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Research article

Dynamic analysis of a firm's green and process innovation under green-based price regulation in a monopoly market with network externality

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Abstract: This paper attempts to explore a firm's green and process innovation under green-based price regulation in a dynamic monopolistic market with network externality. In our work, the product price is linked to the green design quality, and the effects of green performance and network externality on demand structure are explicitly incorporated. Our findings mainly show that (i) in both settings of monopolist decision-making and social planning, there exists a unique saddle point steady-state equilibrium over the whole admissible parameter constellations; (ii) at steady-state equilibrium of the dynamical system, the monopolist's efforts for green and process innovation are lower under monopolist decision-making than that under planning; (iii) the monopolist undersupplies green performance compared with the socially optimal level, while the marginal production cost remains socially efficient; (iv) whether the network scale under monopolist decision-making is larger than that under social planning depends on factors such as the fixed and variable strength of network externalities, but it is independent of the pollutant dynamics.

Keywords: dynamic control; green innovation; network externality; price regulation
Mathematics Subject Classification: 49N90, 37N40

1. Introduction

In modern practice, firms' innovative activities can be carried out in three different directions: product innovation, process innovation, and more recently, emerging green innovation. Product innovation refers to R&D activities that attempt to improve product quality, whereas process innovation typically focuses on reducing marginal production costs. Green innovation, focusing on the improvement of products' green performance, has been a major priority with firms' burgeoning interest in sustainable operations management in recent years [1, 2].

While many researchers have investigated firms' investment portfolios in product and process innovation over the past two decades (see [3], and references therein), theoretical studies on the joint determination of green and process innovation remain scarce. As a pervasive growth strategy in response to rising public pressures for sustainability, firms increasingly prioritize enhancing the green design quality of their products rather than pursuing incremental improvements in conventional product attributes. For instance, Tesla emphasizes battery recycling, energy efficiency, and sustainable materials in electric vehicles; Philips and Midea optimize energy-saving and environmentally friendly designs in home appliances; and Apple and Samsung invest in recyclable materials and low-power modular components for smartphones. These real-world cases demonstrate that improving green design quality, such as sustainable materials, higher energy efficiency, and eco-friendly modularity, often generates more substantial and long-term impacts than traditional product upgrades. This underscores the strategic significance of shifting academic attention from conventional product-process innovation portfolios toward the investigation of green-process innovation decisions.

It is worth noting that the literature on green and process innovation in markets characterized by network externality is particularly limited. Network externality, a demand-side scale effect, refers to the rising or falling utility that a user obtains from using a product as the total number of users grows [4]. According to [5], positive network externality arises when the advantages, or, more precisely, marginal utility, increase with the size of the user base. In sectors like telephony, data communication services, Telex, and over-the-phone facsimile equipment, network externality exists [6]. Owing to the existence of network externality, joining a network benefits not only the individual user but also society.

Since products with network externality can generate overwhelming competitive advantages when firms successfully expand their user base, issues regarding connectivity, compatibility, interoperability, and quality coordination management become central in network economics. Despite ongoing technological advancements resulting in more products exhibiting network externality, only a few studies [7–10] have examined multidimensional innovation portfolios under such conditions. The only available studies on this topic mainly discuss product and process innovation, virtually ignoring green innovation. This ignorance is largely due to the analytical intractability introduced by network externality, namely, that consumers' utility depends not only on intrinsic product attributes but also on network scale. High-tech sectors such as renewable energy equipment and telecommunications, where green product quality continuously improves while costs decrease, often display strong network externality [11]. Consequently, examining firms' green and process innovation strategies in the presence of network externalities has both theoretical and practical relevance.

Moreover, in many of these industries, the price is commonly regulated [12, 13]. Previous studies have demonstrated that introducing innovation activities can significantly alter traditional results concerning price regulation [14]. Therefore, it becomes essential to investigate the innovative activities in network industries under price regulation regime.

As such, this paper develops a theoretical framework to analyse a monopolist's joint decision-making about green and process innovation under network externality and green-performance based price regulation. We focus specifically on these two innovation types for the following reasons: (i) Most crucially, incorporating product innovation makes it challenging to derive explicit solutions; (ii) as discussed above, conventional quality-enhancing product innovation has been extensively studied, while contemporary markets increasingly emphasize green innovation; (iii) restricting attention to green and process innovation enables us to uncover novel conclusions that cannot be obtained from traditional product-process innovation models.

The novel features of this study are fourfold: First, we assume that product price is linked to green performance. Although consumer demand depends on network size, product price remains the primary determinant of purchasing decisions, particularly in markets where the network externality is obvious. Second, we focus on a search good [15], so green performance is assumed to be observable before purchase. To study optimal strategies to improve green performance, we adopt a dynamic model in which green performance is treated as a state variable from both the monopolist's and the social planner's perspectives. Third, the demand function depends on price, green performance, and network size. Beyond intrinsic product value, consumers also derive network value determined by the installed base. Naturally, the following questions come up: How does network externality impact the firm's incentives to innovate its processes and go green? Will complementarity between green and process innovation persist under network externality, as suggested by [16] for quality-based innovation? To answer these questions, following the lead of [16], this paper assumes that the network value is influenced by green performance. Finally, following [17], the dynamics of network size are driven by consumer demand.

The structure of the work is as follows. In Section 2, relevant studies are reviewed. Section 3 develops the basic model. The efforts made under monopoly and government intervention, respectively, are analysed in Sections 4 and 5. The generic solutions are provided in Section 6 using numerical examples. Section 7 discusses the managerial implications of the main outcomes. Section 8 is the summary of this paper. All mathematical proofs can be found in Appendix.

2. Literature review

Our work is mainly related to two streams of literature: Dynamic innovation portfolios, monopolistic decision-making under network externality.

Since the seminal work of [18], a substantial body of research has investigated dynamic innovation portfolios in various settings. Earlier studies mainly focused on identifying the dynamic patterns of product and process innovation, both analytically and numerically, and compared them with established hypotheses in the literature (see [19, 20], and references therein). The work in [21] examined a multiproduct monopolist's optimal investment in product-process innovation, while [22] studied portfolio choices from a product-life-cycle perspective under duopolistic competition. In [23], the author clarified the interplay among dynamic pricing, product and process innovation. Hinloopen et al. [24] conducted a global analysis of the dynamic evolution of innovative activities. Lambertini and Orsini [25] derived a nonlinear innovation-portfolio system and proved the uniqueness of the steady-state equilibrium. Pan and Li [26] investigated how learning-by-doing and knowledge accumulation impact optimal innovation portfolios; similarly, Li and Ni [27] incorporated learning-by-doing into a dynamic product-process innovation framework. Lambertini et al. [28] demonstrated the instability properties of two-dimensional innovative systems. Li [29] further explored product-process innovation for network goods under reference quality effects. Recent studies addressed the pivotal role of consumer behavior in determining dynamic innovation portfolios [30–33]. As mentioned in Section 1, the extant literature on dynamic innovation portfolios mainly focused on product and process innovation rather than on innovative activities aimed at improving green design quality. Our study extends this stream of research by incorporating green innovation into the dynamic portfolio, which is consistent with the sustainability-oriented market practices and fills the theoretical gap in the previous studies.

The literature about the impact of network externality on monopolist decision-making has been a major focus in academics over the past few decades. Empirically, previous studies have shown that network externality can significantly influence the monopolistic operational strategies through its impact on the consumer base (e.g., [34, 35]). Research in this area has traditionally emphasized pricing. For a comprehensive review of earlier work about this subject, see [36]. More recent papers have investigated optimal pricing in markets with local network externality [37–39] and explored how network topology influences pricing strategies [40]. In [41], the authors determined the optimal price for a multiproduct monopolist under a multinomial logit framework. Beyond pricing, several researchers focused on other operational decisions in network industries. Navarro [42] analysed the joint pricing and product innovation for network goods. Hu and Milner [43] studied the impact of network externality on optimal marketing policies for horizontally differentiated products. The marginal contribution of our study to the aforementioned literature lies in integrating dynamic green-process innovation decisions, network externality, and price regulation into a unified monopolistic framework, a setting that, to the best of our knowledge, has not been explored in prior theoretical research.

3. Modelling

3.1. Mathematical formulations

In this subsection, we formally define the basic model mathematically, specifying the symbols used throughout the paper. Consider a market supplied by a monopolistic firm producing a single product and making decisions regarding green innovation (improving green performance of the product) and process innovation (reducing marginal production cost) under green-based price regulation regime over continuous time $t \in [0, +\infty)$. In line with the existing studies (e.g., [44, 45]), the dynamics associated with the green innovation can be described by the equation below:

$$\dot{q}(t) = k(t) - \delta q(t), \tag{3.1}$$

where the control variable $k(t)$ measures the effort for green innovation to improve green performance $q(t)$ over continuous time $t \in [0, +\infty)$; the parameter $\delta \in [0, 1]$ denotes the decay rate of green performance. Accordingly, the cost of employing effort $k(t)$ for green innovation is given by $\frac{\beta}{2}k^2(t)$ (e.g., [45–47]), where β inversely measures the cost efficiency of green innovation.

Notably, in this study, we define green innovation as the demand-oriented enhancement of green design quality. The effort improves the consumers' maximum willingness to pay for the product, capturing the widely observed phenomenon that consumers derive additional utility from purchasing more environmentally friendly products. This setting is consistent with extant empirical evidence showing that consumers value green product attributes such as recyclable materials, low-carbon components, or improved energy efficiency (e.g., [48–50]) and theoretical research in sustainable operations and green product differentiation (e.g., [51–54]). Practical examples include Apple's use of recycled aluminium enclosures and Unilever's fully recyclable packaging, which increase products' perceived environmental value without altering production emissions. Consequently, our model mainly focuses on such market-side effects of greener product design, while process-based emission-reduction activities lie outside the scope of the analysis.

Additionally, as common in previous studies [25, 27, 28], the dynamic equation of the process innovation (lowering marginal production cost) is given by the following form:

$$\dot{c}(t) = -h(t) + \sigma c(t), \tag{3.2}$$

where the control variable $h(t)$ stands for the effort for process innovation to reduce the marginal production cost $c(t)$; the parameter $\sigma \in [0, 1]$ represents the depreciation rate due to the aging of technology. Accordingly, the cost of employing effort $h(t)$ is given by $\frac{\alpha}{2}h^2(t)$, (e.g., [27–29]), where αinversely measures the cost efficiency of process innovation. The interpretation of process innovation as a purely cost reduction activity conforms to common practice in the industrial organization literature [55, 56]. In practice, process innovation often takes the form of adopting more efficient machinery, automating assembly processes, or optimizing production workflows, each of which directly lowers marginal production costs.

Note that, in this paper, similar to prior research [25, 27, 28], the kinematic equations associated with green performance $q(t)$ and marginal production cost $c(t)$ are linear while including exogenous obsolescence (or decay) rates of green performance and productive efficiency. In contrast, the kinematic equations of green performance and marginal production cost are provided, such as in [25], in the form of nonlinearity, i.e., $\dot{q}(t) = [k(t) - \delta]q(t)$ and $\dot{c}(t) = [-h(t) + \sigma]c(t)$, respectively.

Here, we presume that green innovation and process innovation are independent of each other. The rationale behind this assumption is as follows: First, it is consistent with empirical evidence (e.g., [57,58]) as well as with standard practice in the related literature (e.g., [21–23]), where different forms of innovative activities are modelled as distinct and separable decisions. Second, this assumption preserves analytical tractability. Introducing complementarities or spillover effects between the two innovative activities would substantially complicate the dynamic structure of the model, while offering little value to the core objective of the paper, i.e., to examine how firms' incentives for each type of innovation respond to market conditions, rather than to study the interaction between green and process innovation. Third, allowing for cross-effects would not qualitatively change the main insights. Such spillovers, analogous to internal R&D synergies across departments, would simply scale down the equilibrium innovation levels without altering the comparative results across different decision modes. For these reasons, treating the two innovation activities as independent is both empirically plausible and theoretically appropriate for the purpose of this study.

In what follows, we turn to the issue of the monopolist's green-based price regulation. This kind of green-based price regulation is in line with examples of price regulation of monopolistic energy, transportation networks, and telecommunications industries. The regulatory content and method changed from cost-based control, such as the regulation of the various forms of rate-of-return, to the regulation of price, often known in various variations as price caps, beginning in the United Kingdom with the reforms of Telecom. Beesley and Littlechild [59] has effectively outlined the major benefits of the regulation of price. The capacity of the regulation of price to increase productive efficiency has a strong track record (e.g., [60]). Recent years have seen a lot of focus on the market where prices are controlled, presumably in relation to the quality of a product, therefore, it is no longer the true choice variable for enterprises, and competition is essentially based on the quality of a product. Note that price competition plays a limited role and that providers have an incentive to invest in the quality of a product to attract customers in a market with regulated prices (e.g., [61–66], just to mention a few). The literature has dealt with the question of price regulation and innovation, such as [67, 68]. Prieger [68] presented a mechanism for regulated product innovation, and further examined the effects of switching from rate-of-return regulation to price regulation in relation to innovation in US telecommunications. Referring to the work of [69] (footnote 5, P. 420), we presumptively expect a green-based pricing monopolist to offer less environmentally friendly performance than is socially desirable. Through the adoption of minimal green performance criteria, this significant result serves as evidence for the expansion of regulatory controls to green performance (e.g., loss-of-load probability requirements in the power industry and call completion rate in telecommunication). Through the adoption of minimal green performance criteria, this significant result serves as evidence for the expansion of regulatory controls to green performance (e.g., Call completion rate in telecommunications and requirements for loss-of-load probabilities in the power industry). In this work, based on the works of [66, 69], we propose a specific pricing formula where the regulated price is linked to the green performance $q(t)$. Specifically, we consider a linear rule, that is, at any time $t \in [0, +\infty)$, the regulated price $p(t)$ is given by

$$p(t) = a + bq(t), \tag{3.3}$$

where $a > 0$ stands for the fixed price, while $b > 0$ denotes the marginal impact of green performance $q(t)$ on price $p(t)$. It is worth noting that, in the work of [61, 63, 64], the regulated price is exogenous and constant.

Next, we investigate consumers' demand function with network externality. We first analyse the network value of green performance $q(t)$ at continuous time $t \in [0, +\infty)$. According to the works of [4, 16], for a network size $Q(t)$, the network value is given by $[\mu + \vartheta q(t)]Q(t)$, where the parameter $\mu > 0$ indicates the fixed strength of network externality, while the parameter $\vartheta > 0$ stands for the unit variable strength of network externality. Besides, we assume that the consumers are characterized by their intensity of preference $\theta(t)$ for a product of green performance $q(t)$ (where $\theta(t)$ also reflects the consumer's willingness to pay for a product of green performance $q(t)$), and the consumers differ in $\theta(t)$ which is uniformly distributed on the interval $[0, M]$, where M is the potential market size and it is assumed as constant. Each consumer is assumed to consume one unit of the product produced by the monopolist. Thus, at continuous time $t \in [0, +\infty)$, when a type-$\theta(t)$ consumer consumes one unit of the product with green performance $q(t)$ at price $p(t)$, she will receive net utility $\theta(t)q(t) + [\mu + \vartheta q(t)]Q - p(t) \geq 0$. Using the following indifference condition for the consumer with $\underline{\theta}(t)$, namely, $\underline{\theta}(t)q(t) + [\mu + \vartheta q(t)]Q(t) - p(t) = 0$, one can obtain $\underline{\theta}(t) = \frac{p(t)-[\mu+\vartheta q(t)]Q(t)}{q(t)}$, then there must be consumers with $\theta(t) \in [0, \underline{\theta}(t))$ who will not buy any product, while the consumers with $\theta(t) \in (\underline{\theta}(t), M]$ will buy the product. Hence, the consumers' demand function $D(t)$ is given by $D(t) = M - \frac{p(t)-[\mu+\vartheta q(t)]Q(t)}{q(t)}$. Substituting the regulated price (3.3) into above demand function, then we have

$$D(t) = M - b + \vartheta Q(t) - \frac{a - \mu Q(t)}{q(t)}. \tag{3.4}$$

Now, we consider the dynamic equation of network size $Q(t)$. If consumers actually expect the network size to change, their reaction may be different [5]. In particular, if consumers have responsive expectations and anticipate how network sizes will change, then some consumers will enter the network system while others will exit the network system. In this paper, we assume that the numbers of consumers entering the system is given by the following function $R(t) = \xi D(t)$, which means that there is a linear proportional relationship between the numbers of consumers entering the system and their demand size, where the parameter $\xi \geq 0$. Then, similar to the work of [17], the dynamic equation of network size $Q(t)$ can be written as

$$\dot{Q}(t) = \xi D(t) - \eta Q(t), \tag{3.5}$$

where the coefficient $\eta \geq 0$ indicates the exit rate of consumers exiting the network system. Note that, under steady-state, we have $Q(t) = \frac{\xi}{\eta} D(t)$, which means that there is a linear proportional relationship between the network size and demand size. A similar situation occurred in the work of [5], Factually, the work of [5] is worth mentioning due to its status as a recent and fundamental work in which the authors discuss in detail the alternative assumption as to when the network size should be equal or proportional to the demand size in a static setting. By the way, in the works of [10, 70], the authors consider $Q(t)$ as exogenously given.

Substituting demand function (3.4) into dynamic equation of network size $Q(t)$ and rearranging, we have

$$\dot{Q}(t) = \xi(M - b) + (\xi\vartheta - \eta)Q(t) - \frac{\xi[a - \mu Q(t)]}{q(t)}. \tag{3.6}$$

Next, we consider the dynamic equation of pollution stock. The consumption and/or production of final product involves a volume of emissions of pollution equal to $\varepsilon D(t)$, where the coefficient $\varepsilon > 0$ is an exogenous parameter and denotes the emission-to-output ratio. Besides, in the light of [46, 47], we assume that the pollution stock $P(t)$ evolves in a transition law as the following dynamic equation $\dot{P}(t) = \varepsilon D(t) - \kappa P(t)$, while the parameter $\kappa > 0$ stands for the natural decay rate of pollutant stock. Using the demand function (3.4), then the dynamic equation of pollution stock $P(t)$ is given by

$$\dot{P}(t) = \varepsilon[M - b + \vartheta Q(t) - \frac{a - \mu Q(t)}{q(t)}] - \kappa P(t). \tag{3.7}$$

Furthermore, we introduce the damage function caused by pollution emissions. As in [46, 47], interalia, we assume that the pollution damage function $E(t)$ is given by the linear form below:

$$E(t) = \varphi P(t), \tag{3.8}$$

where $\varphi > 0$ represents the damage parameter of pollution stock $P(t)$.

In what follows, we introduce the total cost function. First, it is assumed that the fulfillment of all deterministic customer requests will be ensured and the monopolist does not maintain an inventory. This assumption holds in the context of make-to-order items or situations with short manufacturing lead time. typical examples include the creation of digital goods or software development. Then, given the demand function of consumers, the total cost function $C(t)$ is given by

$$C(t) = [M - b + \vartheta Q(t) - \frac{a - \mu Q(t)}{q(t)}]c(t) + \frac{\beta}{2}k^2(t) + \frac{\alpha}{2}h^2(t). \tag{3.9}$$

To create an incentive for the monopolist to reduce the emission of pollution, we further assume that the social planner controls pollution emissions by levying emission taxes. Let τ denote the pollution tax rate, which is assumed to be constant and exogenously given. Then, the pollution tax levied by the social planner is $\tau\varepsilon[M - b + \vartheta Q(t) - \frac{a-\mu Q(t)}{q(t)}]$. Using regulated price (3.4), the monopolist's net profit function $\pi(t)$ writes

$$\pi(t) = [a - \tau\varepsilon + bq(t) - c(t)][M - b + \vartheta Q(t) - \frac{a - \mu Q(t)}{q(t)}] - \frac{\beta}{2}k^2(t) - \frac{\alpha}{2}h^2(t). \tag{3.10}$$

3.2. Model properties and assumptions

On the basis of the mathematical formulations in subsection 3.1, we now examine the pivotal properties and underlying assumptions of the model. This analysis provides structural insights into the mechanisms driving monopolist's operational decisions, laying the foundation for the subsequent outcomes.

From demand function (3.4), one has that $\frac{\partial D(t)}{\partial q} = \frac{a-\mu Q(t)}{q^2(t)} \geq (<)0$ if and only if $a - \mu Q(t) \geq (<)0$. One can easily obtain the following Property 1 related to the influence of effort for green performance improvement on demand (Proof can be found in Appendix A.1):

Property 1. *For acceptable factors, we have* $\frac{\partial D(t)}{\partial k(t)} \geq (<)0$ *iff* $a - \mu Q(t) \geq (<)0$.

Property 1 means that the demand function $D(t)$ increases with the effort for green performance improvement if and only if $a-\mu Q(t) \geq 0$ over continuous time $t \in [0, +\infty)$. That is to say, when the fixed price is greater than $\mu Q(t)$, as the effort $k(t)$ for green innovation increases, more and more consumers will be attracted to purchase products; when the fixed price is less than $\mu Q(t)$, as the effort $k(t)$ for green innovation increases, more and more consumers will not be attracted to purchase products. Therefore, to expand consumers' demand, in the mechanism design of price regulation, the fixed price a must be greater than $\mu Q(t)$ over continuous time $t \in [0, +\infty)$.

This property about demand function highlights the dual effect of green performance improvement on the demand in regulated industries. On the one hand, a higher level of green performance enlarges the intrinsic market potential by increasing consumers' valuation of environmentally friendly attributes, thereby expanding demand. On the other hand, since the regulated price is explicitly linked to green performance, an improvement in $q(t)$ simultaneously raises the green-performance-based regulated price, dampening demand in the sense of the extensive margin. This trade-off is consistent with several real-world observations: For instance, the home appliances with higher energy efficiency levels often induce price escalation, resulting in lower sales volume of appliances with the highest efficiency ratings [71]; electric vehicles with enhanced environment-friendly attributes may face higher regulated prices, which can lower demand compared with conventional vehicles [72]. It is noteworthy that price regulation de facto makes the implicit mechanism already discussed in the quality–price relationship studies explicit, that is, higher product quality can lead to higher prices and consequently weaker demand for marginal consumers [23, 73, 74]. As such, the nonmonotonic effect captured in Property 1 reflects an economically plausible interaction between regulated pricing and vertically differentiated green attributes.

From demand function (3.4), one can yield that $\frac{\partial D(t)}{\partial Q} = \frac{\mu+\vartheta q(t)}{q(t)} > 0$, which means that the demand function $D(t)$ increases as the network size $Q(t)$ increases. Further, since $D(t) \geq 0$, from demand function (3.4), one can easily obtain $q(t) \geq \frac{a-\mu Q(t)}{M-b+\vartheta Q(t)}$, which means that in a monopoly market with network externality, to assure the economic rationality of the system, green performance $q(t)$ must be greater than or equal to $\frac{a-\mu Q(t)}{M-b+\vartheta Q(t)}$. In other words, in the mechanism design of price regulation, the condition to be satisfied is the fixed price a and the unit variable price b must satisfy the condition $q(t) \geq \frac{a-\mu Q(t)}{M-b+\vartheta Q(t)}$.

Further, we investigate the impact of the effort $k(t)$ on the rate of change in network size $Q(t)$. One can acquire the following property regarding the impact of effort for green performance improvement on the rate of change in network size:

Property 2. *For acceptable factors, we always have* $\frac{\partial \dot{Q}(t)}{\partial k(t)} \geq 0$.

Property 2 implies that, for acceptable factors, at continuous time $t \in [0, +\infty)$, the rate of change in network size $Q(t)$ increases with the effort $k(t)$. Property 2 de facto is consistent with the real-life situation, that is, the increase in the effort $k(t)$ leads to an increase in green performance $q(t)$, thereby attracting more consumers to enter the network.

Accordingly, simple calculations yield Property 3 below, which depicts the influence of effort in green performance improvement on the rate of change in pollution stock (Proof can be found in Appendix A.3):

Property 3. *For acceptable factors, we have* $\frac{\partial \dot{P}(t)}{\partial k(t)} \geq (<)0$ *iff* $a - \mu Q(t) \geq (<)0$.

Property 3 means that, for acceptable factors, the rate of change in pollution stock $P(t)$ increases as the effort for green innovation increases if and only if $a - \mu Q(t) \geq 0$ over continuous time $t \in [0, +\infty)$. In other words, when the fixed price is greater than $\mu Q(t)$, as the effort for green innovation increases, the rate of change in pollution stock $P(t)$ will increase; while when the fixed price is less than $\mu Q(t)$, as the effort for green innovation increases, the rate of change in pollution stock $P(t)$ will decrease. Therefore, in order to reduce the pollution stock, in the mechanism design of price regulation, the fixed price a must be less than $\mu Q(t)$ over continuous time $t \in [0, +\infty)$.

Note that, from Property 1, we have that in order to expand consumers' demand, in the mechanism design of price regulation, the fixed price a must be greater than $\mu Q(t)$, while from Property 3 we have that in order to reduce the pollution stock $P(t)$, in the mechanism design of price regulation, the fixed price a must be less than $\mu Q(t)$. It can be seen that, in a monopoly market with network externality, there is still an eternal contradiction between expanding consumers' demand $D(t)$ and reducing pollution stock $P(t)$.

It is noteworthy that the conditions identified in Property 1 and 3 do not aim to establish a policy paradox, nor do they imply that the intrinsic contradiction is a universal phenomenon in markets with green innovation. Rather, our intention is to claim that such a trade-off arises endogenously in regulatory environments where green attributes raise product price through regulatory rules. Figure 1 illustrates how the key concepts are interconnected under the current model structure. The analysis highlights a mechanism that is inherent to the regulated industries considered in this study, namely, green performance influences environmental outcomes solely through its impact on demand while per-unit emissions are exogenous. Under such conceptual contexts, green performance improvement reduces pollution only if they dampen demand. As Property 1 shows, this demand-dampening effect arises precisely when the regulated price is more responsive to the network-induced valuation ($\mu Q(t)$) than to the baseline consumer valuation (a), formally, when $a < \mu Q(t)$. In such cases, green performance improvement induces the increment in price driven by network externality, which outweighs consumers' intrinsic willingness to pay and ultimately dampen demand. Our model formulations strengthen the contribution to extant studies by identifying how certain regulatory environments may inadvertently create a situation in which environmental goals and consumer adoption incentives diverge. In this setting, the paradox is therefore a direct implication of focusing on a demand-mediated channel of environmental impact, which is characteristic of several regulated markets where product-specific emission intensity is fixed and policy relies on price–quality linkages [75, 76]. The results should thus be interpreted as conditional insights within this regulatory context, rather than as statements about general environmental phenomenon.

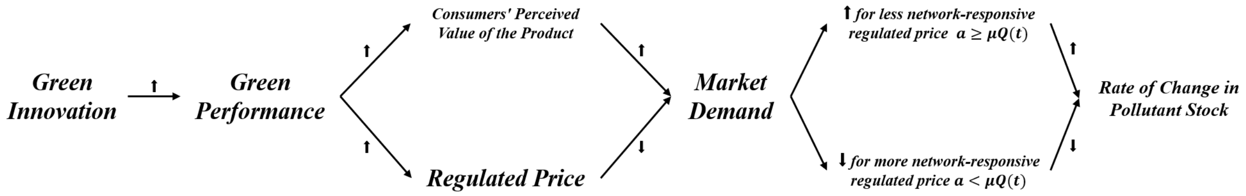

Figure 1. The structural relationships among the key concepts of the model.

Here, it is essential to clarify that the properties identified in this section are structural implications of the model itself rather than the result of any particular proposition. Highlighting this prevents logical misinterpretation and emphasizes that although these outcomes stem from the model's underlying setup, they remain analytically meaningful for understanding the interplay between regulatory constraints and green innovation.

From the profit function (3.10) we find, under the pollution tax levied by the social planner, the regulated price $p(t)$ can be equally seen as the following form: $p(t) = a - \tau\varepsilon + bq(t)$, where $a - \tau\varepsilon$ is equivalent to a reservation price. To reflect the rational behavior of the monopolist firm, we assume that $a - \tau\varepsilon + bq(t) - c(t) \geq 0$. To facilitate the use of this result in next analysis, we summarize this assumption as follows:

Assumption 1. *For admissible parameters, $a - \tau\varepsilon + bq(t) - c(t) \geq 0$ must be met in order to ensure the rational behavior of the monopolist firm over continuous time $t \in [0, +\infty)$.*

Besides, to ensure the presence of the steady-state equilibrium of the problem, the following assumption is introduced:

Assumption 2. *Given acceptable parameters, we assume that: (i) $\rho - \sigma > 0$; (ii) $(\rho + \eta - \xi\vartheta)q(t) - \xi\mu > 0$.*

It is worth mentioning that the above Assumption 2-(i) and (ii) seem to impose significant limitations, but these are justified because otherwise, the marginal production cost and network size may increase (which means that the dynamic control system may not achieve stable state equilibrium). Moreover, even though some outcomes are not predicated on these assumed criteria, the existence of an equilibrium solution of the dynamic system requires that when the monopolist actively participates in green and process innovation, the objective function in the dynamic control system must be bounded. Furthermore, from Assumption 2-(ii), we can obtain that $\rho + \eta - \xi\vartheta \neq 0$.

In the subsequent sections, within the framework of monopolist decision-making and social planning, our analysis focuses on three aspects: (i) Analyzing the steady-state equilibrium of the dynamical control system; (ii) examining the features of steady-state equilibrium, and (iii) discussing the numerical results for the integrated model.

4. Model analysis under monopoly

4.1. Optimal conditions and general results

Under monopoly, the monopolist's goal is to maximize discounted profit flow $\prod$ with respect to control variables $k(t)$ and $h(t)$ under the dynamic constraints (3.1), (3.2), (3.6), and (3.7) over the infinite planning horizon $t \in [0, +\infty)$, that is,

$$\Pi = \max_{k(t),h(t)} \int_0^{+\infty} e^{-\rho t}\{[a - \tau\varepsilon + bq(t) - c(t)][M - b + \vartheta Q(t) - \tfrac{a-\mu Q(t)}{q(t)}] - \tfrac{\beta}{2}k^2(t) - \tfrac{\alpha}{2}h^2(t)\}dt$$
$$s.t. \begin{cases} \dot{q}(t) = k(t) - \delta q(t), \\ \dot{c}(t) = -h(t) + \sigma c(t), \\ \dot{Q}(t) = \xi(M - b) + (\xi\vartheta - \eta)Q(t) - \tfrac{\xi[a-\mu Q(t)]}{q(t)}, \\ \dot{P}(t) = \varepsilon[M - b + \vartheta Q(t) - \tfrac{a-\mu Q(t)}{q(t)}] - \kappa P(t), \\ q(0) = q_0, c(0) = c_0, Q(0) = Q_0, P(0) = P_0, \end{cases} \tag{4.1}$$

where $\rho > 0$ represents the discount rate.

Following optimal control theory [77], the current-value Hamiltonian function H of dynamic optimization problem (4.1) is constructed as in (4.2):

$$\begin{aligned} H = {} & [a - \tau\varepsilon + bq(t) - c(t)][M - b + \vartheta Q(t) - \tfrac{a-\mu Q(t)}{q(t)}] - \tfrac{\beta}{2}k^2(t) - \tfrac{\alpha}{2}h^2(t) + \lambda_1(t)[k(t) - \delta q(t)] \\ & + \lambda_2(t)[-h(t) + \sigma c(t] + \lambda_3(t)\{\xi(M - b) + (\xi\vartheta - \eta)Q(t) - \tfrac{\xi[a-\mu Q(t)]}{q(t)}\} \\ & + \lambda_4(t)\{\varepsilon[M - b + \vartheta Q(t) - \tfrac{a-\mu Q(t)}{q(t)}] - \kappa P(t)\}, \end{aligned} \tag{4.2}$$

where $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, and $\lambda_4(t)$ are shadow prices (co-state variables) associated with the corresponding state variables $q(t)$, $c(t)$, $Q(t)$, and $P(t)$. The economic interpretations of above shadow prices are the marginal benefits that the monopolist obtains from the improvement of green performance, the reduction of marginal production cost, the change of network size, and the change of pollution stock, respectively.

From above Hamiltonian function (4.2), we therefore obtain the first-order conditions with respect to control variables $k(t)$ and $h(t)$, as well as dynamics with respect to co-state variables $\lambda_i(t)$ ($i \in \{1, 2, 3, 4\}$) below, that is,

$$\frac{\partial H}{\partial k(t)} = -\beta k(t) + \lambda_1(t) = 0, \tag{4.3}$$
$$\frac{\partial H}{\partial h(t)} = -\alpha h(t) - \lambda_2(t) = 0, \tag{4.4}$$
$$\begin{aligned} \dot{\lambda}_1(t) &= \rho\lambda_1(t) - \frac{\partial H}{\partial q(t)} \\ &= (\rho + \delta)\lambda_1(t) - b[M - b + \vartheta Q(t) - \frac{a - \mu Q(t)}{q(t)}] - \\ &\quad [a - \tau\varepsilon + bq(t) - c(t) + \xi\lambda_3(t) + \varepsilon\lambda_4(t)]\frac{[a - \mu Q(t)]}{q^2(t)}, \end{aligned} \tag{4.5}$$
$$\dot{\lambda}_2(t) = \rho\lambda_2(t) - \frac{\partial H}{\partial c(t)} \tag{4.6}$$

$$= (\rho-\sigma)\lambda_2(t) + [M - b + \vartheta Q(t) - \frac{a-\mu Q(t)}{q(t)}],$$

$$\dot{\lambda}_3(t) = \rho\lambda_3(t) - \frac{\partial H}{\partial Q(t)} \tag{4.7}$$
$$= [\frac{(\rho+\eta-\xi\vartheta)q(t)-\xi\mu}{q(t)}]\lambda_3(t) - [\vartheta + \frac{\mu}{q(t)}]\times$$
$$[a - \tau\varepsilon + bq(t) - c(t) + \varepsilon\lambda_4(t)],$$

$$\dot{\lambda}_4(t) = \rho\lambda_4(t) - \frac{\partial H}{\partial P(t)} \tag{4.8}$$
$$= (\rho+\kappa)\lambda_4(t).$$

Note that, as time approaches infinity, the discount rate causes the marginal benefit to approach zero, thus obtaining transversality conditions $\lim_{t\to\infty}\lambda_1(t)q(t)e^{-\rho t} = 0$, $\lim_{t\to\infty}\lambda_2(t)c(t)e^{-\rho t} = 0$, $\lim_{t\to\infty}\lambda_3(t)Q(t)e^{-\rho t} = 0$, and $\lim_{t\to\infty}\lambda_4(t)P(t)e^{-\rho t} = 0$ [77], respectively. In fact, above transversality conditions determine the terminal values of the state and/or co-state variables.

In what follows, we provide the kinematic equations describing the evolution of efforts for green and process innovation over time $t \in [0, +\infty)$. For this purpose, we first consider the solution of the dynamic co-state equation (4.7). Due to $P(t) > 0$, using transversal condition $\lim_{t\to\infty}\lambda_4(t)P(t)e^{-\rho t} = 0$, one can obtain $\lambda_4(t) = 0$. Then, solving the equations (4.3) and (4.4) with respect to control variables $k(t)$ and $h(t)$, which can be differentiated totally with respect to time argument t, using $\lambda_4(t) = 0$ and equations (4.5)–(4.7), one can yield the following dynamic equations:

$$\dot{k}(t) = (\rho+\delta)k(t) - \frac{b}{\beta}[M-b+\vartheta Q(t) - \frac{a-\mu Q(t)}{q(t)}] - [a-\tau\varepsilon+bq(t)-c(t)+\xi\lambda_3(t)+\varepsilon\lambda_4(t)]\frac{[a-\mu Q(t)]}{\beta q^2(t)}, \tag{4.9}$$

$$\dot{h}(t) = (\rho-\sigma)h(t) - \frac{1}{\alpha}[M - b + \vartheta Q(t) - \frac{a-\mu Q(t)}{q(t)}]. \tag{4.10}$$

We find from the dynamics (4.9) that the co-state variable $\lambda_3(t)$ is present in this equation, then we impose stationary condition for the kinematic co-state equation (4.7), i.e., $\dot{\lambda}_3(t) = 0$, to determine the dynamic equation $\dot{k}(t)$ as a function of the marginal production cost $c(t)$ and green performance $q(t)$ of the control system in the present study. Since we assume that $(\rho+\eta-\xi\vartheta)q(t) - \xi\mu > 0$, then solving kinematic co-state equation (4.7) under $\dot{\lambda}_3(t) = 0$, and using $\lambda_4(t) = 0$, gives the steady-state co-state variable $\lambda_3(q, c)$ as a function of green performance $q(t)$ and marginal production cost $c(t)$ below:

$$\lambda_3(q, c) = \frac{[a-\tau\varepsilon+bq(t)-c(t)][\mu+\vartheta q(t)]}{(\rho+\eta-\xi\vartheta)q(t)-\xi\mu}. \tag{4.11}$$

From Assumption 1, we have $a - \tau\varepsilon + bq(t) - c(t) \geq 0$. From Assumption 2-(ii), we have $(\rho+\eta-\xi\vartheta)q(t) - \xi\mu > 0$. Then, we can obtain that the co-state variable $\lambda_3(q, c) \geq 0$. Note that, positive shadow price indicates scarcity. The greater the scarcity is, the higher the shadow price will be. Negative shadow price indicates surplus of the products or factors, and the increase in its supply will result in a loss of economic benefits. Further, from the equation (4.11) we have (i) $\frac{\partial\lambda_3(q,c)}{\partial q(t)} = \frac{b[\mu+\vartheta q(t)][(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]-\mu(\rho+\eta)[a-\tau\varepsilon+bq(t)-c(t)]}{[(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]^2}$, which indicates that the shadow price $\lambda_3(q, c)$ increases as the green performance $q(t)$ increases if and only if $\frac{[\mu+\vartheta q(t)][(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}{a-\tau\varepsilon+bq(t)-c(t)} \geq \frac{\mu(\rho+\eta)}{b}$; (ii)

$\frac{\partial \lambda_3(q,c)}{\partial c(t)} = -\frac{\mu+\vartheta q(t)}{(\rho+\eta-\xi\vartheta)q(t)-\xi\mu} < 0$, which means the shadow price $\lambda_3(q,c)$ decreases as the marginal production cost $c(t)$ decreases.

Now, substituting equation (4.11) into kinematic equation (4.9), and using $\lambda_4(t) = 0$ gives the kinematic equation describing evolution of the effort for green innovation below, that is,

$$\dot{k}(t) = (\rho+\delta)k(t) - \frac{b}{\beta}[M - b + \vartheta Q(t) - \frac{a-\mu Q(t)}{q(t)}] - \frac{(\rho+\eta)[a-\mu Q(t)][a-\tau\varepsilon+bq(t)-c(t)]}{\beta[(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]q(t)}. \quad (4.12)$$

Further, noting that the network size $Q(t)$ appears in kinematic equations (4.10) and (4.12), then differentiating these above two kinematic equations with respect to $Q(t)$ gives (i) $\frac{\partial \dot{k}(t)}{\partial Q(t)} = \frac{\mu(\rho+\eta)[a-\tau\varepsilon+bq(t)-c(t)]-b[\mu+\vartheta q(t)][(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}{\beta[(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]q(t)}$, which means the rate of change in effort for green innovation increases as the network size $Q(t)$ if and only if $\frac{[\mu+\vartheta q(t)][(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}{a-\tau\varepsilon+bq(t)-c(t)} \leq \frac{\mu(\rho+\eta)}{b}$; (ii) $\frac{\partial \dot{h}(t)}{\partial Q(t)} = -\frac{\mu+\vartheta q(t)}{\alpha q(t)} < 0$, which means the rate of change in effort for process innovation decreases with the expansion of the network size $Q(t)$.

To provide a more comprehensive examination of the association between the control variables and state variables, we impose the stationary condition to the kinematic co-state equation (3.7), i.e., $\dot{Q}(t) = 0$, then we have the steady-state network size $Q(q)$ as a function of green performance $q(t)$ below:

$$Q(q) = \frac{\xi[a-(M-b)q(t)]}{\xi\mu+(\xi\vartheta-\eta)q(t)}. \quad (4.13)$$

Now, one differentiates above equation (4.13) with respect to $q(t)$ yields $\frac{\partial Q(q)}{\partial q(t)} = -\xi\frac{a(\xi\vartheta-\eta)+\xi\mu(M-b)}{[\xi\mu+(\xi\vartheta-\eta)q(t)]^2}$, which means that the candidate steady-state network size $Q(q)$ decreases as the green performance $q(t)$ increases if and only if $a(\xi\vartheta-\eta)+\xi\mu(M-b) \leq 0$.

Substituting equation (4.13) into the demand function (3.4) and rearranging terms, one can yield the candidate steady-state demand $D(q)$ as a function of green performance $q(t)$ below:

$$D(q) = \frac{\eta[a-(M-b)q(t)]}{\xi\mu+(\xi\vartheta-\eta)q(t)}. \quad (4.14)$$

It is important to acknowledge that, under the steady state of the co-state kinematic equation (3.6), i.e., $\dot{Q}(t) = 0$, the demand function only depends on the green performance $q(t)$. Differentiating the above demand function (4.14) with respect to $q(t)$, we have $\frac{\partial D(q)}{\partial q(t)} = -\frac{\eta[a(\xi\vartheta-\eta)+\xi\mu(M-b)]}{[\xi\mu+(\xi\vartheta-\eta)q(t)]^2}$, which means that the demand $D(q)$ increases as the green performance $q(t)$ increases if and only if $a(\xi\vartheta-\eta)+\xi\mu(M-b) \leq 0$.

Besides, substituting equation (4.13) into the kinematic equation of pollution stock (3.7), and rearranging terms, one can obtain

$$\dot{P}(t) = \frac{\varepsilon\eta[a-(M-b)q(t)]}{\xi\mu+(\xi\vartheta-\eta)q(t)} - \kappa P(t). \quad (4.15)$$

Imposing steady-state condition to the dynamic equation of pollution stock (4.15), i.e., $\dot{P}(t) = 0$, we can obtain the candidate steady-state pollution stock $P(q)$ as a function of green performance $q(t)$, that is,

$$P(q) = \frac{\varepsilon\eta[a-(M-b)q(t)]}{\kappa[\xi\mu+(\xi\vartheta-\eta)q(t)]}. \quad (4.16)$$

Differentiating the above equation (4.16) with respect to $q(t)$ gives $\frac{\partial P(q)}{\partial q(t)} = -\frac{\varepsilon\eta[a(\xi\vartheta-\eta)+\xi\mu(M-b)]}{\kappa[\xi\mu+(\xi\vartheta-\eta)q(t)]^2}$, which means the pollution stock $P(q)$ increases as the green performance $q(t)$ increases if and only if $a(\xi\vartheta - \eta) + \xi\mu(M - b) \leq 0$.

Further, by substituting the equation (4.13) into kinematic equations (4.10) and (4.12), and rearranging, one can obtain the kinematic equations describing the evolution of the efforts for green and process innovation, that is,

$$\dot{k}(t) = (\rho + \delta)k(t) - \frac{b\eta[a - (M - b)q(t)]}{\beta[\xi\mu + (\xi\vartheta - \eta)q(t)]} - \frac{(\rho + \eta)[a(\xi\vartheta - \eta) + \xi\mu(M - b)][a - \tau\varepsilon + bq(t) - c(t)]}{\beta[\xi\mu + (\xi\vartheta - \eta)q(t)][(\rho + \eta - \xi\vartheta)q(t) - \xi\mu]}, \tag{4.17}$$

$$\dot{h}(t) = (\rho - \sigma)h(t) - \frac{\eta[a - (M - b)q(t)]}{\alpha[\xi\mu + (\xi\vartheta - \eta)q(t)]}. \tag{4.18}$$

From the dynamic equations (4.17) and (4.18), one can observe that both green performance $q(t)$ and marginal production cost $c(t)$ are present in the dynamic equation (3.17), while the state variable of green performance $q(t)$ appears in the dynamic equation (4.18). This observation indicates that the effort for green innovation and the effort for process innovation interact with each other over continuous time $t \in [0, +\infty)$. The resultant implications of this particular attribute of kinematic equations (4.17) and (4.18) will become more pronounced in the remaining portion, which is often connected to the stability of the kinematic state-control system below.

Next, we investigate the effect of the effort for green (or process) innovation on the rate of change in effort for process (or green) innovation, respectively. It is easy to obtain Proposition 1 below (Proof can be found in Appendix B.1):

Proposition 1. *Under monopolist decision-making, at each time $t \in [0, +\infty)$, we have (i) $\frac{\partial \dot{k}(t)}{\partial h(t)} \geq (<)0$ iff $\frac{a(\xi\vartheta-\eta)+\xi\mu(M-b)}{\xi\mu+(\xi\vartheta-\eta)q(t)} \leq (>)0$; (ii) $\frac{\partial \dot{h}(t)}{\partial k(t)} \geq (<)0$ iff $a(\xi\vartheta - \eta) + \xi\mu(M - b) \geq (<)0$.*

Proposition 1 shows that, in a monopolistic market with network externality, the impact of the effort for process (or green) innovation on the rate of change in effort for green (or process) innovation depends on specific market conditions. Specifically, (i) the rate of change in effort for green innovation increases with the increase of the effort for process innovation if and only if $\frac{a(\xi\vartheta-\eta)+\xi\mu(M-b)}{\xi\mu+(\xi\vartheta-\eta)q(t)} \leq 0$; (ii) the rate of change in effort for process innovation increases with the increases of the effort for green innovation if and only if $a(\xi\vartheta - \eta) + \xi\mu(M - b) \geq 0$. It is worth noting that the results in Proposition 1 are inconsistent with Proposition 3 in [27] and Proposition 3 in [33], where the authors investigated dynamic optimal control of product and process innovation for a monopolist producing non-network products and showed that the $\dot{k}(t)$ and $\dot{h}(t)$ decrease with $h(t)$ and $k(t)$, respectively.

In the next section, we conduct a comprehensive investigation for the stability property of the steady-state equilibrium under the setting of the monopolist decision-making.

4.2. Steady-state equilibrium and stability analysis

To analyse the stability property of the steady-state equilibrium of state-control system, we first clear up the relationship between the control variables and the state variables. Imposing the stationary conditions on kinematic equations (4.17) and (4.18), i.e., $\dot{k}(t) = 0$ and $\dot{h}(t) = 0$, one can get

$$k^m(q, c) = \frac{1}{\rho + \delta}\{\frac{\eta b[a - (M - b)q(t)]}{\beta[\xi\mu + (\xi\vartheta - \eta)q(t)]} + \frac{(\rho + \eta)[a(\xi\vartheta - \eta) + \xi\mu(M - b)][a - \tau\varepsilon + bq(t) - c(t)]}{\beta[\xi\mu + (\xi\vartheta - \eta)q(t)][(\rho + \eta - \xi\vartheta)q(t) - \xi\mu]}\}, \tag{4.19}$$

$$h^m(q,c) = \frac{\eta[a-(M-b)q(t)]}{\alpha(\rho-\sigma)[\xi\mu+(\xi\vartheta-\eta)q(t)]}. \tag{4.20}$$

For sake of simplicity, we define $\Lambda := \frac{a(\xi\vartheta-\eta)+\xi\mu(M-b)}{[\xi\mu+(\xi\vartheta-\eta)q(t)][(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}$. Differentiating the above candidate steady-state effort $k^m(q,c)$ w.r.t. $c(t)$, we have $\frac{\partial k^m(q,c)}{\partial c(t)} \propto -\Lambda$, which indicates that the candidate steady-state effort for green innovation $k^m(q,c)$ increases with the increase of the marginal production cost $c(t)$ if and only if $\Lambda \leq 0$. Differentiating $h^m(q,c)$ w.r.t. $q(t)$, we have $\frac{\partial h^m(q,c)}{\partial q(t)} \propto -[a(\xi\vartheta-\eta)+\xi\mu(M-b)]$, which means the steady-state effort in process innovation $h^m(q,c)$ increases with the increase of the green performance $q(t)$ if and only if $a(\xi\vartheta-\eta)+\xi\mu(M-b) \leq 0$.

Furthermore, we examine whether the monopolist puts more effort into the green innovation or the process innovation, and under what conditions the monopolist has more effort for green (process) innovation than for process (green) innovation. From the candidate steady-state efforts (4.19) and (4.20), we have

$$k^m(q,c) - h^m(q,c) = \frac{(\rho+\eta)[a(\xi\vartheta-\eta)+\xi\mu(M-b)][a-\tau\varepsilon+bq(t)-c(t)]}{\beta(\rho+\delta)[\xi\mu+(\xi\vartheta-\eta)q(t)][(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}. \tag{4.21}$$

On the basis of Assumption 1, combined with equation (4.21), one can acquire that $k^m(q,c) \geq (<) h^m(q,c)$ if and only if $\Lambda \geq (<) 0$, which implies that the candidate steady-state effort for green innovation is greater (less) than that for process innovation if and only if $\Lambda \geq (<) 0$.

Further, imposing the stationarity condition for dynamic equation (3.2), we have $h^m = \sigma c^m$. Then, equation (4.20) can be rewritten as

$$c^m(q) = \frac{\eta[a-(M-b)q(t)]}{\alpha\sigma(\rho-\sigma)[\xi\mu+(\xi\vartheta-\eta)q(t)]} \tag{4.22}$$

Differentiating the above steady-state marginal production cost $c^m(q)$ w.r.t. $q(t)$, one can yield $\frac{\partial c^m(q)}{\partial q(t)} \propto -[a(\xi\vartheta-\eta)+\xi\mu(M-b)]$, which reflects that the candidate steady-state marginal production cost $c^m(q)$ increases with the green performance $q(t)$ if and only if $a(\xi\vartheta-\eta)+\xi\mu(M-b) \leq 0$.

Substituting equation (4.22) into kinematic equations (4.19) and rearranging terms, we have

$$\begin{aligned} k^m(q) = & \frac{(\rho+\eta)[a(\xi\vartheta-\eta)+\xi\mu(M-b)]}{\beta(\rho+\delta)[\xi\mu+(\xi\vartheta-\eta)q(t)][(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}\{a-\tau\varepsilon+bq(t)-\frac{a-(M-b)q(t)}{\alpha\sigma(\rho-\sigma)[\xi\mu+(\xi\vartheta-\eta)q(t)]} \\ & +\frac{\eta b[a-(M-b)q(t)]}{(\rho+\eta)[a(\xi\vartheta-\eta)+\xi\mu(M-b)][(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}\}. \end{aligned} \tag{4.23}$$

From equation (4.23) we find a highly nonlinear relationship between the candidate steady-state effort for green performance $k^m(q)$ and green performance $q(t)$, it is hard to study the influence of the change in green performance $q(t)$ on steady-state effort for green performance $k^m(q)$ analytically. However, to illustrate the main conclusions drawn from the case that we reviewed, we can turn to numerical methods. We give numbers to various coefficients for quantitative simulation exercises. In Table 1, the default parameter values are shown. The parameter values employed in numerical simulations are consistent with those commonly used in the dynamic optimal control literature (see, e.g., [78–80]). The numerical outcomes remain insensitive under small variations in the neighbourhood of the baseline values, demonstrating the robustness of the results.

Table 1. Baseline parameter values used in the simulation exercises

a	b	δ	η	α	β	ρ	τ	ϑ	M	μ	σ	ϵ	ξ
0.6	0.8	0.6	0.12	0.6	0.25	0.2	0.25	0.2	1.5	0.6	0.12	0.1	0.3

Figure 2 shows how the green performance $q(t)$ impacts the candidate steady-state effort for green innovation $k^m(q)$. From Figure 2, one can see that all else being equal, with the improvement of green performance, the candidate steady-state effort $k^m(q)$ first decreases and then increases. This outcome is summarized in the following Observation 1:

Observation 1. *Given feasible parameters, under monopolist decision-making, as the green performance improves, the candidate steady-state effort for green innovation $k^m(q)$ first decreases and then increases.*

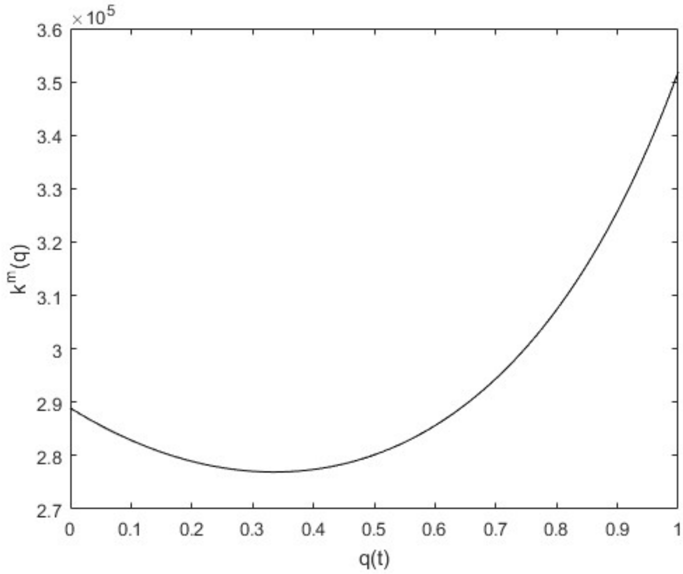

Figure 2. The candidate steady-state effort for green innovation as a function of green performance

Substituting equations (4.13), (4.22), and (4.23) into total cost function (3.9) and profit function (3.10), together with $h^m = \sigma c^m$, we have

$$C(q) = \frac{(2\rho-\sigma)\eta^2[a-(M-b)q(t)]^2}{2\sigma\alpha(\rho-\sigma)^2[\xi\mu+(\xi\vartheta-\eta)q(t)]^2} + \frac{1}{2(\rho+\delta)^2}\{\frac{\eta b[a-(M-b)q(t)]}{\xi\mu+(\xi\vartheta-\eta)q(t)} - \frac{(\rho+\eta)[a(\vartheta-\eta)+\xi\mu(M-b)]\{\alpha\sigma(\rho-\sigma)[a-\tau\varepsilon+bq(t)]-\eta[a-(M-b)q(t)]\}}{\alpha\sigma(\rho-\sigma)[\xi\mu-(\rho+\eta-\xi\vartheta)q(t)][\xi\mu+(\xi\vartheta-\eta)q(t)]^2}\}^2, \tag{4.24}$$

$$\pi(q) = \eta[a-\tau\varepsilon+bq(t) - \frac{\eta[a-(M-b)q(t)]}{\alpha\sigma(\rho-\sigma)[\xi\mu+(\xi\vartheta-\eta)q(t)]}]\frac{[a-(M-b)q(t)]}{\xi\mu+(\xi\vartheta-\eta)q(t)} - \frac{\eta^2}{2\alpha(\rho-\sigma)^2}[\frac{a-(M-b)q(t)}{\xi\mu+(\xi\vartheta-\eta)q(t)}]^2 - \frac{\beta}{2(\rho+\delta)^2}\{\frac{\eta b[a-(M-b)q(t)]}{\beta[\xi\mu+(\xi\vartheta-\eta)q(t)]} - \frac{(\rho+\eta)[a(\vartheta-\eta)+\xi\mu(M-b)]\{\alpha\sigma(\rho-\sigma)[a-\tau\varepsilon+bq(t)]-\eta[a-(M-b)q(t)]\}}{\beta\alpha\sigma(\rho-\sigma)[\xi\mu-(\rho+\eta-\xi\vartheta)q(t)][\xi\mu+(\xi\vartheta-\eta)q(t)]^2}\}^2. \tag{4.25}$$

We find from the equations (4.24) and (4.25) that the total cost function $C(q)$ and profit function $\pi(q)$ are highly nonlinear functions with respect to green performance $q(t)$. It is thus difficult to study the impacts of green performance $q(t)$ on total cost function $C(q)$ and profit function $\pi(q)$ analytically. We resort to numerical methods to show the managerial insights. Using the default parameter values presented in Table 1, the shape of function $C(q)$ and $\pi(q)$ are shown in Figures 3–4 below, respectively.

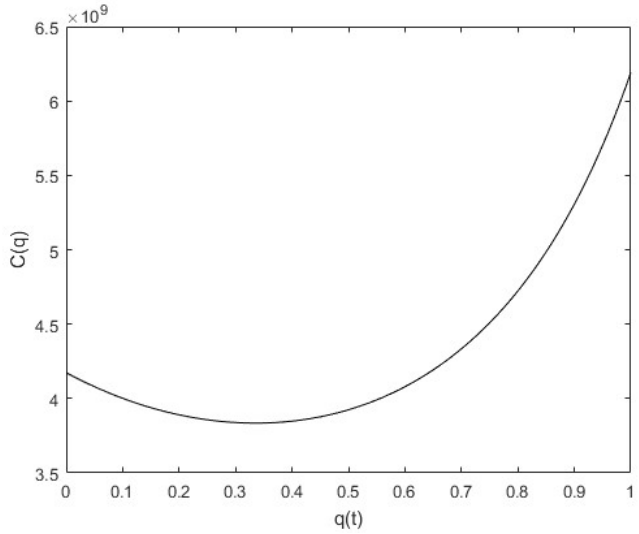

Figure 3. The candidate steady-state total cost as a function of green performance

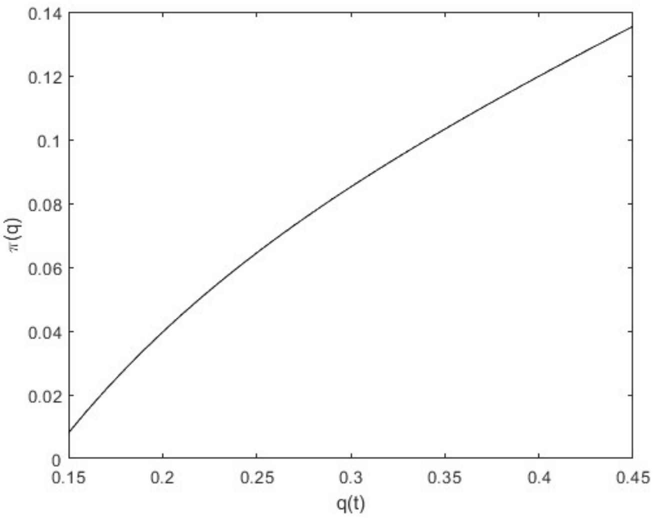

Figure 4. The candidate steady-state profit as a function of green performance

From Figure 3 and 4, one can see that, all else being equal, as the green performance improves, the candidate steady-state total cost first decreases and then increases, while the candidate steady-state profit increases. These two findings are condensed in the following Observation 2:

Observation 2. *Given feasible parameters, under monopolist decision-making, as the green performance improves, the candidate steady-state total cost $C(q)$ first decreases and then increases, while the candidate steady-state profit $\pi(q)$ increases.*

Combining dynamic constraints (3.1)–(3.2) with the kinematic equations (4.17)–(4.18), the dynamic state-control system of four coupled differential equations can be obtained:

$$\begin{cases} \dot{k}(t) = (\rho+\delta)k(t) - \frac{b\eta[a-(M-b)q(t)]}{\beta[\xi\mu+(\xi\vartheta-\eta)q(t)]} - \frac{(\rho+\eta)[a(\xi\vartheta-\eta)+\xi\mu(M-b)][a-\tau\varepsilon+bq(t)-c(t)]}{\beta[\xi\mu+(\xi\vartheta-\eta)q(t)][(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}, \\ \dot{h}(t) = (\rho-\sigma)h(t) - \frac{\eta[a-(M-b)q(t)]}{\alpha[\xi\mu+(\xi\vartheta-\eta)q(t)]}, \\ \dot{q}(t) = k(t) - \delta q(t), \\ \dot{c}(t) = -h(t) + \sigma c(t). \end{cases} \tag{4.26}$$

Solving the above dynamic state-control system (4.26) under the steady-state conditions $\dot{k}(t) = \dot{h}(t) = \dot{q}(t) = \dot{c}(t) = 0$, one can obtain the steady-state equilibrium $\{k^m, h^m, q^m, c^m\}$. Accordingly, we have Proposition 2 as shown below (Proof can be seen in Appendix B.2):

Proposition 2. *Under monopolist decision-making, $\{k^m, h^m, q^m, c^m\}$ is the unique saddle-point steady-state equilibrium over the whole admissible parameter range, in which $k^m = \delta q^m$, $h^m = \sigma c^m$, $q^m = -\frac{A_2}{4A_1} + \frac{1}{2}\sqrt{\frac{A_2^2}{4A_1^2} - \frac{2A_3}{3A_1} + \Delta^m} + \frac{1}{2}\sqrt{\frac{A_2^2}{2A_1^2} - \frac{4A_3}{3A_1} - \Delta^m - \frac{-\frac{A_2^3}{A_1^3} + \frac{4A_2A_3}{A_1^2} - \frac{8A_4}{A_1}}{4\sqrt{\frac{A_2^2}{4A_1^2} - \frac{2A_3}{3A_1} + \Delta^m}}}$, and $c^m = \frac{\eta[a-(M-b)q^m]}{\alpha\sigma(\rho-\sigma)[\mu+(\vartheta-\eta)q^m]}$, where A_1, A_2, A_3, A_4, and Δ^m are presented in Appendix B.2.*

Substituting q^m in Proposition 2 into (4.3) and (4.16), we have $Q^m = \frac{a-(M-b)q^m}{\xi\mu+(\xi\vartheta-\eta)q^m}$ and $P^m = \frac{\varepsilon\eta[a-(M-b)q^m]}{\kappa[\xi\mu+(\xi\vartheta-\eta)q^m]} = \frac{\varepsilon\eta Q^m - q^m}{\kappa}$, respectively.

In this subsection, we examine the stability of steady-state equilibrium from the perspective of the monopolistic decision. The social welfare implications will be considered in the next section as a complement to our analysis.

5. Social welfare implications

In this section, the method for evaluating the efficiency of both the efforts in green and process innovation are consistent with [69], which are also utilized in the works of [25, 27, 28], just to mention a few. We will evaluate the social incentive to improve green performance and reduce marginal production cost. The instantaneous social welfare $sw(t)$ defined as the sum of the profit $\pi(t)$, consumer surplus $cs(t)$, tax revenue $\tau\epsilon D(t)$, and environmental damage $-E(t)$, that is, $sw(t) = \pi(t) + cs(t) + \tau\varepsilon D(t) - E(t)$. The consumer surplus $cs(t)$ is given by the following formula:

$$cs(t) = \int_{a+bq(t)}^{Mq(t)+[\mu+\vartheta q(t)]Q(t)} \{M - \frac{a + bq(t) - [\mu+\vartheta q(t)]Q(t)}{q(t)}\}d\omega = \frac{1}{2q(t)}\{[\mu+\vartheta q(t)]Q(t) - a + (M-b)q(t)\}^2. \tag{5.1}$$

Following equations (3.10) and (5.1), one can obtain the following social welfare function:

$$\begin{aligned} sw(t) = {} & \tfrac{1}{2q(t)}\{[\mu+\vartheta q(t)]Q(t) - a + (M-b)q(t)\}^2 + \tfrac{1}{q(t)}[a + bq(t) - c(t)]\{[\mu+\vartheta q(t)]Q(t) - a \\ & + (M-b)q(t)\} - \tfrac{\beta}{2}k^2(t) - \tfrac{\alpha}{2}h^2(t) - \varphi P(t). \end{aligned} \tag{5.2}$$

Under social planning, the goal of the social planner is to find equilibrium efforts $k(t)$ and $h(t)$ over continuous time $t \in [0, +\infty)$, such that the discounted social welfare flow SW is maximized, namely,

$$SW = \max_{k(t),h(t)} \int_0^\infty e^{-\rho t}\{\tfrac{1}{q(t)}[a + bq(t) - c(t)][(\mu + \vartheta q(t))Q(t) - a + (M - b)q(t)] - \tfrac{\beta}{2}k^2(t) - \tfrac{\alpha}{2}h^2(t) - \varphi P(t) + \tfrac{1}{2q(t)}[(\mu + \vartheta q(t))Q(t) - a + (M - b)q(t)]^2\}dt$$

$$s.t.\begin{cases} \dot{q}(t) = k(t) - \delta q(t), \\ \dot{c}(t) = -h(t) + \sigma c(t), \\ \dot{Q}(t) = \xi(M - b) + (\xi\vartheta - \eta)Q(t) - \frac{\xi[a-\mu Q(t)]}{q(t)}, \\ \dot{P}(t) = \varepsilon[M - b + \vartheta Q(t) - \frac{a-\mu Q(t)}{q(t)}] - \kappa P(t), \\ q(0) = q_0, c(0) = c_0, Q(0) = Q_0, P(0) = P_0, \end{cases} \tag{5.3}$$

where $\rho > 0$ is the constant discount factor.

According to the maximum principle, we can construct the current-value Hamiltonian function of the dynamic optimization problem (5.3) as follows:

$$\begin{aligned} H = &\tfrac{1}{2q(t)}[(M - b)q(t) + (\mu + \vartheta q(t))Q(t) - a]^2 + \tfrac{1}{q(t)}[a + bq(t) - c(t)][(M - b)q(t) + (\mu + \vartheta q(t))Q(t) - a] \\ &- \tfrac{\beta}{2}k^2(t) - \tfrac{\alpha}{2}h^2(t) - \varphi P(t) + \omega_1(t)[k(t) - \delta q(t)] + \omega_2(t)[-h(t) + \sigma c(t] + \omega_3(t)\{\xi(M - b)+ \\ &(\xi\vartheta - \eta)Q(t) - \tfrac{\xi[a-\mu Q(t)]}{q(t)}\} + \omega_4(t)\{\varepsilon[M - b + \vartheta Q(t) - \tfrac{a-\mu Q(t)}{q(t)}] - \kappa P(t)\}, \end{aligned} \tag{5.4}$$

where $\omega_i(t)$ ($i \in \{1, 2, 3, 4\}$) are shadow prices (co-state variables) corresponding to state variables $q(t)$, $c(t)$, $Q(t)$, and $P(t)$, respectively.

From Hamiltonian function (5.4), one can obtain the first-order conditions with respect to control variables $k(t)$ and $h(t)$, as well as the dynamic equations with respect to co-state variable $\omega_i(t)$ ($i \in \{1, 2, 3, 4\}$), respectively:

$$\frac{\partial H}{\partial k(t)} = -\beta k(t) + \omega_1(t) = 0, \tag{5.5}$$

$$\frac{\partial H}{\partial h(t)} = -\alpha h(t) - \omega_2(t) = 0, \tag{5.6}$$

$$\begin{aligned} \dot{\omega}_1(t) &= \rho\omega_1(t) - \frac{\partial H}{\partial q(t)} \\ &= (\rho + \delta)\omega_1(t) + \frac{1}{2q^2(t)}\{(M - b)q(t) + [(\mu + \vartheta q(t)]Q(t) - a\}\{a + (M + b)q(t) + [\mu + \vartheta q(t)] \\ &\quad Q(t) - 2c(t)]\} - [Mq(t) + (\mu + \vartheta q(t))Q(t) - c(t)]\frac{M + \vartheta Q(t)}{q(t)} + \frac{b[a + bq(t) - c(t)]}{q(t)} \\ &\quad - [\xi\omega_3(t) + \varepsilon\omega_4(t)]\frac{[a - \mu Q(t)]}{q^2(t)}, \end{aligned} \tag{5.7}$$

$$\begin{aligned} \dot{\omega}_2(t) &= \rho\omega_2(t) - \frac{\partial H}{\partial c(t)} \\ &= (\rho - \sigma)\omega_2(t) + [M - b + \vartheta Q(t) - \frac{a - \mu Q(t)}{q(t)}], \end{aligned} \tag{5.8}$$

$$\dot{\omega}_3(t) = \rho\omega_3(t) - \frac{\partial H}{\partial Q(t)} \tag{5.9}$$

$$= [\frac{(\rho+\eta-\xi\vartheta)q(t)-\xi\mu}{q(t)}]\omega_3(t) - \{Mq(t)+[\mu+\vartheta q(t)]Q(t)-c(t)+\varepsilon\omega_4(t)\}\frac{[\mu+\vartheta q(t)]}{q(t)},$$

$$\dot{\omega}_4(t) = \rho\omega_4(t) - \frac{\partial H}{\partial P(t)} = (\rho+\kappa)\omega_4(t)+\varphi, \tag{5.10}$$

where the transversal conditions are given by $\lim_{t\to\infty}\omega_1(t)q(t)e^{-\rho t} = 0$, $\lim_{t\to\infty}\omega_2(t)c(t)e^{-\rho t} = 0$, $\lim_{t\to\infty}\omega_3(t)Q(t)e^{-\rho t} = 0$, and $\lim_{t\to\infty}\omega_4(t)P(t)e^{-\rho t} = 0$.

In what follows, we provide kinematic equations that describe the evolution of the efforts for green and process innovation over continuous time $t \in [0, +\infty)$. We first consider the solution of equation (5.10). Solving equation (5.10) with respect to co-state variable $\omega_4(t)$ and using transversal condition $\lim_{t\to\infty}\omega_4(t)P(t)e^{-\rho t} = 0$, we have

$$\omega_4(t) = -\frac{\varphi}{\rho+\kappa}. \tag{5.11}$$

Based on Assumption 2, solving co-state equation (5.9) under $\dot{\omega}_3(t) = 0$ w.r.t. $\omega_3(t)$, and using (5.11), we have

$$\omega_3(t) = \frac{[\mu+\vartheta q(t)]\{(\rho+\kappa)[Mq(t)+(\mu+\vartheta q(t))Q(t)-c(t)]-\varepsilon\varphi\}}{(\rho+\kappa)[(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}. \tag{5.12}$$

Differentiating equations (5.5) and (5.6) with respect to time t, combined with equations (5.11)–(5.12), (5.7)–(5.9), one can obtain the following dynamics regarding the control variables $k(t)$ and $h(t)$:

$$\dot{k}(t) = (\rho+\delta)k(t) + \frac{1}{2\beta q^2(t)}\{(M-b)q(t)+(\mu+\vartheta q(t))Q(t)-a\}\{(M+b)q(t)+[\mu+\vartheta q(t)]Q(t) - 2c(t)+a]\} - \frac{1}{\beta q(t)}\{[M+\vartheta Q(t)][Mq(t)+(\mu+\vartheta q(t))Q(t)-c(t)] - b[a+bq(t)-c(t)]\} - \frac{a-\mu Q(t)}{\beta q^2(t)}\{\frac{(\rho+\kappa)[Mq(t)+(\mu+\vartheta q(t))Q(t)-c(t)][\mu+\vartheta q(t)]-\varepsilon\varphi(\rho+\eta)q(t)}{(\rho+\kappa)[(\rho+\eta-\xi\vartheta)q(t)-\xi\mu]}\} - \frac{\varphi}{\beta(\rho+\kappa)}, \tag{5.13}$$

$$\dot{h}(t) = (\rho-\sigma)h(t) - \frac{1}{\alpha}[M-b+\vartheta Q(t) - \frac{a-\mu Q(t)}{q(t)}]. \tag{5.14}$$

In this section, we'll primarily examine the stability characteristics of the steady-state equilibrium under social planning. In light of this, we have the subsequent Proposition 3 (Proof can be seen in Appendix B.3):

Proposition 3. *Under social planning, $\{k^s, h^s, q^s, c^s\}$ is the unique saddle-point steady-state equilibrium over the whole admissible parameter range, in which $k^s = \delta q^s$, $h^s = \sigma c^s$, $q^s = -\frac{B_2}{4B_1} + \frac{1}{2}\sqrt{\frac{B_2^2}{4B_1^2} - \frac{2B_3}{3B_1} + \Delta^s} + \frac{1}{2}\sqrt{\frac{B_2^2}{2B_1^2} - \frac{4B_3}{3B_1} - \Delta^s - \frac{-\frac{B_2^3}{B_1^3}+\frac{4B_2B_3}{B_1^2}-\frac{8B_4}{B_1}}{4\sqrt{\frac{B_2^2}{4B_1^2}-\frac{2B_3}{3B_1}+\Delta^s}}}$, and $c^s = \frac{\eta[a-(M-b)q^s]}{\alpha\sigma(\rho-\sigma)[\xi\mu+(\xi\vartheta-\eta)q^s]}$, where B_0, B_1, B_2, B_3, and Δ^s are presented in Appendix B.3.*

Substituting q^s in Proposition 3 into (5.13) and (5.14), we have $Q^s = \frac{a-(M-b)q^s}{\xi\mu+(\xi\vartheta-\eta)q^s}$ and $P^s = \frac{\varepsilon\eta Q^s - q^s}{\kappa}$.

The analysis so far concentrates on the saddle-point property of the steady-state equilibrium under monopolist decision-making and social planning, respectively. Nevertheless, from Proposition 2 and

3, we find that the expressions of steady-state equilibrium are too complicated to investigate whether the green performance (or marginal production cost) is higher (or lower) under monopolist decision-making than that under social planning. In the next section, numerical experiments are performed to further investigate this problem.

6. Numerical experiments

Since $k^m = \delta q^m$, $h^m = \sigma c^m$, $k^s = \delta q^s$, and $h^s = \sigma c^s$, we only offer numerical simulations in the neighbourhoods of efforts $\{k^m, h^m\}$ and $\{k^s, h^s\}$. We analyse numerical solutions with the help of MATLAB 2025. Figure 5 shows the numerical results in the neighbourhoods of $\{k^m, h^m\}$ and $\{k^s, h^s\}$ against time t, where the dotted line represents the numerical solutions in the neighbourhoods of $\{k^m, h^m\}$, while the solid line represents the numerical solutions in the neighbourhoods of $\{k^s, h^s\}$.

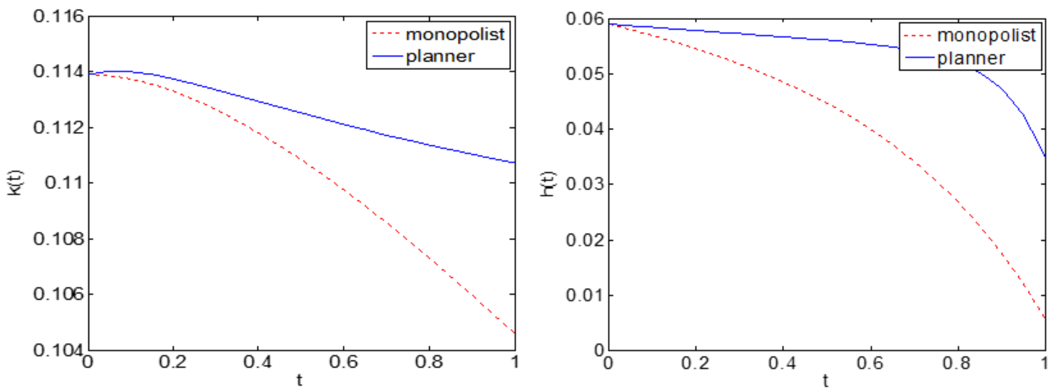

Figure 5. The evolutionary trajectories in the neighbourhoods of $\{k^m, h^m\}$ and $\{k^s, h^s\}$ against time t.

From Figure 5, as expected, in the neighbourhoods of steady-state equilibrium, both the efforts $\{k^s, h^s\}$ are higher than $\{k^m, h^m\}$, which means that, compared with the social planner, in the neighbourhoods of steady-state equilibrium, the monopolist has an insufficient-effort problem. Due to in the neighbourhoods of steady-state equilibrium, both the efforts $\{k^s, h^s\}$ are higher than the efforts $\{k^m, h^m\}$, then we have $q^s > q^m$, $c^s > c^m$. Since $p(t) = a + bq(t)$, we have $p^s > p^m$. Further, we have $Q^s - Q^m = -\frac{\xi[a(\xi\vartheta-\eta)+\xi\mu(M-b)](q^s-q^m)}{[\mu\xi+(\xi\vartheta-\eta)q^s][\xi\mu+(\xi\vartheta-\eta)q^m]}$, which means $Q^s - Q^m \geq (<)0$ if and only if $a(\xi\vartheta - \eta) + \xi\mu(M - b) \leq (>)0$. We summarize the above results in Observation 3 below:

Observation 3. *In the neighbourhoods of steady-state equilibrium, (i) the efforts for green and process innovation are lower under monopolist decision-making than that under planning; (ii) while the price is regulated, the price is lower under monopolist decision than that under social planning; (iii) the monopolist undersupplies green performance compared with the socially optimal level, while the marginal production cost is socially efficient; and (iv) the network size is higher (lower) under social planning than that under monopolist decision-making iff* $a(\xi\vartheta - \eta) + \xi\mu(M - b) \leq (>)0$.

7. Discussion

This section discusses the managerial implications of our main outcomes, compares our results with findings in previous literature, and illustrates their practical relevance through real-world examples. Our analysis yields several insights that are directly relevant for managers in network industries.

Under-investment problem in innovative activities

The comparison between profit-seeking and socially optimal outcomes highlights that there exists under-investment problem in green innovation aimed at improving product's green performance, while the process innovation remains socially efficient. The downward distortion in innovative supply has been documented in extensive studies. In the earlier works [69, 81], researchers have demonstrated in static frameworks that product innovation is typically undersupplied, whereas process innovation is socially efficient. The informative work in [25] replicated these results in a dynamic setting. In contrast, Cellini and Lambertini [55] showed that process innovation may become socially inefficient with a positive probability, implying potential conflicts between private and social incentives. More recent studies incorporating consumer-side behavioral factors (e.g., [29, 82]) further revealed that under-investment may arise simultaneously in both product and process dimensions. Our study supports the classical finding that process innovation is socially efficient, while demonstrating that green innovation aimed at improving green design quality continues to suffer from under-investment, even in the presence of network externality and price regulation. The policy implications behind this result is: Firms, when acting independently, tend to prioritize short-term profitability over long-term environmental performance, thereby providing a clear rationale for targeted policy interventions such as subsidies or tax credits. On the contrary, the social efficiency of process innovation, departing from studies claiming multidimensional under-investment, indicates that firms have adequate incentives to invest in production-efficiency improvements without additional regulatory pressure. A practical illustration is provided by government subsidy programs for electric vehicles: These subsidies incentivize firms to invest more in green design features [83], such as battery recycling and sustainable materials, helping to mitigate the under-investment problem in green innovation while leaving process innovation incentives largely unaffected [84].

Managing network scale

Our results reveal that the social optimality of network scale is jointly determined by several key factors: The strength of fixed and variable network externality, consumer entry and exit rates, regulated fixed and variable prices, and overall market potential. Notably, whether the steady-state network scale exceeds the socially optimal level is irrelevant with the pollutant dynamics, suggesting that network size should be managed through market conditions and regulatory instruments rather than environmental considerations. A practical illustration can be seen in mobile telecommunications: regulators and operators coordinate the expansion of 5G networks by controlling pricing, spectrum allocation, and user entry incentives, ensuring the network grows toward a socially optimal scale while maintaining efficiency [5]. This finding complements the literature on green innovation [2] by demonstrating that environmental factors do not directly constrain network growth, which provides a clear guidance for regulators seeking to coordinate network expansion with innovation incentives.

The impact of green performance

We examine the long-run influence of green performance on a monopolist's innovation incentives, total cost, and firm profitability. As the green performance improves, green innovation effort initially decreases and then rises; total cost first decreases and then increases; while profit gradually grows. These findings suggest that higher green performance can enhance profitability, in spite of rising total cost. Moreover, while improvement in green performance may temporarily reduce effort for green innovation, further enhancements can stimulate both effort and profits. In other words, investing in green design not only supports environmental goals but also contributes to long-term financial performance, even in the absence of stringent environmental regulation. This is consistent with previous studies demonstrating the positive impact of green performance on operational outcomes [85, 86]. As an example, Apple's investments in energy-efficient and recyclable smartphone components initially slow innovation in other green features, but continued improvement in green performance ultimately increases both innovation effort and long-term profitability, demonstrating that environmental and financial objectives can coincide [87].

Steady-state saddle-point equilibrium

The stability analysis identifies the conditions under which firms' dynamic decisions converge to a steady state. This is crucial for practitioners aiming at anticipating long-run innovation behavior or evaluating the risk of oscillatory or unstable investment patterns, which may arise in complex nonlinear dynamical systems [28, 88].

8. Concluding remarks

This work investigates the dynamic optimal control problems of a monopolist firm's efforts for green and process innovation in a market exhibiting network externality. The main characteristics of this work are: (i) the price is linked to the green performance; (ii) the demand function of consumers depends on price, green performance, and network size. This paper aims to examine the monopolist firm's efforts for green and process innovation, explore the relationship between these two innovation efforts, and analyse the saddle-point property of the steady-state equilibrium under the monopolist firm decision and social planner decision, respectively. Our main findings indicate that the system admits unique saddle-point steady-state equilibrium over the whole admissible parameter range in the setting of monopolist decision-making and social planning, respectively. Furthermore, as we find, in the neighbourhoods of steady-state equilibrium, the monopolist's efforts for green and process innovation are lower under monopolist firm decision than those under social planner decision. While the price is regulated, the price will be higher in the case of social planner decision than that under the monopolist decision; the monopolist firm undersupplies green performance as compared to the social planner decision, while the marginal production cost is socially efficient. At the same time, the network size is higher (lower) under social planner decision than that under monopolist firm decision if and only if $a(\xi\vartheta - \eta) + \xi\mu(M - b) \leq (>) 0$.

Future research naturally arises on the basis of our work. One is to keep the model considered in this paper and to analyse the monopolist firm's efforts for the green and process innovation relating to knowledge accumulation along the lines of [27–29]. The second one is to analyse the dynamics of firms' green and process innovation in the Cournot duopoly along the lines of [22, 89, 90]. Third, along the lines investigated by [5, 10], one can consider the monopolist's decision regarding green and process innovation, where the network size should be proportional to or equal to the demand size in a dynamic setting.

Author contributions

Huiquan Li: Conceptualization and writing original draft; Huiquan Li and Lijia Ge: Programming; Lijia Ge and Junfang Xi: Validation and revising; Shoude Li: Methodology and supervision.

Use of Generative-AI tools declaration

In the preparation of this work, the authors used Generative AI tools such as ChatGPT to assist in improving the clarity of the language. All AI-assisted content was carefully reviewed and revised by the authors, who take full responsibility for the final version of the manuscript.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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