Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 5(1): 507–521.
DOI:10.3934/math.2020034
Received: 09 October 2019
Accepted: 29 November 2019
Published: 06 December 2019

Research article

Complex solitons in the conformable (2+1)-dimensional Ablowitz-Kaup-Newell-Segur equation

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Abstract: In this paper, we study on the conformable (2+1)-dimensional Ablowitz-KaupNewell-Segur equation in order to show the existence of complex combined dark-bright soliton solutions. To this purpose an effective method which is the sine-Gordon expansion method is used. The 2D and 3D surfaces under some suitable values of parameters are also plotted.

Keywords: conformable (2+1)-dimensional Ablowitz-Kaup-Newell-Segur equation; sine Gordon expansion method; complex soliton solutions
Mathematics Subject Classification: 35Qxx, 35C08, 35L05

1. Introduction

Various natural phenomena, especially in physics, are commonly modelled by nonlinear differential equations. For a better understanding of the behaviors of these equations and the properties of the corresponding solutions, many researchers have improved various methods such as the homotopy perturbation method [1,2], the homotopy analysis method [3,4], expfunction method [5,6], $\exp(-\Omega(\xi))$ expansion function method [7], extended sinh- Gordon equation expansion method [8–11], Hirota' s bilinear method [12], Bäcklund transformation [13]. By using the sine-Gordon expansion method we will investigate the solutions of the conformable (2+1)-dimensional Ablowitz-Kaup-Newell-Segur equation. The Ablowitz-KaupNewell-Segur(AKNS) water wave equation which is playing a fundamental role in physics [14]. A number of methods have been used for searching explicit solutions to the AKNS equation. Like for, the inverse scattering transformation, the modified simple equation method, the Hirota' s bilinear method, the ansatz method, the bilinear Bäcklund transformation [15–22]. However, only recently some authors have attempted to solve the

Ablowitz-KaupNewell-Segur equation with fractional derivative [23,24]. Nowadays, the researchers study on fractional calculus and improved new operators which are known as the Caputo, the Riemann−Liouville, the Caputo−Fabrizio, the Atangana−Baleanu derivatives. Fractional order models are better describe the real-world problems and thus they are used in engineering and applied sciences. The scientists have proposed many mathematical tools to fractional models recently, they can be seen in [35–61]. The conformable fractional operator overcome some limitations of other fractional operators and provides basic properties of classical calculus such as derivative of the quotient of two functions, the chain rule, the product of two functions, Rolle's theorem, mean value theorem. The application of the conformable derivatives is simpler and very efficient. Furthermore, it allows us better understand behaviors of pysical phenomenon.

In this paper, we study the Conformable (2+1)-dimensional AKNS (CAKNS) water wave equation with a perturbation parameterρ,

$$4\frac{\partial^{2\alpha}\varphi}{\partial x^{\alpha}\partial t^{\alpha}}+\frac{\partial^{4\alpha}\varphi}{\partial x^{3\alpha}\partial t^{\alpha}}+8\frac{\partial^{\alpha}\varphi}{\partial x^{\alpha}}\frac{\partial^{2\alpha}\varphi}{\partial x^{\alpha}\partial y^{\alpha}}+4\frac{\partial^{2\alpha}\varphi}{\partial x^{2\alpha}}\frac{\partial^{\alpha}\varphi}{\partial y^{\alpha}}-\rho\frac{\partial^{2\alpha}\varphi}{\partial x^{2\alpha}}=0,\ \ 0<\alpha\leq 1, \tag{1.1}$$

where α denotes the conformable derivative respect to x, y, t.

The paper is organized as follows. The definition and some properties of conformable derivative are given in section 2, the main structure of the sine-Gordon expansion method (SGEM)is given in section 3. We will give application the SGEM to the mention equation in section 4. Conclusions are given in the last section 5.

2. Preliminary remarks on conformable derivative

Definition: Let $h : [0, \infty) \longrightarrow R$ be a given function, the conformable derivative of h of order α is defined as,

$$L_{\alpha}(h)(t) = \lim_{\varepsilon\to 0} \frac{h(t+\varepsilon t^{1-\alpha})-h(t)}{\varepsilon},$$

for all $t > 0, \alpha \in (0, 1)$[25].

Theorem: Let L_{α} be the derivative operator with order α and $\alpha \in (0, 1) and h, k$ be α- differentiable at a point $t > 0$.Then [25,26], we have the following

i . $L_{\alpha}(ah + bk) = aL_{\alpha}(h) + bL_{\alpha}(k), \forall a,\ b \in R.$

ii . $L_{\alpha}(t^{p}) = pt^{p-\alpha}, \forall p \in R.$

iii . $L_{\alpha}(hk) = hL_{\alpha}(g) + kL_{\alpha}(f).$

iv . $L_{\alpha}(\frac{h}{k}) = \frac{kL_{\alpha}(h)-hL_{\alpha}(k)}{k^{2}}.$

v . $L_{\alpha}(\lambda) = 0$, for all constant functions $h(t) = \lambda$.

vi . If h is differentiable then $L_{\alpha}(h)(t) = t^{1-\alpha}\frac{dh}{dt}(t)$.

3. The Sine-Gordon expansion method

We will give general structure of the SGEM in this section.
Let us consider the (2+1)-dimensional sine-Gordon equation is given by [27–34];

$$\varphi_{xx}^{2\alpha} + \varphi_{yy}^{2\alpha} - \varphi_{tt}^{2\alpha} = \eta^2 sin(\varphi), \tag{3.1}$$

where $\varphi = \varphi(x, y, t), \eta$ is a real constant. Using the wave transform $\varphi = \varphi(x, y, t) = U(\psi)$, $\psi = cos\theta\frac{x^\alpha}{\alpha} + sin\theta\frac{y^\alpha}{\alpha} + c\frac{t^\alpha}{\alpha}$ to to Eq.(2), we can find the nonlinear ordinary differential equations,

$$U'' = \frac{\eta^2}{(1 - c^2)} sin(U), \tag{3.2}$$

where $U = U(\psi), \psi$ *and* c are the amplitude and velocity of the travelling waves, respectively. We integrate Eq.(3) then we obtain as follows;

$$\left[\left(\frac{U}{2}\right)'\right]^2 = \frac{\eta^2}{(1 - c^2)} sin^2\left(\frac{U}{2}\right) + K, \tag{3.3}$$

where $K$is the integration constant. Substituting $K = 0, w(\psi) = \frac{U}{2} and b^2 = \frac{\eta^2}{(1-c^2)}$ in Eq.(4), gives

$$w' = bsin(w). \tag{3.4}$$

Setting $b = 1$ in Eq.(5) gives

$$w' = sin(w). \tag{3.5}$$

Solving Eq.(6) via separation of variables, we obtain

$$sin(w(\psi)) = sech(\psi), \tag{3.6}$$

$$cos(w(\psi)) = tanh(\psi), \tag{3.7}$$

Suppose that the nonlinear fractional differential equation is given in the more general form;

$$P(\varphi_x^\alpha, \varphi_t^\alpha, \varphi_y^\alpha, \varphi_{xx}^{2\alpha}, \varphi_x^\alpha \varphi_{xy}^{2\alpha}, \varphi_{xt}^{2\alpha}, \varphi_{xxy,.....}^{3\alpha}), \tag{3.8}$$

where and $\alpha \in (0, 1]$ is the order of the conformable derivative. To obtain the solutions of Eq.(9), we suppose the following expressions

$$U(\psi) = \sum_{i=1}^{n} tanh^{i-1}(\psi)[B_i sech(\psi) + A_i tanh(\psi)] + A_0. \tag{3.9}$$

$$U(w) = \sum_{i=1}^{n} cos^{i-1}(w)[B_i sin(w) + A_i cos(w)] + A_0. \tag{3.10}$$

Applying the homogeneous balance principle between the highest power nonlinear term and highest derivative in the nonlinear ordinary differential equation(NODE), we determine the value of n . Putting Eq.(11) and its consecutive derivatives into the NODE, we obtain a polynomial equation with $sin^i(w)cos^j(w)$ Using some trigonometric properties to the polynomial equation, it is obtained an algebraic equation system by equating to zero the same power summation of coefficients. With aid of the computation programme, we solve the equation system to obtain the $A_i, B_i,$ *and* c values. Substituting the A_i, B_i, c values into Eq.(10), we get the new travelling wave solutions to the Eq.(9).

4. Application of SGEM

In this section, we will give application of the SGEM to conformable(2+1)-dimensional Ablowitz-Kaup-Newell-Segurwater wave equation. Let us consider the Eq.(1). Putting the following wave transformation into Eq.(1)

$$\varphi(x,y,t) = U(\psi), \psi = cos\theta\frac{x^{\alpha}}{\alpha} + sin\theta\frac{y^{\alpha}}{\alpha} + c\frac{t^{\alpha}}{\alpha}, \tag{4.1}$$

So that by using the conformable derivative properties,the Eq.(1) is converted into

$$(4ccos\theta - \rho cos^2\theta)U^{''} + (12sin\theta cos^2\theta)U^{'}U^{''} + (ccos^3\theta)U^{iv} = 0, \tag{4.2}$$

Integrating once Eq.(13) with respect to ψ , we obtain

$$(4ccos\theta - \rho cos^2\theta)U^{'} + (6sin\theta cos^2\theta)(U^{'})^2 + (ccos^3\theta)U^{'''} = 0, \tag{4.3}$$

We transform $U^{'} = V$, it can be written as,

$$(4ccos\theta - \rho cos^2\theta)V + (6sin\theta cos^2\theta)(V)^2 + (ccos^3\theta)V^{''} = 0, \tag{4.4}$$

Using the homogenous balance principle between $V^{''} and V^2$, we obtain

$$n = 2 \tag{4.5}$$

For the value $n = 2$ Eq.(11) take the form,

$$V(w) = B_1 sin(w) + A_1 cos(w) + B_2 cos(w)sin(w) + A_2 cos^2(w) + A_0, \tag{4.6}$$

Differentiating Eq.(17) twice, yields

$$\begin{aligned} V^{''}(w) = {} & B_1 cos^2(w)sin(w) - B_1 sin^3(w) - 2A_1 sin^2(w)cos(w) + \\ & B_2 cos^3(w)sin(w) - 5B_2 sin^3(w)cos(w) - 4A_2 cos^2(w)sin^2(w) + \\ & 2A_2 sin^4(w), \end{aligned} \tag{4.7}$$

Substituting Eqs.(17–18) into Eq.(15), we obtain a trigonometric function with different degrees. Equating to zero all sum of coefficients of the same power of the trigonometric functions,we get the following algebraic equation system.

$constants : 4ccos[\theta]A_0 - \rho cos[\theta]^2 A_0 + 6cos[\theta]^2 sin[\theta]A_0^2 + 6cos[\theta]^2 sin[\theta]B_1^2 + 2ccos[\theta]^3 A_2 = 0$

$Cos[w] : 4ccos[\theta]A_1 - \rho cos[\theta]^2 A_1 + 12cos[\theta]^2 sin[\theta]A_0 A_1 + 12cos[\theta]^2 sin[\theta]A_1 A_2 = 0,$

$Cos[w]Sin[w]^2 : -2ccos[\theta]^3 A_1 - 12cos[\theta]^2 sin[\theta]A_1 A_2 + 12cos[\theta]^2 sin[\theta]B_1 B_2 = 0$

$Cos[w]^2 : 6cos[\theta]^2 sin[\theta]A_1^2 + 4ccos[\theta]A_2 - \rho cos[\theta]^2 A_2 - 2ccos[\theta]^3 A_2 + 12cos[\theta]^2 sin[\theta]A_0 A_2 + 6cos[\theta]^2 sin[\theta]A_2^2 - 6cos[\theta]^2 sin[\theta]B_1^2 = 0$

$Cos[w]^2 Sin[w]^2 : -6ccos[\theta]^3 A_2 - 6cos[\theta]^2 sin[\theta]A_2^2 + 6cos[\theta]^2 sin[\theta]B_2^2 = 0$

$Sin[w] : 4ccos[\theta]B_1 - \rho cos[\theta]^2 B_1 - ccos[\theta]^3 B_1 + 12cos[\theta]^2 sin[\theta]A_0 B_1 = 0$

$Cos[w]^2 Sin[w] : 2ccos[\theta]^3 B_1 + 12cos[\theta]^2 sin[\theta]A_2 B_1 + 12cos[\theta]^2 sin[\theta]A_1 B_2 = 0$

$Cos[w]Sin[w] : 12cos[\theta]^2 sin[\theta]A_1 B_1 + 4ccos[\theta]B_2 - \rho cos[\theta]^2 B_2 + ccos[\theta]^3 B_2 +$

$12cos[\theta]^2 sin[\theta]A_0B_2 + 12cos[\theta]^2 sin[\theta]A_2B_2 = 0$
$Cos[w]Sin[w]^3 : -6ccos[\theta]^3B_2 - 12cos[\theta]^2 sin[\theta]A_2B_2 = 0$
The solution of this set of equation gives the coefficients of Eq.(10).So that we have the following cases:
Case 1:
$A_0 = -A_2;\ B_1 = 0;\ A_1 = 0;\ B_2 = iA_2;\ \rho = -\frac{2(4+cos[\theta]^2)sin[\theta]A_2}{cos[\theta]^2};\ c = -\frac{2sin[\theta]A_2}{cos[\theta]}$, gives the solution in the form,

$$\varphi_1(x,y,t) = A_2(-iSech[f(x,y,t)] - Tanh[f(x,y,t)]) \tag{4.8}$$

where $f(x,y,t) = \frac{x^\alpha cos[\theta]+y^\alpha sin[\theta]}{\alpha} - \frac{2t^\alpha sin[\theta]A_2}{\alpha cos[\theta]}$
Case 2:
$A_0 = -\frac{2A_2}{3};\ B_1 = 0;\ A_1 = 0;\ B_2 = -iA_2;\ \rho = \frac{2(-4+cos[\theta]^2)sin[\theta]A_2}{cos[\theta]^2};\ c = -\frac{2sin[\theta]A_2}{cos[\theta]}$, gives the solution

$$\varphi_2(x,y,t) = \frac{1}{3}A_2(f(x,y,t) + 3iSech[f(x,y,t)] - 3Tanh[f(x,y,t)]) \tag{4.9}$$

Case 3:
$A_0 = \frac{\rho cos[\theta]^2}{12sin[\theta]-3cos[\theta]^2 sin[\theta]};\ A_1 = 0;\ A_2 = \frac{\rho cos[\theta]^2}{2(-4+cos[\theta]^2)sin[\theta]};\ B_1 = 0;\ B_2 = \frac{i\rho cos[\theta]^2}{2(-4+cos[\theta]^2)sin[\theta]};\ c = -\frac{\rho cos[\theta]}{-4+cos[\theta]^2}$,enable to write the solution as,

$$\varphi_3(x,y,t) = \frac{\rho cos[\theta]^2(g(x,y,t) + 3iSech[g(x,y,t)] - 3Tanh[g(x,y,t)])}{6(-4+cos[\theta]^2)sin[\theta]} \tag{4.10}$$

where $g(x,y,t) = \frac{x^\alpha cos[\theta]+y^\alpha sin[\theta]}{\alpha} - \frac{t^\alpha \rho cos[\theta]}{\alpha(-4+cos[\theta]^2)}$
Case 4:
$A_0 = \frac{ccos[\theta]}{2sin[\theta]};\ A_1 = 0;\ A_2 = -\frac{ccos[\theta]}{2sin[\theta]};\ B_1 = 0;\ B_2 = \frac{iccos[\theta]}{2sin[\theta]};\ \rho = \frac{c(4+cos[\theta]^2)}{cos[\theta]}$, gives,

$$\varphi_4(x,y,t) = \frac{c}{2}cot[\theta](-iSech[\psi] + Tanh[\psi]) \tag{4.11}$$

Case 5:
$A_0 = \frac{ccos[\theta]}{3sin[\theta]};\ A_1 = 0;\ A_2 = -\frac{ccos[\theta]}{2sin[\theta]};\ B_1 = 0;\ B_2 = -\frac{iccos[\theta]}{2sin[\theta]};\ \rho = \frac{4c}{cos[\theta]} - ccos[\theta]$,

$$\varphi_5(x,y,t) = -\frac{c}{6}cosec[\theta](\psi - 3iSech[\psi] - \frac{1}{2}Tanh[\psi]) \tag{4.12}$$

Case 6:
$A_1 = 0;\ A_2 = \frac{-3A_0}{2};\ B_1 = 0;\ B_2 = -\frac{3iA_0}{2};\ \rho = -\frac{3(-4+cos[\theta]^2)sin[\theta]A_0}{cos[\theta]^2};\ c = \frac{3sin[\theta]A_0}{cos[\theta]}$, enable to write the solution as,

$$\varphi_6(x,y,t) = -\frac{1}{2}A_0(h(x,y,t) - 3iSech[h(x,y,t)] - 3Tanh[h(x,y,t)]) \tag{4.13}$$

where $h(x,y,t) = \frac{x^\alpha cos[\theta]+y^\alpha sin[\theta]}{\alpha} + \frac{3t^\alpha sin[\theta]A_0}{\alpha cos[\theta]}$,

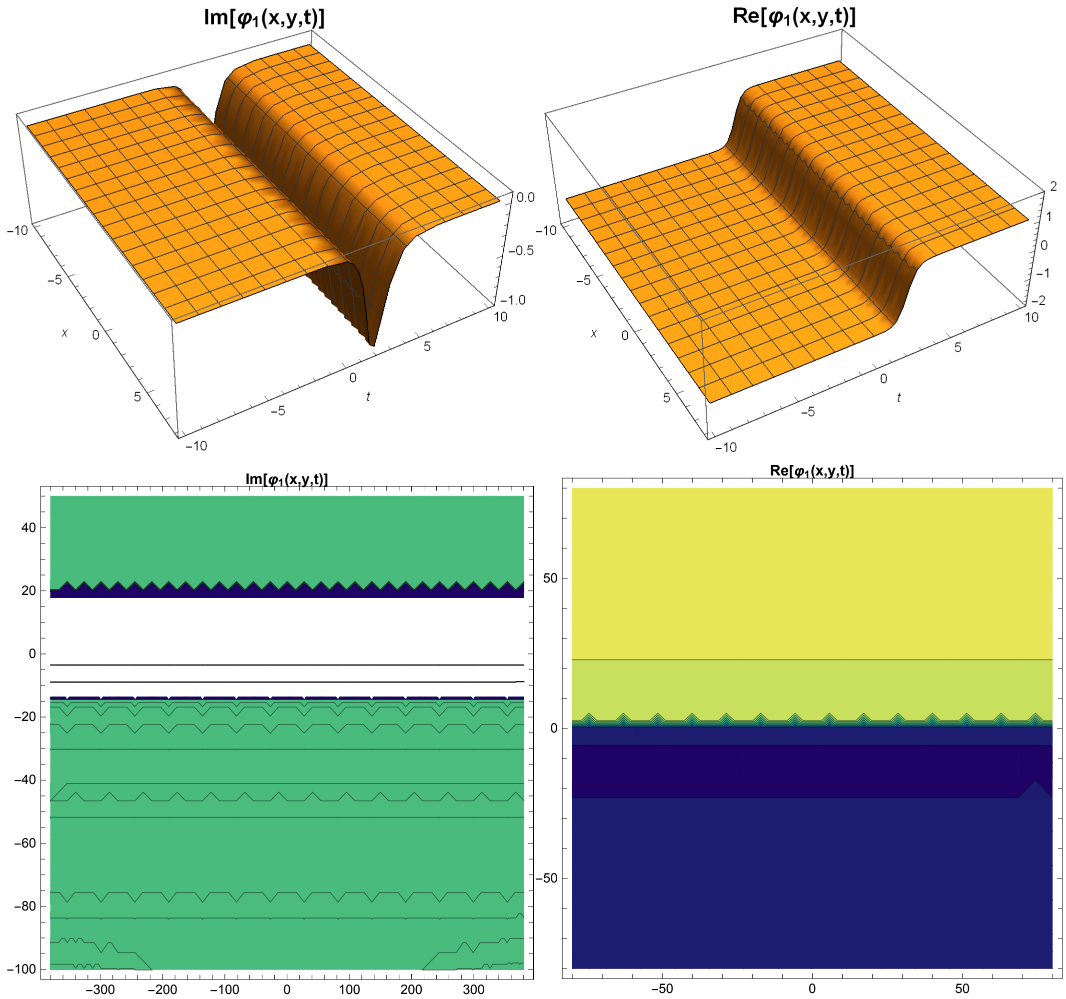

Figure 1. The 3D and contour plots of $\varphi_1(x, y, t)$ when $\alpha = 0.9, A_2 = 1, \theta = \frac{\pi}{2}$.

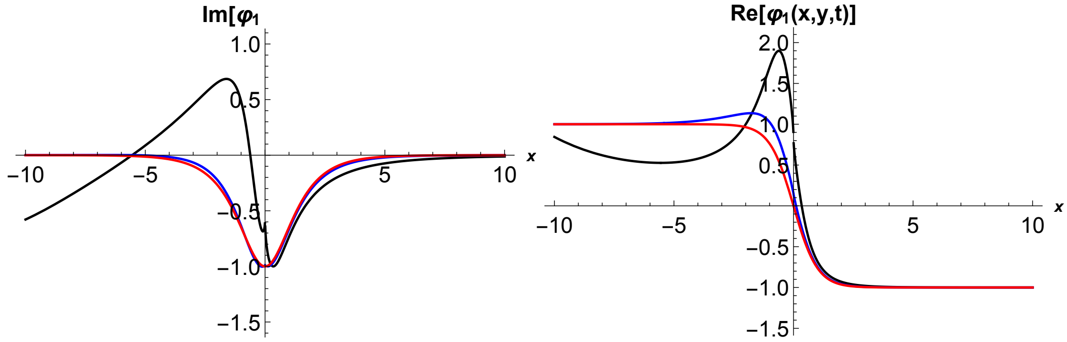

Figure 2. The 2D- plots of $\varphi_1(x, y, t)$ when $\alpha = 0.5, 0.9, 1, A_2 = 1, \theta = \frac{\pi}{2}, t = 1$.

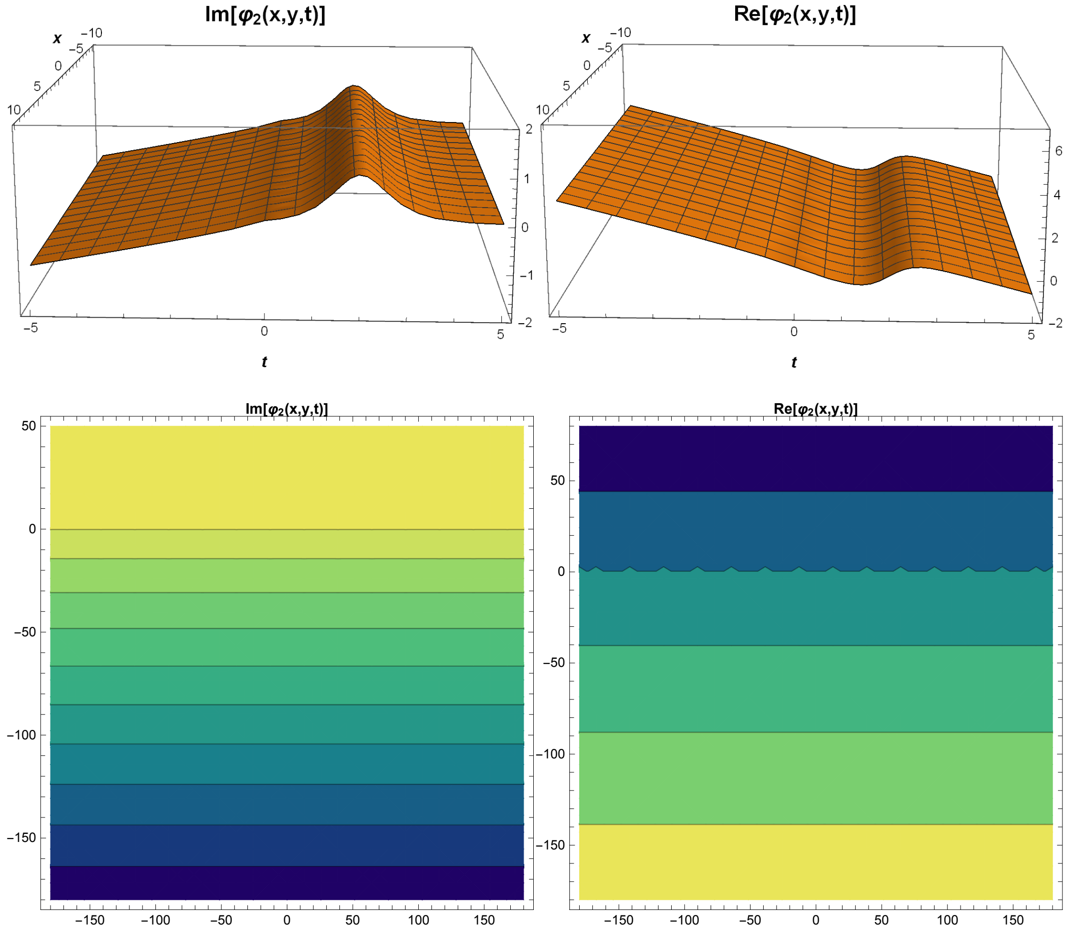

Figure 3. The 3D and contour plots of $\varphi_2(x, y, t)$ when $\alpha = 0.9, A_2 = 1, \theta = \frac{\pi}{2}$.

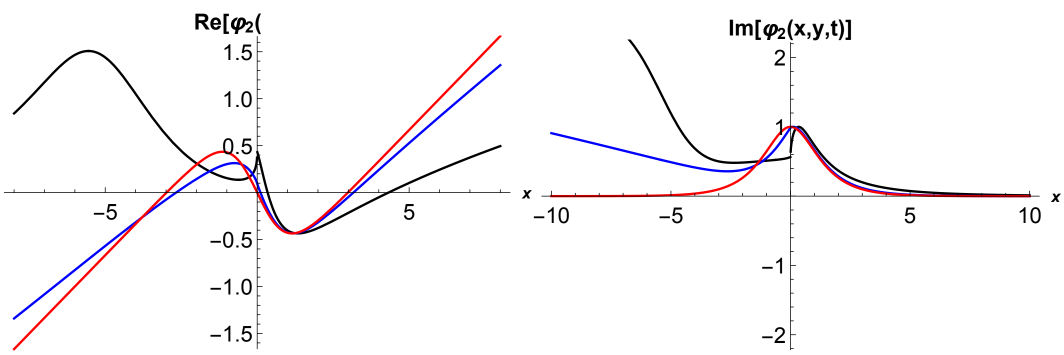

Figure 4. The 2D- plots of $\varphi_2(x, y, t)$ when $\alpha = 0.5, 0.9, 1, A_2 = 1, \theta = \frac{\pi}{2}, t = 1$.

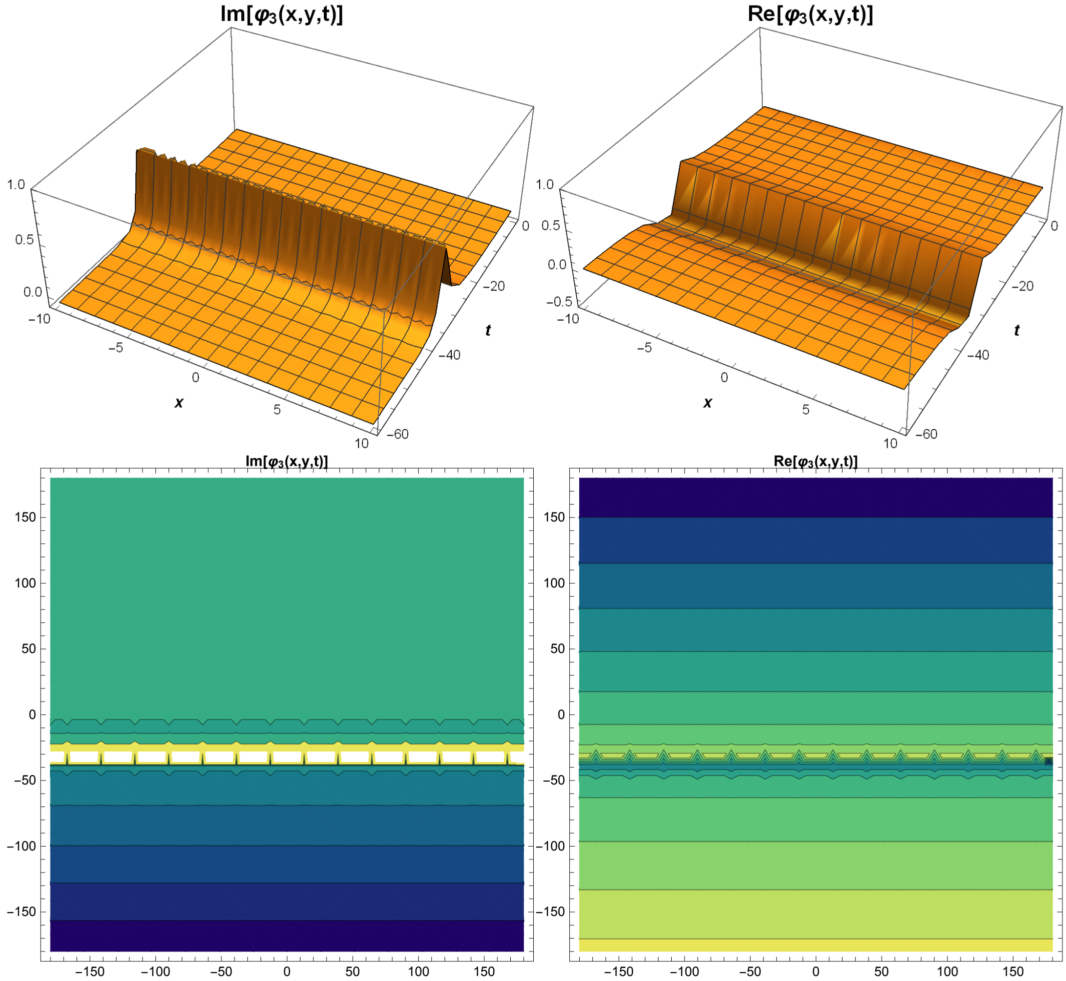

Figure 5. The 3D and contour plots of $\varphi_3(x, y, t)$ when $\alpha = 0.9, \rho = 0.5, \theta = \frac{\pi}{2}$.

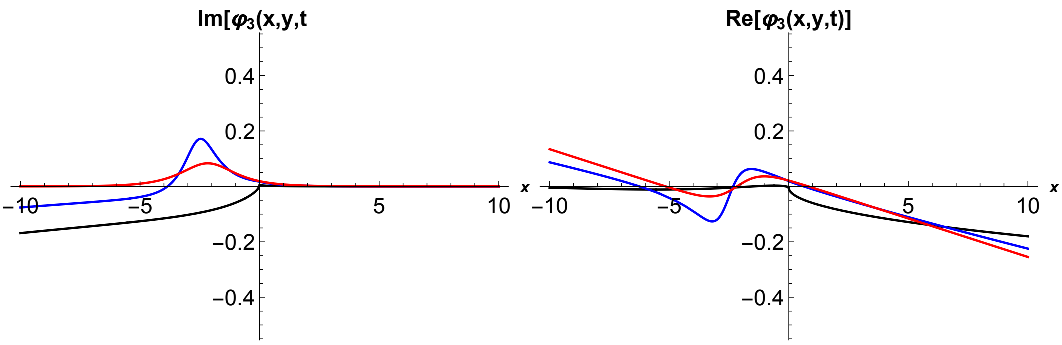

Figure 6. The 2D- plots of $\varphi_3(x, y, t)$ when $\alpha = 0.5, 0.9, 1, \rho = 0.5, \theta = \frac{\pi}{2}, t = 1$.

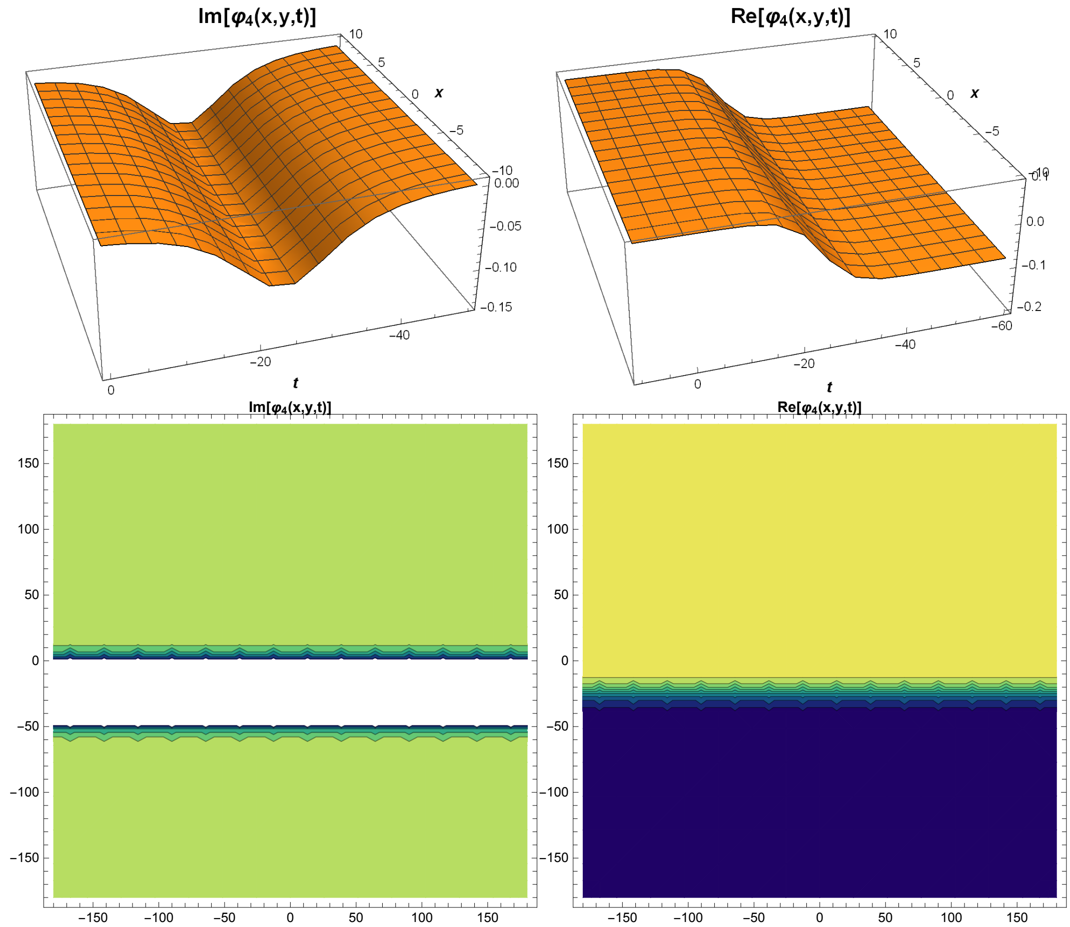

Figure 7. The 3D and contour plots of $\varphi_4(x, y, t)$ when $\alpha = 0.9, c = 0.17, \theta = \frac{\pi}{2}$.

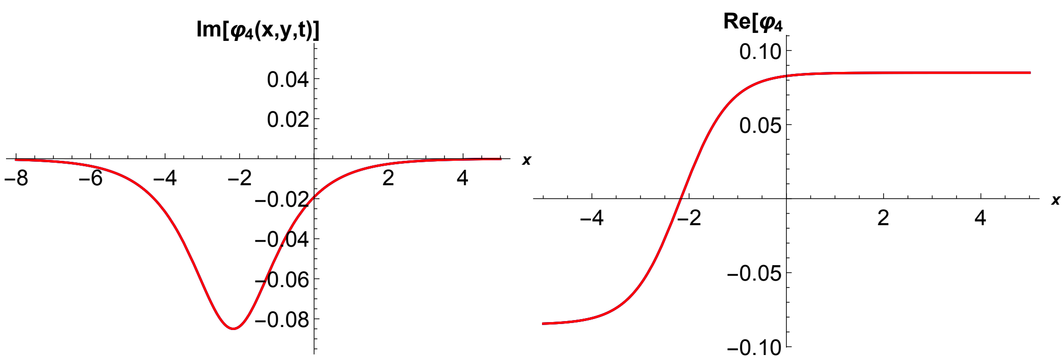

Figure 8. The 2D- plots of $\varphi_4(x, y, t)$ when $\alpha = 0.5, 0.9, 1, c = 0.17, \theta = \frac{\pi}{2}, t = 1$.

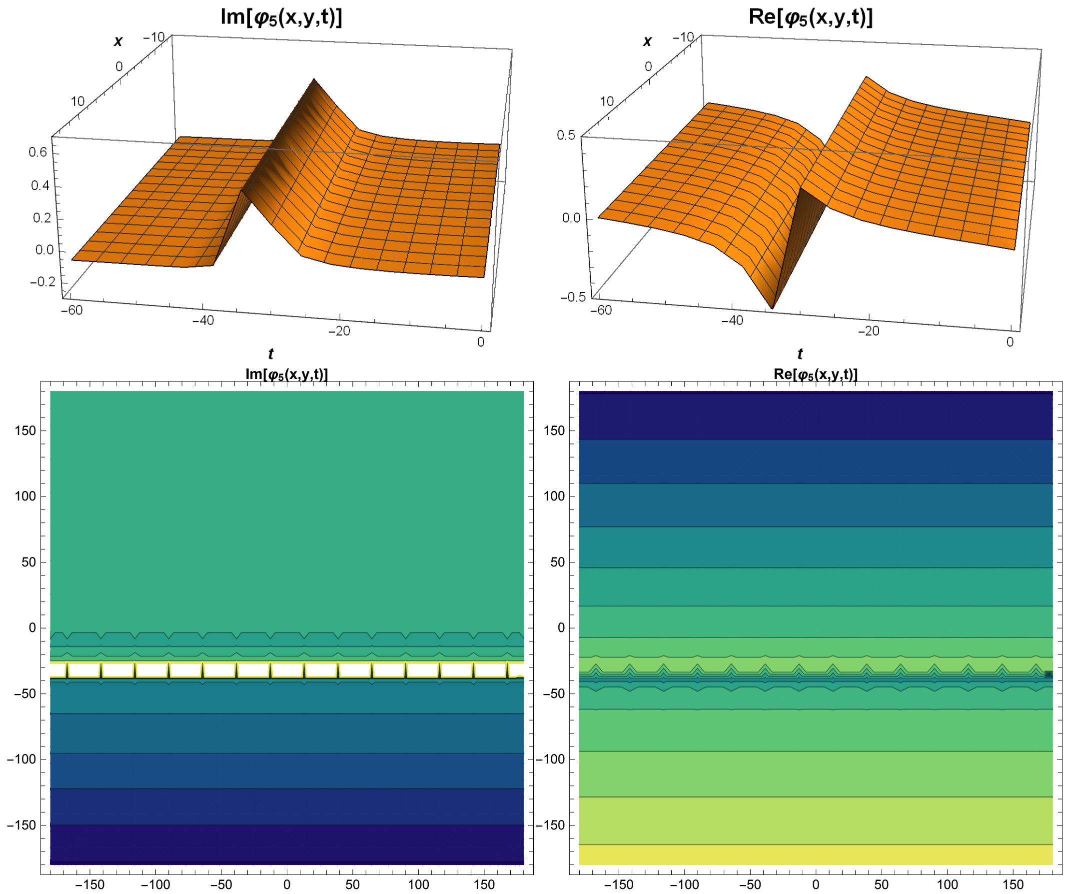

Figure 9. The 3D and contour plots of $\varphi_5(x, y, t)$ when $\alpha = 0.9, c = 0.17, \theta = \frac{\pi}{2}$.

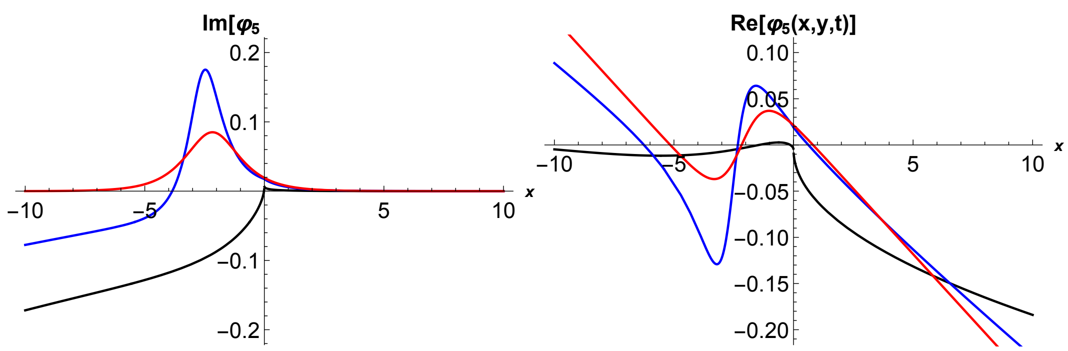

Figure 10. The 2D- plots of $\varphi_5(x, y, t)$ when $\alpha = 0.5, 0.9, 1, c = 0.17, \theta = \frac{\pi}{2}, t = 1$.

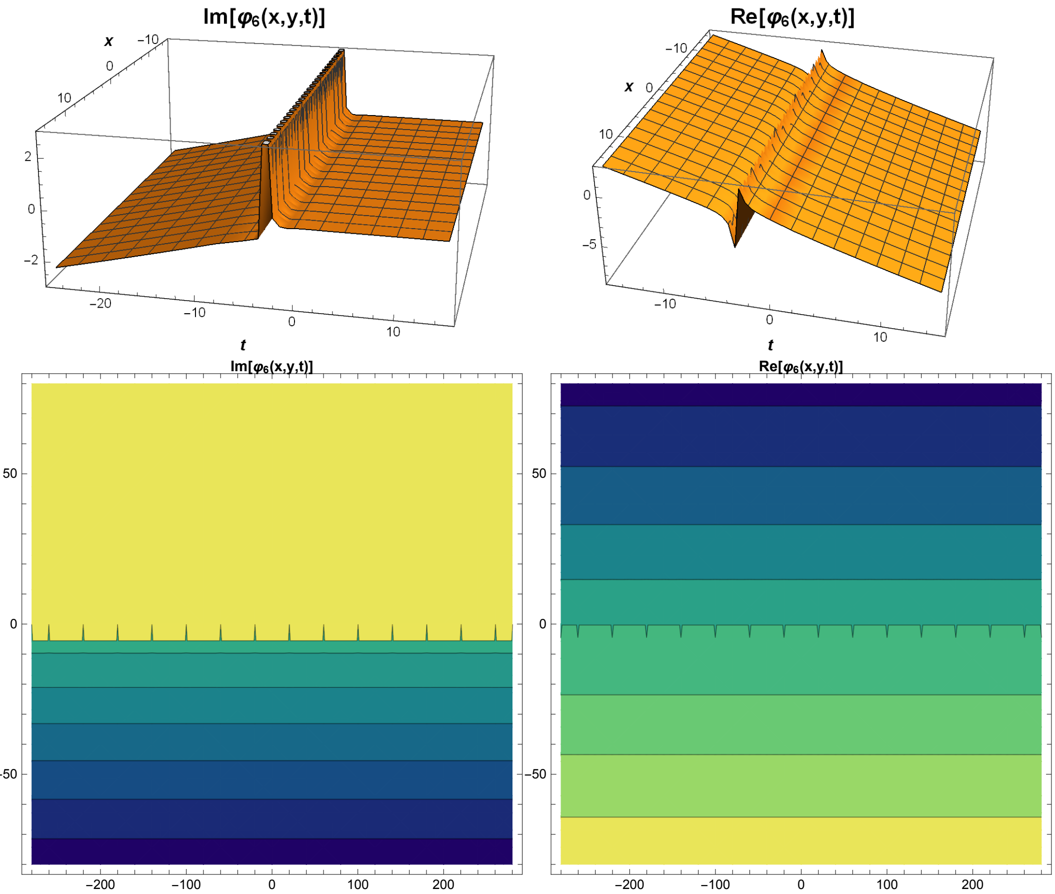

Figure 11. The 3D and contour plots of $\varphi_6(x, y, t)$ when $\alpha = 0.9, A_0 = 0.5, \theta = \frac{\pi}{2}$.

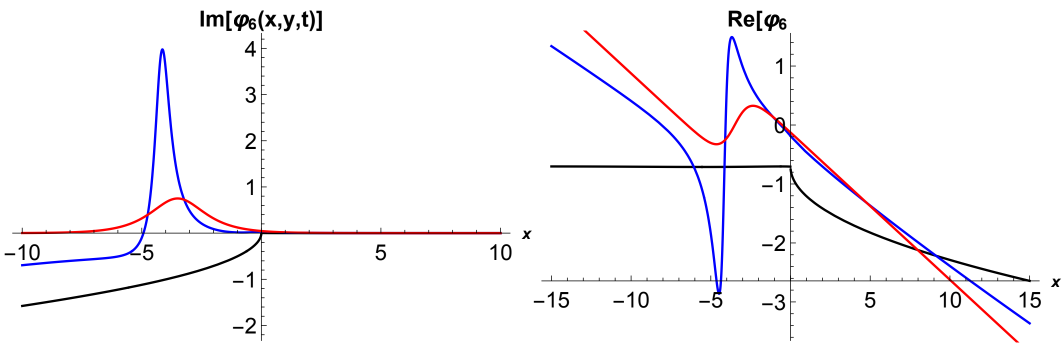

Figure 12. The 2D- plots of $\varphi_6(x, y, t)$ when $\alpha = 0.5, 0.9, 1, A_0 = 0.5, \theta = \frac{\pi}{2}, t = 1$.

5. Conclusion

In this paper, SGEM has been applied to the conformable (2+1)-dimensional AKNS water wave equation. We have found complex combined dark-bright soliton solutions. The two- and three-dimensional surfaces of all solutions obtained by SGEM under the suitable values of parameters were plotted by showing the main characteristic physical properties of the solutions. The all 2D graphics are observed when the fractional order $\alpha = 0.5, 0.9, 1$ respectively. When $\alpha = 0.5$ in Figures 2, 4, 6, 10, 12, the solutions behave unstable, in Figure 8 the solution for all α values same behavior. It is also observed that Figures 1, 3, 5, 7, 9, 11 have presented travelling wave behaviours for the governing model. It is seen that fractional order approaches to $\alpha = 1$ the solutions act similar behaviors to integer order. Moreover, the difference of the method is giving different new solutions to have powerful nonlinearity differential equations which have not analytical solutions. It can be also observed that the obtained results may be helpful to better understand water wave propagation, especially in ocean wave dynamics. To the best of our knowledge, these entirely new solutions to the conformable (2+1)-dimensional AKNS have been firstly submitted to the literature. We will develop the above approach to use the differential equations have powerful nonlinearity future analysis.

Conflict of interest

Authors declare that there is no conflict of interest in this paper.

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