



Research article

On some stochastic differential equations with jumps subject to small positives coefficients

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Abstract: We provide a large deviation principle for jumps and stochastic diffusion processes, according to a viscosity coefficient (ε) and a small scaling parameter (δ) both going at the same rate. To do so we have to come up with estimates on the *moment Lyapunov function* trajectories.

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1. Introduction

Specially, let us consider ε (viscosity coefficient) and δ_ε (scaling parameter, written as a function of ε) both be non-negatives, such that

$$\mathbf{H.1} \quad \lim_{\varepsilon \rightarrow 0} \frac{\delta_\varepsilon}{\varepsilon} := \gamma \quad (\gamma \in \mathbb{R}_+^*).$$

Our concern is the following d -dimensional stochastic differential equations (SDE) :

$$X_t^{x,\varepsilon,\delta_\varepsilon} - x = \sqrt{\varepsilon} \int_0^t \sigma \left(\frac{X_s^{x,\varepsilon,\delta_\varepsilon}}{\delta_\varepsilon} \right) dW_s + \frac{\varepsilon}{\delta_\varepsilon} \int_0^t b \left(\frac{X_s^{x,\varepsilon,\delta_\varepsilon}}{\delta_\varepsilon} \right) ds + \int_0^t c \left(\frac{X_s^{x,\varepsilon,\delta_\varepsilon}}{\delta_\varepsilon} \right) ds + L_t^{\varepsilon,\delta_\varepsilon}, \quad x \in \mathbb{R}^d, \quad (1.1)$$

where $\{W_t : t \geq 0\}$ is a d -dimensional standard Brownian motion and $L^{\varepsilon,\delta_\varepsilon} := \{L_t^{\varepsilon,\delta_\varepsilon} : t \geq 0\}$ is a Poisson point process with continuous compensator, independent of W , both defined on a given filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})$ with $\mathbb{F} := \{\mathcal{F}_t : t \geq 0\}$ being the \mathbb{P} -completion of the filtration \mathcal{F} . More precisely, we assume that $L^{\varepsilon,\delta_\varepsilon}$ takes the form:

$$L_t^{\varepsilon,\delta_\varepsilon} := \int_0^t \int_{\mathbb{R}^d} k \left(\frac{X_{s-}^{x,\varepsilon,\delta_\varepsilon}}{\delta_\varepsilon}, y \right) (\varepsilon N^{\varepsilon^{-1}}(dsdy) - \nu(dy)ds), \quad t \geq 0 \quad (1.2)$$

where k is a given predictable function (see [10], Chap. IV Sect. 9), and N is a Poisson random counting measure on \mathbb{R}^d with mean Lèvy measure (or intensity measure) ν . The coefficients b, c, σ are subject to suitable conditions.

The family of equation (1.1) subject to a combination effect of homogenization and large deviations, is a classical problem which goes back to Paolo Baldi [1] at the end of 20'th century. Such a problem, when $L_t^{\varepsilon, \delta_\varepsilon} \equiv 0$, has been extensively investigated by Freidlin and Sowers [9]. They have shown three classical regimes depending on the relative rate at which the small viscosity coefficient ε and the scaling parameter δ_ε tend to zero. They have provided some effective rate functions associated to a large deviation principle (LDP) for SDE which have been used as direct applications to wavefront propagation. Recently, we have focused the problem (1.1) in the sense that homogenization dominates (see [4]), at the same time, we have provided *lower* and *upper* bounds of LDP for SDE which allows jumps processes. The purpose of the present article is to carry out the program outlined in [4] regarding the following case: The small coefficients (ε and δ_ε) go at the same rate, which is technically more difficult. Indeed, in this case, the logarithm moment generating function of $X_T^{x, \varepsilon, \delta_\varepsilon}$, noted $g_{T,x}^\varepsilon$ below, may not have an explicit value limit. Thanks to Baxendale and Stroock [2], we overcome this challenge by using the *superior* and *inferior* limits of the function $g_{T,x}^\varepsilon$.

Definition 1. Let $X^{x, \varepsilon, \delta}$ be a \mathbb{R}^d -valued random variable and let $\mathbb{P}_{\varepsilon, \delta}$ denote its distribution on the Borel subsets of \mathbb{R}^d , that is, $\mathbb{P}_{\varepsilon, \delta}(A) = \mathbb{P}(X^{x, \varepsilon, \delta} \in A)$. The family $\{X^{x, \varepsilon, \delta}; \varepsilon, \delta \geq 0\}$ satisfies an LDP if there exists a lower semi-continuous function $I : \mathbb{R}^d \rightarrow [0, +\infty]$ such that

- for each open set $A \subseteq \mathbb{R}^d$ $\liminf_{\varepsilon \rightarrow 0} \varepsilon \log \mathbb{P}(X^{x, \varepsilon, \delta_\varepsilon} \in A) \geq -\inf_{x \in A} I(x)$,
- for each closed set $B \subseteq \mathbb{R}^d$ $\limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbb{P}(X^{x, \varepsilon, \delta_\varepsilon} \in B) \leq -\inf_{x \in B} I(x)$.

I is called the rate function for the LDP. A rate function is good if for each $a \in \mathbb{R}_+$, $\{x : I(x) \leq a\}$ is compact.

Our program will be as follows. In Section (2) we set up some notation, make precisely our hypothesis and state the main result. In Section (3) we give an overview of some existing characterizations of large deviations, comparing them with ours, and we give the outline of the proofs.

Finally, we wish to say that the subject related to this topic, besides those of the references mentioned earlier can be found in [12, 15] and references therein.

2. Preliminaries

2.1. Notations and formulation

Denote expectation with respect to \mathbb{P} by \mathbb{E} and the gradient operator by ∇ . We have already defined $\langle \cdot, \cdot \rangle$ as the standard Euclidean inner product on \mathbb{R}^d , and $\|\cdot\|$ as the associated norm. Let $C_p(\mathbb{R}^d, \mathbb{R}^d)$ be the collection of continuous mapping from \mathbb{R}^d into \mathbb{R}^d which are periodic of period 1 in each coordinate of the argument and let $\|\cdot\|_{C_p(\mathbb{R}^d, \mathbb{R}^d)}$ be the associated sup norm. Let \mathbb{T}^d be the d -dimensional torus of size 1, and let $\|\cdot\|_{C(\mathbb{T}^d, \mathbb{R}^d)}$ be the standard sup norm on $C(\mathbb{T}^d, \mathbb{R}^d)$, the space of continuous mapping from \mathbb{T}^d into \mathbb{R}^d . Also we define $\mathcal{P}(\mathbb{T}^d)$ as the collection of all probability measure on \mathbb{T}^d .

The $\{\sigma_i : 1 \leq i \leq d\}$ in (1.1) are assumed to be in $C_p(\mathbb{R}^d, \mathbb{R}^d)$, and we also assume that

$$\kappa := \inf \left\{ \sum_{i=1}^d \langle \theta, \sigma_i(x) \rangle^2 : x \in \mathbb{R}^d, \theta \in \mathbb{R}^d, \|\theta\| = 1 \right\} > 0. \tag{2.1}$$

We assume that b, c in (1.1) are in $C_p(\mathbb{R}^d, \mathbb{R}^d)$.

We now turn our attention to the Poisson part. We first consider a Poisson random measure $N^{\varepsilon^{-1}}(\cdot, \cdot)$ on $[0, T] \times \mathbb{R}^d$ defined on the space probability $(\Omega, \mathcal{F}, \mathbb{P})$, with Lèvy measure $\varepsilon^{-1}\nu$ such that the standard integrability condition holds:

$$\int_{\mathbb{R}^d \setminus \{0\}} (1 \wedge |y|^2) \nu(dy) < +\infty. \tag{2.2}$$

The compensator of $\varepsilon N^{\varepsilon^{-1}}$ is thus the deterministic measure $\varepsilon \hat{N}^{\varepsilon^{-1}}(dtdy) := dt\nu(dy)$ on $[0, T] \times \mathbb{R}^d$. In this paper we shall be interested in *Poisson point process* of class (QL), namely a point process whose counting measure has continuous compensator (see Ikeda and Watanabe [10]). More precisely, in light of the representation theorem of the Poisson point process ([10], Chap. II, Theorem 7.4), we shall assume that $L^{\varepsilon, \delta_\varepsilon}$ is a pure jump process of the following form :

$$L_t^{\varepsilon, \delta_\varepsilon} := \int_0^t \int_{\mathbb{R}^d \setminus \{0\}} k\left(\frac{\cdot}{\delta_\varepsilon}, y\right)(s) (\varepsilon N^{\varepsilon^{-1}}(dsdy) - \nu(dy)ds), \quad t \geq 0,$$

where k is $C_p(\mathbb{R}^d \times \mathbb{R}^d, \mathbb{R}^d)$ with respect to first variable, integrable with respect to $dtdy$, so that the counting measure of $L^{\varepsilon, \delta_\varepsilon}$, denoted by $N_{L^{\varepsilon, \delta_\varepsilon}}(dtdy)$ takes the form :

$$N_{L^{\varepsilon, \delta_\varepsilon}}((0, t] \times A) := \int_0^t \int_{\mathbb{R}^d} \mathbf{1}_A \left(k\left(\frac{\cdot}{\delta_\varepsilon}, y\right)(s) \right) \varepsilon N^{\varepsilon^{-1}}(dsdy) = \sum_{0 \leq s \leq t} \mathbf{1}_{\{\Delta L_s^{\varepsilon, \delta_\varepsilon} \in A\}}, \tag{2.3}$$

and its compensator is therefore $\hat{N}_{L^{\varepsilon, \delta_\varepsilon}}(dtdy) := k\left(\frac{\cdot}{\delta_\varepsilon}, y\right)(s) \mathbb{E}[N_{L^{\varepsilon, \delta_\varepsilon}}(dtdy)] = k\left(\frac{\cdot}{\delta_\varepsilon}, y\right)(s) \nu(dy)dt$, and hence continuous, *i.e.*, $L^{\varepsilon, \delta_\varepsilon}$ has continuous statistic.

The Markov processes $X^{\varepsilon, \delta_\varepsilon}$ that we consider include jump processes and diffusion. Next let's write down its generator on twice continuously differentiable functions with compact support by

$$\begin{aligned} \mathcal{L}_{\varepsilon, \delta_\varepsilon} \phi(x) := & \frac{\varepsilon}{2} \sum_{i,j=1}^d a_{ij} \left(\frac{x}{\delta_\varepsilon} \right) \frac{\partial^2 \phi(x)}{\partial x_i \partial x_j} + \frac{\varepsilon}{\delta_\varepsilon} \sum_{i=1}^d b_i \left(\frac{x}{\delta_\varepsilon} \right) \frac{\partial \phi(x)}{\partial x_i} + \sum_{i=1}^d c_i \left(\frac{x}{\delta_\varepsilon} \right) \frac{\partial \phi(x)}{\partial x_i} \\ & + \frac{1}{\varepsilon} \int_{\mathbb{R}^d} \left[\phi \left(x + \varepsilon k \left(\frac{x}{\delta_\varepsilon}, y \right) \right) - \phi(x) - \varepsilon \sum_{i=1}^d k_i \left(\frac{x}{\delta_\varepsilon}, y \right) \frac{\partial \phi(x)}{\partial x_i} \right] \nu(dy), \quad x \in \mathbb{R}^d, \end{aligned} \tag{2.4}$$

where the matrix $a := (a_{ij})$ is factored as $a := \sigma \sigma^*$, and $*$ denotes the transpose. We set

$$\mathcal{L}_\gamma := \frac{1}{2} \sum_{i,j=1}^d a_{ij}(x) \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^d b_i(x) \frac{\partial}{\partial x_i} + \gamma \sum_{i=1}^d c_i(x) \frac{\partial}{\partial x_i}, \quad x \in \mathbb{R}^d, \tag{2.5}$$

and require the followings :

$$\mathbf{H.2} \left\{ \begin{array}{l} \text{(Global Lipschitz) there exists a constant } C_1 \text{ such that for any } \zeta := \sigma_i, b, c, 1 \leq i \leq d, \text{ and } k : \\ \text{i)} \quad \|\zeta(x') - \zeta(x)\| + \int_{\mathbb{R}^d} \|k(x', y) - k(x, y)\| \nu(dy) \leq C_1 \|x' - x\|, \quad \forall x', x \in \mathbb{R}^d. \\ \text{(Growth) there exists constant } C_2 \text{ such that for any } \zeta := \sigma_i, b, c, 1 \leq i \leq d, \text{ and } k : \\ \text{ii)} \quad \|\zeta(x)\|^2 + \int_{\mathbb{R}^d} \|k(x, y)\|^2 \nu(dx) \leq C_2 (1 + \|x\|^2), \quad \forall x \in \mathbb{R}^d. \end{array} \right.$$

By requirement there exists a \mathcal{L}_γ -diffusion with jumps on \mathbb{R}^d and by periodicity assumption on the coefficients such a process induces process \bar{X} which has both a diffusion component and a jump component on the d -dimensional torus \mathbb{T}^d , moreover the \mathcal{L}_γ -diffusion-part process is ergodic. We denote by m_γ its unique invariant measure. To move the SDE (1.1) to the torus \mathbb{T}^d , we define the pull-back $\bar{X}_t^{\varepsilon, \delta_\varepsilon} := \frac{1}{\delta_\varepsilon} X_{(\delta_\varepsilon/\sqrt{\varepsilon})^2 t}^{x, \varepsilon, \delta_\varepsilon}$ which satisfies the SDE :

$$\bar{X}_t^{\varepsilon, \delta_\varepsilon} - \frac{x}{\delta_\varepsilon} = \int_0^t \sigma(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) d\bar{W}_s^\varepsilon + \int_0^t b(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) ds + \frac{\delta_\varepsilon}{\varepsilon} \int_0^t c(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) ds + \bar{L}_t^{\varepsilon, \delta_\varepsilon}, \tag{2.6}$$

where

$$\left\{ \begin{array}{l} \bar{W}_t^\varepsilon := \frac{\sqrt{\varepsilon}}{\delta_\varepsilon} W_{(\delta_\varepsilon/\sqrt{\varepsilon})^2 t} \text{ is Brownian motion} \\ \bar{L}_t^{\varepsilon, \delta_\varepsilon} := \frac{\varepsilon}{\delta_\varepsilon} \int_0^t \int_{\mathbb{R}^d} k(\bar{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y) \left(N^{(\delta_\varepsilon/\varepsilon)^2}(dsdy) - \left(\frac{\delta_\varepsilon}{\varepsilon}\right)^2 \nu(dy) ds \right), t \geq 0. \end{array} \right.$$

The infinitesimal generator $\bar{\mathcal{L}}_{\varepsilon, \delta_\varepsilon}$ of the diffusion component associated to the process $\bar{X}^{\varepsilon, \delta_\varepsilon}$ is

$$\bar{\mathcal{L}}_{\varepsilon, \delta_\varepsilon} := \frac{1}{2} \sum_{i,j=1}^d a_{ij}(x) \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^d b_i(x) \frac{\partial}{\partial x_i} + \frac{\delta_\varepsilon}{\varepsilon} \sum_{i=1}^d c_i(x) \frac{\partial}{\partial x_i}, \quad x \in \mathbb{T}^d, \tag{2.7}$$

This generator tends to \mathcal{L}_γ defined in (2.5), as $\varepsilon \rightarrow 0$.

2.2. Usual formulas

The proof of the main Theorem 4 (below) relies on explicit calculation of the logarithm moments of $X^{\varepsilon, \delta_\varepsilon}$ and the followings *Girsanov's formula* and *Itô's formula*. Before proceeding, let us introduce some space.

For E locally compact, let $\mathcal{H}^2(T, \lambda)$ be the linear space of all equivalence classes of mappings $F : [0, T] \times E \times \Omega \rightarrow \mathbb{R}$ which coincide almost everywhere with respect to $dt \otimes d\lambda \otimes d\mathbb{P}$ and which satisfy the following conditions :

- F is predictable;
- $\int_0^T \int_{E \setminus \{0\}} \mathbb{E}(|F(t, z)|^2) dt \lambda(dz) < +\infty$.

We endow $\mathcal{H}^2(T, \lambda)$ with the inner product $\langle F, G \rangle_{T, \lambda} := \int_0^T \int_{E \setminus \{0\}} \mathbb{E}(F(t, z)G(t, z)) dt \lambda(dz)$. Then, it is well know that $(\mathcal{H}^2(T, \lambda); \langle \cdot, \cdot \rangle_{T, \lambda})$ is a real separable Hilbert space.

Let N_p be a Poisson random measure on $\mathbb{R}_+ \times (E \setminus \{0\})$ with intensity measure λ , according to a given \mathcal{F}_T -adapted, σ -finite point process p which is independent of the Brownian motion W . Let \tilde{N}_p be the associated compensated Poisson random measure. Now we have (see, D. Applebaum [3] Chapter 5, Section 2)

Lemma 2 (Girsanov's formula). *Let X be a Lévy process such that e^X is a martingale, i.e.,*

$$X_t = \int_0^t b(s)ds + \int_0^t \sigma(s)dW_s + \int_0^t \int_E H(s, z)\tilde{N}_p(dsdz) + \int_0^t \int_E K(s, z)N_p(dsdz),$$

with

$$b(t) = -\frac{1}{2}\sigma^2(t) - \int_E (e^{H(t,z)} - 1 - H(t,z))\lambda(dz) - \int_E (e^{K(t,z)} - 1)\lambda(dz), \mathbb{P} - a.s.$$

We suppose that there exists $C > 0$ such that

$$|K(t, z)| \leq C, \quad \forall t \geq 0, \forall z \in E.$$

For $L \in \mathcal{H}^2(T, \lambda)$ we define

$$M_t := \int_0^t \int_{z \neq 0} L(s, z)\tilde{N}(dsdz).$$

Set

$$U(t, z) = (e^{H(t,z)} - 1)\mathbf{1}_{\{\|z\| < 1\}} + (e^{K(t,z)} - 1)\mathbf{1}_{\{\|z\| \geq 1\}}$$

and we suppose that

$$\int_0^T \int_{\{\|z\| \leq 1\}} (e^{H(s,z)} - 1)^2 \lambda(dz)ds < +\infty.$$

Finally, we define

$$B_t = W_t - \int_0^t \sigma(s)ds \quad \text{and} \quad N_t = M_t - \int_0^t \int_{z \neq 0} L(s, z)U(s, z)\lambda(dz)ds, \quad 0 \leq t \leq T.$$

Let \mathbf{Q} be the probability measure on (Ω, \mathcal{F}_T) defined as:

$$\frac{d\mathbf{Q}}{d\mathbb{P}} := e^{X_T}.$$

Then under \mathbf{Q} , B_t is a Brownian motion and N_t is a \mathbf{Q} -martingale.

Next define a d -dimensional semi-martingale $Y_t := (Y_1, \dots, Y_d)$ by

$$Y_t^i = Y_0^i + M_t^i + A_t^i + \int_0^t \int_E f_i(s, z, \cdot)\tilde{N}_p(dsdz) + \int_0^t \int_E g_i(s, z, \cdot)N_p(dsdz), \quad i := 1, \dots, d \quad (2.8)$$

where

- M_t is locally continuous square integrable (\mathcal{F}_t) -martingale and $M_0 := 0$;
- A_t is a continuous (\mathcal{F}_t) -adapted process whose almost all sample functions are of bounded variation on each finite interval and $A_0 := 0$;

- g is (\mathcal{F}_t) -predictable and for $t > 0$, $\int_0^t \int_E \|g(s, z, \cdot)\| \lambda(dz) ds < +\infty$ a.s.;
- f is (\mathcal{F}_t) -predictable and for $t > 0$, $\int_0^t \int_E \|f(s, z, \cdot)\|^2 \lambda(dz) ds < +\infty$ a.s.

We have (see, for example Ikeda and Watanabe [10] Theorem 5.1)

Lemma 3 (Itô's formula). *Let F be a function of class C^2 on \mathbb{R}^d and Y_t a d -dimensional semi-martingale given in (2.8). Then the stochastic process $F(Y_t)$ is also a (\mathcal{F}_t) -semi-martingale and the following formula holds :*

$$\begin{aligned} F(Y_t) - F(Y_0) &= \sum_{i=1}^d \int_0^t \frac{\partial F}{\partial x_i}(Y_s) dM_s^i + \sum_{i=1}^d \int_0^t \frac{\partial F}{\partial x_i}(Y_s) A_s^i ds + \frac{1}{2} \sum_{i,j=1}^d \int_0^t \frac{\partial^2 F}{\partial x_i \partial x_j}(Y_s) d\langle M^i, M^j \rangle_s \\ &+ \int_0^t \int_E [F(Y_{s-} + g(s, z, \cdot)) - F(Y_{s-})] N_p(dz ds) \\ &+ \int_0^t \int_E [F(Y_{s-} + f(s, z, \cdot)) - F(Y_{s-})] \tilde{N}_p(dz ds) \\ &+ \int_0^t \int_E \left[F(Y_{s-} + f(s, z, \cdot)) - F(Y_{s-}) - \sum_{i=1}^d f_i(s, z, \cdot) \frac{\partial F}{\partial x_i}(Y_s) \right] \lambda(dz) ds. \end{aligned}$$

2.3. The main result

A version of the next theorem (without jumps) can be seen in Baxendale and Stroock [2] (Corollary 1.12). This is a very long one and we now have all the necessary arguments to be able to copy it, *mutatis mutandis*, by replacing it back in our frame. For this reason, we will content ourselves with the statement of the theorem, referring the careful reader to the aforementioned paper.

Before proceeding, we give some definitions. Denote by $C^\infty(\mathbb{T}^d)$ the space of functions of class C^∞ on \mathbb{T}^d . For all function $\phi \in C^\infty(\mathbb{T}^d)$ and all probability measure $\mu \in \mathcal{P}(\mathbb{T}^d)$, let us set

$$\begin{aligned} b_\mu(\phi) &:= \int_{\mathbb{T}^d} (I + \nabla\phi)(x) b(x) \mu(dx), & c_\mu(\phi) &:= \int_{\mathbb{T}^d} (I + \nabla\phi)(x) c(x) \mu(dx), \\ k_\mu(\phi, y) &:= \int_{\mathbb{T}^d} (I + \nabla\phi)(x) k(x, y) \mu(dx), & \bar{c}_\mu(\phi) &:= c_\mu(\phi) - \int_{\mathbb{R}^d} k_\mu(\phi, y) \nu(dy), \\ a_\mu(\phi) &:= \int_{\mathbb{T}^d} (I + \nabla\phi)(x) a(x) (I + \nabla\phi)(x) \mu(dx), & \Delta_{a_\mu}(\phi) &:= \frac{1}{2} \int_{\mathbb{T}^d} \text{Tr}\{\nabla\phi(x) a(x) \nabla\phi(x)\} \mu(dx). \end{aligned}$$

Define

$$\begin{aligned} \bar{\mathcal{J}}_\gamma(\theta) &:= \inf_{\phi \in C^\infty(\mathbb{T}^d)} \sup_{\mu \in \mathcal{P}(\mathbb{T}^d)} \left\{ \frac{1}{2} \langle \theta, a_\mu(\phi) \theta \rangle + \left\langle \bar{c}_\mu(\phi) + \frac{1}{\gamma} (\Delta_{a_\mu} + b_\mu)(\phi), \theta \right\rangle \right. \\ &\quad \left. + \int_{\mathbb{R}^d} \int_{\mathbb{T}^d} \left(e^{\langle k(z, y), \theta \rangle} - 1 \right) \mu(dz) \nu(dy) \right\}. \end{aligned}$$

Now fix $\mu \in \mathcal{P}(\mathbb{T}^d)$, fix also $\phi \in C^\infty(\mathbb{T}^d)$, by assumption on a , the matrix $a_\mu(\phi)$ is strictly positive-definite. Then letting $a_\mu^{-1}(\phi)$ be its inverse matrix, we define the norm $\|\theta\|_{a_\mu^{-1}(\phi)} := \sqrt{\langle \theta, a_\mu^{-1}(\phi) \theta \rangle}$ for all

$\theta \in \mathbb{R}^d$. Next, we define

$$Q_{a_\mu(\phi), \nu} := \left\{ w \in \mathbb{R}^d : w = a_\mu(\phi)u \text{ for some } u \in \mathbb{R}^d \right\} + S_\nu,$$

where S_ν denotes the support of ν . Let $\text{con}(Q_{a_\mu(\phi), \nu})$ be the smallest convex cone that contains $Q_{a_\mu(\phi), \nu}$ and define

$$T_{a_\mu(\phi), \nu} := \left\{ \bar{c}_\mu(\phi) \right\} + \text{con}(Q_{a_\mu(\phi), \nu}). \quad (2.9)$$

Let $\text{ri}(A)$ denotes the relative interior of a set A . In light of the characterization lemma (see Ikeda and Watanabe [10], Lemma 10.2.3) to the effective domain of $\mathcal{J}(\theta) := \inf_{\theta' \in \mathbb{R}^d} \left\{ \langle \theta, \theta' \rangle - \bar{\mathcal{J}}(\theta') \right\}$, we have

$$\text{ri} \left(\inf_{\phi \in C^\infty(\mathbb{T}^d)} \sup_{\mu \in \mathcal{P}(\mathbb{T}^d)} T_{a_\mu(\phi), \nu} \right) \equiv \text{ri}(\text{dom} \mathcal{J}(\theta)).$$

By the requirements (2.1) and (2.2), it is well know that

$$\mathcal{J}_\gamma(\theta) := \sup_{\phi \in C^\infty(\mathbb{T}^d)} \inf_{\mu \in \mathcal{P}(\mathbb{T}^d)} \left\{ \frac{1}{2} \left\| \theta - \bar{c}_\mu(\phi) - \frac{1}{\gamma} (\Delta_{a_\mu} + b_\mu)(\phi) \right\|_{a_\mu^{-1}(\phi)}^2 + \int_{\mathbb{R}^d} \int_{\mathbb{T}^d} \varrho \left(\frac{\|\theta\|}{\|k(z, y)\|} \right) \mu(dz) \nu(dy) \right\}$$

where $\varrho(r) := r \log r - r + 1$, $r \in (0, +\infty)$.

Finally, let us define, for $T > 0$ and $x \in \mathbb{R}^d$,

$$g_{T,x}^\varepsilon(\theta) := \varepsilon \log \mathbb{E} \left\{ \exp \left(\frac{1}{\varepsilon} \langle \theta, X_T^{x, \varepsilon, \delta_\varepsilon} \rangle \right) \right\}, \quad \varepsilon > 0, \theta \in \mathbb{R}^d. \quad (2.10)$$

Now we state our main result.

Theorem 4 (Main result). *For all $T > 0$, we assume that the hypothesis (H.1) and (H.2) hold true. Suppose that*

$$g_\gamma^{T,x}(\theta) := \lim_{\varepsilon \rightarrow 0} g_{T,x}^\varepsilon(\theta) = \lim_{\varepsilon \rightarrow 0} \varepsilon \log \mathbb{E} \left\{ \exp \frac{1}{\varepsilon} (\theta, X_T^{x, \varepsilon, \delta_\varepsilon}) \right\}$$

exists uniformly with respect to $x \in \mathbb{R}^d$. In addition

$$g_\gamma^{T,x}(\theta) := \langle x, \theta \rangle + T \bar{\mathcal{J}}_\gamma(\theta).$$

Then we have

$$\begin{aligned} \liminf_{\varepsilon \rightarrow 0} g_{T,x}^\varepsilon(\theta) &\geq \langle x, \theta \rangle + T \left(\bar{Q}_\gamma(\theta) + \alpha_{1,\gamma}(\theta) + \alpha_{2,\gamma}(\theta) \right) \\ \limsup_{\varepsilon \rightarrow 0} g_{T,x}^\varepsilon(\theta) &\leq \langle x, \theta \rangle + T \left(\bar{Q}_\gamma(\theta) + \beta_{1,\gamma}(\theta) + \beta_{2,\gamma}(\theta) \right) \end{aligned}$$

where

$$\begin{aligned}\bar{Q}_\gamma(\theta) &:= \frac{1}{2} \langle \theta, a_{m_\gamma}(\phi)\theta \rangle + \left\langle \bar{c}_{m_\gamma}(\phi) + \frac{1}{\gamma} (\Delta_{a_{m_\gamma}} + b_{m_\gamma})(\phi), \theta \right\rangle \\ &\quad + \int_{\mathbb{R}^d} \int_{\mathbb{T}^d} \left(e^{\langle k(x,y), \theta \rangle} - 1 \right) m_\gamma(dx) \nu(dy) \\ \alpha_{1,\gamma}(\theta) &:= \inf \left\{ \gamma \int_{\mathbb{R}^d} \int_{\mathbb{T}^d} \|k(x,y) \nabla \Psi_\theta(x)\| m_\gamma(dx) \nu(dy) : \Psi_\theta \in C^\infty(\mathbb{T}^d) \right\} \\ \alpha_{2,\gamma}(\theta) &:= \inf \left\{ \frac{\gamma^2}{2} \int_{\mathbb{T}^d} \|\sigma^*(x) \nabla \Psi_\theta(x)\|^2 m_\gamma(dx) : \Psi_\theta \in C^\infty(\mathbb{T}^d) \right\} \\ \beta_{1,\gamma}(\theta) &:= \inf_{\Psi_\theta \in C^\infty(\mathbb{T}^d)} \sup_{\mu \in \mathcal{P}(\mathbb{T}^d)} \left\{ \gamma \int_{\mathbb{R}^d} \int_{\mathbb{T}^d} \|k(x,y) \nabla \Psi_\theta(x)\| \mu(dx) \nu(dy) \right\} \\ \beta_{2,\gamma}(\theta) &:= \inf_{\Psi_\theta \in C^\infty(\mathbb{T}^d)} \sup_{\mu \in \mathcal{P}(\mathbb{T}^d)} \left\{ \frac{\gamma^2}{2} \int_{\mathbb{T}^d} \|\sigma^*(x) \nabla \Psi_\theta(x)\|^2 \mu(dx) \right\},\end{aligned}$$

and we have used Ψ_θ to denote the unique $u \in C^\infty(\mathbb{T}^d)$ which satisfies

$$\mathcal{L}_\gamma u = Q(\cdot, \theta) - \bar{Q}_\gamma(\theta) \quad \text{and} \quad \int_{\mathbb{T}^d} u(x) m_\gamma(dx) = 0,$$

and with

$$\begin{aligned}Q(x, \theta) &:= \frac{1}{2} \langle \theta, (I + \nabla \phi)(x) \sigma(x) \rangle^2 + \int_{\mathbb{R}^d} \langle \theta, (I + \nabla \phi)(x) (c(x) - k(x, y)) \rangle \nu(dy) \\ &\quad + \frac{1}{\gamma} \left\langle \theta, \frac{1}{2} \text{Tr} \{ \nabla \phi a \nabla \phi \}(x) + (I + \nabla \phi)(x) b(x) \right\rangle + \int_{\mathbb{R}^d} \left(e^{\langle k(x,y), \theta \rangle} - 1 \right) \nu(dy), \quad \phi \in C^\infty(\mathbb{T}^d).\end{aligned}$$

Finally, set

$$I_\gamma^{T,x}(\eta) := \sup_\theta \{ \langle \theta, \eta \rangle - g_\gamma^{T,x}(\theta) \} = \sup_\theta \{ \langle \theta, \eta - x \rangle - T \bar{\mathcal{J}}_\gamma(\theta) \} = T \mathcal{J} \left(\frac{\eta - x}{T} \right).$$

Then for every set Γ on $\mathcal{B}(\mathbb{R}^d)$,

$$\begin{aligned}\liminf_{\varepsilon \rightarrow 0} \varepsilon \log \mathbb{P} \left\{ X_T^{x,\varepsilon,\delta_\varepsilon} \in \Gamma \right\} &\geq - \inf_{\eta \in \overset{\circ}{\Gamma}} I_\gamma^{T,x}(\eta) \\ \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbb{P} \left\{ X_T^{x,\varepsilon,\delta_\varepsilon} \in \Gamma \right\} &\leq - \inf_{\eta \in \bar{\Gamma}} I_\gamma^{T,x}(\eta).\end{aligned}$$

3. Outline of the LDP characterization

3.1. Overview and discussion

Freidlin-Wentzell's theory is interested in the behaviour of a stochastic process (seen as the Brownian perturbation of an ordinary differential equation (ODE)). For the description and the background for ODE over-amortised by a Brownian martingale, we refer to [5, 8, 13] and the papers therein. At the

same time, non-linear stochastic evolution equations have been studied in various literatures. However, there are still few results on the large deviation for stochastic evolution equations with jumps (see, for example [12, 15]).

In all the references listed above the question is: What do SDE's trajectories look like when the viscosity parameter ε is small? The answer is twofold depending on the time scales considered: Finite time interval or increasingly large time scale. Another question can also be considered: "When the process is ergodic and has an invariant distribution μ_ε , what is the asymptotic behaviour of μ_ε when $\varepsilon \rightarrow 0$?"

This is the question in all the references available for us in this direction, with the one important exception given by the paper [9] (see also [1]). In [9] large deviations problems arising in stochastic homogenization are discussed. The scope of our approach based on the LDP with combination of homogenization, differs substantially from that of [9].

Our version of SDE contains a substantial novelty, which makes it possible to handle the Freidlin-Wentzell's large deviation arising in stochastic homogenization for non-linear evolution equations with Poisson jumps and Brownian motions. To explain this modified construction in the most transparent way, we took the "jump component" in a relatively simple form, that is, we consider Poisson point process of class (QL).

There are several *toolbox* for the large deviations (see, for example [6]). The way we choose allows us to characterize the LDP with help of the analysis of the logarithmic moment generating function.

We refer to Baxendale et al. [2] for the characterization and the general results on the LDP with help of the analysis of the logarithmic moment generating function. However, on this way we meet substantial difficulties already when we try to prove that the LDP holds. Initially the corresponding rate function (this one is denoted by I) is identified as the Legendre transform (or convex conjugate) of the logarithmic moment generating function defined, when the limit exists, as

$$g_\gamma^{T,x}(\theta) = \lim_{\varepsilon \rightarrow 0} \varepsilon \log \mathbb{E} \left\{ \exp \frac{1}{\varepsilon} \langle \theta, X_T^{x,\varepsilon,\delta_\varepsilon} \rangle \right\}.$$

Subsequently, the Cameron-Martin-Girsanov transformation is used to find an alternative expression for I . Although from the standpoint of large deviation theory it is the rate function I which is of paramount interest, it is the logarithmic moment generating function $g_\gamma^{T,x}$ which is most important for the analysis of Lyapunov exponents. Indeed, one sees that $g_\gamma^{T,x}$ is here what is called the *moment Lyapunov function*.

3.2. LDP results

The proof of the main result can be conducted in six steps, which we outline below.

Proof. (Theorem 4)

Step 1: (calculation of the *moment Lyapunov function*).

The calculation of this limit is inspired by Freidlin [8], Lemma 2.1 (without jumps) Section 7.2.

For any $\theta \in \mathbb{R}^d$

$$\lim_{\varepsilon \rightarrow 0} g_{T,x}^\varepsilon(\theta) = g_\gamma^{T,x}(\theta) = \langle x, \theta \rangle + T \overline{\mathcal{J}}_y(\theta),$$

exists uniformly in $x \in \mathbb{R}^d$, with

$$\begin{aligned} \bar{\mathcal{J}}_\gamma(\theta) = & \inf_{\phi \in C^\infty(\mathbb{T}^d)} \sup_{\mu \in \mathcal{P}(\mathbb{T}^d)} \left\{ \frac{1}{2} \langle \theta, a_\mu(\phi) \theta \rangle + \left\langle \bar{c}_\mu(\phi) + \frac{1}{\gamma} (\Delta_{a_\mu} + b_\mu)(\phi), \theta \right\rangle \right. \\ & \left. + \int_{\mathbb{R}^d} \int_{\mathbb{T}^d} \left(e^{\langle k(z, y), \theta \rangle} - 1 \right) \mu(dz) \nu(dy) \right\} \end{aligned}$$

being the largest eigenvalue of the differential operator

$$\begin{aligned} L_{\theta, \gamma} := & \frac{1}{\gamma^2} \mathcal{L}_\gamma + \frac{1}{\gamma} \sum_{i, j=1}^d a_{i, j}(x) \theta_i \frac{\partial}{\partial x_i} + \sum_{i=1}^d \left(\frac{1}{\gamma} b_i + c_i \right) (x) \theta_i + \frac{1}{2} \sum_{i, j=1}^d \theta_i^* a_{i, j}(x) \theta_j \\ & + \int_{\mathbb{R}^d} \sum_{i=1}^d \left\{ k_i(x, y) \theta_i + \frac{1}{\gamma} k_i(x, y) \frac{\partial}{\partial x_i} \right\} \nu(dy) + \int_{\mathbb{R}^d} \left(e^{\langle k(x, y), \theta \rangle} - 1 \right) \nu(dy). \end{aligned}$$

Step 2: (change of probability measure).

We set $\hat{X}_t^{\varepsilon, \delta_\varepsilon} := \frac{1}{\delta_\varepsilon} X_t^{x, \varepsilon, \delta_\varepsilon}$, by the Itô's formula we have for all $\phi \in C^\infty(\mathbb{T}^d)$

$$\begin{aligned} \tilde{X}_t^{\varepsilon, \delta_\varepsilon} &= X_t^{x, \varepsilon, \delta_\varepsilon} + \delta_\varepsilon \left[\phi(\hat{X}_t^{\varepsilon, \delta_\varepsilon}) - \phi\left(\frac{x}{\delta_\varepsilon}\right) \right] \\ &= x + \int_0^t (I + \nabla \phi)(\hat{X}_s^{\varepsilon, \delta_\varepsilon}) \left[c(\hat{X}_s^{\varepsilon, \delta_\varepsilon}) - \int_{\mathbb{R}^d} k(\hat{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y) \nu(dy) \right] ds \\ &\quad + \sqrt{\varepsilon} \int_0^t (I + \nabla \phi)(\hat{X}_s^{\varepsilon, \delta_\varepsilon}) \sigma(\hat{X}_s^{\varepsilon, \delta_\varepsilon}) dW_s \\ &\quad + \frac{\varepsilon}{\delta_\varepsilon} \int_0^t \left[\frac{1}{2} \text{Tr} \{ \nabla \phi(\hat{X}_s^{\varepsilon, \delta_\varepsilon}) a(\hat{X}_s^{\varepsilon, \delta_\varepsilon}) \nabla \phi(\hat{X}_s^{\varepsilon, \delta_\varepsilon}) \} + (I + \nabla \phi)(\hat{X}_s^{\varepsilon, \delta_\varepsilon}) b(\hat{X}_s^{\varepsilon, \delta_\varepsilon}) \right] ds \\ &\quad + \delta_\varepsilon \int_0^t \int_{\mathbb{R}^d} \left[\phi(\hat{X}_{s-}^{\varepsilon, \delta_\varepsilon} + \varepsilon k(\hat{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y)) - \phi(\hat{X}_{s-}^{\varepsilon, \delta_\varepsilon}) \right] N^{\varepsilon^{-1}}(dy ds) \\ &\quad + \varepsilon \int_0^t \int_{\mathbb{R}^d} k(\hat{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y) N^{\varepsilon^{-1}}(dy ds). \end{aligned}$$

Let us define for $z \in \mathbb{T}^d$

$$H^{\varepsilon, \varphi}(z) := \varphi(z + \varepsilon k(z, \cdot)) - \varphi(z) \quad \forall \varphi \in C^\infty(\mathbb{T}^d).$$

So, we are going to consider the logarithm moment generating function of $X^{x, \varepsilon, \delta_\varepsilon}$, $g_{T, x}^\varepsilon$ (2.10). By

Girsanov’s formula, we have

$$\begin{aligned}
 g_{T,x}^\varepsilon(\theta) = & \langle x, \theta \rangle + \varepsilon \log \widetilde{\mathbb{E}} \left[\exp \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \left(\frac{1}{2} \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left\langle \theta, (I + \nabla\phi) \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \sigma \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \right\rangle^2 ds \right. \right. \\
 & + \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left\langle \theta, (I + \nabla\phi) \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \left[c \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) - \int_{\mathbb{R}^d} k \left(\overline{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y \right) \nu(dy) \right] \right\rangle ds \\
 & + \frac{\delta_\varepsilon}{\varepsilon} \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left\langle \theta, \frac{1}{2} \text{Tr} \{ \nabla\phi \left(\hat{X}_s^{\varepsilon, \delta_\varepsilon} \right) a \left(\hat{X}_s^{\varepsilon, \delta_\varepsilon} \right) \nabla\phi \left(\hat{X}_s^{\varepsilon, \delta_\varepsilon} \right) \} + (I + \nabla\phi) \left(\hat{X}_s^{\varepsilon, \delta_\varepsilon} \right) b \left(\hat{X}_s^{\varepsilon, \delta_\varepsilon} \right) \right\rangle ds \\
 & + \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left(e^{\left\{ \frac{\delta_\varepsilon}{\varepsilon} \left\langle \theta, H^{\varepsilon, \phi} \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, y \right) \right\rangle \right\}} - 1 \right) \nu(dy) ds \\
 & \left. + \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left(e^{\left\langle \theta, k \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, y \right) \right\rangle} - 1 \right) \nu(dy) ds - \frac{\delta_\varepsilon}{\varepsilon} \left(\phi \left(\overline{X}_t^{\varepsilon, \delta_\varepsilon} \right) - \phi \left(\frac{x}{\delta_\varepsilon} \right) \right) \right\} \right]
 \end{aligned} \tag{3.1}$$

where $\widetilde{\mathbb{E}}$ is the expectation operator with respect to the probability $\widetilde{\mathbb{P}}$ defined as

$$\begin{aligned}
 \frac{d\widetilde{\mathbb{P}}}{d\mathbb{P}} := & \exp \left(\left(\frac{\delta_\varepsilon}{\varepsilon} \right) \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left\langle \theta, (I + \nabla\phi) \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \sigma \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \right\rangle dW_s \right. \\
 & \left. - \frac{1}{2} \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left\langle \theta, (I + \nabla\phi) \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \sigma \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \right\rangle^2 ds \right) \\
 & \times \exp \left(\int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left\langle \theta, k \left(\overline{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y \right) \right\rangle N^{(\delta_\varepsilon/\varepsilon)^2}(dy ds) \right. \\
 & \left. - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left(e^{\left\langle \theta, k \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, y \right) \right\rangle} - 1 \right) \nu(dy) ds \right) \\
 & \times \exp \left(\frac{\delta_\varepsilon}{\varepsilon} \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left\langle \theta, H^{\varepsilon, \phi} \left(\overline{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y \right) \right\rangle N^{(\delta_\varepsilon/\varepsilon)^2}(dy ds) \right. \\
 & \left. - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left(e^{\left\{ \frac{\delta_\varepsilon}{\varepsilon} \left\langle \theta, H^{\varepsilon, \phi} \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, y \right) \right\rangle \right\}} - 1 \right) \nu(dy) ds \right).
 \end{aligned}$$

Step 3: (ergodicity).

Let us set, for all $z \in \mathbb{T}^d$, for all $\theta \in \mathbb{R}^d$:

$$\begin{aligned}
 \Phi(z, \theta) := & \frac{1}{2} \left\langle \theta, (I + \nabla\phi(z)) \sigma(z) \right\rangle^2 + \int_{\mathbb{R}^d} \left\langle (I + \nabla\phi(z)) (c(z) - k(z, y)), \theta \right\rangle \nu(dy) \\
 & + \frac{1}{\gamma} \left\langle \theta, \frac{1}{2} \text{Tr} \{ \nabla\phi(z) a(z) \nabla\phi(z) \} + (I + \nabla\phi(z)) b(z) \right\rangle + \int_{\mathbb{R}^d} \left(e^{\langle k(z, y), \theta \rangle} - 1 \right) \nu(dy).
 \end{aligned} \tag{3.2}$$

and let us set $\Psi_\theta \in C^\infty(\mathbb{T}^d)$ be the unique solution of

$$\mathcal{L}_\gamma \Psi_\theta = \Phi - \int_{\mathbb{T}^d} \Phi(z, \theta) m_\gamma(dz), \quad \text{which satisfies} \quad \int_{\mathbb{T}^d} \Psi_\theta(z) m_\gamma(dz) = 0.$$

Such a solution Ψ_θ must exist again by the assumptions on the coefficients (see, for instance, Pardoux et al. [11]). So applying Itô formula to $\left(\frac{\delta_\varepsilon}{\sqrt{\varepsilon}}\right)^2 \Psi_\theta(\bar{X})$, we have

$$\begin{aligned} \left(\frac{\delta_\varepsilon}{\sqrt{\varepsilon}}\right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \Phi(\bar{X}_s^{\varepsilon, \delta_\varepsilon}, \theta) ds &= T \int_{\mathbb{T}^d} \Phi(z, \cdot) m_\gamma(dz) + \left(\frac{\delta_\varepsilon}{\sqrt{\varepsilon}}\right)^2 \left[\Psi_\theta(\bar{X}_{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T}^{\varepsilon, \delta_\varepsilon}) - \Psi_\theta\left(\frac{x}{\delta_\varepsilon}\right) \right] \\ &\quad - \left(\frac{\delta_\varepsilon}{\sqrt{\varepsilon}}\right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \sigma(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) \nabla \Psi_\theta(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) dW_s \\ &\quad - \frac{\delta_\varepsilon^2}{\varepsilon} \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left(\frac{\delta_\varepsilon}{\varepsilon} - \gamma\right) c(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) \nabla \Psi_\theta(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) ds \\ &\quad + \frac{\delta_\varepsilon^3}{\varepsilon^2} \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} k(\bar{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y) \nabla \Psi_\theta(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) \nu(dy) ds \\ &\quad - \left(\frac{\delta_\varepsilon}{\sqrt{\varepsilon}}\right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left[\Psi_\theta(\bar{X}_{s-}^{\varepsilon, \delta_\varepsilon} + \varepsilon k(\bar{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y)) - \Psi_\theta(\bar{X}_{s-}^{\varepsilon, \delta_\varepsilon}) \right] N^{(\delta_\varepsilon/\varepsilon)^2}(dy ds). \end{aligned} \tag{3.3}$$

Then putting (3.3) into the formula (3.1), we have

$$\begin{aligned} g_{T,x}^\varepsilon(\theta) &= \langle x, \theta \rangle + T \int_{\mathbb{T}^d} \Phi(z, \theta) m(dz) + \varepsilon \log \widetilde{\mathbb{E}} \left[\exp \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon}\right)^2 \left(\Psi_\theta(\bar{X}_{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T}^{\varepsilon, \delta_\varepsilon}) - \Psi_\theta\left(\frac{x}{\delta_\varepsilon}\right) \right) \right. \right. \\ &\quad \left. \left. - \frac{\delta_\varepsilon}{\varepsilon} \left(\phi(\hat{X}_t^{\varepsilon, \delta_\varepsilon}) - \phi\left(\frac{x}{\delta_\varepsilon}\right) \right) \right. \right. \\ &\quad \left. \left. + \left(\frac{\delta_\varepsilon}{\varepsilon}\right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left(e^{\left\{ \frac{\delta_\varepsilon}{\varepsilon} \langle \theta, H^{\varepsilon, \phi}(\bar{X}_s^{\varepsilon, \delta_\varepsilon}, y) \rangle \right\}} - 1 \right) \nu(dy) ds \right. \right. \\ &\quad \left. \left. - \left(\frac{\delta_\varepsilon}{\varepsilon}\right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \sigma(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) \nabla \Psi_\theta(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) dW_s \right. \right. \\ &\quad \left. \left. - \left(\frac{\delta_\varepsilon}{\varepsilon}\right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left(\frac{\delta_\varepsilon}{\varepsilon} - \gamma\right) c(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) \nabla \Psi_\theta(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) ds \right. \right. \\ &\quad \left. \left. + \left(\frac{\delta_\varepsilon}{\varepsilon}\right)^3 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} k(\bar{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y) \nabla \Psi_\theta(\bar{X}_s^{\varepsilon, \delta_\varepsilon}) \nu(dy) ds \right. \right. \\ &\quad \left. \left. - \left(\frac{\delta_\varepsilon}{\varepsilon}\right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} H^{\varepsilon, \Psi_\theta}(\bar{X}_{s-}^{\varepsilon, \delta_\varepsilon}) N^{(\delta_\varepsilon/\varepsilon)^2}(dy ds) \right\} \right]. \end{aligned} \tag{3.4}$$

Step 4: (upper bound).

First, we have

$$\begin{aligned}
 & \left| \log \left[\widetilde{\mathbb{E}} \left(\exp \left\{ - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \sigma \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \nabla \Psi_\theta \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) dW_s \right\} \right) \right] \right| \\
 & \leq \left| \log \left[\widetilde{\mathbb{E}} \left(\exp \left\{ - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \sigma \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \nabla \Psi_\theta \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) dW_s \right\} \right) \right. \right. \\
 & \quad \times \exp \left\{ - \frac{1}{2} \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^4 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left\| \sigma \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \nabla \Psi_\theta \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \right\|^2 ds \right\} \\
 & \quad \left. \times \exp \left\{ \frac{1}{2} \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^4 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left\| \sigma \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \nabla \Psi_\theta \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \right\|^2 ds \right\} \right] \right| \leq \frac{T}{\varepsilon} \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \frac{\beta_{2,\gamma}(\theta)}{\gamma^2}.
 \end{aligned}$$

Secondly, we can see

$$\left| \log \left[\widetilde{\mathbb{E}} \left(\exp \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^3 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} k \left(\overline{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y \right) \nabla \Psi_\theta \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \nu(dy) ds \right\} \right) \right] \right| \leq \frac{T}{\varepsilon} \frac{\delta_\varepsilon}{\varepsilon} \frac{\beta_{1,\gamma}(\theta)}{\gamma}.$$

Thirdly, we observe

$$\begin{aligned}
 & \widetilde{\mathbb{E}} \left[\exp \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \left(\Psi_\theta \left(\overline{X}_{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T}^{\varepsilon, \delta_\varepsilon} \right) - \Psi_\theta \left(\frac{x}{\delta_\varepsilon} \right) \right) - \frac{\delta_\varepsilon}{\varepsilon} \left(\phi \left(\overline{X}_{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T}^{\varepsilon, \delta_\varepsilon} \right) - \phi \left(\frac{x}{\delta_\varepsilon} \right) \right) \right. \right. \\
 & \quad - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left(\frac{\delta_\varepsilon}{\varepsilon} - \gamma \right) c \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \nabla \Psi_\theta \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) ds \\
 & \quad - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} H^{\varepsilon, \Psi_\theta} \left(\overline{X}_{s-}^{\varepsilon, \delta_\varepsilon} \right) N^{(\delta_\varepsilon/\varepsilon)^2}(dy) ds \\
 & \quad \left. \left. + \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left(e \left\{ \frac{\delta_\varepsilon}{\varepsilon} \left\langle \theta, H^{\varepsilon, \phi} \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, y \right) \right\rangle \right\} - 1 \right) \nu(dy) ds \right\} \right] \\
 & \leq \exp \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 K_1 + \frac{\delta_\varepsilon}{\varepsilon} K_2 + \frac{T}{\varepsilon} \left(\frac{\delta_\varepsilon}{\varepsilon} - \gamma \right) K_3 + T \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 K_4 + T \frac{\delta_\varepsilon}{\varepsilon} K_5 \right\}.
 \end{aligned} \tag{3.5}$$

In fact, we notice that

$$\begin{aligned} & \left| \log \left[\widetilde{\mathbb{E}} \left(\exp \left\{ - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} H^{\varepsilon, \Psi_\theta} \left(\overline{X}_{s-}^{\varepsilon, \delta_\varepsilon} \right) N^{(\delta_\varepsilon/\varepsilon)^2} (dy ds) \right\} \right) \right] \right| \\ & \leq \left| \log \left(\exp \left\{ - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} H^{\varepsilon, \Psi_\theta} \left(\overline{X}_{s-}^{\varepsilon, \delta_\varepsilon}, y \right) N^{(\delta_\varepsilon/\varepsilon)^2} (dy ds) \right. \right. \right. \\ & \quad \left. \left. - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left[e \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 H^{\varepsilon, \Psi_\theta} \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, y \right) \right\} - 1 \right] \nu(dy) ds \right\} \right. \\ & \quad \left. \times \exp \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left[e \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 H^{\varepsilon, \Psi_\theta} \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, y \right) \right\} - 1 \right] \nu(dy) ds \right\} \right) \right| \\ & \leq \exp \left\{ \frac{T}{\varepsilon} \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \left\| H^{\varepsilon, \Psi_\theta} \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, \cdot \right) \right\|_{C^\infty(\mathbb{T}^d, \mathbb{R}^d)} + o(1) \right\}. \\ & \leq \exp \left\{ T \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \left\| k \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, \cdot \right) \right\|_{C^\infty(\mathbb{T}^d, \mathbb{R}^d)} + o(1) \right\}, \end{aligned}$$

and

$$\begin{aligned} & \left| \widetilde{\mathbb{E}} \left[\exp \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left(e \left\{ \frac{\delta_\varepsilon}{\varepsilon} \left\langle \theta, H^{\varepsilon, \phi} \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, y \right) \right\rangle \right\} - 1 \right) \nu(dy) ds \right\} \right] \right| \\ & \leq \exp \left\{ T \left(\frac{\delta_\varepsilon}{\varepsilon} \right) \left\| k \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, \cdot \right) \right\|_{C^\infty(\mathbb{T}^d, \mathbb{R}^d)} + o(1) \right\}. \end{aligned}$$

From (3.5) we have

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} \varepsilon \log \widetilde{\mathbb{E}} & \left[\exp \left\{ \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \left(\Psi_\theta \left(\overline{X}_{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T}^{\varepsilon, \delta_\varepsilon} \right) - \Psi_\theta \left(\frac{x}{\delta_\varepsilon} \right) \right) - \frac{\delta_\varepsilon}{\varepsilon} \left(\phi \left(\overline{X}_{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T}^{\varepsilon, \delta_\varepsilon} \right) - \phi \left(\frac{x}{\delta_\varepsilon} \right) \right) \right. \right. \\ & \quad - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \left(\gamma - \frac{\delta_\varepsilon}{\varepsilon} \right) c \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) \nabla \Psi_\theta \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon} \right) ds \\ & \quad - \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} H^{\varepsilon, \Psi_\theta} \left(\overline{X}_{s-}^{\varepsilon, \delta_\varepsilon} \right) N^{(\delta_\varepsilon/\varepsilon)^2} (dy ds) \\ & \quad \left. \left. + \left(\frac{\delta_\varepsilon}{\varepsilon} \right)^2 \int_0^{(\sqrt{\varepsilon}/\delta_\varepsilon)^2 T} \int_{\mathbb{R}^d} \left(e \left\{ \frac{\delta_\varepsilon}{\varepsilon} \left\langle \theta, H^{\varepsilon, \phi} \left(\overline{X}_s^{\varepsilon, \delta_\varepsilon}, y \right) \right\rangle \right\} - 1 \right) \nu(dy) ds \right\} \right] \rightarrow 0. \end{aligned}$$

Then, for all $\theta \in \mathbb{R}^d$, $T > 0$, we have

$$\limsup_{\varepsilon \rightarrow 0} g_{T,x}^\varepsilon(\theta) \leq \langle \theta, x \rangle + T \left(\overline{Q}_\gamma(\theta) + \beta_{1,\gamma}(\theta) + \beta_{2,\gamma}(\theta) \right). \tag{3.6}$$

Step 5: (lower bound).

As it has already been seen previously, we then have

$$\liminf_{\varepsilon \rightarrow 0} g_{T,x}^\varepsilon(\theta) \geq \langle \theta, x \rangle + T \left(\bar{Q}_\gamma(\theta) + \alpha_{1,\gamma}(\theta) + \alpha_{2,\gamma}(\theta) \right). \tag{3.7}$$

Step 6: (Fenchel-Legendre transform).

Let us set for $\theta \in \mathbb{R}^d$, X_1 be a Gaussian random vector with logarithm moment generating function

$$\bar{\Lambda}_{1,\gamma}(\theta) := \inf_{\phi \in C^\infty(\mathbb{T}^d)} \sup_{\mu \in \mathcal{P}(\mathbb{T}^d)} \left\{ \frac{1}{2} \langle \theta, a_\mu(\phi) \theta \rangle + \left\langle \bar{c}_\mu(\phi) + \frac{1}{\gamma} (\Delta_{a_\mu} + b_\mu)(\phi), \theta \right\rangle \right\}.$$

and X_2 be a stationary Poisson process on \mathbb{R}^d independent of X_1 with logarithm moment generating function

$$\bar{\Lambda}_2(\theta) := \sup_{\mu \in \mathcal{P}(\mathbb{T}^d)} \left\{ \int_{\mathbb{R}^d} \int_{\mathbb{T}^d} \left(e^{\langle k(z,y), \theta \rangle} - 1 \right) \mu(dz) \nu(dy) \right\}.$$

We notice that

$$\bar{\mathcal{J}}_\gamma(\theta) := \bar{\Lambda}_{1,\gamma}(\theta) + \bar{\Lambda}_2(\theta).$$

Let $\Lambda_1(\theta)$ and $\Lambda_2(\theta)$ denote respectively the Fenchel-Legendre transform of $\bar{\Lambda}_1$ and $\bar{\Lambda}_2$, we have

$$\Lambda_{1,\gamma}(\theta) := \sup_{\phi \in C^\infty(\mathbb{T}^d)} \inf_{\mu \in \mathcal{P}(\mathbb{T}^d)} \left\{ \frac{1}{2} \left\| \theta - \bar{c}_\mu(\phi) - \frac{1}{\gamma} (\Delta_{a_\mu} + b_\mu)(\phi) \right\|_{a_\mu^{-1}(\phi)}^2 \right\}$$

and

$$\Lambda_2(\theta) := \inf_{\mu \in \mathcal{P}(\mathbb{T}^d)} \left\{ \int_{\mathbb{T}^d} \int_{\mathbb{R}^d} \varrho \left(\frac{\|\theta\|}{\|k(x,y)\|} \right) \nu(dy) \mu(dx) \right\}.$$

Since $\bar{\mathcal{J}}(\theta)$ is the logarithm moment generating of $X_1 + X_2$, it follows that its Fenchel-Legendre transform is

$$\mathcal{J}_\gamma(\theta) := \Lambda_{1,\gamma}(\theta) + \Lambda_2(\theta).$$

For the final assertion of the Theorem 4, we can observe that \mathcal{J}_γ is convex.

□

We also need to know that $X_t^{x,\varepsilon,\delta_\varepsilon}$ is exponentially tight in the path space. Indeed, let $\mathcal{D}([0, T], \mathbb{R}^d)$ be the space of functions that map $[0, T]$ into \mathbb{R}^d , which are right continuous and having left hand limits. $\mathcal{D}([0, T], \mathbb{R}^d)$ is metricated by the Skorohod metric, with respect to which it is complete and separable. Then, as in Proposition 3.2 [9], we have

Remark 5. We assume that the hypothesis (H.1) and (H.2) hold true. For any fixed $T > 0$, $x \in \mathbb{R}^d$ and $\alpha \in (0, 1/2)$,

$$\limsup_{\varrho \rightarrow \infty} \limsup_{\varepsilon \rightarrow 0} \varepsilon \log \mathbb{P} \left\{ \left\| X_T^{x,\varepsilon,\delta_\varepsilon} \right\|_{\mathcal{D}^\alpha([0,T],\mathbb{R}^d)} \geq \varrho \right\} = -\infty.$$

To complete this part, we point out that the previous proof also provides the LDP in the path space $\mathcal{D}([0, T], \mathbb{R}^d)$. Let us consider some definitions :

$$S_{0,T}(\varphi) := \begin{cases} \int_0^T \mathcal{J}(\dot{\varphi}(s)) ds & \text{if } \varphi \in \mathcal{D}([0, T], \mathbb{R}^d) \text{ and } \varphi(0) = x \\ +\infty & \text{else.} \end{cases}$$

Since the function \mathcal{J} is convex we can show that

$$\inf_{\substack{\varphi \in \mathcal{D}([0, T], \mathbb{R}^d) \\ \varphi(0)=x, \varphi(T)=z}} \int_0^T \mathcal{J}(\dot{\varphi}(s)) ds := T \mathcal{J}\left(\frac{z-x}{T}\right).$$

So we have

Remark 6. For all $T > 0$, we assume that the hypothesis **(H.1)** and **(H.2)** hold true. Then for every $x \in \mathbb{R}^d$, the family $\{X_T^{x,\varepsilon,\delta_\varepsilon} : \varepsilon > 0\}$ of $\mathcal{D}([0, T], \mathbb{R}^d)$ -valued random variables has a large deviations principle with good rate function $S_{0,T}(\varphi)$ for all $\varphi \in \mathcal{D}([0, T], \mathbb{R}^d)$.

4. Conclusion

We have derived the general expression of the action functional for stochastic processes with jumps. In particular, we generalize the Regime 2 considered in Freidlin-Sowers [9] for some classical SDE, as well as the version in Baldi [1] for a family of measures on a topological vector space. Furthermore, we have analysed a general LDP arising in stochastic homogenization and we have extended the result in [4].

Before finishing, we notice that the large deviations established in this paper generalizes Schilder's theorem, which is useful both to probabilist who are interested in the trajectories consequences of stochastic process and to statisticians who are interested in the weights of small balls given by the Wiener measure. Such results can have implications in both random optimization and Bayesian statistics when studying trajectories of *posterior* density estimates or regression problems. For example, one may consult the following works to see how highly theoretical probabilistic developments can rebound on unexpected statistical applications [7, 14].

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Conflict of interest

The authors declare no conflict of interest.

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