsilon2(t))| + \frac{4\sqrt{2}}{9}|E_3(\Upsilon2(t))| + \frac{8}{17}|E_4(\Upsilon2(t))| + \frac{24}{25}|E_5(\Upsilon2(t))| + \\ &\frac{24\sqrt{2}}{41}|E_6(\Upsilon2(t))| + \frac{4\sqrt{3}}{7}|E_7(\Upsilon2(t))| + \frac{8\sqrt{3}}{19}|E_8(\Upsilon2(t))| + \frac{\sqrt{3}}{2}|E_9(\Upsilon2(t))| + \\ &\frac{2\sqrt{6}}{7}|E_{10}(\Upsilon2(t))| + \frac{3}{5}|E_{11}(\Upsilon2(t))| + \frac{2}{5}\sqrt{6}|E_{12}(\Upsilon2(t))| + \frac{\sqrt{3}}{2}|E_{13}(\Upsilon2(t))| + \\ &\frac{6\sqrt{2}}{11}|E_{14}(\Upsilon2(t))| + \frac{4\sqrt{3}}{7}|E_{15}(\Upsilon2(t))| + \frac{4\sqrt{6}}{11}|E_{16}(\Upsilon2(t))| + \frac{\sqrt{12288}}{152}|E_{17}(\Upsilon2(t))| + \\ &\frac{12}{13}|E_{18}(\Upsilon2(t))| + \frac{\sqrt{12288}}{112}|E_{19}(\Upsilon2(t))| + \frac{\sqrt{24576}}{176}|E_{20}(\Upsilon2(t))| + \frac{\sqrt{18432}}{136}|E_{21}(\Upsilon2(t))| + \\ &\frac{24}{25}|E_{22}(\Upsilon2(t))|, \\ &= 1(4t) + \frac{2\sqrt{6}}{7}(4t) + \frac{4\sqrt{2}}{9}(4) + \frac{8}{17}(4t-4) + \frac{24}{25}(4t-4) + \frac{24\sqrt{2}}{41}(18t^2 - 30t + 14) + \\ &\frac{4\sqrt{3}}{7}(4t-4) + \frac{8\sqrt{3}}{19}(4t-4) + \frac{\sqrt{3}}{2}(12t-8) + \frac{2\sqrt{6}}{7}(4t-4) + \frac{3}{5}(4t-4) + \\ &\frac{2\sqrt{6}}{5}(8t-4) + \frac{\sqrt{3}}{2}(4t-4) + \frac{6}{11}\sqrt{2}(36t^2 - 72t + 36) + \frac{4\sqrt{3}}{7}(4) + \frac{4\sqrt{6}}{11}(4t-4) + \\ &\frac{\sqrt{12288}}{152}(4t-4) + \frac{12}{13}(4t-4) + \frac{\sqrt{12288}}{112}(4t-4) + \frac{\sqrt{24576}}{176}(4t-4) + \frac{\sqrt{18432}}{136}(4t - \\ &4) + \frac{24}{25}(36t^2 - 76t + 40), \end{aligned}$$

$$\Longrightarrow GA_M(\Upsilon_2) = \left(\frac{146884}{5525} + \frac{1848296\sqrt{2}}{69003} - \frac{1550\sqrt{3}}{133} - \frac{2176\sqrt{6}}{385}\right) + t\left(\frac{-315728}{5525} - \frac{414096\sqrt{2}}{7667} + \frac{2120\sqrt{3}}{133} + \frac{3232\sqrt{6}}{385}\right) + t^2\left(\frac{864}{25} + \frac{13608\sqrt{2}}{451}\right).$$

□

2.3. Results for third type of Dominating David Derived Network

In this section, we calculate the multiplicative topological degree based indices for third type of Dominating David Derived network $D_3(t)$ of dimension t.

Theorem 2.3.1. Consider the third type of Dominating David Derived network $\Upsilon_3 \cong D_3(t)$ for $t \in \mathbb{N}$. The first and second multiplicative Zagreb indices are equal to

$$II_1(\Upsilon_3) = 384t(-1 + 3t)(22 + t(-59 + 45t)),$$

$$II_2(\Upsilon_3) = 32768t^2(9t - 5)(11 + 9t(2t - 3)).$$

Proof. Let Υ_3 be the third type of Dominating David Derived network. The Υ_3 has $18t^2 - 6t$ vertices of degree 2 and $45t^2 - 59t + 22$ vertices of degree 4. The edge set of $D_3(t)$ is divided into three partitions based on the degrees of end vertices. Table 5, shows such an edge partition of $D_3(t)$. Thus from (1.1) is follows that,

$$II_1(\Upsilon_3) = \prod_{r \in V(\Upsilon_3)} (d_r)^2.$$

By using vertex partition, we get

Table 5. Edge partition of third type of Dominating David Derived network ($D_3(t)$) based on degrees of end vertices of each edge.

(d_r, d_s) where $rs \in E(\Upsilon_3)$	Number of edges
$(2, 2)$	$4t$
$(2, 4)$	$36t^2 - 20t$
$(4, 4)$	$72t^2 - 108t + 44$

$$II_1(\Upsilon_3) = (2)^2(18t^2 - 6t) \times (4)^2(45t^2 - 59t + 22),$$

$$II_1(\Upsilon_3) = 4(18t^2 - 6t) \times 16(45t^2 - 59t + 22),$$

By making some calculations, we have

$$\Longrightarrow II_1(\Upsilon_3) = 384t(-1 + 3t)(22 + t(-59 + 45t)).$$

Also from (1.2), we have

$$II_2(\Upsilon_3) = \prod_{rs \in E(\Upsilon_3)} (d_r \times d_s).$$

By using Table 5 edge partitions, we get

$$\begin{aligned} II_2(\Upsilon_3) &= 4|E_1(\Upsilon 3(t))| \times 8|E_2 \Upsilon 3(t)| \times 16|E_3 \Upsilon 3(t)|, \\ &= 4(4t) \times 8(36t^2 - 20t) \times 16(72t^2 - 108t + 44), \end{aligned}$$

By making some calculations, we have

$$\Longrightarrow II_2(\Upsilon_3) = 32768t^2(9t-5)(11+9t(2t-3)).$$

□

Now we calculate some advanced topological indices named as first Hyper-Zagreb index and second Hyper-Zagreb index of a graph Υ_3.

Theorem 2.3.2. Let $\Upsilon_3 \cong D_3(t)$ be the third type of Dominating David Derived network, then

$$HII_1(\Upsilon_3) = 2359296t^2(9t-5)(11+9t(2t-3)),$$

$$HII_2(\Upsilon_3) = 16777216t^2(9t-5)(11+9t(2t-3)).$$

Proof. The outcome is obtained by using the edge partition provided in Table 5. The result is from (1.3)

$$HII_1(\Upsilon_3) = \prod_{pq\in E(\Upsilon_3)} (d_p + d_q)^2.$$

$$\begin{aligned} HII_1(\Upsilon_3) &= 16|E_1(\Upsilon 2(t))| \times 36|E_2(\Upsilon 2(t))| \times 64|E_3(\Upsilon 2(t))|, \\ &= 16(4t) \times 36(36t^2-20t) \times 64(72t^2-108t+44), \end{aligned}$$

By doing some calculations, we get

$$\Longrightarrow HII_1(\Upsilon_3) = 2359296t^2(9t-5)(11+9t(2t-3)).$$

Also from (1.4),

$$HII_2(\Upsilon_3) = \prod_{pq\in E(\Upsilon_3)} (d_p \times d_q)^2.$$

$$HII_2(\Upsilon_3) = 16|E_1(\Upsilon 3(t))| \times 64|E_2(\Upsilon 3(t))| \times 256|E_3(\Upsilon 3(t))|,$$

$$HII_2(\Upsilon_3) = 16(4t) \times 64(36t^2-20t) \times 256(72t^2-108t+44),$$

By doing some calculations, we get

$$\Longrightarrow HII_2(\Upsilon_3) = 16777216t^2(9t-5)(11+9t(2t-3)).$$

□

Now the first and second Universal-Zagreb indices are defined as.

Theorem 2.3.3. Let $\Upsilon_3 \cong D_3(t)$ be the third type of Dominating David Derived network, then

$$MZ_1^a(\Upsilon_3) = 3^a 64^{1+a} t^2(9t-5)(11+9t(2t-3)),$$

$$MZ_2^a(\Upsilon_3) = 8^{2+3a} t^2(9t-5)(11+9t(2t-3)).$$

Proof. The outcome is obtained by using the edge partition provided in Table 5. The result is from (1.5),

$$MZ_1^a(\Upsilon_3) = \prod_{pq\in E(G)} (d_p + d_q)^a.$$

$$\begin{aligned} MZ_1^a(\Upsilon_3) &= (4)^a|E_1(\Upsilon 3(t))| \times (6)^a|E_2(\Upsilon 3(t))| \times (8)^a|E_3(\Upsilon 3(t))|, \\ &= 4^a(4t) \times 6^a(36t^2 - 20t) \times 8^a(72t^2 - 108t + 44), \end{aligned}$$

By doing some calculation, we get

$$\Longrightarrow MZ_1^a(\Upsilon_3) = 3^a 64^{1+a} t^2(9t - 5)(11 + 9t(2t - 3)).$$

Thus from (1.6),

$$MZ_2^a(\Upsilon_3) = \prod_{pq\in E(\Upsilon_3)} (d_p \times d_q)^a.$$

$$\begin{aligned} MZ_2^a(\Upsilon_3) &= (4)^a|E_1(\Upsilon 3(t))| \times (8)^a|E_2(\Upsilon 3(t))| \times (16)^a|E_3(\Upsilon 3(t))|, \\ &= 4^a(4t) \times 8^a(36t^2 - 20t) \times 16^a(72t^2 - 108t + 44), \end{aligned}$$

$$\Longrightarrow MZ_2^a(\Upsilon_3) = 8^{2+3a} t^2(9t - 5)(11 + 9t(2t - 3)).$$

□

The sum and product connectivity of multiplicative indices are described as follows.

Theorem 2.3.4. Let $\Upsilon_3 \cong D_3(t)$ be the third type of Dominating David Derived network of type 3, then

$$SCII(\Upsilon_3) = \frac{8}{\sqrt{3}} t^2(9t - 5)(11 + 9t(2t - 3)),$$

$$PCII(\Upsilon_3) = 2\sqrt{2} t^2(9t - 5)(11 + 9t(2t - 3)).$$

Proof. The outcome is obtained by using the edge partition provided in Table 5. The result is from (1.7),

$$SCII(\Upsilon_3) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_p + d_q}}.$$

$$\begin{aligned} SCII(\Upsilon_3) &= \frac{1}{2}|E_1(\Upsilon 3(t))| \times \frac{1}{\sqrt{6}}|E_2(\Upsilon 3(t))| \times \frac{1}{\sqrt{8}}|E_3(\Upsilon 3(t))|, \\ &= \frac{1}{2}(4t) \times \frac{1}{\sqrt{6}}(36t^2 - 20t) \times \frac{1}{\sqrt{8}}(72t^2 - 108t + 44), \end{aligned}$$

By doing some calculation, we get

$$\Longrightarrow SCII(\Upsilon_3) = \frac{8}{\sqrt{3}} t^2(9t - 5)(11 + 9t(2t - 3)).$$

Also from (1.8),

$$PCII(\Upsilon_3) = \prod_{pq\in E(\Upsilon_3)} \frac{1}{\sqrt{d_p \times d_q}}.$$

$$\begin{aligned} PCII(\Upsilon_3) &= \frac{1}{2}|E_1(\Upsilon3(t))| \times \frac{1}{\sqrt{8}}|E_2(\Upsilon3(t))| \times \frac{1}{4}|E_3(\Upsilon3(t))|, \\ &= \frac{1}{2}(4t) \times \frac{1}{\sqrt{8}}(36t^2 - 20t) \times \frac{1}{\sqrt{16}}(72t^2 - 108t + 44), \end{aligned}$$

By doing some calculations we get

$$\Longrightarrow PCII(\Upsilon_3) = 2\sqrt{2}t^2(9t - 5)(11 + 9t(2t - 3)).$$

□

The Multiple atom-bond connectivity index and Multiple Geometric-Arithmetic index are calculated as follows.

Theorem 2.3.5. Let $\Upsilon_3 \cong D_3(t)$ be the third type of Dominating David Derived network, then

$$\begin{aligned} ABC_M(\Upsilon_3) &= \frac{1}{192}(-144\sqrt{23} + 48\sqrt{35} - 24\sqrt{71} + 96\sqrt{78} - 24\sqrt{95} - 24\sqrt{134} - 12\sqrt{143} - \\ &\quad 6\sqrt{254} + 24\sqrt{318} + 30\sqrt{510} + 32\sqrt{573}) + \frac{1}{192}t(168\sqrt{14} + 144\sqrt{23} + 48\sqrt{35} + \\ &\quad 24\sqrt{71} - 264\sqrt{78} + 24\sqrt{95} + 24\sqrt{134} + 12\sqrt{143} + 6\sqrt{254} - 90\sqrt{318} - 57\sqrt{510} + \\ &\quad 32\sqrt{573}) + \frac{1}{192}t^2(216\sqrt{78} + 54\sqrt{318} + 27\sqrt{510}), \end{aligned}$$

$$\begin{aligned} GA_M(\Upsilon_3) &= (36t^2 - 60t + 36) + \frac{8}{17}(4t - 4) + \frac{8}{9}\sqrt{2}(4t - 4) + \frac{10}{9}\sqrt{2}(4t + 4) + \\ &\quad \frac{2}{3}\sqrt{2}(8t - 8) + \frac{2}{3}\sqrt{2}(12t - 12) + \frac{4}{5}(36t^2 - 60t + 16) + \frac{4}{5}(36t^2 - 44t + 16). \end{aligned}$$

Proof. The outcome is obtained by using the edge partition provided in Table 6. The result is from (1.9).

Table 6. Edge partition of third type of Dominating David Derived network ($D_3(t)$) based on degrees product of end vertices of each edge.

(M_r, M_s) where $rs \in E(\Upsilon_3)$	Number of edges	(M_r, M_s) where $rs \in E(\Upsilon_3)$	Number of edges
(8,8)	4t	(64,64)	8t
(8,64)	4t+4	(64,128)	8t-8
(8,128)	4t-4	(64,256)	$36t^2 - 60t + 16$
(16,32)	12t-12	(128,128)	4t-4
(16,64)	$36t^2 - 44t + 16$	(128,256)	4t+4
(16,128)	4t-4	(256,256)	$36t^2 - 76t + 40$
(32,256)	4t-4		

$$ABC_M(\Upsilon_3) = \sum_{rs\in E(\Upsilon_3)} \sqrt{\frac{M_r + M_s - 2}{M_r \times M_s}}.$$

$$\begin{aligned}
ABC_M(\Upsilon_3) &= \sqrt{\frac{7}{32}}|E_1(\Upsilon3(t))| + \sqrt{\frac{35}{256}}|E_2(\Upsilon3(t))| + \sqrt{\frac{67}{512}}|E_3(\Upsilon3(t))| + \sqrt{\frac{23}{256}}|E_4(\Upsilon3(t))| + \\
&\sqrt{\frac{39}{512}}|E_5(\Upsilon3(t))| + \sqrt{\frac{71}{1024}}|E_6(\Upsilon3(t))| + \sqrt{\frac{143}{4096}}|E_7(\Upsilon3(t))| + \sqrt{\frac{63}{2048}}|E_8(\Upsilon3(t))| + \\
&\sqrt{\frac{95}{4096}}|E_9(\Upsilon3(t))| + \sqrt{\frac{159}{8192}}|E_{10}(\Upsilon3(t))| + \sqrt{\frac{127}{8192}}|E_{11}(\Upsilon3(t))| + \\
&\sqrt{\frac{191}{16384}}|E_{12}(\Upsilon3(t))| + \sqrt{\frac{255}{32768}}|E_{13}(\Upsilon3(t))|, \\
&= \sqrt{\frac{7}{32}}(4t) + \sqrt{\frac{35}{256}}(4t+4) + \sqrt{\frac{67}{512}}(4t-4) + \sqrt{\frac{23}{256}}(12t-12) + \sqrt{\frac{39}{512}}(36t^2 - \\
&44t + 16) + \sqrt{\frac{71}{1024}}(4t-4) + \sqrt{\frac{143}{4096}}(4t-4) + \sqrt{\frac{63}{2048}}(8t) + \sqrt{\frac{95}{4096}}(8t-8) + \\
&\sqrt{\frac{159}{8192}}(36t^2 - 60t + 16) + \sqrt{\frac{127}{8192}}(4t-4) + \sqrt{\frac{191}{16384}}(4t+4) + \\
&\sqrt{\frac{255}{32768}}(36t^2 - 76t + 40),
\end{aligned}$$

By doing some calculations, we get

$$\begin{aligned}
\Longrightarrow ABC_M(\Upsilon_3) &= \frac{1}{192}(-144\sqrt{23} + 48\sqrt{35} - 24\sqrt{71} + 96\sqrt{78} - 24\sqrt{95} - 24\sqrt{134} - 12\sqrt{143} - \\
&6\sqrt{254} + 24\sqrt{318} + 30\sqrt{510} + 32\sqrt{573}) + \frac{1}{192}t(168\sqrt{14} + 144\sqrt{23} + 48\sqrt{35} + \\
&24\sqrt{71} - 264\sqrt{78} + 24\sqrt{95} + 24\sqrt{134} + 12\sqrt{143} + 6\sqrt{254} - 90\sqrt{318} - \\
&57\sqrt{510} + 32\sqrt{573}) + \frac{1}{192}t^2(216\sqrt{78} + 54\sqrt{318} + 27\sqrt{510}).
\end{aligned}$$

Also from (1.10),

$$GA_M(\Upsilon_3) = \sum_{rs\in E(\Upsilon_3)} \frac{2\sqrt{M_r \times M_s}}{(M_r + M_s)}.$$

$$\begin{aligned}
GA_M(\Upsilon_3) &= (1)|E_1(\Upsilon3(t))| + \frac{4\sqrt{2}}{9}|E_2(\Upsilon3(t))| + \frac{8}{17}|E_3(\Upsilon3(t))| + \frac{2\sqrt{2}}{3}|E_4(\Upsilon3(t))| + \frac{4}{5}|E_5(\Upsilon3(t))| + \\
&\frac{4\sqrt{2}}{9}|E_6(\Upsilon3(t))| + \frac{\sqrt{8192}}{144}|E_7(\Upsilon3(t))| + (1)|E_8(\Upsilon3(t))| + \frac{\sqrt{8192}}{96}|E_9(\Upsilon3(t))| + \\
&\frac{4}{5}|E_{10}(\Upsilon3(t))| + (1)|E_{11}(\Upsilon3(t))| + (\frac{\sqrt{32768}}{192})|E_{12}(\Upsilon3(t))| + (1)|E_{13}(\Upsilon3(t))|, \\
&= (1)|(4t) + \frac{4\sqrt{2}}{9}(4t+4) + \frac{8}{17}(4t-4)| + \frac{2\sqrt{2}}{3}(12t-12) + \frac{4}{5}(36t^2 - 44t + 16) +
\end{aligned}$$

$$\frac{4\sqrt{2}}{9}(4t-4)+\frac{\sqrt{8192}}{144}(4t-4)+(1)(8t)+\frac{\sqrt{8192}}{96}(8t-8)+\frac{4}{5}(36t^2-60t+16)+$$
$$(1)(4t-4)+\frac{\sqrt{32768}}{192}(4t+4)+(1)(36t^2-76t+40),$$

$$\begin{aligned}\Longrightarrow GA_M(\Upsilon_3) \;=\; & (36t^2-60t+36)+\frac{8}{17}(4t-4)+\frac{8}{9}\sqrt{2}(4t-4)+\frac{10}{9}\sqrt{2}(4t+4)+\\ & \frac{2}{3}\sqrt{2}(8t-8)+\frac{2}{3}\sqrt{2}(12t-12)+\frac{4}{5}(36t^2-60t+16)+\frac{4}{5}(36t^2-44t+16).\end{aligned}$$

□

The graphical representations of topological indices of these networks are depicted in Figures 6 and 7 for certain values of t. By varying the different values of t, the graphs are increasing. These graphs show the accuracy of the results.

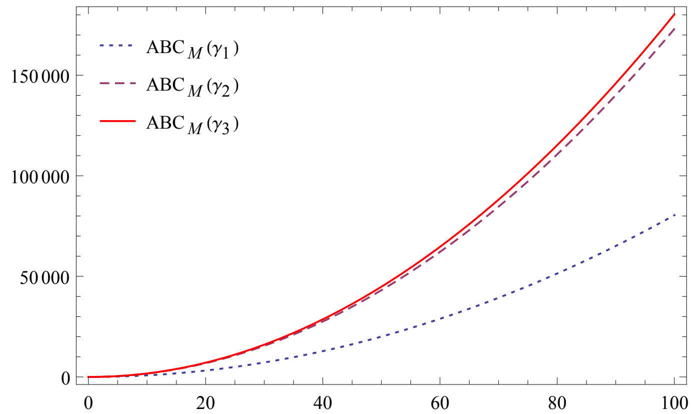

Figure 6. Comparison of ABC_M index for γ_1, γ_2 and γ_3.

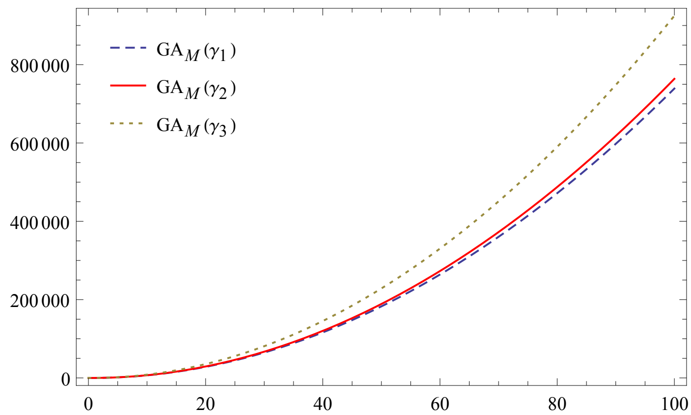

Figure 7. Comparison of GA_M index for γ_1, γ_2 and γ_3.

3. Conclusion

In this article, we computed degree-based indices for some derived graphs of HC_n graph. We also computed certain degree-based polynomials such as Multiplicative Zagreb, Hyper Zagreb, Universal

Zagreb, Sum and Product connectivity of Multiplicative indices, Multiple Atom-Bond connectivity index and Multiple Geometric Arithmetic index for three types of Dominating David Derived networks. We also gave index comparison of these networks. Almost all indices increase with increase in t. These facts may be useful for people working in computer science and chemistry who encounter honeycomb networks. Finding expressions of derived graphs like these is an open problem for many other topological indices.

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Conflict of interest

The authors declare no conflict of interest.

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