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Research article

Identification of the source term in Navier-Stokes system with incomplete data

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Abstract: The aim of this work is to get instantaneous information at fixed instant T on pollution term in Navier-Stokes system in which the initial condition is incomplete. The best method which can solve this problem is the sentinel method; It allows estimating the pollution term at which we look for information independently of the missing term that we do not want to identify. So, we prove the existence of such instantaneous sentinel by solving a problem of controllability with constraint on the control.

Keywords: controllability; control optimal; Navier-Stokes system; instantaneous sentinel; pollution term
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1. Introduction

Theoretical hydrodynamics has long attracted the attention of scientists working in a variety of specialized fields. The mathematical model of a viscous fluids governed by the basic Navier-Stokes equations had to serve as a space goat ([5, 18]), answering for all the accumulated absurdities of the theory of ideals fluids as well as accounting for the lifting force, the drag, the turbulent wake and many others things.

In the modeling of the Navier-Stokes type, the source terms as well as the initial or boundary conditions may be unknown.

The sentinel method introduced by J. L. Lions [4] is adapted to the estimation of this incomplete or unknown data, in the Navier-Stokes type or in the problems governed by parabolic system in general, for example, pollution in river or a lake. So Since the introduction of the sentinel method many authors developed several applications, such as in environment, in ecology ([6–8, 14, 17–19]).

J. Velin [12] consider a Navier-Stokes system with missing initial data condition and perturbation distributed term, he use a discriminating distributed sentinel with constraints to characterize the pollution term in the interval $[0, T]$ (see [10, 13]), his result is based on an adapted distributed Carlemen Inequality permitting to revisit a study investigated by O. Nakoulima [21] but the difficult problem is to characterize this pollution at a fixed time and that what we want to do.

Let Ω be a non-empty open bounded set of $\mathbb{R}^2$ with sufficiently smooth boundary Γ, $\omega \subset \Omega$ be a non-empty bounded open subset. Denote by $Q = \Omega \times [0, T]$ and $\Sigma = \Gamma \times]0, T[$. Let $y = y(x, t)$ be the solution of the Navier-Stokes system

$$\frac{\partial y}{\partial t} + y\nabla y - \Delta y + \nabla p = f \;\text{ in }\; Q, \tag{1.1}$$

and

$$\operatorname{div} y = 0 \;\text{ in }\; Q, \tag{1.2}$$

the initial conditions

$$y(0) = g \;\text{ in }\; \Omega, \tag{1.3}$$

the boundary condition

$$y = 0 \;\text{ on }\; \Sigma, \tag{1.4}$$

which simulates the transportation of a flow $y(x, t)$ which is submitted to the pressure $p(x, t)$.

Let us introduce the spaces V and H, which are usual in the analysis of Navier-Stokes systems

$$V = \left\{ y \in (H_0^1(\Omega))^2, \operatorname{div} y = 0 \right\} \tag{1.5}$$

$$H = \left\{ y \in (L^2(\Omega))^2, \operatorname{div} y = 0,\; y.\nu = 0 \;\text{ on } \Gamma \right\}, \tag{1.6}$$

where ν is the unit exterior normal to Γ.

If $f \in L^2(0, T; H)$ and $g \in H$, the system (1.1)–(1.4) has an unique solution such that

$$y \in (0, T, V) \cap L^\infty(0, T, H) \tag{1.7}$$

$$y' \in L^2(0, T, V'). \tag{1.8}$$

Remark 1.1. We have $y \in L^2(0, T; V)$ and $y' \in L^2(0, T; V')$, then y is a continuous function from $[0, T]$ to H.

We are interested in systems with data that are not completely known, for example the source term

$$f = \xi + \lambda\widehat{\xi}$$

as well as the initial conditions

$$g = y_0 + \sum_{i=1}^{N} \tau_i \widehat{y}_0^i$$

where ξ and y_0 are given. However, the terms $\lambda\widehat{\xi}$ is unknown function so-called pollution term, λ is a small real parameter.

The functions $\widehat{y_0^i}$, $1 \leq i \leq N$, are linearly independent in H and belongs to a vector subspace of N dimension, which we denote by

$$G = \left\{\widehat{y_0^1}, \widehat{y_0^2}, ..., \widehat{y_0^N}\right\}.$$

The parameters τ_i, $1 \leq i \leq N$, are unknown, supposed small so-called missing term.

The question is to obtain information on the pollution term not affected by the missing term of the initial data in the system

$$\frac{\partial y}{\partial t} + y\nabla y - \Delta y + \nabla p = \xi + \lambda \widehat{\xi} \ \text{ in } \ Q, \tag{1.9}$$

and

$$\operatorname{div} y = 0 \ \text{ in } \ Q, \tag{1.10}$$

the initial conditions

$$y(0) = y_0 + \sum_{i=1}^{N} \tau_i \widehat{y_0^i} \ \text{ in } \ \Omega, \tag{1.11}$$

the boundary condition

$$y = 0 \ \text{ on } \ \Sigma. \tag{1.12}$$

There are two possible approaches to this problem, one is more classical and uses, the least square method (see G. Chavent [3]), but the problem in this method that the pollution and the missing term play the same role, so we cannot separate them.

The other method is the sentinel method which is used to study systems of incomplete data. The notion permits to distinguish and to analyses two types of incomplete data, the pollution term at which we look for information independently of the missing term that we do not want to identify.

Typically, the Lions sentinel is a linear functional sensitive to certain parameters we are trying to evaluate, and insensitive to others which do not interest us.

So we show that this functional can be associated to our system and allows to characterize the pollution term.

In this paper we study this system with incomplete initial data, we use the instantaneous sentinel concept [21], which relies on the following three objects: Some state equation, some observation function and some control function to be determined.

2. Setting of the problem

Let $y(x, t, \lambda, \tau) = y(\lambda, \tau)$ with $\tau = (\tau_1, \tau_2, ..., \tau_N)$, be the unique solution of the problem (1.9)–(1.12). We denote by

$$y(x, T; \lambda, \tau) = y_{obs}, \forall x \in O. \tag{2.1}$$

An observation which is a measure of the concentration of the pollution taken at the fixed time T and on a non-empty open subset $O \subset \Omega$ called observatory.

Let h be some function in $L^2(O)$, for any control function $u \in L^2(\omega)$, we introduce the functional $\mathcal{S}(\lambda, \tau)$ as follows:

$$\mathcal{S}(\lambda, \tau) = \int_\omega u y(x, T; \lambda, \tau) dx + \int_O h y(x, T; \lambda, \tau) dx. \tag{2.2}$$

$$\mathcal{S}(\lambda, \tau) = \int_\Omega \left[h \chi_O y(x, T; \lambda, \tau) + u \chi_\omega y(x, T; \lambda, \tau)\right] dx, \tag{2.3}$$

where χ_O and χ_ω are the characteristic functions for the open sets O and ω respectively, such that

$$\chi_O : L^2(\Omega) \longrightarrow L^2(O)$$

$$\chi_\omega : L^2(\Omega) \longrightarrow L^2(\omega).$$

Definition 2.1. Let $\mathcal{S}$ is a real function (2.3) depending only on the parameters λ and τ. S is said a sentinel defined by h if the following conditions is satisfied:

$$\frac{\partial \mathcal{S}}{\partial \tau}(\lambda, \tau)\bigg|_{\lambda=0,\tau=0} = 0 \tag{2.4}$$

i.e.,

$$\frac{\partial \mathcal{S}}{\partial \tau_i}(\lambda, \tau)\bigg|_{\lambda=0,\tau_i=0} = 0, \; 1 \le i \le N.$$

There exists a control $u \in L^2(\omega)$ such that:

$$\|u\|_{L^2(\omega)} = \min_{\alpha \in U} \|\alpha\| \tag{2.5}$$

where $U = \left\{ \alpha \in L^2(\omega), \text{ such that } \dfrac{\partial \mathcal{S}}{\partial \tau}(\lambda, \tau)\bigg|_{\lambda=0,\tau=0} = 0 \right\}$.

Remark 2.2. 1) Condition (2.4) express insensitivity of $\mathcal{S}$ with respect to small variations of τ and assume the existence of the derivate.

2) According to (2.5) which consists in an optimal criterion of selection for (2.4).

3) Lions sentinel corresponds to the case $\omega = O$; if we choose $u = -h$, then (2.4) holds, so that problem (2.5) admits a unique solution, may have an interest only if $u \neq -h$.

4) We extend the method of sentinel to the case of observation and control having their supports in two different open sets, we assumed that $O \cap \omega \neq \emptyset$.

We consider the function y_0 and p_0 which solve the problem (1.9)–(1.12) for $\lambda = 0$ and $\tau_i = 0$

$$\frac{\partial y_0}{\partial t} + y_0 \nabla y_0 - \Delta y_0 + \nabla p_0 = \xi \quad \text{in} \quad Q, \tag{2.6}$$

and

$$\operatorname{div} y_0 = 0 \;\text{ in }\; Q, \tag{2.7}$$

the initial conditions

$$y_0(0) = y_0 \;\text{ in }\; \Omega, \tag{2.8}$$

the boundary condition

$$y_0 = 0 \;\text{ on }\; \Sigma. \tag{2.9}$$

We consider the function y_{τ_i} defined by $y_{\tau_i} = \dfrac{\partial y}{\partial \tau_i}(0,0)$, which is the unique solution of the problem

$$\frac{\partial y_{\tau_i}}{\partial t} + y_0 \nabla y_{\tau_i} + y_{\tau_i} \nabla y_0 - \Delta y_{\tau_i} + \nabla p_{\tau_i} = 0 \;\text{ in }\; Q, \tag{2.10}$$

and

$$\mathrm{div}\, y_{\tau_i} = 0 \ \text{ in } \ Q, \tag{2.11}$$

the initial conditions

$$y_{\tau_i}(0) = \widehat{y}_0^i \ \text{ in } \ \Omega, \tag{2.12}$$

the boundary condition

$$y_{\tau_i} = 0 \ \text{ on } \ \Sigma. \tag{2.13}$$

Remark 2.3. The condition (2.4) holds if and only if

$$\int_\Omega (h\chi_O + u\chi_\omega)\, y_{\tau_i}(T)dx = 0, \quad 1 \leq i \leq N. \tag{2.14}$$

In order to transform this equation, we introduce the classical adjoint state

$$-\frac{\partial q}{\partial t} + (y_0\nabla)\, q + q\,(\nabla y_0) - \Delta q + \nabla\pi = 0 \quad \text{in} \quad Q \ , \tag{2.15}$$

and

$$\mathrm{div}\, q = 0 \ \text{ in } \ Q, \tag{2.16}$$

the initial conditions

$$q(T) = h\chi_O + u\chi_\omega \ \text{ in } \ \Omega, \tag{2.17}$$

the boundary condition

$$q = 0 \ \text{ on } \ \Sigma. \tag{2.18}$$

Theorem 2.4. *Let $q = (q_1, q_2)$ be the solution to the backward problem (2.15)–(2.18), then the existence of an instantaneous sentinel insensitive to the missing data is equivalent to the null-controllability problem*

$$\int_\Omega q(0)\widehat{y}_0^i dx = 0, \ \ 1 \leq i \leq N \tag{2.19}$$

i.e

$$q(0) \in G^\perp$$

where $G^\perp$ is the orthogonal of G in $L^2(\Omega)$.

Proof. Multiplying (2.15) by y_{τ_i}, and integrating by parts over Ω, we find

$$\int_\Omega q(0)y_{\tau_i}(0)dx - \int_\Omega q(T)y_{\tau_i}(T)dx = 0, \quad 1 \leq i \leq N,$$

then

$$\int_\Omega q(0)\widehat{y}_0^i dx - \int_\Omega (h\chi_O + u\chi_\omega)\, y_{\tau_i}(T)dx = 0, \ \ 1 \leq i \leq N,$$

thanks to (2.14), we have:

$$\int_\Omega q(0)\widehat{y}_0^i dx = 0, \ \ 1 \leq i \leq N.$$

Thus, we obtain

$$q(0) \perp \widehat{y}_0^i, \ \ 1 \leq i \leq N$$

□

3. Optimal control problem

In this section we are interested to solve the problem (2.5), so we consider the optimization problem

$$\min_{u\in M} \|u\|^2_{L^2(\omega)}, \tag{3.1}$$

with $M = \{u \in L^2(\omega)$ such hat, we have (2.4) and $\int_\Omega q(0)\widehat{y}_0^i dx = 0,\ 1 \leq i \leq N$ where q is the solution of (2.15)–(2.18)$\}$.

Lemma 3.1. *The problem (3.1) admits in unique solution.*

Proof. The set M is a non-empty, closed and convex set. The mapping

$$v \to \|v\|^2_{L^2(\omega)}$$

is continuous, coercive and strictly convex, therefore, the problem (3.1) admits an unique solution denoted by $\widehat{v} \in M$ which satisfies

$$\left\|\widehat{v}\right\|_{L^2(\omega)} \leq \|v\|_{L^2(\omega)},\ \forall v \in M.$$

□

4. Characterization of optimal control

To characterize the optimal control, let us introduce q_0 by

$$-\frac{\partial q_0}{\partial t} + y_0\nabla q_0 + q_0\nabla y_0 - \Delta q_0 + \nabla\pi_1 \quad = \quad 0 \quad \text{in} \quad Q\ , \tag{4.1}$$

and

$$\operatorname{div} q_0 = 0 \ \text{ in } \ Q, \tag{4.2}$$

the initial conditions

$$q_0(T) = h\chi_O \ \text{ in } \ \Omega, \tag{4.3}$$

the boundary condition

$$q_0 = 0 \ \text{ on } \ \Sigma. \tag{4.4}$$

And define $z = z(u)$ as the solution of.

$$-\frac{\partial z}{\partial t} + y_0\nabla z + z\nabla y_0 - \Delta z + \nabla\pi_2 \quad = \quad -\nabla\Psi_2 \quad \text{in} \quad Q\ , \tag{4.5}$$

and

$$\operatorname{div} z = 0 \ \text{ in } \ Q, \tag{4.6}$$

the initial conditions

$$z(T) = u\chi_\omega \ \text{ in } \ \Omega, \tag{4.7}$$

the boundary condition

$$z = 0 \ \text{ on } \ \Sigma. \tag{4.8}$$

Then

$$q = q_0 + z = q_0 + z(u), \quad \pi = \pi_1 + \pi_2$$

we want to find u such that

$$\int_\Omega z(0;u)\widehat{y}_0^i dx = -\int_\Omega q_0(0)\widehat{y}_0^i dx, \quad 1 \le i \le N. \tag{4.9}$$

We define ρ as the solution of

$$-\frac{\partial \rho}{\partial t} + y_0 \nabla \rho + \rho \nabla y_0 - \Delta \rho + \nabla \sigma \;=\; 0 \quad \text{in} \quad Q\ , \tag{4.10}$$

and

$$\operatorname{div} \rho = 0 \ \text{ in } \ Q, \tag{4.11}$$

the initial conditions

$$\rho(0) = \sum_{i=1}^{N} \alpha_i \widehat{y}_0^i \ \text{ in } \ \Omega, \tag{4.12}$$

the boundary condition

$$\rho = 0 \ \text{ on } \ \Sigma, \tag{4.13}$$

where α_i is not determined. Let ξ is the solution of the system

$$-\frac{\partial \xi}{\partial t} + y_0 \nabla \xi + \xi \nabla y_0 - \Delta \xi + \nabla r \;=\; 0 \quad \text{in} \quad Q\ , \tag{4.14}$$

and

$$\operatorname{div} \xi = 0 \ \text{ in } \ Q, \tag{4.15}$$

the initial conditions

$$\xi(T) = \rho\,(T)\chi_\omega \ \text{ in } \ \Omega, \tag{4.16}$$

the boundary condition

$$\xi = 0 \ \text{ on } \ \Sigma. \tag{4.17}$$

And we want to determine $\alpha = \{\alpha_1, \alpha_2, ..., \alpha_N\} \in \mathbb{R}^N$ such that

$$\int_\Omega \xi(0)\widehat{y}_0^i dx = -\int_\Omega q_0(0)\widehat{y}_0^i dx, \quad 1 \le i \le N.$$

We introduce the linear operator Λ by

$$\Lambda\alpha = \left\{ \int_\Omega \xi(0)\widehat{y}_0^1 dx, \int_\Omega \xi(0)\widehat{y}_0^2 dx, ..., \int_\Omega \xi(0)\widehat{y}_0^N dx \right\}. \tag{4.18}$$

Then

$$\Lambda \in \mathcal{L}(\mathbb{R}^N, \mathbb{R}^N),$$

and

$$\Lambda\alpha = \left\{ \int_\Omega \alpha\widehat{y}_0^1 dx, \int_\Omega \alpha\widehat{y}_0^2 dx, ..., \int_\Omega \alpha\widehat{y}_0^N dx \right\}.$$

Theorem 4.1. *According to the unique continuation theorem of Mizohata [4], we have at least one sentinel given by*

$$\mathcal{S}(\lambda,\tau)=\int_{\Omega}(h\chi_{O}+\rho(T)\chi_{\omega})y(x,T;\lambda,\tau)dx,$$

where ρ *is the solution of (4.10)–(4.13), so*

$$u=\rho(T)\chi_{\omega}$$

is the solution of (2.4)–(2.5).

Proof. We multiply (4.10) by $\widetilde{\rho}$ corresponding to $\widetilde{\alpha}$, and we integrate by parts. We obtain

$$\langle\Lambda\alpha,\widetilde{\alpha}\rangle=\int_{\omega}\rho(T)\widetilde{\rho}(T)dx. \tag{4.19}$$

Therefore, Λ is a symmetric and positive matrix. Let us now set

$$\|\alpha\|_F=\left(\int_{\omega}\rho(T)^2dx\right)^{1/2}. \tag{4.20}$$

And let $y_i(x,t)$ the solution of

$$\begin{cases}\dfrac{\partial y_i}{\partial t}+(y_0\nabla)\,y_i-\Delta y_i+\nabla p_i &= 0 \quad \text{in} \quad Q,\\ \operatorname{div} y_i &= 0 \quad \text{in} \quad Q,\\ y_i(0) &= \widehat{y}_0^i \quad \text{in} \quad \Omega,\\ y_i &= 0 \quad \text{on} \quad \Sigma.\end{cases}$$

We define in this way a norm on the space F of the functions α, where the Hilbert space F is the completion of smooth functions for the norm (4.20) (indeed if $\|\alpha\|_F=0$ then $\rho=0$ on ω and according to the unique continuation theorem of Mizohata $\rho=0$ on Q so that $\alpha=0$). Then if F' denotes the dual of F, we have

$$\Lambda: F\longrightarrow F' \text{ is an isomorphism.}$$

Therefore, the equation

$$\Lambda\alpha=-\left\{\int_{\Omega}q(0)\widehat{y}_0^1dx,\int_{\Omega}q(0)\widehat{y}_0^2dx,...,\int_{\Omega}q(0)\widehat{y}_0^Ndx\right\} \tag{4.21}$$

admits a unique solution if

$$-\int_{\Omega}q(0)\widehat{y}_0^idx\in F',\quad 1\le i\le N. \tag{4.22}$$

We set

$$\beta=\left\{\int_{\Omega}q(0)\widehat{y}_0^1dx,\int_{\Omega}q(0)\widehat{y}_0^2dx,...,\int_{\Omega}q(0)\widehat{y}_0^Ndx\right\}$$

then the solution of (4.21) is given by:

$$\alpha=-\Lambda^{-1}\beta.$$

If we multiplying (4.1) by ρ, and integrating over Q we obtain

$$u=\rho(T)\chi_{\omega} \tag{4.23}$$

is the solution of (2.5), (2.19). □

Remark 4.2. The space F is identical to $\mathbb{R}^N$, and its norm is equivalent to the Euclidian norm. Then, Λ is a isomorphism from $\mathbb{R}^N$ to $\mathbb{R}^N$.

4.1. Identification of the pollution term

Let $\mathcal{S}_{obs}$ be the measured sentinel corresponding to the state of the system on the observatory O at the time T,

$$\mathcal{S}_{obs}(\lambda,\tau) = \int_\Omega (h\chi_O + u\chi_\omega) y_{obs}(x,T;\lambda,\tau)dx. \tag{4.24}$$

Theorem 4.3. *The pollution term is estimated as follows:*

$$\int_0^T \int_\Omega q(h)\lambda\widehat{\xi}dxdt = \mathcal{S}_{obs}(\lambda,\tau) - \mathcal{S}(0,0), \tag{4.25}$$

where $\mathcal{S}(0,0)$ is the sentinel corresponding to the state $y(x,T;0,0)$.

Proof. We have

$$\mathcal{S}_{obs}(\lambda,\tau) = \mathcal{S}(0,0) + \lambda \frac{\partial \mathcal{S}}{\partial \lambda}(\lambda,\tau_i)\bigg|_{\lambda=0,\tau_i=0} + \circ(\lambda,\tau_i), \text{ for } \lambda,\ \tau \text{ small.} \tag{4.26}$$

And

$$\frac{\partial \mathcal{S}}{\partial \lambda}(\lambda,\tau) = \int_\Omega (h\chi_O + u\chi_\omega) y_\lambda dx, \tag{4.27}$$

where y_λ defined by $y_\lambda = \dfrac{\partial y}{\partial \lambda}(0,0)$ (which depends only on $\widehat{\xi}$ and the other known data) is the unique solution of

$$\frac{\partial y_\lambda}{\partial t} + y_0 \nabla y_\lambda + y_\lambda \nabla y_0 - \Delta y_\lambda + \nabla p_\lambda = \widehat{\xi} \ \text{ in } \ Q, \tag{4.28}$$

and

$$\operatorname{div} y_\lambda = 0 \ \text{ in } \ Q, \tag{4.29}$$

the initial conditions

$$y_\lambda(0) = 0 \ \text{ in } \ \Omega, \tag{4.30}$$

the boundary condition

$$y_\lambda = 0 \ \text{ on } \ \Sigma. \tag{4.31}$$

And

$$\lambda \frac{\partial \mathcal{S}}{\partial \lambda}(\lambda,\tau)\bigg|_{\lambda=0,\tau=0} = \mathcal{S}_{obs}(\lambda,\tau) - \mathcal{S}(0,0). \tag{4.32}$$

We designate as $q(h)$ the unique solution of (2.15)–(2.18) depending on h.

Multiply (2.15) by y and integrate by part, we obtain

$$\int_\Omega (h\chi_O + u\chi_\omega) y_\lambda dx = \int_0^T \int_\Omega q(h)\lambda\widehat{\xi}dxdt, \tag{4.33}$$

therefore, the pollution term can be characterized

$$\int_0^T \int_\Omega q(h)\lambda\widehat{\xi}dxdt = \mathcal{S}_{obs}(\lambda,\tau) - \mathcal{S}(0,0).$$

□

Conflict of interest

The authors declare no conflict of interest.

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