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Research article

Mixed lump and soliton solutions for a generalized (3+1)-dimensional Kadomtsev-Petviashvili equation

Yu-Lan Ma[1] **and Bang-Qing Li**[2,*]

[1] School of Mathematics and Statistics, Beijing Technology and Business University, Beijing, 100048, China

[2] School of Computer and Information Engineering, Beijing Technology and Business University, Beijing, 100048, China

* **Correspondence:** Email: libq@th.btbu.edu.cn.

Abstract: Under investigation is a generalized (3+1)-dimensional Kadomtsev-Petviashvili equation which can be used to describe nonlinear wave propagation in dissipative media. Via the bilinear transformation method, the mixed lump and soliton solutions are obtained for the equation. The asymptotic behavior of the mixed solutions are analyzed. Furthermore, the fusion and fission behaviors of the lump and soliton are observed for the first time. The lump and soliton can merge into a whole soliton over time, or, on the contrary, the soliton may differentiate into a lump and a new soliton. During the processes, the amplitude of the lump will greatly vary, while the amplitude of the soliton will change slightly.

Keywords: generalized (3+1)-dimensional Kadomtsev-Petviashvili equation; bilinear transformation method; mixed lump and soliton solution; asymptotic behavior; fusion and fission
Mathematics Subject Classification: 35Q51, 35Q53, 37K40

1. Introduction

As we know, nonlinearity is a large class of essential phenomena of the world, and the soliton theory plays a critical part in nonlinear science [1–12]. Lump, sometimes called as rogue wave, is a special form of solitons, which has been observed in the deep ocean [13, 14], water wave experiments in tank [15, 16], and optical fibers experiments [17]. Lump is generally localized for space and time variables, and has a bigger amplitude being several times than ones of its surrounding waves. Lump would be harmful, disastrous, and even destructive for some nonlinear systems, such as ocean and water engineering. But lump may be used to amplify signals in other systems, such as in ferrite magnetic materials and optical fibers. It is significant to predict and find where and when it appears and disappears. The research on the lump solution has drawn more and more attention [18–36].

Interaction behaviors between solitons are meaningful in physic and its applications, because they will affect the wave propagation, such as elastic and non-elastic collisions [9, 37–40], nonlinear superposition effects [41,42], fusion and fission phenomena [43,44].

In this work, we investigate a generalized (3+1)-dimensional Kadomtsev-Petviashvili equation, which reads

$$(u_t + 6uu_x + u_{xxx})_x + \alpha\left(u_{yy} + u_{zz}\right) = 0, \tag{1.1}$$

where u stands for a normalized physical quantity depending on spatial variable (x, y, z) and temporal variable t, and α is a equation parameter.

Equation (1.1) was initially considered to model the nonlinear wave propagations in dissipative media [45–47]. When $\alpha < 0$, Eq. (1.1) is called as KPI-type equation, and when $\alpha > 0$, Eq. (1.1) is called as KPII-type equation. Because of the importance in theory and application, Eq. (1.1) has been extensively investigated by various methods. In Ref. [48], the integrability and Painlevé test was discussed, and the one-soliton and two-soliton solutions and four classes of specific three-soliton solutions were explicitly presented to Eq. (1.1) with $\alpha = \pm 3$. In Ref. [49], analytical breather solution was obtained via the bilinear transformation method for Eq. (1.1) with $\alpha = -1$. Then, rogue wave solution was attained as a long wave homoclinic limit of the breathers. In Refs. [50,51], the traveling wave solutions were discussed to Eq. (1.1) with $\alpha = -3$. However, novel fusion and fission dynamics of mixed lump and soliton solution has not been reported for the equation (1.1) so far.

Recently, a method was proposed to calculate the lump solution by extending the bilinear method [52–56]. Its key idea is to construct proper polynomial functions in the bilinear form. We obtain the mixed lump and soliton solutions of Eq. (1.1) with $\alpha < 0$ via the method. Furthermore, the fusion and fission behaviors between the lump and soliton are first observed.

2. Bilinear form of Equation (1.1)

In this section, we will give the following theorem derived from the bilinear theory.

Theorem 1. The bilinear form of Eq. (1.1) is

$$\left[D_x D_t + 6u_0 D_x^2 + D_x^4 + \alpha\left(D_y^2 + D_z^2\right)\right](f \cdot f) = 0, \tag{2.1}$$

where D is the bilinear operator defined in Ref. [57] (also see Refs. [40,41]).

Proof. Firstly, we are able to introduce a transformation to Eq. (1.1) as

$$u = u_0 + 2(\ln f)_{xx}, \tag{2.2}$$

where $f > 0$ is a real function of x, y, z and t, u_0 is an arbitrary real constant.

From (2.2), it is seen that

$$u_t = 2(\ln f)_{xxt}, u_x = 2(\ln f)_{xxx}, u_{xxx} = 2(\ln f)_{xxxxx}, u_{yy} = 2(\ln f)_{xxyy}, u_{zz} = 2(\ln f)_{xxzz}. \tag{2.3}$$

Substituting (2.2) and (2.3) into Eq. (1.1) and integrating once with respect to x and letting the integral constant be zero, it follows

$$(\ln f)_{xxt} + 6u_0(\ln f)_{xxx} + 12(\ln f)_{xx}(\ln f)_{xxx} + (\ln f)_{xxxxx} + \alpha\left[(\ln f)_{xyy} + (\ln f)_{xzz}\right] = 0. \tag{2.4}$$

Integrating once again with respect to x and letting the integral constant be zero, it yields

$$(\ln f)_{xt} + 6u_0(\ln f)_{xx} + 6(\ln f)_{xx}(\ln f)_{xx} + (\ln f)_{xxxx} + \alpha\left[(\ln f)_{yy} + (\ln f)_{zz}\right] = 0. \tag{2.5}$$

Noticing that

$$(\ln f)_{xt} = \frac{1}{2}\frac{D_x D_t (f\cdot f)}{f^2}, 6u_0(\ln f)_{xx} = \frac{6u_0}{2}\frac{D_x^2 (f\cdot f)}{f^2}, \tag{2.6}$$

$$(\ln f)_{xxxx} + 6(\ln f)_{xx}(\ln f)_{xx} = \frac{1}{2}\frac{D_x^4 (f\cdot f)}{f^2}, \tag{2.7}$$

$$(\ln f)_{yy} = \frac{1}{2}\frac{D_y^2 (f\cdot f)}{f^2}, (\ln f)_{zz} = \frac{1}{2}\frac{D_z^2 (f\cdot f)}{f^2}, \tag{2.8}$$

and substituting (2.6)-(2.8) into Eq. (2.5), we find Eq. (2.5) is right. Thus, the Theorem 1 is proved.

3. Mixed lump and soliton solutions

In this section, we first obtain the lump solution for Eq. (1.1) from the bilinear form (2.1). Then the mixed lump and soliton solutions will be attained.

3.1. Lump solution

According to the idea in Refs. [52, 53], we set f in the bilinear form (2.1) as

$$f = h_0 + \xi_1^2 + \xi_2^2, \tag{3.1}$$

where $\xi_i = a_i x + b_i y + c_i z + d_i t,\ i = 1, 2.$

Substituting (3.1) into (2.1), collecting the terms with the same power of $(\xi_1^2 - \xi_2^2), \xi_1\xi_2, \xi_1^0\xi_2^0$, and letting their coefficients be zero, we get a set of equations. When $a_1^2 + a_2^2 \neq 0$ and $(a_1b_2 - a_2b_1)^2 + (a_1c_2 - a_2c_1)^2 \neq 0$, we have

$$\begin{cases} d_1 = -6u_0a_1 + \frac{\alpha}{a_1^2+a_2^2}\left[a_1\left(b_2^2 + c_2^2 - b_1^2 - c_1^2\right) - 2a_2\left(b_1b_2 + c_1c_2\right)\right], \\ d_2 = -6u_0a_2 - \frac{\alpha}{a_1^2+a_2^2}\left[a_2\left(b_2^2 + c_2^2 - b_1^2 - c_1^2\right) + 2a_1\left(b_1b_2 + c_1c_2\right)\right], \\ h_0 = \frac{-3\left(a_1^2+a_2^2\right)^3}{\alpha\left[(a_1b_2-a_2b_1)^2+(a_1c_2-a_2c_1)^2\right]}, \alpha < 0. \end{cases} \tag{3.2}$$

Therefore, the lump solution of Eq. (1.1) can be obtained as follow

$$u(x,y,z,t) = u_0 + \frac{4\left[h_0\left(a_1^2 + a_2^2\right) + \left(a_2^2 - a_1^2\right)\left(\xi_1^2 - \xi_2^2\right) - 4a_1a_2\xi_1\xi_2\right]}{\left(h_0 + \xi_1^2 + \xi_2^2\right)^2}, \tag{3.3}$$

where $\xi_i = a_i x + b_i y + c_i z + d_i t,\ i = 1, 2.$

We give the plots of the solution (3.3) in six coordinates, namely, the x-y-u, the x-z-u, the y-z-u, the x-t-u, the y-t-u and the z-t-u coordinates (see Figure 1). The lump wave is localized in all the spaces and time directions. In fact, for the solution (3.3), it is seen to prove

$$\lim_{\forall x,y,z,t\to\pm\infty} u(x,y,z,t) = u_0.$$

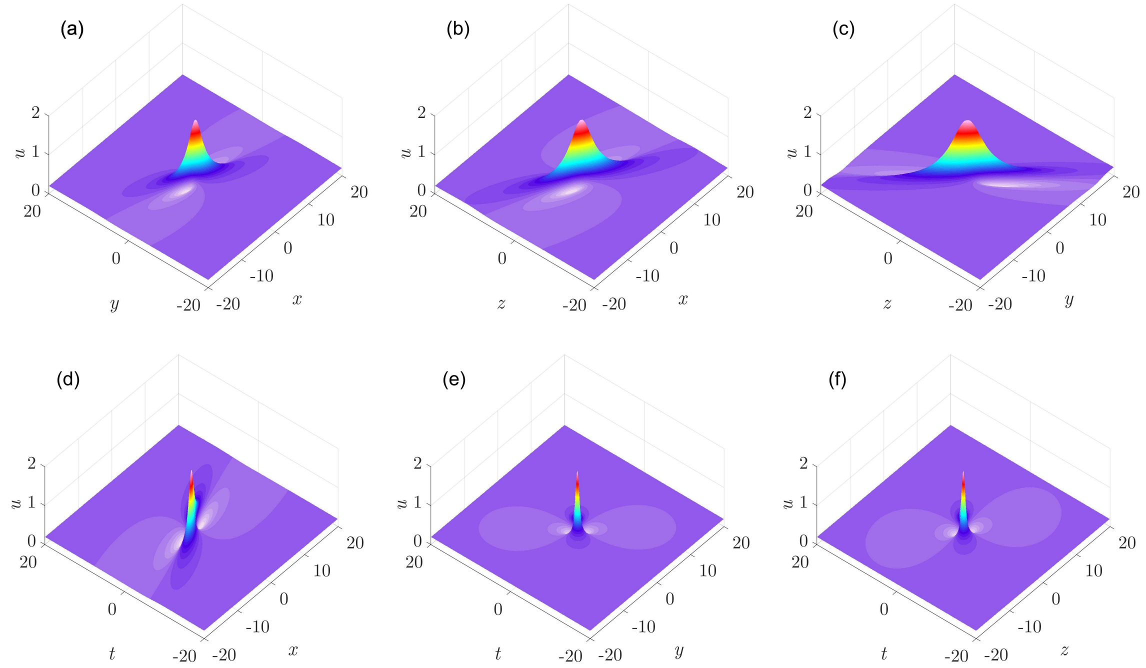

Figure 1. The plots of the lump solution (3.3) in different coordinates. The parameter settings are as follows: $u_0 = 0.2, a_1 = 1.0, a_2 = 0.2, b_1 = 1.0, b_2 = 1.9, c_1 = 1.2, c_2 = 1.3, \alpha = -0.3$, (a) the x-y-u coordinate with $z = t = 0$; (b) the x-z-u coordinate with $y = t = 0$; (c) the y-z-u coordinate with $x = t = 0$; (d) the x-t-u coordinate with $y = z = 0$; (d) the y-t-u coordinate with $x = z = 0$; (f) the z-t-u coordinate with $x = y = 0$.

3.2. Mixed lump and soliton solutions

We set f in the bilinear form (2.1) as

$$f = h_0 + \xi_1^2 + \xi_2^2 + \mathrm{e}^{\xi_3}, \tag{3.4}$$

where $\xi_i = a_i x + b_i y + c_i z + d_i t$, a_i, b_i, c_i and d_i $(i = 1, 2, 3)$ are constants to be determined later.

Substituting (3.4) into (2.1), collecting the terms with the same power of $\left(\xi_1^2 + \xi_2^2\right)\mathrm{e}^{\xi_3}$, $\xi_1^2 - \xi_2^2, \xi_1\xi_2, \xi_1\mathrm{e}^{\xi_3}$, $\xi_2\mathrm{e}^{\xi_3}, \mathrm{e}^{\xi_3}$ and $\xi_1^0\xi_2^0$, and letting their coefficients be zero, we get a set of equations. Solving this set of equations will yields two sets of solutions with respect to a_i, b_i, c_i, d_i $(i = 1, 2, 3)$ and h_0 as follows

Case I:

$$\begin{cases} a_1 = a_3 = (-1)^j \sqrt[4]{-\frac{\alpha}{3}\left(b_2^2 + c_2^2\right)}, j = 0, 1, a_2 = 0, \\ b_1 = b_3 = b \neq 0, c_1 = c_3 = c \neq 0, \\ d_1 = -6u_0 a_1 + \dfrac{\alpha}{a_1}\left(b_2^2 + c_2^2 - b^2 - c^2\right), d_2 = -\dfrac{2\alpha}{a_1}\left(bb_2 + cc_2\right), \\ d_3 = -6u_0 a_1 - a_1^3 - \frac{\alpha\left(b^2+c^2\right)}{a_1}, h_0 = 1, \end{cases} \tag{3.5}$$

Case II:

$$
\begin{cases}
a_1 = (-1)^j \sqrt[4]{-\frac{\alpha c^2}{3b^2}\left(b_2^2 + c_2^2\right)}, j = 0, 1, a_2 = 0, a_3 = \frac{ba_1}{c}, \\
b_1 = c_1 = c \neq 0, b_3 = c_3 = b \neq 0, \\
d_1 = -6u_0 a_1 + \frac{\alpha}{a_1}\left(b_2^2 + c_2^2 - 2b^2\right), d_2 = -\frac{2\alpha c}{a_1}(b_2 + c_2), \\
d_3 = -\frac{6u_0 b a_1}{c} - \left(\frac{ba_1}{c}\right)^3 - \frac{2\alpha bc}{a_1}, h_0 = \frac{c^2}{b^2},
\end{cases}
\tag{3.6}
$$

where $b \neq 0, c \neq 0$ are arbitrary real constants, b_2 and c_2 are arbitrary real constants and satisfy $b_2^2 + c_2^2 \neq 0$.

Thus, we are able to obtain two mixed solutions for Eq. (1.1) corresponding to (3.5) and (3.6), respectively,

$$
u_1(x, y, z, t) = u_0 + 2\sqrt{-\frac{\alpha\left(b_2^2 + c_2^2\right)}{3}}\frac{\left(2 - 2\left(\xi_1^2 - \xi_2^2\right) + 3e^{\xi_3} - 4\xi_1 e^{\xi_3} + \left(\xi_1^2 + \xi_2^2\right)e^{\xi_3}\right)}{\left(h_0 + \xi_1^2 + \xi_2^2 + e^{\xi_3}\right)^2}, \tag{3.7}
$$

with $\xi_i = a_i x + b_i y + c_i z + d_i t$, a_i, b_i, c_i and d_i $(i = 1, 2, 3)$ are given by (3.5), and

$$
u_2(x, y, z, t) = u_0 + 2\sqrt{-\frac{\alpha c^2\left(b_2^2 + c_2^2\right)}{3b^2}}\frac{\left(\frac{2c^2}{b^2} - 2\left(\xi_1^2 - \xi_2^2\right) + 3e^{\xi_3} - \frac{4b}{c}\xi_1 e^{\xi_3} + \frac{b^2}{c^2}\left(\xi_1^2 + \xi_2^2\right)e^{\xi_3}\right)}{\left(h_0 + \xi_1^2 + \xi_2^2 + e^{\xi_3}\right)^2}, \tag{3.8}
$$

$\xi_i = a_i x + b_i y + c_i z + d_i t$, h_0 a_i, b_i, c_i and d_i $(i = 1, 2, 3)$ are given by (3.6).

The mixed lump and soliton solutions (3.7) and (3.8) involve exponential function and rational function, which mathematically represents lump and soliton, respectively. In Figure 2, the plots of the solution (3.7) are figured in all the six coordinates.

Remark: The mixed lump and soliton solutions (3.7) and (3.8) are also named as lump-kink solutions.

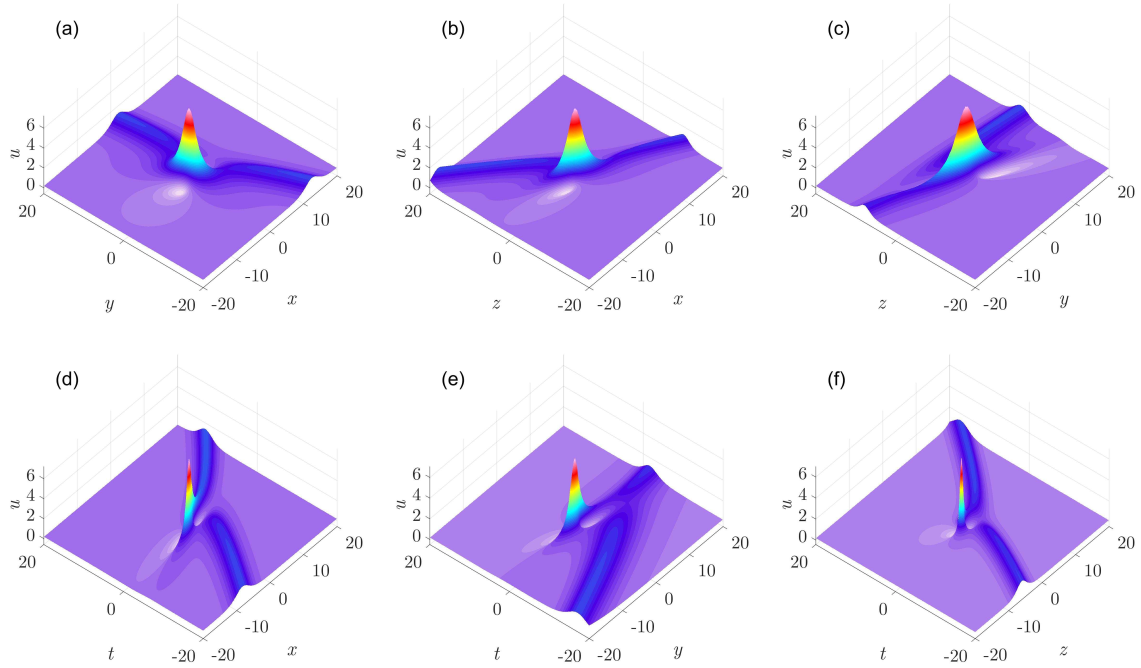

Figure 2. The plots of the mixed lump and soliton solution (3.7) in different coordinates. The parameter settings are as follows: $u_0 = 0.2, a_2 = 0, b = 0.2, b_2 = 0.8, c = 1, c_2 = 1.2, h_0 = 1, \alpha = -0.5$, (a) the x-y-u coordinate with $z = t = 0$; (b) the x-z-u coordinate with $y = t = 0$; (c) the y-z-u coordinate with $x = t = 0$; (d) the x-t-u coordinate with $y = z = 0$; (d) the y-t-u coordinate with $x = z = 0$; (f) the z-t-u coordinate with $x = y = 0$.

3.3. Asymptotic behavior of the mixed solutions

Asymptoticity is an important concept that depicts the global characteristics of a system [58, 59]. We take the solution (3.7) under $t \to \infty$ as the example to discuss the asymptotic behavior of the mixed solutions.

When $(x, y, z) \to (x_0, y_0, z_0)$, $t \to +\infty$ and $d_3 > 0$ to (3.7), we are able to derive

$$\left(h_0 + \xi_1^2 + \xi_2^2 + e^{\xi_3}\right)^2 = O\left(\left(e^{\xi_3}\right)^2\right),$$

$$\left(2 - 2\left(\xi_1^2 - \xi_2^2\right) + 3\mathrm{e}^{\xi_3} - 4\xi_1\mathrm{e}^{\xi_3} + \left(\xi_1^2 + \xi_2^2\right)\mathrm{e}^{\xi_3}\right) = O\left(\left(\xi_1^2 + \xi_2^2\right)\mathrm{e}^{\xi_3}\right).$$

Thereby, we have

$$\lim_{t\to+\infty} \frac{\left(\xi_1^2 + \xi_2^2\right)e^{\xi_3}}{e^{2\xi_3}} = \lim_{t\to+\infty} \frac{\left(\xi_1^2 + \xi_2^2\right)}{e^{\xi_3}} = 0.$$

When $(x, y, z) \to (x_0, y_0, z_0)$, $t \to +\infty$ and $d_3 < 0$, we can derive

$$\left(h_0 + \xi_1^2 + \xi_2^2 + e^{\xi_3}\right)^2 = O\left(\left(\xi_1^2 + \xi_2^2\right)^2\right),$$

$$\left(2 - 2\left(\xi_1^2 - \xi_2^2\right) + 3\mathrm{e}^{\xi_3} - 4\xi_1\mathrm{e}^{\xi_3} + \left(\xi_1^2 + \xi_2^2\right)\mathrm{e}^{\xi_3}\right) = O\left(\left(\xi_1^2 + \xi_2^2\right)^2\right).$$

Thus, we know

$$\lim_{t\to+\infty}\frac{O\left(\xi_1^2+\xi_2^2\right)}{O\left(\left(\xi_1^2+\xi_2^2\right)^2\right)}=0.$$

Consequently, the mixed solution (3.7) will lead

$$\lim_{\substack{(x,y,z)\to(x_0,y_0,z_0),\\ t\to+\infty}} u_1(x,y,z,t)=u_0+2\sqrt{-\frac{\alpha\left(b_2^2+c_2^2\right)}{3}}\frac{O\left(\xi_1^2+\xi_2^2\right)}{O\left(\left(\xi_1^2+\xi_2^2\right)^2\right)}=u_0. \tag{3.9}$$

Similarly, we have

$$\lim_{\substack{(x,y,z)\to(x_0,y_0,z_0),\\ t\to-\infty}} u_1(x,y,z,t)=u_0. \tag{3.10}$$

Thereby, it is seen that

$$\lim_{\substack{(x,y,z)\to(x_0,y_0,z_0),\\ t\to\infty}} u_1(x,y,z,t)=\lim_{\substack{(x,y,z)\to(x_0,y_0,z_0),\\ t\to\infty}} u_2(x,y,z,t)=u_0. \tag{3.11}$$

Besides, the solitons, involved in the mixed solutions, are global. This feature is different from one of the lump. We give graphically the asymptotic feature of the solitons. In Figure 3, the solitons will hold its profile, and its amplitude will tend to a stable value which is determined by the settings.

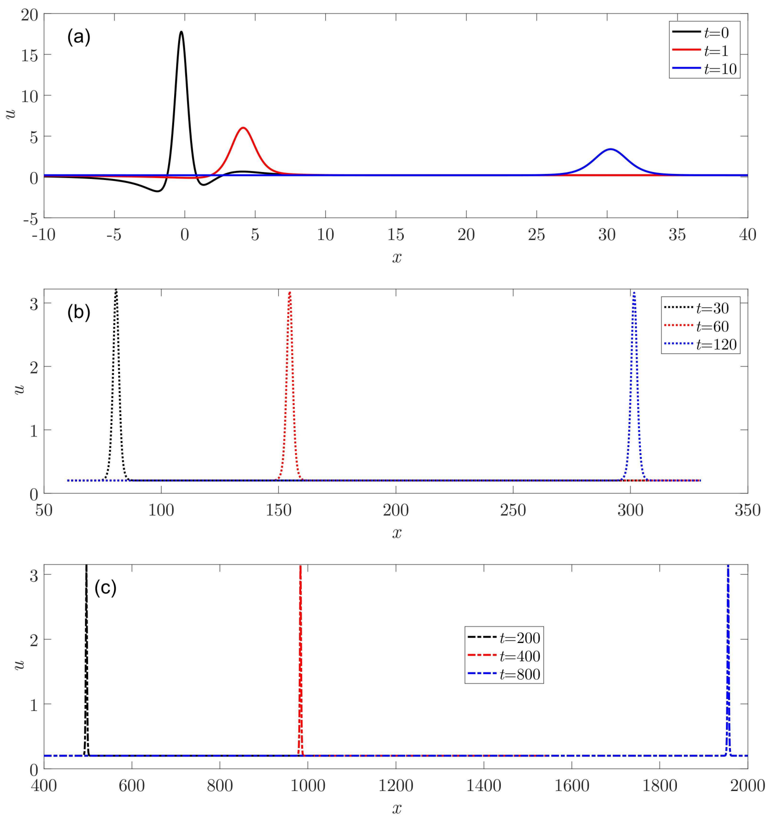

Figure 3. The asymptotic behavior of the mixed solution over time. The plots are given by the mixed solution (3.7). The settings are as follows: $y = z = 0, \alpha = -0.5, u_0 = 0.2, j = 0, b = 0.6, b_2 = 2, c = 0.6, c_2 = 3$, and different t.

4. Fusion and fission dynamics between lump and soliton

Without loss of generality, we just discuss the fusion and fission dynamics of the mixed solutions (3.7) and (3.8) with $z = 0$ in the x-y-u coordinate.

4.1. Fusion of lump and soliton

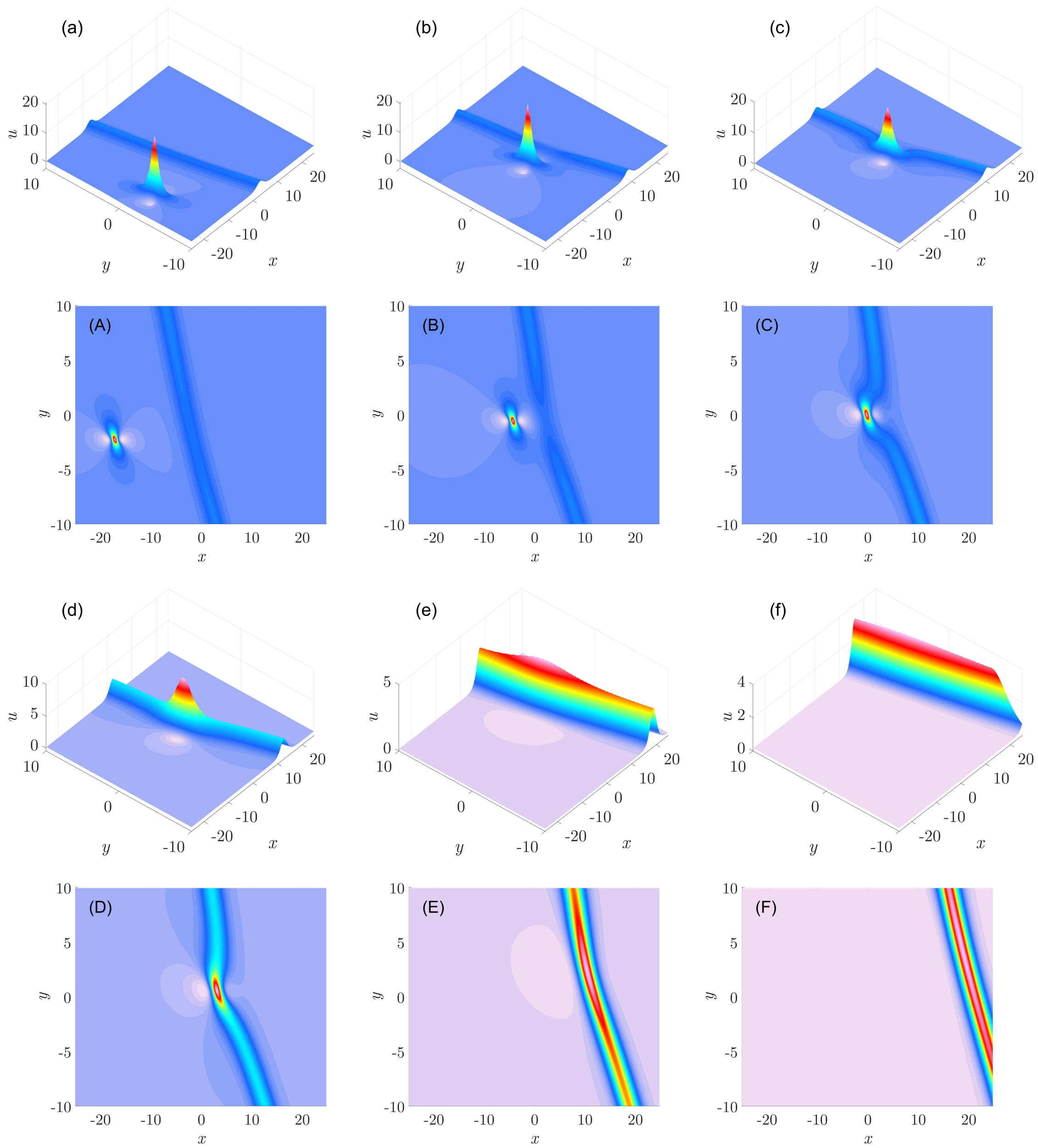

Figure 4. The fusion behavior between the lump and soliton over time. The plots are given by the mixed solution (3.7). The settings are as follows: $z = 0, \alpha = -0.5, u_0 = 0.2, j = 0, b = 0.6, b_2 = 2, c = -1, c_2 = 2.5$, the other parameters are determined by (3.5), and different t: (a) $t = -10$; (b) $t = -5$; (c) $t = -2.5$; (d) $t = 0$; (e) $t = 2.5$; (f) $t = 5$.

We first unearth the fusion behavior between the lump and soliton for the equation (1.1). By setting $z = 0, \alpha = -0.5, u_0 = 0.2, j = 0, b = 0.6, b_2 = 2, c = -1$ and $c_2 = 2.5$ in the mixed solution (3.7), and letting the time variable t vary from $t = -10$ to $t = 5$, we are able to observe the fusion between the lump and soliton over time. A series of plots are given to demonstrate the fusion behavior (see Figure 4). In detail, the lump and soliton all move from the negative to the positive direction of the x-axis during the process. As $t = -10$, the lump and soliton are completely separated. As t varies form -10 to 0, they are gradually approach. At $t = 0$, the lump and soliton are together, but their amplitudes are greatly different. With the further increase of time, their amplitudes are getting closer and closer until the lump and soliton completely merge into a soliton.

In addition, during the fusion process between the lump and soliton, it is very clear that the amplitude of the lump obviously decreases (from about 20 to 3). However, the amplitude of the soliton gradually increases. It means that the energy of the lump is transmitted into the soliton. The amplitude evolution of the soliton is illustrated in Figure 5.

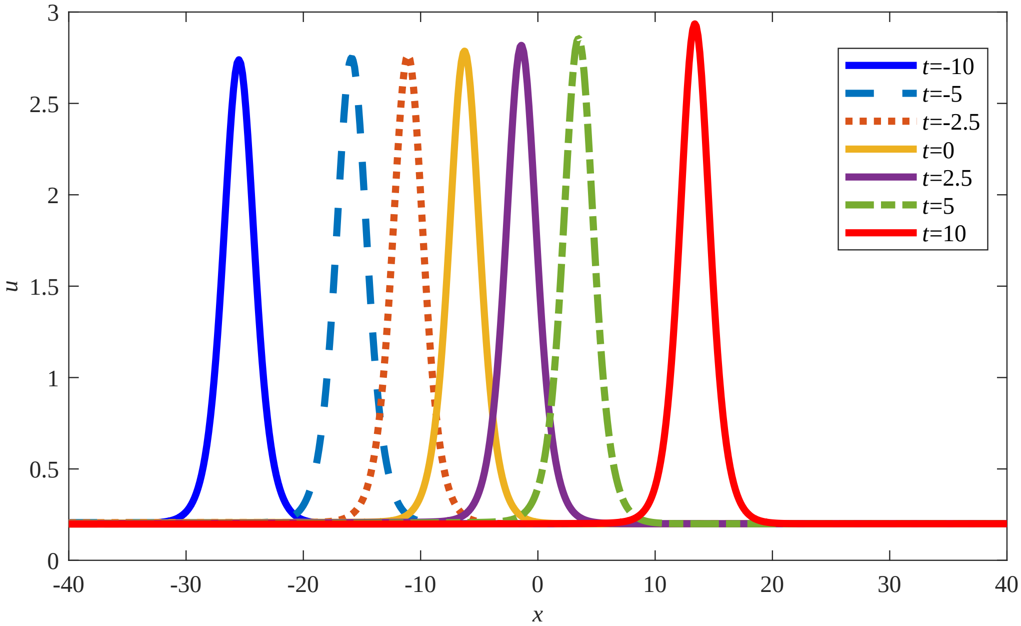

Figure 5. The amplitude evolution of the soliton over time. The plots are given by the mixed solution (3.7). The settings are as follows: $y = 25, z = 0, \alpha = -0.5, u_0 = 0.2, j = 0, b = 0.6, b_2 = 2, c = -1, c_2 = 2.5$, and different t.

4.2. *Fission of lump and soliton*

The behavior corresponding to the fusion is fission. Now, we investigate the fission behavior between the lump and soliton via the mixed solution (3.8) by a similar way used in the previous subsection.

By $z = 0, \alpha = -0.5, u_0 = 0.2, j = 0, b = 1, b_2 = 1, c = 0.2$ and $c_2 = 1$ in the mixed solution (3.8), and letting the time variable t vary from $t = -1$ to $t = 1$, we are able to observe the fission behavior over time at the seven values ($t = -1, -0.5, -0.25, 0, 0.25$ and 0.5, respectively). During the process, the lump is gradually separated from the soliton, and are thrown farther and farther away. Simultaneously, the amplitude of the lump increases rapidly, and the amplitude of the soliton decreases gradually. More details can be found in Figures 6 and 7.

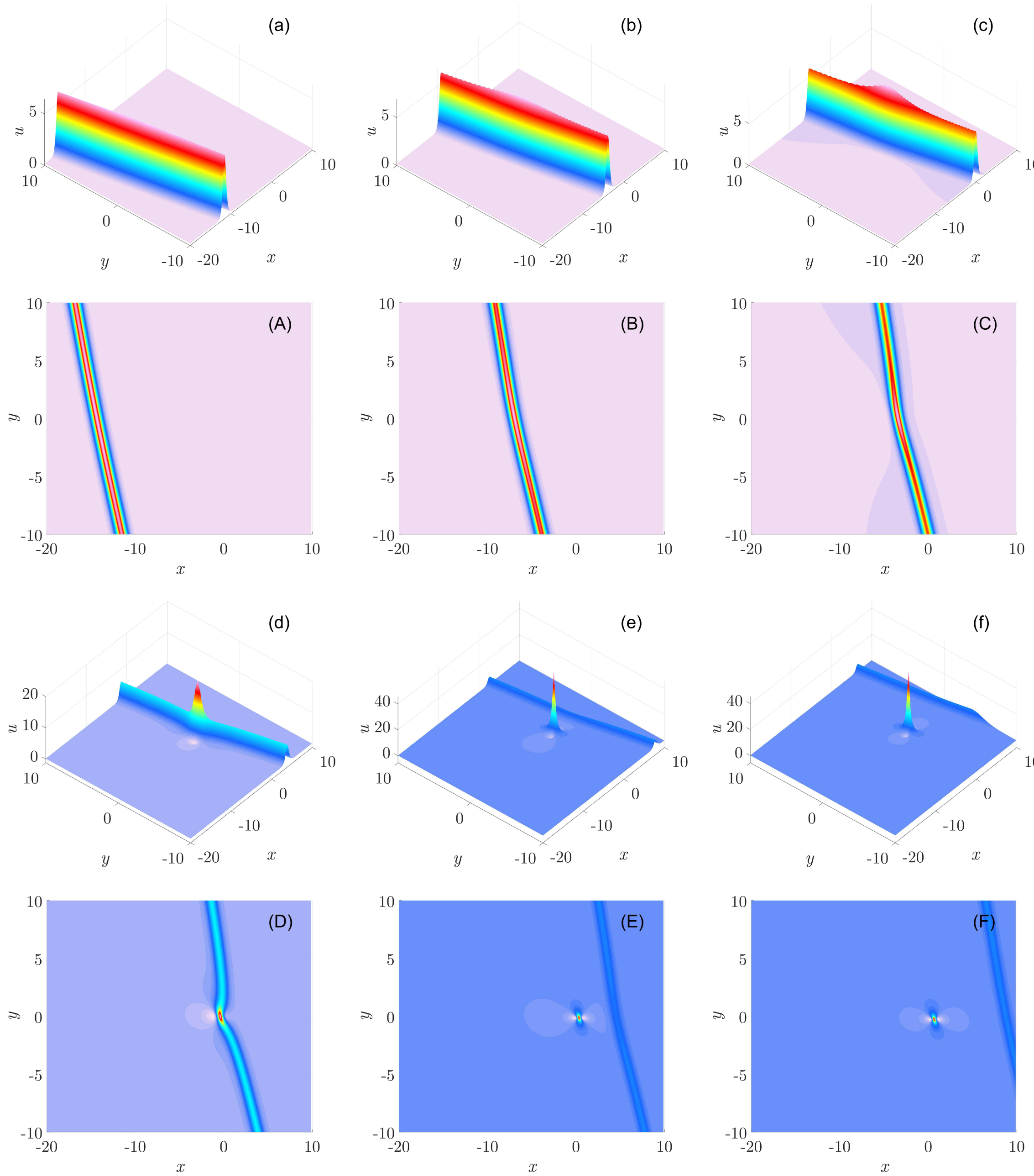

Figure 6. The fission behavior between the lump and soliton over time. The plots are given by the mixed solution (3.8). The settings are as follows: $z = 0, \alpha = -0.5, u_0 = 0.2, j = 0, b = 1, b_2 = 1, c = 0.2, c_2 = 1$, the other parameters are determined by (3.6), and different t: (a) $t = -1$; (b) $t = -0.5$; (c) $t = -0.25$; (d) $t = 0$; (e) $t = 0.25$; (f) $t = 0.5$.

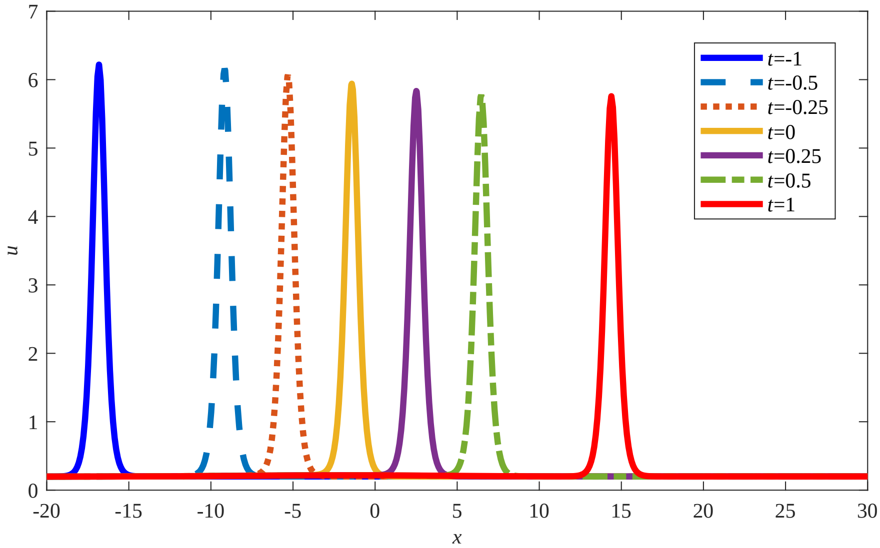

Figure 7. The amplitude evolution of the soliton over time. The plots are given by the mixed solution (3.8). The settings are as follows: $y = 10, z = 0, \alpha = -0.5, u_0 = 0.2, j = 0, b = 1, b_2 = 1, c = 0.2, c_2 = 1$, and different t.

5. Conclusions

The (3+1)-dimensional Kadomtsev-Petviashvili equation (1.1) is widely used to depict the nonlinear wave propagation in diverse dissipative media. The lump and soliton are two classical types of nonlinear waves. In this work, the main attention is focused on the mixed lump and soliton solutions and their dynamics for the equation.

Starting from the bilinear transformation of the equation (1.1), through properly constructing the polynomial functions in the bilinear forms, the lump solution was first obtained, then two mixed lump and soliton solutions were constructed under the equation parameter $\alpha < 0$. The mixed solutions are fundamental for the further study of the interaction behaviors between the lump and soliton.

Based on the mixed solutions, the asymptotic behavior of the mixed solutions are analyzed. Furthermore, novel fusion and fission behaviors between the lump and soliton were observed for the first time. The lump and soliton can merge into a whole soliton, or, on the contrary, the soliton may be differentiated into a lump and a new soliton. During the processes, the amplitude of the lump will greatly vary, while the amplitude of the soliton will change slightly. Considering the importance of the lump and soliton in physics and its applications, these new observations are valuable to increase understanding of the equation and can be used to explain interesting interaction phenomena between different nonlinear waves.

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Conflict of interest

The authors declare that they have no competing interests in this paper.

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