Electronic
Research Archive

https://www.aimspress.com/journal/era

ERA, 34(6): 3713–3735.
DOI: 10.3934/era.2026168
Received: 22 March 2026
Revised: 22 April 2026
Accepted: 27 April 2026
Published: 30 April 2026

Research article

Command-filter-based predefined-time adaptive tracking control for nonlinear systems with time-varying state constraints and input saturation

Sen Wang, Changqi Jing, Chenxuan Sheng, Huai Liu and Guobao Liu*

School of Electrical and Automation Engineering, Nanjing Normal University, Nanjing 210023, China

* **Correspondence:** Email: lgb5566778@163.com.

Abstract: This paper investigates the adaptive tracking control problem for a class of uncertain nonlinear systems subject to time-varying state constraints and input saturation. To avoid the repeated differentiation encountered in conventional recursive design, a command-filter-based predefined-time adaptive control scheme is developed. Fuzzy logic systems (FLSs) are employed to approximate the unknown nonlinear functions, while a fuzzy observer is constructed to estimate the unmeasurable states. In addition, an auxiliary compensation mechanism is introduced to reduce the influence of filtering errors, and a smooth saturation decomposition together with an auxiliary signal is incorporated to handle the mismatch between the designed control input and the actual actuator output caused by input saturation. By combining recursive design with a time-varying barrier Lyapunov function (BLF), it is shown that all closed-loop signals remain bounded, the prescribed state constraints are not violated, and the tracking error enters a small neighborhood of the origin within a predefined time. Finally, simulation results based on an RLC circuit example are provided to demonstrate the effectiveness of the proposed method.

Keywords: adaptive tracking control; command-filtered design; predefined-time stability; time-varying state constraints; input saturation; fuzzy observer

1. Introduction

Finite-time and fixed-time control have received sustained attention in nonlinear control systems due to their attractive transient-performance characteristics [1–4]. Finite-time control can drive the system states to the desired region in finite time, which makes it appealing in applications requiring fast response, such as autonomous vehicles, robotic systems, and aerospace systems [5–7]. However, the settling time in finite-time control generally depends on the initial conditions, which limits its applicability when the initial state cannot be specified in advance. To overcome this drawback, fixed-time control was developed to guarantee convergence within a uniform upper bound independent of the initial conditions [8–10]. Although fixed-time control improves the predictability of the transient process, the convergence bound is still determined implicitly by the controller parameters.

Predefined-time control further extends this line of research by allowing the convergence time to be assigned explicitly in advance [11–15]. This feature is particularly useful in control tasks with prescribed transient-performance requirements. In [16], predefined-time full-state constrained control was investigated for a mobile robot system with delay. For an RLC circuit, a predefined-time command-filtered control strategy was studied in [17]. In [18], a practical predefined-time stability criterion was established for strict-feedback nonlinear systems with uncertainties and disturbances. These results indicate that predefined-time control provides a flexible framework for balancing transient-speed requirements and closed-loop robustness in nonlinear systems.

Another issue that frequently arises in recursive adaptive control is the so-called explosion of complexity. In conventional backstepping design, repeated differentiation of virtual control laws becomes increasingly cumbersome as the system order grows, which limits practical implementation. Dynamic surface control (DSC) has been widely employed to alleviate this problem by introducing first-order filters [19]. Nevertheless, the filter dynamics inevitably introduce filtering errors, and these errors may affect the achievable control performance if they are not explicitly compensated. To address this issue, command-filter-based design has been developed and successfully applied to various nonlinear control problems [20–22]. Compared with standard DSC, command filtering provides a more convenient way to construct recursive controllers and to incorporate compensation signals for mitigating filter-induced mismatch.

In addition to the above challenges, practical nonlinear systems are often subject to actuator limitations and state constraints. Input saturation is one of the most common actuator nonlinearities and may significantly deteriorate system performance or even lead to instability if ignored in the controller design [23–25]. For instance, input saturation has been considered in multi-agent consensus control and nonlinear adaptive control problems [26]. On the other hand, state constraints are also essential in many engineering systems for safety, physical feasibility, and performance preservation. Barrier Lyapunov functions (BLFs) have been shown to be effective in handling constrained-control problems [27–30]. In particular, time-varying BLFs provide a useful tool for enforcing prescribed state bounds that may change over time, which is desirable in practice when the allowable operating region is not fixed.

Motivated by the above observations, the considered problem is technically challenging because it involves the simultaneous treatment of unavailable full-state information, unknown nonlinearities, time-varying state constraints, actuator saturation, and filter-induced errors caused by the command-filter technique. In addition, predefined-time tracking performance is required in the presence of these factors, which further increases the difficulty of controller design. To address these challenges, a command-filter-based predefined-time adaptive tracking control scheme is developed for uncertain nonlinear systems by combining fuzzy approximation, fuzzy state observation, recursive design, and time-varying BLF techniques.

Compared with related existing results, the main contributions of this paper are summarized as follows:

1) A command-filter-based predefined-time adaptive tracking control framework is developed for uncertain nonlinear systems with time-varying state constraints and actuator saturation. By

integrating a time-varying BLF with fuzzy approximation and fuzzy state observation, the proposed method guarantees constraint satisfaction and ensures that the tracking error enters a small neighborhood of the origin within a predefined time.

2) A command-filter-based controller construction together with an auxiliary compensation mechanism is introduced to avoid repeated differentiation and explicitly handle filter-induced errors in the recursive design, which facilitates the implementation of the predefined-time control scheme under time-varying state constraints.
3) A smooth saturation decomposition together with an auxiliary signal is employed to alleviate the mismatch caused by actuator saturation, which improves the practical implementability of the proposed control scheme under asymmetric input saturation.

2. Problem formulation and preliminaries

2.1. System descriptions

Consider the following output-feedback nonlinear system:

$$\begin{cases} \dot{x}_i = x_{i+1} + g_i(x) + w_i(t), \quad 1 \le i \le n-1, \\ \dot{x}_n = u(\upsilon) + g_n(x) + w_n(t), \\ y = x_1, \end{cases} \tag{2.1}$$

where $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ denotes the system state vector, and $y \in \mathbb{R}$ is the measurable output. The functions $g_i(x)$ are unknown smooth nonlinearities, and the external disturbances $w_i(t)$ satisfy $|w_i(t)| \le \bar{w}_i, i = 1, \ldots, n$. The actuator input is subject to asymmetric saturation and is described by

$$u(\upsilon) = \operatorname{sat}(\upsilon) = \begin{cases} F_h, & \upsilon(t) > F_h, \\ \upsilon(t), & -F_d \le \upsilon(t) \le F_h, \\ -F_d, & \upsilon(t) < -F_d, \end{cases} \tag{2.2}$$

where $F_h > 0$ and $F_d > 0$ are saturation bounds, and $\upsilon(t)$ denotes the designed control input.

Assumption 1. *The reference signal y_r is continuous and bounded. Then, there exists a compact set Ω_{y_r} such that $\Omega_{y_r} = \left\{[y_r, \dot{y}_r]^T : y_r^2 + \dot{y}_r^2 \le \bar{y}_{r0}\right\}$, where $\bar{y}_{r0} > 0$ is a constant.*

Assumption 2. *Each unknown function $g_i(\cdot)$ satisfies the Lipschitz condition $|g_i(G) - g_i(H)| \le \mathcal{X}_i\|G - H\|$, for any $G, H \in \mathbb{R}^n$, where $\mathcal{X}_i > 0$ are constants, $i = 1, \ldots, n$.*

Assumption 3. *For any given constant $K_{b_1} > 0$, there exist positive constants $R_0, R_1, \ldots, R_n$ such that the reference signal y_r and its derivatives up to order n satisfy $|y_r(t)| \le R_0 < K_{b_1}$, and $|y_r^{(i)}(t)| \le R_i, i = 1, 2, \ldots, n$.*

Lemma 1. *([27]) For any $c_d \in \mathbb{R}$ and $\epsilon > 0$, the following inequality holds:*

$$0 \le |c_d| - c_d \tanh\left(\frac{c_d}{\epsilon}\right) \le 0.2785\epsilon.$$

Lemma 2. *([27]) To address the asymmetric saturated input in* (2.2)*, let* $sat(\upsilon) = \Psi(\upsilon) + \Xi(\upsilon)$ *and define its smooth function as follows:*

$$\Psi(v) = r\tanh\left(\frac{\upsilon(t) - \jmath}{r}\right) + \bar{r},$$

where $\Xi(\upsilon)$ *is a bounded term,* $|\Xi(\upsilon)| \leq U$*.* $r = \frac{F_h + F_d}{2}$*,* $\bar{r} = \frac{F_h - F_d}{2}$*,* $\jmath = -r\,\mathrm{artanh}(-\bar{r}/r)$.

Definition 1. *([11]) Let the origin be the equilibrium point of the nonlinear system described by* $\dot{x}(t) = f(x)$*. For any* $t \geq T_d$*, if there exist constants* $\epsilon > 0$ *and* $T_d > 0$ *such that* $\|x(x_0, t)\| \leq \epsilon$*, then the equilibrium point of the system is considered practical predefined time stable (PPTS).*

Remark 1. *In conventional finite-time control, the settling time usually depends on the initial conditions. In fixed-time control, although the settling time becomes independent of the initial conditions, its upper bound is still implicitly determined by the controller parameters. In contrast, predefined-time control allows the convergence time to be explicitly assigned in advance, which is more suitable for practical engineering applications with prescribed transient-performance requirements.*

Lemma 3. *([11]) For a predefined time* $T_d > 0$*, suppose that there exists a positive definite Lyapunov function* $V(x)$ *for the nonlinear system* (2.1) *such that* $\dot{V} \leq -\frac{\pi}{\mu T_d} V^{1+\frac{\mu}{2}} - \frac{\eta_0 \pi}{\mu T_d} V^{1-\frac{\mu}{2}} + \Delta$*, where* $0 < \mu < 1$*,* $\eta_0 = \sqrt{2}$*, and* $\Delta \geq 0$ *is a constant. Then, the origin of the system is PPTS with the predefined time* T_d*. Moreover, the system trajectory ultimately enters the bounded set* $\Omega \triangleq \left\{V(x) : V(x) \leq \frac{2\mu T_d \Delta}{\pi}\right\}$.

Lemma 4. *([12]) For any* $P_1 \geq 0$*,* $P_2 \geq 0$*, and* $c_p > 0$*, one has*

$$P_1^{c_p}(P_2 - P_1) \leq \frac{1}{1 + c_p}\left(P_2^{1+c_p} - P_1^{1+c_p}\right).$$

Lemma 5. *([12]) For a bounded function* b_i*, the following inequality holds:*

$$-\sum_{i=1}^{n} \frac{b_i^2}{h_i} \leq -\left(\sum_{i=1}^{n} \frac{b_i^2}{2h_i}\right)^{\hbar} - \frac{1}{n^{k-1}}\left(\sum_{i=1}^{n} \frac{b_i^2}{2h_i}\right)^{k} + C_f,$$

where $h_i > 0$*,* $k > 1$*,* $0 < \hbar < 1$*, and* $C_f = (1 - \hbar)\hbar^{\frac{\hbar}{1-\hbar}} + \sum_{i=1}^{n}\left(\frac{\bar{b}_i^2}{2h_i}\right)^{k}$*, with* $\bar{b}_i$ *being positive constants satisfying* $|b_i| \leq \bar{b}_i$.

Lemma 6. *([16]) For* $q_i \in \mathbb{R}$*,* $i = 1, \ldots, n$*, the following inequalities hold:*

$$\left(\sum_{i=1}^{n} |q_i|\right)^{\tilde{\imath}} \leq \sum_{i=1}^{n} |q_i|^{\tilde{\imath}}, \quad \left(\sum_{i=1}^{n} |q_i|\right)^{r} \leq n^{r-1} \sum_{i=1}^{n} |q_i|^{r},$$

where $0 < \tilde{\imath} \leq 1$ *and* $r \geq 1$.

Lemma 7. *([17]) For any* $X \in \mathbb{R}^n$*,* $Y \in \mathbb{R}^n$*,* $q > 0$*,* $c_q > 0$*, and* $\omega > 0$*, one has*

$$|X|^{q}|Y|^{c_q} \leq \frac{q}{q + c_q}\omega|X|^{q+c_q} + \frac{c_q}{q + c_q}\omega^{-\frac{q}{c_q}}|Y|^{q+c_q}.$$

2.2. Fuzzy logic systems

FLSs have been widely used to approximate unknown nonlinear functions in nonlinear control systems [27]. Due to their universal approximation capability for continuous nonlinear functions on compact sets and their linearly parameterized form, FLSs are well-suited for the subsequent adaptive-law design and Lyapunov-based stability analysis under the output-feedback framework.

R^l: If χ_1 is A_1^l, χ_2 is $A_2^l, \ldots$, and χ_n is A_n^l, then

$$\beta \quad is \quad B^l, \quad l = 1, 2, \ldots, N_r,$$

where $\chi = [\chi_1, \chi_2, \ldots, \chi_n]^T$ is the input vector, β is the output of the FLS, and N_r denotes the number of fuzzy rules. The membership functions associated with the fuzzy sets A_j^l and B^l are denoted by $\mu_{A_j^l}(\chi_j)$ and $\mu_{B^l}(\beta)$, respectively.

Then, the FLS can be written as

$$\beta(\chi) = \frac{\sum_{l=1}^{N_r} \bar{\beta}_l \prod_{j=1}^{n} \mu_{A_j^l}(\chi_j)}{\sum_{l=1}^{N_r} \prod_{j=1}^{n} \mu_{A_j^l}(\chi_j)}, \tag{2.3}$$

where $\bar{\beta}_l = \max_{\beta \in \mathbb{R}} \mu_{B^l}(\beta)$.

Define $\psi(\chi) = [\psi_1(\chi), \ldots, \psi_{N_r}(\chi)]^T$, $W = \left[\bar{\beta}_1, \ldots, \bar{\beta}_{N_r}\right]^T$, $\psi_l(\chi) = \frac{\prod_{j=1}^{n} \mu_{A_j^l}(\chi_j)}{\sum_{l=1}^{N_r} \prod_{j=1}^{n} \mu_{A_j^l}(\chi_j)}$. Then, (2.3) can be rewritten as

$$\beta(\chi) = W^T \psi(\chi). \tag{2.4}$$

Lemma 8. *([27]) Given that $f(\chi)$ is a continuous function on the compact set Ω, for all $\varepsilon_f > 0$, there exists an FLS that satisfies*

$$\sup_{\chi \in \Omega} \left| f(\chi) - W^T \psi(\chi) \right| \le \varepsilon_f.$$

2.3. Design of the fuzzy observer

Since only the output signal y is measurable in (2.1), a fuzzy state observer is introduced to estimate the unmeasured states.

According to Lemma 8, each unknown nonlinear function $g_i(x)$ can be approximated by an FLS as

$$g_i(\hat{x}) = W_i^{*T} \psi_i(\hat{x}) + \epsilon_i, \tag{2.5}$$

where $\psi_i(\hat{x})$ is the fuzzy basis function vector, W_i^* is the ideal weight vector, $\hat{x}$ is the estimated state vector, and ϵ_i is the approximation error satisfying $|\epsilon_i| \le \bar{\epsilon}_i$, where $\bar{\epsilon}_i > 0$ are unknown constants.

Define $\tilde{g}_i = g_i(x) - g_i(\hat{x}), i = 1, \ldots, n$. Then, by combining (2.1) and (2.5), one obtains

$$\begin{cases} \dot{x}_i = x_{i+1} + W_i^{*T} \psi_i(\hat{x}) + \tilde{g}_i + \epsilon_i + w_i(t), \\ \dot{x}_n = u(v) + W_n^{*T} \psi_n(\hat{x}) + \tilde{g}_n + \epsilon_n + w_n(t), \\ y = x_1. \end{cases} \tag{2.6}$$

The fuzzy state observer is designed as

$$\begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} + \hat{W}_i^T \psi_i(\hat{x}) + l_i(y - \hat{x}_1), \\ \dot{\hat{x}}_n = u(\upsilon) + \hat{W}_n^T \psi_n(\hat{x}) + l_n(y - \hat{x}_1), \\ \hat{y} = \hat{x}_1, \end{cases} \tag{2.7}$$

where $l_i > 0$ are design parameters, and $\hat{W}_i$ denotes the estimate of W_i^*. Define the estimation error $\tilde{W}_i = W_i^* - \hat{W}_i$.

The observation error is defined as

$$\varepsilon = x - \hat{x} = [\varepsilon_1, \ldots, \varepsilon_n]^T. \tag{2.8}$$

By using (2.6)–(2.8), the observation-error dynamics can be derived as

$$\dot{\varepsilon} = A\varepsilon + \sum_{i=1}^{n} B_i \tilde{W}_i^T \psi_i(\hat{x}) + \tilde{G} + E + D, \tag{2.9}$$

where $B_i = [\underbrace{0, \ldots, 0, 1}_{i}, 0, \ldots, 0]^T$, $\tilde{G} = \begin{bmatrix} \tilde{g}_1 \\ \tilde{g}_2 \\ \vdots \\ \tilde{g}_n \end{bmatrix}$, $A = \begin{bmatrix} -l_1 & & & \\ -l_2 & & I_{n-1} & \\ \vdots & & & \\ -l_n & 0 & \cdots & 0 \end{bmatrix}$, $E = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$, $D = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$.

Remark 2. *The parameter l_i is selected appropriately to construct the matrix A as a strictly Hurwitz matrix, guaranteeing the existence of a symmetric matrix S that satisfies the equation $A^T S + SA = -Q$ for every positive definite matrix $Q = Q^T$.*

Choose the following Lyapunov function:

$$V_0 = \varepsilon^T S \varepsilon. \tag{2.10}$$

The time derivative of V_0 is

$$\dot{V}_0 = -\varepsilon^T Q \varepsilon + 2\varepsilon^T S \left(\sum_{i=1}^{n} B_i \tilde{W}_i^T \psi_i(\hat{x}) + \tilde{G} + E + D \right). \tag{2.11}$$

Using $0 < \psi_i^T(\hat{x})\psi_i(\hat{x}) \leq 1$, Assumption 2, and Young's inequality, one has

$$2\varepsilon^T S \Big(\sum_{i=1}^{n} B_i \tilde{W}_i^T \psi_i(\hat{x}) + \tilde{G} + E + D \Big) \leq \sum_{i=1}^{n} \mathcal{X}_i^2 \|\varepsilon\|^2 + \sum_{i=1}^{n} \tilde{W}_i^T \tilde{W}_i + \|E\|^2 + \|\bar{D}\|^2 + (3+n)\|S\|^2\|\varepsilon\|^2, \tag{2.12}$$

where $\bar{D} = [\bar{w}_1, \ldots, \bar{w}_n]^T$.

Combining (2.11) and (2.12) yields

$$\dot{V}_0 \leq -\Lambda_0 \|\varepsilon\|^2 + \sum_{i=1}^{n} \tilde{W}_i^T \tilde{W}_i + N_0, \tag{2.13}$$

where $\Lambda_0 = \lambda_{\min}(Q) - (3+n)\|S\|^2 - \sum_{i=1}^{n} \mathcal{X}_i^2$, and $N_0 = \|E\|^2 + \|\bar{D}\|^2$.

2.4. The predefined-time filter

To avoid the repeated differentiation involved in the recursive controller design, an auxiliary predefined-time filter is introduced as

$$\begin{cases} \dot{\lambda}_i = -c_1\sigma_i^{1+\mu} - c_2\sigma_i^{1-\mu}\tanh\left(\dfrac{c_2\sigma_i^{2-\mu}}{\tau_i}\right) - \rho_i\sigma_i, \\ \lambda_i(0) = \alpha_i(0), \end{cases} \tag{2.14}$$

where $\sigma_i = \lambda_{i+1,c} - \alpha_i$ represents the filtering error; $\lambda_{i+1,c}(t) = \lambda_i(t)$ and α_i denote the output and input of the filter, respectively; $c_1 = \frac{\eta\pi}{\vartheta\mu T_d}$, $c_2 = \frac{\bar{\eta}\pi}{\bar{\vartheta}\mu T_d}$, with $\eta = (2n)^{\mu/2}$, $\bar{\eta} = \sqrt{2}$, $\vartheta = 2^{1+\frac{\mu}{2}}$, $\bar{\vartheta} = 2^{1-\frac{\mu}{2}}$, $\mu = \mu_1/\mu_2$, and μ_1 and μ_2 satisfy $\mu_1 < \mu_2$; T_d denotes the predefined time, which is a positive design parameter; and $\tau_i > 0$ and $\rho_i > 1/2$ are designed parameters.

Theorem 1. *The filtering error σ_i enters the bounded set $\Omega_1 = \left\{\sigma = [\sigma_1, \ldots, \sigma_{n-1}]^T : V_\sigma \leq \frac{2\mu T_d \Delta_\sigma}{\pi}\right\}$ within the predefined time T_d, where $V_\sigma = \sum_{\varsigma=1}^{n-1} \frac{1}{2}\sigma_\varsigma^2$.*

Proof. Consider the Lyapunov function

$$V_\sigma = \sum_{\varsigma=1}^{n-1} \frac{1}{2}\sigma_\varsigma^2. \tag{2.15}$$

From (2.14), one has $\sigma_\varsigma = \lambda_{\varsigma+1,c} - \alpha_\varsigma$, and therefore

$$\begin{aligned} \dot{V}_\sigma &= \sum_{\varsigma=1}^{n-1} \sigma_\varsigma\dot{\sigma}_\varsigma = \sum_{\varsigma=1}^{n-1} \sigma_\varsigma\left(\dot{\lambda}_{\varsigma+1,c} - \dot{\alpha}_\varsigma\right) \\ &= \sum_{\varsigma=1}^{n-1}\left[- c_1|\sigma_\varsigma|^{2+\mu} - c_2|\sigma_\varsigma|^{2-\mu}\tanh\left(\frac{c_2|\sigma_\varsigma|^{2-\mu}}{\tau_\varsigma}\right) - \rho_\varsigma\sigma_\varsigma^2 - \sigma_\varsigma\dot{\alpha}_\varsigma\right]. \end{aligned} \tag{2.16}$$

From the recursive controller construction, together with the boundedness of the reference signal and the adaptive estimates, one can conclude that $\dot{\alpha}_\varsigma$ is bounded. Hence, there exists a positive constant $\bar{\alpha}_\varsigma$ such that $|\dot{\alpha}_\varsigma| \leq \bar{\alpha}_\varsigma, \varsigma = 1, \ldots, n-1$. Then, by Young's inequality,

$$-\sigma_\varsigma\dot{\alpha}_\varsigma \leq \frac{1}{2}\sigma_\varsigma^2 + \frac{1}{2}\bar{\alpha}_\varsigma^2. \tag{2.17}$$

Moreover, by Lemma 1, one has $-c_2|\sigma_\varsigma|^{2-\mu}\tanh\left(\frac{c_2|\sigma_\varsigma|^{2-\mu}}{\tau_\varsigma}\right) \leq -c_2|\sigma_\varsigma|^{2-\mu} + 0.2785\tau_\varsigma$. Substituting this inequality and (2.17) into (2.16), and using $\rho_\varsigma > 1/2$, yields

$$\dot{V}_\sigma \leq -\sum_{\varsigma=1}^{n-1} c_1|\sigma_\varsigma|^{2+\mu} - \sum_{\varsigma=1}^{n-1} c_2|\sigma_\varsigma|^{2-\mu} + \Delta_\sigma, \tag{2.18}$$

where $\Delta_\sigma = \sum_{\varsigma=1}^{n-1}\left(\frac{1}{2}\bar{\alpha}_\varsigma^2 + 0.2785\tau_\varsigma\right)$.

By Lemma 6 and the definitions of c_1 and c_2, one obtains

$$\dot{V}_\sigma \leq -\frac{\pi}{\mu T_d}V_\sigma^{1+\frac{\mu}{2}} - \frac{\bar{\eta}\pi}{\mu T_d}V_\sigma^{1-\frac{\mu}{2}} + \Delta_\sigma. \tag{2.19}$$

Then, according to Lemma 3, the filtering error σ_i enters the bounded set $\Omega_1 = \left\{\sigma = [\sigma_1, \ldots, \sigma_{n-1}]^T : V_\sigma \leq \frac{2\mu T_d\Delta_\sigma}{\pi}\right\}$ within the predefined time T_d. The proof is complete.

3. Controller design and stability analysis

In this section, a command-filter-based predefined-time adaptive control scheme is developed for system (2.1). To facilitate the recursive design, we define the following coordinate transformation:

$$\begin{cases} s_1 = y - y_r, \\ s_i = \hat{x}_i - \lambda_{i,c},\ i = 2, \ldots, n-1, \\ s_n = \hat{x}_n - \lambda_{n,c} - \chi_u, \end{cases} \tag{3.1}$$

where s_1 denotes the tracking error and χ_u is an auxiliary signal introduced to compensate for the actuator saturation effect.

Define the filter compensation signal ν_i as

$$\begin{cases} \dot{\nu}_1 = \nu_2 + (\lambda_{2,c} - \alpha_1) - a_1|\nu_1|^{1+\mu}\mathrm{sgn}(\nu_1) - a_2|\nu_1|^{1-\mu}\mathrm{sgn}(\nu_1), \\ \dot{\nu}_i = \nu_{i+1} + (\lambda_{i+1,c} - \alpha_i) - \nu_{i-1} - a_1|\nu_i|^{1+\mu}\mathrm{sgn}(\nu_i) - a_2|\nu_i|^{1-\mu}\mathrm{sgn}(\nu_i), \\ \dot{\nu}_n = -\nu_{n-1} - a_1|\nu_n|^{1+\mu}\mathrm{sgn}(\nu_n) - a_2|\nu_n|^{1-\mu}\mathrm{sgn}(\nu_n), \end{cases} \tag{3.2}$$

where $a_1 = \frac{\pi}{\mu T_d 2^{\mu/2}}$, $a_2 = \frac{\bar{\eta}\pi}{\mu T_d 2^{1-\mu/2}}$.

Then, define the compensated error variables as

$$z_i = s_i - \nu_i, \quad i = 1, 2, \ldots, n. \tag{3.3}$$

For notational simplicity, define $\bar{x}_i = [\hat{x}_1, \ldots, \hat{x}_i]^T \in \mathbb{R}^i, i = 1, \ldots, n$.

3.1. Predefined-time adaptive backstepping control design

Step 1: From (2.6)–(2.8), (3.1), and (3.3), the derivative of z_1 is given by

$$\dot{z}_1 = s_2 + \lambda_{2,c} + \varepsilon_2 + W_1^{*T}\psi_1(\hat{x}) + \tilde{g}_1 + \epsilon_1 + w_1 - \dot{y}_r - \dot{\nu}_1. \tag{3.4}$$

The barrier Lyapunov function is constructed as

$$V_1 = V_0 + \frac{1}{2}\log\frac{K_{b_1}^2(t)}{K_{b_1}^2(t) - z_1^2} + \frac{1}{2}\tilde{W}_1^T\tilde{W}_1 + \frac{1}{2}\tilde{M}_1^2, \tag{3.5}$$

where $|z_1| < K_{b_1}(t)$, $\tilde{M}_1 = M_1^* - \hat{M}_1$, and $M_1^* = \|W_1^*\|^2$.

The time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 = {}& \dot{V}_0 + \frac{z_1}{K_{b_1}^2(t) - z_1^2}(\varepsilon_2 + \tilde{g}_1 + \epsilon_1 + w_1) - \tilde{W}_1^T\dot{\hat{W}}_1 + \frac{z_1}{K_{b_1}^2(t) - z_1^2}\left(-W_1^{*T}\psi_1(\bar{x}_1) + W_1^{*T}\psi_1(\hat{x})\right) \\ & + \frac{z_1}{K_{b_1}^2(t) - z_1^2}\left(s_2 + \lambda_{2,c} - \frac{\dot{K}_{b_1}(t)z_1}{K_{b_1}(t)} + \hat{W}_1^T\psi_1(\bar{x}_1)\right) - \tilde{M}_1\dot{\hat{M}}_1 + \frac{z_1}{K_{b_1}^2(t) - z_1^2}\left(\tilde{W}_1^T\psi_1(\bar{x}_1) - \dot{y}_r - \dot{\nu}_1\right). \end{aligned} \tag{3.6}$$

By Young's inequality and Assumption 2, one obtains

$$\frac{z_1}{K_{b_1}^2(t) - z_1^2}(\varepsilon_2 + \tilde{g}_1 + \epsilon_1 + w_1) \le \frac{2z_1^2}{\left(K_{b_1}^2(t) - z_1^2\right)^2} + \frac{1}{2}\|\varepsilon\|^2 + \frac{\mathcal{X}_1^2}{2}\|\varepsilon\|^2 + \frac{1}{2}\|\epsilon\|^2 + \frac{1}{2}\bar{w}_1^2, \tag{3.7}$$

$$\frac{z_1}{K_{b_1}^2(t)-z_1^2}\left(W_1^{*T}\psi_1(\hat{x})-W_1^{*T}\psi_1(\bar{x}_1)\right)\leq\frac{2z_1^2M_1^*}{\left(K_{b_1}^2(t)-z_1^2\right)^2}+\frac{1}{2}. \tag{3.8}$$

Substituting (2.13) and (3.6)–(3.8) yields

$$\begin{aligned}\dot{V}_1\leq&-\Lambda_1\|\varepsilon\|^2+N_1+\tilde{W}_1^T\left[\frac{z_1\psi_1(\bar{x}_1)}{K_{b_1}^2(t)-z_1^2}-\dot{\hat{W}}_1\right]+\frac{z_1}{K_{b_1}^2(t)-z_1^2}\left[\frac{2z_1}{K_{b_1}^2(t)-z_1^2}+\frac{2z_1\hat{M}_1}{K_{b_1}^2(t)-z_1^2}+s_2+\alpha_1\right.\\&\left.+(\lambda_{2,c}-\alpha_1)+\hat{W}_1^T\psi_1(\bar{x}_1)-\dot{y}_r-\dot{\nu}_1-\frac{\dot{K}_{b_1}(t)z_1}{K_{b_1}(t)}\right]+\sum_{i=1}^{n}\tilde{W}_i^T\tilde{W}_i+\tilde{M}_1\left[\frac{2z_1^2}{\left(K_{b_1}^2(t)-z_1^2\right)^2}-\dot{\hat{M}}_1\right],\end{aligned}\tag{3.9}$$

where $\Lambda_1=\Lambda_0-\frac{1}{2}-\frac{\mathcal{X}_1^2}{2}$, $N_1=N_0+\frac{1}{2}\|E\|^2+\frac{1}{2}\bar{w}_1^2+\frac{1}{2}$.

We design the virtual controller α_1, the adaptation laws $\dot{\hat{W}}_1$, $\dot{\hat{M}}_1$, and the compensation signal $\dot{\nu}_1$ as

$$\begin{aligned}\alpha_1=&-c_1\frac{\operatorname{sgn}(z_1)|z_1|^{1+\mu}}{\left(K_{b_1}^2(t)-z_1^2\right)^{\mu/2}}-c_2\frac{\operatorname{sgn}(z_1)|z_1|^{1-\mu}}{\left(K_{b_1}^2(t)-z_1^2\right)^{-\mu/2}}\tanh\left(\frac{c_2\left(\frac{z_1^2}{K_{b_1}^2(t)-z_1^2}\right)^{1-\mu/2}}{\tau_1}\right)-\hat{W}_1^T\psi_1(\bar{x}_1)+\dot{y}_r\\&-\frac{2z_1}{K_{b_1}^2(t)-z_1^2}-\frac{2z_1\hat{M}_1}{K_{b_1}^2(t)-z_1^2}+\frac{\dot{K}_{b_1}(t)z_1}{K_{b_1}(t)}-a_1|\nu_1|^{1+\mu}\operatorname{sgn}(\nu_1)-a_2|\nu_1|^{1-\mu}\operatorname{sgn}(\nu_1),\end{aligned}\tag{3.10}$$

$$\dot{\nu}_1=(\lambda_{2,c}-\alpha_1)+\nu_2-a_1|\nu_1|^{1+\mu}\operatorname{sgn}(\nu_1)-a_2|\nu_1|^{1-\mu}\operatorname{sgn}(\nu_1),\tag{3.11}$$

$$\dot{\hat{W}}_1=\frac{z_1\psi_1(\bar{x}_1)}{K_{b_1}^2(t)-z_1^2}-4\hat{W}_1,\tag{3.12}$$

$$\dot{\hat{M}}_1=\frac{2z_1^2}{\left(K_{b_1}^2(t)-z_1^2\right)^2}-\delta c_1\hat{M}_1^{1+\mu}-\bar{\delta}c_2\hat{M}_1^{1-\mu},\tag{3.13}$$

where $\delta=2+\mu$, $\bar{\delta}=2-\mu$.

Remark 3. *From* (3.10)–(3.13)*, the hyperbolic tangent function is introduced to ensure that the derivative of* α_1 *remains well-defined as* $z_1\to 0$*, thereby avoiding control singularity in the recursive design.*

Substituting (3.10)–(3.13) into (3.9), and using Lemma 1 together with Young's inequality $4\tilde{W}_1^T\hat{W}_1\leq-2\tilde{W}_1^T\tilde{W}_1+2W_1^{*T}W_1^*$, gives

$$\begin{aligned}\dot{V}_1\leq&-c_1\left(\frac{z_1^2}{K_{b_1}^2(t)-z_1^2}\right)^{1+\frac{\mu}{2}}-c_2\left(\frac{z_1^2}{K_{b_1}^2(t)-z_1^2}\right)^{1-\frac{\mu}{2}}+\bar{N}_1+\frac{z_1z_2}{K_{b_1}^2(t)-z_1^2}\\&+\sum_{i=2}^{n}\tilde{W}_i^T\tilde{W}_i+\delta c_1\tilde{M}_1\hat{M}_1^{1+\mu}+\bar{\delta}c_2\tilde{M}_1\hat{M}_1^{1-\mu}-\Lambda_1\|\varepsilon\|^2-\tilde{W}_1^T\tilde{W}_1,\end{aligned}\tag{3.14}$$

where $\bar{N}_1=N_1+0.2785\tau_1+2W_1^{*T}W_1^*$.

Step *i* ($2 \le i \le n-1$): Differentiating z_i yields

$$\dot{z}_i = \dot{s}_i - \dot{v}_i = s_{i+1} + \lambda_{i+1,c} + \hat{W}_i^T \psi_i(\hat{x}) + l_i(y - \hat{x}_1) - \dot{\lambda}_{i,c} - \dot{v}_i. \tag{3.15}$$

We choose the Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2}\log\frac{K_{b_i}^2(t)}{K_{b_i}^2(t) - z_i^2} + \frac{1}{2}\tilde{W}_i^T\tilde{W}_i + \frac{1}{2}\tilde{M}_i^2, \tag{3.16}$$

where $|z_i| < K_{b_i}(t)$, $\tilde{M}_i = M_i^* - \hat{M}_i$, and $M_i^* = \|W_i^*\|^2$.

Then, the time derivative of V_i is

$$\begin{aligned}\dot{V}_i = \dot{V}_{i-1} &+ \frac{z_i}{K_{b_i}^2(t) - z_i^2}\left(-W_i^{*T}\psi_i(\bar{x}_i) + W_i^{*T}\psi_i(\hat{x})\right) - \tilde{W}_i^T\dot{\hat{W}}_i + \frac{z_i}{K_{b_i}^2(t) - z_i^2}\left(l_i(y - \hat{x}_1) - \tilde{W}_i^T\psi_i(\hat{x})\right)\\ &- \tilde{M}_i\dot{\hat{M}}_i + \frac{z_i}{K_{b_i}^2(t) - z_i^2}\left(s_{i+1} + \lambda_{i+1,c} - \dot{v}_i - \dot{\lambda}_{i,c}\right) + \frac{z_i}{K_{b_i}^2(t) - z_i^2}\left(\tilde{W}_i^T\psi_i(\bar{x}_i) + \hat{W}_i^T\psi_i(\bar{x}_i) - \frac{\dot{K}_{b_i}(t)z_i}{K_{b_i}(t)}\right).\end{aligned} \tag{3.17}$$

By Young's inequality, one has

$$\frac{z_i}{K_{b_i}^2(t) - z_i^2}\left(-W_i^{*T}\psi_i(\bar{x}_i) + W_i^{*T}\psi_i(\hat{x})\right) \le \frac{2z_i^2 M_i^*}{\left(K_{b_i}^2(t) - z_i^2\right)^2} + \frac{1}{2}, \tag{3.18}$$

$$\frac{z_i}{K_{b_i}^2(t) - z_i^2}\left(-\tilde{W}_i^T\psi_i(\hat{x}) + l_i(y - \hat{x}_1)\right) \le \frac{z_i^2}{\left(K_{b_i}^2(t) - z_i^2\right)^2} + \frac{1}{2}\tilde{W}_i^T\tilde{W}_i + \frac{1}{2}l_i^2\|\varepsilon\|^2. \tag{3.19}$$

Then, we design the virtual controller α_i, the adaptation laws $\dot{\hat{W}}_i$, $\dot{\hat{M}}_i$, and the compensation signal $\dot{v}_i$ as

$$\begin{aligned}\alpha_i = &- c_1\frac{\mathrm{sgn}(z_i)|z_i|^{1+\mu}}{\left(K_{b_i}^2(t) - z_i^2\right)^{\mu/2}} - c_2\frac{\mathrm{sgn}(z_i)|z_i|^{1-\mu}}{\left(K_{b_i}^2(t) - z_i^2\right)^{-\mu/2}}\tanh\left(\frac{c_2\left(\frac{z_i^2}{K_{b_i}^2(t) - z_i^2}\right)^{1-\mu/2}}{\tau_i}\right) - \frac{2z_i\hat{M}_i}{K_{b_i}^2(t) - z_i^2}\\ &- \frac{z_i}{K_{b_i}^2(t) - z_i^2} - \frac{K_{b_i}^2(t) - z_i^2}{K_{b_{i-1}}^2(t) - z_{i-1}^2}z_{i-1} - a_1|v_i|^{1+\mu}\mathrm{sgn}(v_i) - a_2|v_i|^{1-\mu}\mathrm{sgn}(v_i) - v_{i-1}\\ &- \hat{W}_i^T\psi_i(\bar{x}_i) + \dot{\lambda}_{i,c} + \frac{\dot{K}_{b_i}(t)z_i}{K_{b_i}(t)},\end{aligned} \tag{3.20}$$

$$\dot{v}_i = (\lambda_{i+1,c} - \alpha_i) + v_{i+1} - v_{i-1} - a_1|v_i|^{1+\mu}\mathrm{sgn}(v_i) - a_2|v_i|^{1-\mu}\mathrm{sgn}(v_i), \tag{3.21}$$

$$\dot{\hat{W}}_i = \frac{z_i\psi_i(\bar{x}_i)}{K_{b_i}^2(t) - z_i^2} - 5\hat{W}_i, \tag{3.22}$$

$$\dot{\hat{M}}_i = \frac{2z_i^2}{\left(K_{b_i}^2(t) - z_i^2\right)^2} - \delta c_1\hat{M}_i^{1+\mu} - \bar{\delta}c_2\hat{M}_i^{1-\mu}. \tag{3.23}$$

Substituting (3.18)–(3.23) into (3.17), and using Lemma 1 together with Young's inequality $5\tilde{W}_i^T\hat{W}_i \le -\frac{5}{2}\tilde{W}_i^T\tilde{W}_i + \frac{5}{2}W_i^{*T}W_i^*$, one obtains

$$\begin{aligned}\dot{V}_i \le & -c_1\sum_{\varsigma=1}^{i}\left(\frac{z_\varsigma^2}{K_{b_\varsigma}^2(t)-z_\varsigma^2}\right)^{1+\frac{\mu}{2}} - c_2\sum_{\varsigma=1}^{i}\left(\frac{z_\varsigma^2}{K_{b_\varsigma}^2(t)-z_\varsigma^2}\right)^{1-\frac{\mu}{2}} + \sum_{\varsigma=i+1}^{n}\tilde{W}_\varsigma^T\tilde{W}_\varsigma - \sum_{\varsigma=1}^{i}\tilde{W}_\varsigma^T\tilde{W}_\varsigma \\ & - \Lambda_i\|\varepsilon\|^2 + \sum_{\varsigma=1}^{i}\delta c_1\tilde{M}_\varsigma\hat{M}_\varsigma^{1+\mu} + \sum_{\varsigma=1}^{i}\bar{\delta}c_2\tilde{M}_\varsigma\hat{M}_\varsigma^{1-\mu} + \frac{z_iz_{i+1}}{K_{b_i}^2(t)-z_i^2} + \bar{N}_i,\end{aligned} \tag{3.24}$$

where $\bar{N}_i = \frac{5}{2}W_i^{*T}W_i^* + 0.2785\tau_i + \bar{N}_{i-1} + \frac{1}{2}$, $\Lambda_i = \Lambda_{i-1} - \frac{1}{2}l_i^2$.

Step n: According to Lemma 2, the derivative of z_n is

$$\dot{z}_n = \Xi(\upsilon) + \hat{W}_n^T\psi_n(\hat{x}) + l_n(y-\hat{x}_1) - \dot{\lambda}_{n,c} + \chi_u + \upsilon - \dot{\nu}_n, \tag{3.25}$$

where the auxiliary signal χ_u is designed as $\dot{\chi}_u = -\chi_u - \upsilon + \Psi(\upsilon)$.

We choose the Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2}\log\frac{K_{b_n}^2(t)}{K_{b_n}^2(t)-z_n^2} + \frac{1}{2}\tilde{W}_n^T\tilde{W}_n + \frac{1}{2}\tilde{M}_n^2, \tag{3.26}$$

where $|z_n| < K_{b_n}(t)$, $\tilde{M}_n = M_n^* - \hat{M}_n$, and $M_n^* = \|W_n^*\|^2$.

Then, the derivative of V_n is

$$\begin{aligned}\dot{V}_n = & \dot{V}_{n-1} + \frac{z_n}{K_{b_n}^2(t)-z_n^2}\left(W_n^{*T}\psi_n(\hat{x}) - W_n^{*T}\psi_n(\bar{x}_n)\right) \\ & + \frac{z_n}{K_{b_n}^2(t)-z_n^2}\left(-\tilde{W}_n^T\psi_n(\hat{x}) + l_n(y-\hat{x}_1)\right) - \tilde{W}_n^T\dot{\hat{W}}_n \\ & + \frac{z_n}{K_{b_n}^2(t)-z_n^2}\left(\chi_u + \upsilon - \dot{\lambda}_{n,c} - \dot{\nu}_n + \Xi(\upsilon)\right) - \tilde{M}_n\dot{\hat{M}}_n \\ & + \frac{z_n}{K_{b_n}^2(t)-z_n^2}\left(\tilde{W}_n^T\psi_n(\bar{x}_n) - \frac{\dot{K}_{b_n}(t)z_n}{K_{b_n}(t)} + \hat{W}_n^T\psi_n(\bar{x}_n)\right).\end{aligned} \tag{3.27}$$

By Young's inequality, it follows that

$$\frac{z_n}{K_{b_n}^2(t)-z_n^2}\left(-W_n^{*T}\psi_n(\bar{x}_n) + W_n^{*T}\psi_n(\hat{x})\right) \le \frac{1}{2} + \frac{2z_n^2M_n^*}{\left(K_{b_n}^2(t)-z_n^2\right)^2}, \tag{3.28}$$

$$\frac{z_n}{K_{b_n}^2(t)-z_n^2}\left(-\tilde{W}_n^T\psi_n(\hat{x}) + l_n(y-\hat{x}_1)\right) \le \frac{z_n^2}{\left(K_{b_n}^2(t)-z_n^2\right)^2} + \frac{1}{2}\tilde{W}_n^T\tilde{W}_n + \frac{1}{2}l_n^2\|\varepsilon\|^2, \tag{3.29}$$

$$\frac{z_n}{K_{b_n}^2(t)-z_n^2}\Xi(\upsilon) \le \frac{z_n^2}{2\left(K_{b_n}^2(t)-z_n^2\right)^2} + \frac{U^2}{2}. \tag{3.30}$$

We design the actual controller υ, the adaptation laws $\dot{\hat{W}}_n$, $\dot{\hat{M}}_n$, and the compensation signal $\dot{\nu}_n$ as

$$\upsilon = -\nu_{n-1} - a_1|\nu_n|^{1+\mu}\text{sgn}(\nu_n) - a_2|\nu_n|^{1-\mu}\text{sgn}(\nu_n) - \frac{K_{b_n}^2(t) - z_n^2}{K_{b_{n-1}}^2(t) - z_{n-1}^2} z_{n-1} - \hat{W}_n^T \psi_n(\bar{x}_n)$$
$$- c_1 \frac{\text{sgn}(z_n)|z_n|^{1+\mu}}{\left(K_{b_n}^2(t) - z_n^2\right)^{\mu/2}} - c_2 \frac{\text{sgn}(z_n)|z_n|^{1-\mu}}{\left(K_{b_n}^2(t) - z_n^2\right)^{-\mu/2}} \tanh\left(\frac{c_2\left(\frac{z_n^2}{K_{b_n}^2(t)-z_n^2}\right)^{1-\mu/2}}{\tau_n}\right)$$
$$- \frac{3z_n}{2\left(K_{b_n}^2(t) - z_n^2\right)} - \frac{2z_n\hat{M}_n}{K_{b_n}^2(t) - z_n^2} + \frac{\dot{K}_{b_n}(t)z_n}{K_{b_n}(t)} - \chi_u + \dot{\lambda}_{n,c}, \tag{3.31}$$

$$\dot{\nu}_n = -\nu_{n-1} - a_1|\nu_n|^{1+\mu}\text{sgn}(\nu_n) - a_2|\nu_n|^{1-\mu}\text{sgn}(\nu_n), \tag{3.32}$$

$$\dot{\hat{W}}_n = \frac{z_n\psi_n(\bar{x}_n)}{K_{b_n}^2(t) - z_n^2} - 5\hat{W}_n, \tag{3.33}$$

$$\dot{\hat{M}}_n = \frac{2z_n^2}{\left(K_{b_n}^2(t) - z_n^2\right)^2} - \delta c_1 \hat{M}_n^{1+\mu} - \bar{\delta} c_2 \hat{M}_n^{1-\mu}. \tag{3.34}$$

Substituting (3.28)–(3.34) into (3.27), and using Young's inequality $5\tilde{W}_n^T\hat{W}_n \le -\frac{5}{2}\tilde{W}_n^T\tilde{W}_n + \frac{5}{2}W_n^{*T}W_n^*$, together with Lemma 1, one has

$$\dot{V}_n \le -\sum_{\varsigma=1}^{n} c_1\left(\frac{z_\varsigma^2}{K_{b_\varsigma}^2(t) - z_\varsigma^2}\right)^{1+\frac{\mu}{2}} - \sum_{\varsigma=1}^{n} c_2\left(\frac{z_\varsigma^2}{K_{b_\varsigma}^2(t) - z_\varsigma^2}\right)^{1-\frac{\mu}{2}}$$
$$- \Lambda_n\|\varepsilon\|^2 + \bar{N}_n - \sum_{\varsigma=1}^{n}\tilde{W}_\varsigma^T\tilde{W}_\varsigma + \sum_{\varsigma=1}^{n}\delta c_1\tilde{M}_\varsigma\hat{M}_\varsigma^{1+\mu} + \sum_{\varsigma=1}^{n}\bar{\delta}c_2\tilde{M}_\varsigma\hat{M}_\varsigma^{1-\mu}, \tag{3.35}$$

where $\Lambda_n = \Lambda_{n-1} - \frac{1}{2}l_n^2$, $\bar{N}_n = \bar{N}_{n-1} + 0.2785\tau_n + \frac{5}{2}W_n^{*T}W_n^* + \frac{1}{2} + \frac{U^2}{2}$.

For the above control design, a schematic diagram of the developed method is shown in Figure 1.

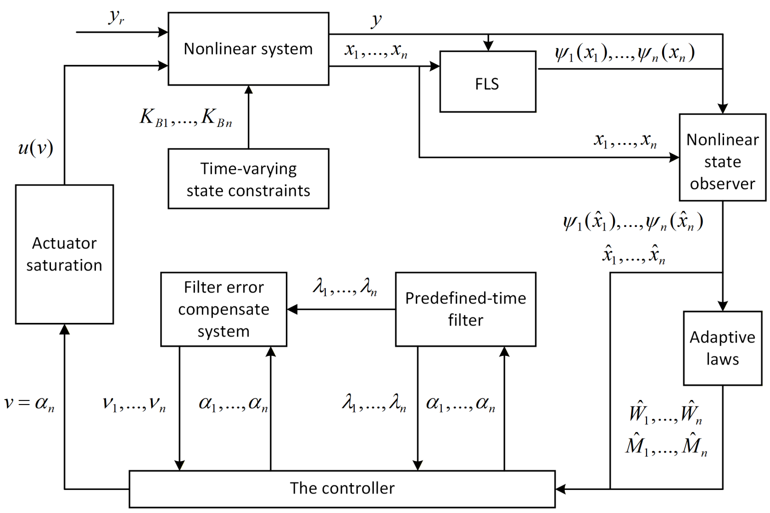

Figure 1. Schematic diagram of the developed control scheme.

Theorem 2. *For the auxiliary compensation system* (3.2)*, under Assumptions 1–3 and the proposed adaptive control scheme, the compensation signal ν_i enters the bounded set $\Omega_2 = \left\{\nu = [\nu_1, \ldots, \nu_n]^T : V_\nu \leq \frac{2\mu T_d \Delta_\nu}{\pi}\right\}$ within the predefined time T_d.*

Proof. Consider the Lyapunov function

$$V_\nu = \sum_{\varsigma=1}^{n} \frac{1}{2}\nu_\varsigma^2. \tag{3.36}$$

With the help of (3.2), the expression for the time derivative of V_ν is obtained as follows:

$$\dot{V}_\nu = \sum_{\varsigma=1}^{n} \nu_\varsigma \dot{\nu}_\varsigma \leq \sum_{\varsigma=1}^{n} \left(\nu_\varsigma(\lambda_{\varsigma+1,c} - \alpha_\varsigma) - \frac{2\pi}{\mu T_d 2^{1+\frac{\mu}{2}}} \nu_\varsigma^{2+\mu} - \frac{\bar{\eta}\pi}{\mu T_d 2^{1-\frac{\mu}{2}}} \nu_\varsigma^{2-\mu} \right). \tag{3.37}$$

Based on Lemma 7, one has

$$\nu_\varsigma(\lambda_{\varsigma+1} - \alpha_\varsigma) \leq \Gamma_\sigma \nu_\varsigma \leq \frac{\pi}{\mu T_d}\left(\frac{1}{2}\nu_\varsigma^2\right)^{1+\frac{\mu}{2}} + \frac{(1+\mu)\Gamma_\sigma}{2+\mu}\left(\frac{\pi(2+\mu)}{\Gamma_\sigma \mu T_d 2^{1-\frac{\mu}{2}}}\right)^{\frac{1}{-1-\mu}}, \tag{3.38}$$

where $\Gamma_\sigma > 0$.

Substituting (3.38) into (3.37), we have

$$\begin{aligned} \dot{V}_\nu &\leq \sum_{\varsigma=1}^{n} \left(-\frac{\pi}{\mu T_d}\Big(\frac{1}{2}\nu_\varsigma^2\Big)^{1+\frac{\mu}{2}} - \frac{\bar{\eta}\pi}{\mu T_d}\Big(\frac{1}{2}\nu_\varsigma^2\Big)^{1-\frac{\mu}{2}} + \frac{(1+\mu)\Gamma_\sigma}{2+\mu}\left(\frac{\pi(2+\mu)}{\Gamma_\sigma \mu T_d 2^{1-\frac{\mu}{2}}}\right)^{\frac{1}{-1-\mu}} \right) \\ &\leq -\frac{\pi}{\mu T_d} V_\nu^{1+\frac{\mu}{2}} - \frac{\bar{\eta}\pi}{\mu T_d} V_\nu^{1-\frac{\mu}{2}} + \Delta_\nu, \end{aligned} \tag{3.39}$$

where $\Delta_\nu = \sum_{\varsigma=1}^{n} \frac{(1+\mu)\Gamma_\sigma}{2+\mu}\left(\frac{\pi(2+\mu)}{\Gamma_\sigma \mu T_d 2^{1-\frac{\mu}{2}}}\right)^{\frac{1}{-1-\mu}}$.

According to Lemma 3, it can be proved that the filter compensation signal ν_i converges to the bounded set Ω_2 within the predefined time T_d.

Theorem 3. *Consider the uncertain nonlinear system* (2.1)*. Under Assumptions 1–3, together with Theorems 1 and 2, and Lemmas 1–8, the fuzzy observer* (2.7)*, predefined-time filter* (2.14)*, virtual and actual control laws* (3.10)*,* (3.20)*, and* (3.31)*, adaptation laws* (3.12)*,* (3.13)*,* (3.22)*,* (3.23)*,* (3.33)*,* (3.34)*, and compensation system* (3.2) *guarantee that:*

i. *the prescribed state constraints are not violated for all $t \geq 0$;*
ii. *the tracking error enters a small neighborhood of the origin no later than the predefined time T_d;*
iii. *all signals in the closed-loop system remain bounded.*

Proof. According to Lemma 4, for each $\varsigma = 1, \ldots, n$, one has

$$\begin{aligned} \tilde{M}_\varsigma \hat{M}_\varsigma^{1+\mu} &\leq \frac{1}{\delta}\left(M_\varsigma^{*\delta} - \hat{M}_\varsigma^{\delta}\right) \leq \frac{1}{\delta}\left(2M_\varsigma^{*\delta} - \tilde{M}_\varsigma^{\delta}\right), \\ \tilde{M}_\varsigma \hat{M}_\varsigma^{1-\mu} &\leq \frac{1}{\bar{\delta}}\left(M_\varsigma^{*\bar{\delta}} - \hat{M}_\varsigma^{\bar{\delta}}\right) \leq \frac{1}{\bar{\delta}}\left(2M_\varsigma^{*\bar{\delta}} - \tilde{M}_\varsigma^{\bar{\delta}}\right). \end{aligned} \tag{3.40}$$

According to Lemma 2 in [27], one has

$$-\frac{z_i^2}{K_{b_i}^2(t)-z_i^2}\leq -\log\frac{K_{b_i}^2(t)}{K_{b_i}^2(t)-z_i^2}. \tag{3.41}$$

Substituting (3.40) and (3.41) into (3.35), and choosing the design parameters such that $\Lambda_n \geq 2\lambda_{\max}(S)$, yields

$$\begin{aligned}\dot{V}_n \leq & -\sum_{\varsigma=1}^{n} c_1\left(\log\frac{K_{b_\varsigma}^2(t)}{K_{b_\varsigma}^2(t)-z_\varsigma^2}\right)^{1+\frac{\mu}{2}}-\sum_{\varsigma=1}^{n} c_2\left(\log\frac{K_{b_\varsigma}^2(t)}{K_{b_\varsigma}^2(t)-z_\varsigma^2}\right)^{1-\frac{\mu}{2}}\\ & -\sum_{\varsigma=1}^{n}\tilde{W}_\varsigma^T\tilde{W}_\varsigma-2\varepsilon^T S\varepsilon-\sum_{\varsigma=1}^{n}c_1\tilde{M}_\varsigma^{2+\mu}-\sum_{\varsigma=1}^{n}c_2\tilde{M}_\varsigma^{2-\mu}+N_n,\end{aligned} \tag{3.42}$$

where $N_n=\sum_{\varsigma=1}^{n}c_1M_\varsigma^{*\,2+\mu}+\sum_{\varsigma=1}^{n}c_2M_\varsigma^{*\,2-\mu}+\bar{N}_n$.

Furthermore, applying Lemma 5 gives

$$\begin{aligned}&-2\varepsilon^T S\varepsilon\leq -(\varepsilon^T S\varepsilon)^{1-\frac{\mu}{2}}-(\varepsilon^T S\varepsilon)^{1+\frac{\mu}{2}}+C_\varepsilon,\\ &-\tilde{W}_\varsigma^T\tilde{W}_\varsigma\leq -\left(\frac{1}{2}\tilde{W}_\varsigma^T\tilde{W}_\varsigma\right)^{1-\frac{\mu}{2}}-\left(\frac{1}{2}\tilde{W}_\varsigma^T\tilde{W}_\varsigma\right)^{1+\frac{\mu}{2}}+C_W,\end{aligned} \tag{3.43}$$

where $C_\varepsilon=\frac{\mu}{2}\left(1-\frac{\mu}{2}\right)^{\frac{2-\mu}{\mu}}+\bar{\varepsilon}^{1+\mu/2}$, $C_W=\frac{\mu}{2}\left(1-\frac{\mu}{2}\right)^{\frac{2-\mu}{\mu}}+\left(\frac{\bar{W}_\varsigma^2}{2}\right)^{1+\mu/2}$, and $\bar{\varepsilon}$ and $\bar{W}_\varsigma$ are positive constants satisfying $\|\varepsilon\|\leq\bar{\varepsilon}$, $\|\tilde{W}_\varsigma\|\leq\bar{W}_\varsigma$.

Substituting (3.43) into (3.42), and choosing the design parameters such that $\mu T_d\geq\max\{\eta\pi,\bar{\eta}\pi\}$, one obtains

$$\begin{aligned}\dot{V}_n\leq & -\frac{\eta\pi}{\mu T_d}\Bigg[(\varepsilon^T S\varepsilon)^{1+\frac{\mu}{2}}+\sum_{\varsigma=1}^{n}\left(\frac{1}{2}\log\frac{K_{b_\varsigma}^2(t)}{K_{b_\varsigma}^2(t)-z_\varsigma^2}\right)^{1+\frac{\mu}{2}}+\sum_{\varsigma=1}^{n}\left(\frac{1}{2}\tilde{W}_\varsigma^T\tilde{W}_\varsigma\right)^{1+\frac{\mu}{2}}+\sum_{\varsigma=1}^{n}\left(\frac{1}{2}\tilde{M}_\varsigma^2\right)^{1+\frac{\mu}{2}}\Bigg]\\ & -\frac{\bar{\eta}\pi}{\mu T_d}\Bigg[(\varepsilon^T S\varepsilon)^{1-\frac{\mu}{2}}+\sum_{\varsigma=1}^{n}\left(\frac{1}{2}\log\frac{K_{b_\varsigma}^2(t)}{K_{b_\varsigma}^2(t)-z_\varsigma^2}\right)^{1-\frac{\mu}{2}}+\sum_{\varsigma=1}^{n}\left(\frac{1}{2}\tilde{W}_\varsigma^T\tilde{W}_\varsigma\right)^{1-\frac{\mu}{2}}+\sum_{\varsigma=1}^{n}\left(\frac{1}{2}\tilde{M}_\varsigma^2\right)^{1-\frac{\mu}{2}}\Bigg]+\bar{N},\end{aligned} \tag{3.44}$$

where $\bar{N}=N_n+C_\varepsilon+\sum_{\varsigma=1}^{n}C_{W,\varsigma}$.

Then, by Lemma 6, it follows that

$$\begin{aligned}\dot{V}_n\leq & -\frac{\pi}{\mu T_d}\left[\varepsilon^T S\varepsilon+\sum_{\varsigma=1}^{n}\frac{1}{2}\log\frac{K_{b_\varsigma}^2(t)}{K_{b_\varsigma}^2(t)-z_\varsigma^2}+\sum_{\varsigma=1}^{n}\frac{1}{2}\tilde{W}_\varsigma^T\tilde{W}_\varsigma+\sum_{\varsigma=1}^{n}\frac{1}{2}\tilde{M}_\varsigma^2\right]^{1+\frac{\mu}{2}}\\ & -\frac{\bar{\eta}\pi}{\mu T_d}\left[\varepsilon^T S\varepsilon+\sum_{\varsigma=1}^{n}\frac{1}{2}\log\frac{K_{b_\varsigma}{}^2(t)}{K_{b_\varsigma}^2(t)-z_\varsigma^2}+\sum_{\varsigma=1}^{n}\frac{1}{2}\tilde{W}_\varsigma^T\tilde{W}_\varsigma+\sum_{\varsigma=1}^{n}\frac{1}{2}\tilde{M}_\varsigma^2\right]^{1-\frac{\mu}{2}}+\bar{N}\\ = & -\frac{\pi}{\mu T_d}V_n^{1+\frac{\mu}{2}}-\frac{\bar{\eta}\pi}{\mu T_d}V_n^{1-\frac{\mu}{2}}+\bar{N}.\end{aligned} \tag{3.45}$$

According to (3.45) and Lemma 3, the closed-loop system is practically predefined-time stable. Specifically, for all $t \geq T_d$, the trajectory of the closed-loop system is guaranteed to converge to a small neighborhood of the origin defined by the residual set $\Omega_3 = \left\{V_n([\varepsilon, z, \tilde{W}, \tilde{M}]) : V_n \leq \frac{2\mu T_d \bar{N}}{\pi}\right\}$. This implies that the observation errors ε_i, the compensated errors z_i, and the adaptive estimation errors $\tilde{W}_i$ and $\tilde{M}_i$ remain within a neighborhood of zero for all $t \geq T_d$. Since $\tilde{W}_i = W_i^* - \hat{W}_i$ and $\tilde{M}_i = M_i^* - \hat{M}_i$ with W_i^* and M_i^* being constants, the boundedness of the adaptive laws $\hat{W}_i$ and $\hat{M}_i$ is also ensured.

Furthermore, by Theorems 1 and 2, the filter errors σ_i and the compensation signals ν_i are bounded. Given that $s_i = z_i + \nu_i$, it can be concluded that the tracking errors s_i also converge to a small neighborhood of the origin within the predefined time T_d. Consequently, based on the recursive design of the controller and filter definitions, all remaining closed-loop signals, including $\alpha_i, \lambda_i, \hat{x}_i, x_i, \chi_u$, and υ, are practically predefined-time stable. Therefore, all signals in the closed-loop system remain bounded and converge to their respective residual sets within the predefined time T_d.

Subsequently, a further analysis is conducted to ensure that the state variables do not violate the prescribed constraint boundaries. From the aforementioned proof, it can be concluded that there exist positive constants Γ_{ν_i}, Γ_{λ_i}, and Γ_{ε_i} such that $|\nu_i| < \Gamma_{\nu_i}$, $|\lambda_{i+1,c}| < \Gamma_{\lambda_i}$, $|\varepsilon_i| < \Gamma_{\varepsilon_i}$, and $|\sigma_i| < \Gamma_{\sigma_i}$. Based on (3.1) and (3.3), we have $|x_1| = |z_1| + |y_r| + |\nu_1|$. In light of Assumption 3, the following inequality holds: $|x_1| \leq K_{b_1}(t) + R_0 + \Gamma_{\nu_1}$. Let $K_{B_1}(t) = K_{b_1}(t) + R_0 + \Gamma_{\nu_1}$, and then we can get $|x_1| < K_{B_1}(t)$. By combining (2.8), (3.1), and (3.3), it follows that $|x_2| = |z_2| + |\lambda_{2,c}| + |\nu_2| + |\varepsilon_2| \leq K_{b_2}(t) + \Gamma_{\lambda_1} + \Gamma_{\nu_2} + \Gamma_{\varepsilon_2}$. Let $K_{B_2}(t) = K_{b_2}(t) + \Gamma_{\lambda_1} + \Gamma_{\nu_2} + \Gamma_{\varepsilon_2}$. Then we can get $|x_2| < K_{B_2}(t)$. Likewise, it can be proved that $|x_i| < K_{B_i}(t), i = 1, 2, ..., n$.

Remark 4. *Two possible extensions of the present work may be of interest for future research. First, replacing the proportional-type fuzzy observer with a proportional-integral-type, one may further improve the estimation performance [31]. Besides, extending the present continuous-time design to an event-triggered implementation is also worth investigating for reducing communication and computation burden [32, 33].*

4. Simulation results

To verify the effectiveness of the proposed control scheme, simulation studies are carried out on an RLC circuit system [17]. The corresponding dynamic model is described by:

$$\begin{cases} \dot{\wp}_q = \dfrac{1}{L}\Im_L + w_1(t), \\ \dot{\Im}_L = u(\upsilon) - \dfrac{1}{C}\wp_q - \dfrac{R}{L}\Im_L + w_2(t), \end{cases} \tag{4.1}$$

where $\wp_q$ denotes the capacitor charge, $\Im_L$ denotes the inductor current, and $w_1(t)$ and $w_2(t)$ are unknown bounded disturbances. The system parameters are listed in Table 1.

Table 1. System parameters of the RLC circuit.

Parameters	Value
Inductance	$L = 1\,\text{H}$
Capacitance	$C = 10\,\text{F}$
Resistance	$R = 1\,\Omega$

The actuator is subject to asymmetric saturation, and the actual control input is described by

$$u(\upsilon) = \begin{cases} 2, & \upsilon(t) > 2, \\ \upsilon(t), & -1 \le \upsilon(t) \le 2, \\ -1, & \upsilon(t) < -1. \end{cases} \tag{4.2}$$

Let $y = x_1 = \wp_q$, $x_2 = \Im_L$. Then, the RLC circuit model (4.1) can be rewritten in the form of (2.1) as

$$\begin{cases} \dot{x}_1 = \dfrac{1}{L} x_2 + g_1(x) + w_1(t), \\ \dot{x}_2 = u(\upsilon) + g_2(x) + w_2(t), \end{cases} \tag{4.3}$$

where $g_1(x) = 0$, $g_2(x) = -\frac{1}{C}x_1 - \frac{R}{L}x_2$, and the disturbances are selected as $w_1(t) = 0.1\sin(x_1) + 0.05\sin(t)$, $w_2(t) = 0.075\cos(0.5t)$.

To approximate the unknown nonlinear functions, the following Gaussian membership functions are selected:

$$\begin{aligned} &\mu_{A_j^1} = e^{-\frac{(\hat{x}_j+1.5)^2}{2}}, \mu_{A_j^2} = e^{-\frac{(\hat{x}_j+1)^2}{2}}, \mu_{A_j^3} = e^{-\frac{(\hat{x}_j+0.5)^2}{2}}, \\ &\mu_{A_j^4} = e^{-\frac{\hat{x}_j^2}{2}}, \mu_{A_j^5} = e^{-\frac{(\hat{x}_j-0.5)^2}{2}}, \mu_{A_j^6} = e^{-\frac{(\hat{x}_j-1)^2}{2}}, \\ &\mu_{A_j^7} = e^{-\frac{(\hat{x}_j-1.5)^2}{2}}, j = 1, 2. \end{aligned}$$

The reference signal is chosen as $y_r = 0.2\sin\left(\frac{\pi t}{4}\right)$. The initial conditions are selected as $x_1(0) = 0$, $x_2(0) = 0$, $\hat{M}_1(0) = 0.15$, $\hat{M}_2(0) = 0.2$, $\hat{W}_1(0) = 0.1$, $\hat{W}_2(0) = 0.2$. The main design parameters are given in Table 2.

Table 2. Control parameters.

Parameters	Value	Parameters	Value
τ_1	0.001	l_1	10
τ_2	0.001	l_2	12
μ_1	6	ρ_1	100
μ_2	25	T_d	4
$K_{B_1}(t)$	$0.2\sin t + 1.5$	$K_{B_2}(t)$	$0.5\sin t + 2$

The simulation results under the nominal condition are shown in Figures 2–5. Figure 2 presents the trajectories of the output y, the reference signal y_r, and the observer state $\hat{x}_1$. It can be seen that the system output tracks the reference signal well while remaining within the prescribed bound. Figure 3 shows the trajectories of x_2 and $\hat{x}_2$, which verifies the effectiveness of the fuzzy observer. The actual actuator input $u(\upsilon)$ and the designed control input υ are shown in Figure 4, and the adaptive estimates are shown in Figure 5. These results confirm that the proposed controller achieves satisfactory tracking performance while respecting the actuator saturation and state constraints.

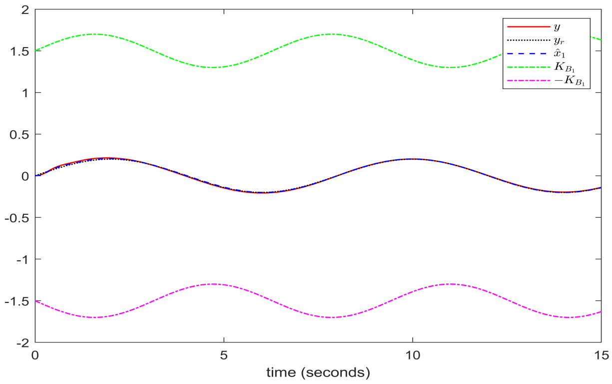

Figure 2. Trajectories of y, y_r, and $\hat{x}_1$ under the prescribed constraint.

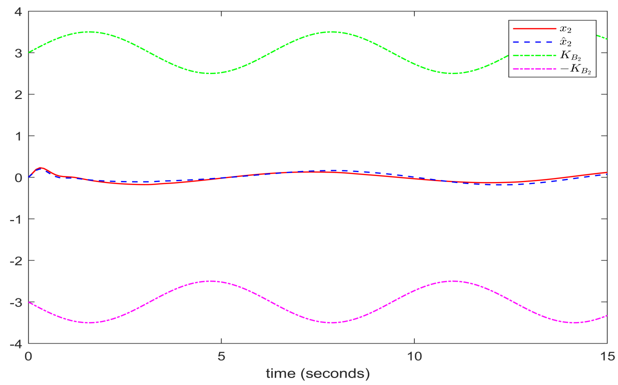

Figure 3. Trajectories of x_2 and $\hat{x}_2$ under the prescribed constraint.

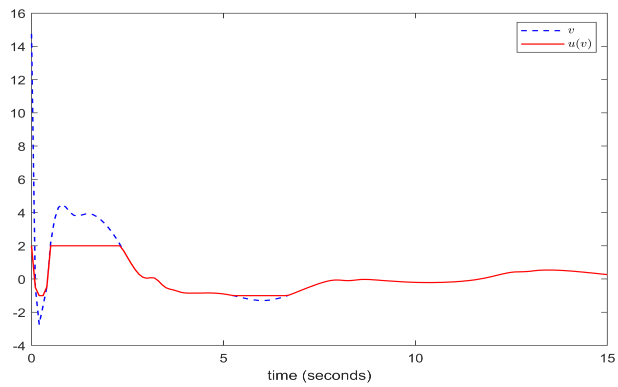

Figure 4. Trajectories of the actual actuator input $u(\upsilon)$ and the designed control input υ.

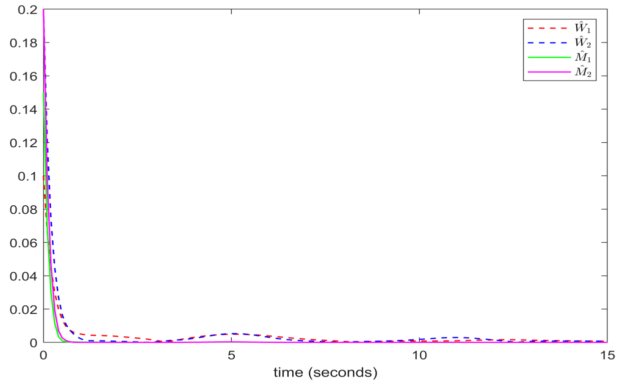

Figure 5. Trajectories of the adaptive estimates.

To further illustrate that the proposed predefined-time control method is insensitive to the initial conditions, a set of comparative simulations is performed by varying only the initial states, while keeping all controller parameters unchanged. The following three cases are considered: Case 1: $[x_1(0), x_2(0)]^T = [0, 0]^T$; Case 2: $[x_1(0), x_2(0)]^T = [0.15, 0.1]^T$; Case 3: $[x_1(0), x_2(0)]^T = [0.2, 0.15]^T$.

The corresponding results for the tracking error s_1 and the filtering error σ_1 are shown in Figures 6 and 7, respectively. It can be observed that, under different initial conditions, both the tracking error and the filtering error converge rapidly before the prescribed predefined time $T_d = 4$. This demonstrates that the proposed control strategy is insensitive to the initial conditions and possesses the desired predefined-time convergence property.

In addition, to evaluate the effect of the design parameters on the control performance, comparative simulations are performed under different predefined times T_d and different values of μ, while fixing the initial conditions at $x_1(0) = 0$, $x_2(0) = 0$. The following parameter settings are considered: Case 4: $T_d = 4$; Case 5: $T_d = 7$; Case 6: $T_d = 10$; Case 7: $\mu = 0.16$; Case 8: $\mu = 0.24$; Case 9: $\mu = 0.32$. For Cases 4–6, the parameter $\mu = 0.24$ is kept unchanged. For Cases 7–9, the predefined time is fixed at $T_d = 4$.

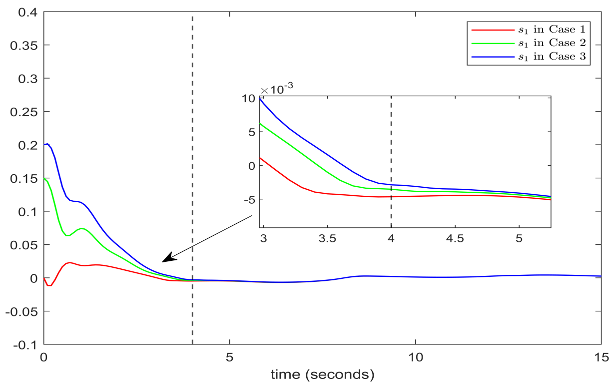

Figure 6. Evolution of the tracking error s_1 under different initial conditions.

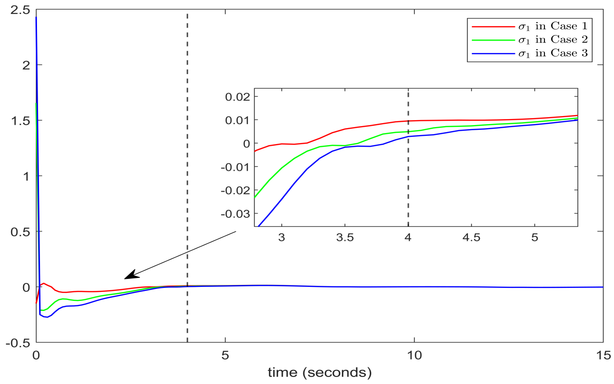

Figure 7. Evolution of the filtering error σ_1 under different initial conditions.

Figures 8 and 9 depict the evolution of the tracking error s_1 under different values of T_d and μ, respectively. The results show that, for all tested parameter settings, the tracking error enters a small neighborhood of the origin before the prescribed predefined time. Moreover, when the other parameters are fixed, smaller values of T_d and μ generally lead to faster convergence and smaller transient fluctuations, which agrees well with the theoretical analysis.

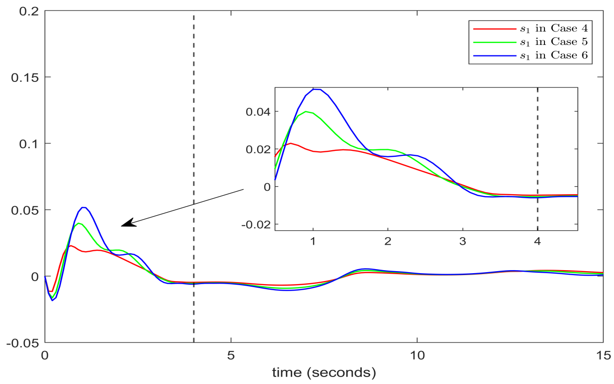

Figure 8. Trajectories of the tracking error s_1 under different predefined times T_d.

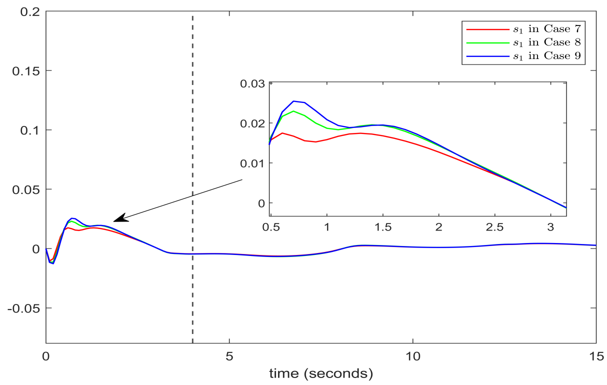

Figure 9. Trajectories of the tracking error s_1 under different values of μ.

Compared with many existing studies that consider only part of the above issues, the present example is used to validate the proposed method under the simultaneous presence of output-feedback estimation, time-varying state constraints, asymmetric actuator saturation, and predefined-time tracking requirements.

Overall, the simulation results confirm that the proposed method achieves predefined-time tracking while maintaining the state trajectories within the prescribed constraint bounds.

5. Conclusions

This paper has investigated the adaptive tracking control problem for a class of uncertain nonlinear systems subject to time-varying state constraints and input saturation. By combining fuzzy approximation, fuzzy state observation, command-filter-based recursive design, and a time-varying barrier Lyapunov function, a predefined-time adaptive control scheme has been developed. An auxiliary compensation mechanism has been introduced to reduce the effect of filtering errors, and a smooth saturation decomposition together with an auxiliary signal have been incorporated to handle the mismatch caused by actuator saturation. It has been shown that all closed-loop signals remain bounded, the prescribed state constraints are not violated, and the tracking error enters a small neighborhood of the origin within the predefined time. Simulation results based on an RLC circuit example have verified the effectiveness of the proposed method. The present results may provide useful guidance for nonlinear engineering systems in which only output information is available, while state constraints, actuator saturation, and prescribed transient-performance requirements must be handled simultaneously. Typical application scenarios include circuit systems, robotic systems, and other actuator-limited nonlinear plants. Future work will further investigate more complex practical settings and experimental validation.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare there are no conflicts of interest.

References

1. J. Yu, P. Shi, J. Liu, C. Lin, Neuroadaptive finite-time control for nonlinear MIMO systems with input constraint, *IEEE Trans. Cybern.*, **52** (2020), 6676–6683. https://doi.org/10.1109/TCYB.2020.3032530
2. E. Cong, X. Zhan, L. Zhu, Accuracy-preassigned fixed-time synchronization of inertial neural networks with time-varying leakage delays and proportional delays, *Electron. Res. Arch.*, **33** (2025), 5897–5915. https://doi.org/10.3934/era.2025262
3. W. Li, X. Chen, L. Ji, J. Song, S. Li, Finite-time formation control for multi-agent systems with dynamic leader and input saturation, *Electron. Res. Arch.*, **33** (2025), 7699–7716. https://doi.org/10.3934/era.2025340

4. X. Wang, B. Niu, H. Wang, X. Zhao, W. Chen, Prescribed performance-based finite-time consensus technology of nonlinear multiagent systems and application to FDPs, *IEEE Trans. Circuits Syst. II Express Briefs*, **70** (2022), 591–595. https://doi.org/10.1109/TCSII.2022.3176313
5. Z. Liang, Z. Wang, J. Zhao, X. Ma, Fast finite-time path-following control for autonomous vehicle via complete model-free approach, *IEEE Trans. Ind. Inf.*, **19** (2022), 2838–2846. https://doi.org/10.1109/TII.2022.3165630
6. M. Li, J. Zhang, S. Li, F. Wu, Adaptive finite-time fault-tolerant control for the full-state-constrained robotic manipulator with novel given performance, *Eng. Appl. Artif. Intell.*, **125** (2023), 106650. https://doi.org/10.1016/j.engappai.2023.106650
7. W. Liu, Quantized output feedback control for nonlinear systems under parameter criteria, *Automatica*, **187** (2026), 112922. https://doi.org/10.1016/j.automatica.2026.112922
8. S. Sui, C. P. Chen, S. Tong, A novel full errors fixed-time control for constraint nonlinear systems, *IEEE Trans. Autom. Control*, **68** (2022), 2568–2575. https://doi.org/10.1109/TAC.2022.3200962
9. F. Wei, L. Zhang, B. Niu, G. Zong, Adaptive decentralized fixed-time neural control for constrained strong interconnected nonlinear systems with input quantization, *Int. J. Robust Nonlinear Control*, **34** (2024), 9899–9927. https://doi.org/10.1002/rnc.7497
10. K. Lu, Z. Liu, H. Yu, C. P. Chen, Y. Zhang, Adaptive fuzzy inverse optimal fixed-time control of uncertain nonlinear systems, *IEEE Trans. Fuzzy Syst.*, **30** (2021), 3857–3868. https://doi.org/10.1109/TFUZZ.2021.3132151
11. H. Xu, D. Yu, Y. J. Liu, Observer-based fuzzy adaptive predefined time control for uncertain nonlinear systems with full-state error constraints, *IEEE Trans. Fuzzy Syst.*, **32** (2023), 1370–1382. https://doi.org/10.1109/TFUZZ.2023.3321669
12. L. Feng, C. Zhang, M. Abdel-Aty, J. Cao, F. E. Alsaadi, Prescribed-time trajectory tracking control for a class of nonlinear system, *Electron. Res. Arch.*, **32** (2024), 6535–6552. https://doi.org/10.3934/era.2024305
13. Y. Pan, Y. Chen, H. Liang, Event-triggered predefined-time control for full-state constrained nonlinear systems: A novel command filtering error compensation method, *Sci. China Technol. Sci.*, **67** (2024), 2867–2880. https://doi.org/10.1007/s11431-023-2607-8
14. W. Liu, H. Zhao, H. Shen, S. Xu, J. H. Park, Command-filter based predefined-time control for state-constrained nonlinear systems subject to preassigned performance metrics, *IEEE Trans. Autom. Control*, **69** (2024), 7801–7807. https://doi.org/10.1109/TAC.2024.3397468
15. B. Ning, Q. L. Han, M. Luan, G. Wen, X. Ge, X. M. Zhang, et al., An overview of distributed fixed-time and prescribed-time optimization of multi-agent systems, *Sci. China Inf. Sci.*, **69** (2026), 121202. https://doi.org/10.1007/s11432-025-4647-9
16. J. Chen, C. Hua, D. Mu, Y. Zhang, Predefined-time full-state constrained control for mobile robot systems with deferred constraints, *IEEE Trans. Ind. Electron.*, **72** (2024), 2968–2976. https://doi.org/10.1109/TIE.2024.3433532
17. H. Wang, Z. Meng, J. Qiao, S. Liu, Command filtered-based adaptive predefined-time control for uncertain nonlinear systems with applications to RLC circuit, *IEEE Trans. Circuits Syst. I Regul. Pap.*, **71** (2024), 4792–4801. https://doi.org/10.1109/TCSI.2024.3431276

18. S. Sui, C. P. Chen, S. Tong, Command filter-based predefined time adaptive control for nonlinear systems, *IEEE Trans. Autom. Control*, **69** (2024), 7863–7870. https://doi.org/10.1109/TAC.2024.3399998

19. D. Swaroop, J. K. Hedrick, P. P. Yip, J. C. Gerdes, Dynamic surface control for a class of nonlinear systems, *IEEE Trans. Autom. Control*, **45** (2000), 1893–1899. https://doi.org/10.1109/TAC.2000.880994

20. Y. X. Li, Command filter adaptive asymptotic tracking of uncertain nonlinear systems with time-varying parameters and disturbances, *IEEE Trans. Autom. Control*, **67** (2021), 2973–2980. https://doi.org/10.1109/TAC.2021.3089626

21. J. Liu, Q. G. Wang, J. Yu, Command-filter-approximator-based adaptive control for uncertain nonlinear systems and its application in PMSMs, *IEEE Trans. Syst. Man Cybern.: Syst.*, **53** (2023), 6828–6835. https://doi.org/10.1109/TSMC.2023.3287480

22. X. Zhang, W. Zhang, J. Cao, H. Liu, Observer-based command filtered adaptive fuzzy control for fractional-order MIMO nonlinear systems with unknown dead zones, *Expert Syst. Appl.*, **255** (2024), 124623. https://doi.org/10.1016/j.eswa.2024.124623

23. C. Wen, J. Zhou, Z. Liu, H. Su, Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance, *IEEE Trans. Autom. Control*, **56** (2011), 1672–1678. https://doi.org/10.1109/TAC.2011.2122730

24. Y. F. Gao, X. M. Sun, C. Wen, W. Wang, Adaptive tracking control for a class of stochastic uncertain nonlinear systems with input saturation, *IEEE Trans. Autom. Control*, **62** (2016), 2498–2504. https://doi.org/10.1109/TAC.2016.2600340

25. K. Yong, M. Chen, Y. Shi, Q. Wu, Flexible performance-based robust control for a class of nonlinear systems with input saturation, *Automatica*, **122** (2020), 109268. https://doi.org/10.1016/j.automatica.2020.109268

26. P. R. Ali, M. Rehan, W. Ahmed, A. Basit, I. Ahmed, A novel output feedback consensus control approach for generic linear multi-agent systems under input saturation over a directed graph topology, *ISA Trans.*, **148** (2024), 128–139. https://doi.org/10.1016/j.isatra.2024.02.029

27. Y. Hu, W. Liu, B. Ma, Event-trigger-based composite adaptive fuzzy control for nonlinear time-varying state constraint systems with asymmetric input saturation, *Eur. J. Control*, **75** (2024), 100892. https://doi.org/10.1016/j.ejcon.2023.100892

28. Y. Hwang, C. M. Kang, W. Kim, Robust nonlinear control using barrier lyapunov function under lateral offset error constraint for lateral control of autonomous vehicles, *IEEE Trans. Intell. Transp. Syst.*, **23** (2020), 1565–1571. https://doi.org/10.1109/TITS.2020.3023617

29. Y. Wang, X. Wang, C. Liu, H. Zhang, H. Zhou, X. Zhang, et al., State-constrained control strategy for safe navigation trajectory tracking of hovercraft based on improved barrier lyapunov function, *Ocean Eng.*, **303** (2024), 117791. https://doi.org/10.1016/j.oceaneng.2024.117791

30. M. Wang, X. Dong, X. Ren, Adaptive output feedback control for a multi-motor driving system with completely tracking errors constraint, in *2021 33rd Chinese Control and Decision Conference (CCDC)*, IEEE, (2021), 2672–2677. https://doi.org/10.1109/CCDC52312.2021.9601779

31. G. Tan, Z. Wang, Z. Shi, Proportional–integral state estimator for quaternion-valued neural networks with time-varying delays, *IEEE Trans. Neural Networks Learn. Syst.*, **34** (2021), 1074–1079. https://doi.org/10.1109/TNNLS.2021.3103979

32. X. M. Zhang, Q. L. Han, B. Zhang, X. Ge, Monotonically-increasing-function-based event-triggered sampling scheme for stabilization of networked nonlinear systems, *Sci. China Inf. Sci.*, **69** (2026), 152201. https://doi.org/10.1007/s11432-025-4644-x

33. X. M. Zhang, Q. L. Han, X. Ge, D. Ding, B. Ning, B. Zhang, An overview of recent advances in event-triggered control, *Sci. China Inf. Sci.*, **68** (2025), 161201. https://doi.org/10.1007/s11432-024-4437-9

AIMS Press