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ERA, 34(6): 3678–3695.
DOI: 10.3934/era.2026166
Received: 04 February 2026
Revised: 15 April 2026
Accepted: 23 April 2026
Published: 30 April 2026

Research article

Dynamics of a non-autonomous competitive system under simultaneous impulsive pollution and anthropogenic disturbances

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Abstract: In this paper, the permanence and extinction of a class of non-autonomous competitive population systems existing in an impulsively polluted environment are investigated. By establishing a time-varying differential equation model that couples population competition, pollutant dynamics, and discrete impulsive disturbances, we employ the comparison theorem of impulsive differential equations and analytical methods to derive sufficient criteria guaranteeing population persistence and global extinction. Furthermore, by constructing a Lyapunov function, we obtain conditions for the global attractivity of the system solutions. The theoretical analysis indicates that the ultimate fate of the population is jointly determined by the time-varying growth rates, competition intensities, period of impulsive disturbances, and pollutant toxicity levels. This study provides a quantitative theoretical basis for assessing the ecological risks of sporadic pollution events in competitive communities.

Keywords: non-autonomous competitive population system; impulsive effect; permanence; extinction; global attractivity

1. Introduction

Environmental pollution is one of the most severe ecological challenges facing the world today. Its stress effects on biological populations extend beyond the physiological levels of individual and profoundly affect community structure and ecosystem stability. Research indicates that the extinction of a single species may trigger a chain reaction, leading to the extinction involving 10–30 other plants, insects, and higher animals dependent on it for survival [1]. Therefore, in population dynamics research, quantitatively characterizing the impacts of environmental pollution and using such characterization to predict species persistence and extinction are of great theoretical and practical significance [2–5].

Early studies often treated environmental stressors and human management impacts as continuous processes. However, real-world disturbances are typically discontinuous and abrupt. This is reflected not only in impulsive pollution inputs, such as industrial emissions, agricultural pesticide applications, and accidental spills, but also in widespread direct human interventions in ecosystems, such as fishing, periodic pesticide spraying, and sudden habitat destruction. A common characteristic of these external pulse disturbances is that they exert sudden and intense instantaneous effects on population size or environmental conditions within an extremely short period, thereby forcing abrupt changes in the trajectories of population dynamics [6–8]. Consequently, classical continuous differential equations are difficult to use for accurately describing such phenomena, whereas impulsive differential equations provide a more suitable theoretical framework for modeling. Since Liu et al. [9] pioneered the introduction of impulsive effects into environmenal pollution modeling, research in this direction has continued to deepen. Scholars have successively explored the effects of impulsive toxin inputs on single-species systems [10, 11], stage-structured systems [12], predator-prey systems [13], competitive systems [14], and food chain systems [15]. By employing a comparison theorem of impulsive differential equations and analytical methods, they have discussed population persistence and extinction. More recent studies have further integrated stochastic disturbances into impulsive pollution models [16–20]. For example, Liu and Bai [18] proposed and studied a cooperative model; by taking white noises, Markovian switching, and Levy jumps into account, Wang et al. [19] analyzed a stochastic service-resource mutualism model, while Meng et al. [20] investigated the conditions for microbial survival in a stochastic chemostat model. These studies mark a shift in the field toward more complex and realistic modeling of uncertainty.

In addition to impulsive models, spatial heterogeneity has also been recognized as a key factor influencing population dynamics in polluted environments. In recent years, partial differential equation (PDE) models, particularly reaction-diffusion systems, have been widely used to investigate the interaction between toxicants and biological populations in spatially heterogeneous environments. These models incorporate spatial dispersal and environmental gradients, providing a more realistic description of population distribution and evolution. For example, Ma and Zhang [21] studied the global attractor and threshold dynamics of a reaction-diffusion population model in a polluted environment. Ma et al. [22] further analyzed the dynamical behavior and optimal control of a toxicant population system with diffusion effects. In addition, Deng and Huang [23] proposed a toxicant-taxis reaction-diffusion model, revealing complex spatiotemporal dynamics driven by toxicant gradients. These studies demonstrate that spatial structure plays an important role in determining population persistence and extinction.

While spatial models provide crucial insights into pattern formation and regional persistence, the analytical derivation of explicit persistence-extinction thresholds is often more tractable in models focusing on temporal dynamics, especially when addressing the intricate interplay of multiple time-varying factors.

In traditional population dynamics research, autonomous systems are commonly used as foundational models describing population size fluctuations, typically assuming parameters such as birth rates, death rates, and the intensity of intraspecific and interspecific interactions as constants. However, biological populations in nature exist within continuously changing environments. Environmental factors such as temperature, light exposure, precipitation, and resource availability exhibit distinct periodic variations (e.g., seasonal shifts, climatic fluctuations, or habitat alterations) caused by human activity. These variations directly or indirectly influence key population processes such as birth and death rates, competitive capacity, and migration behavior, causing model parameters to vary over time rather than remain fixed. Consequently, employing non-autonomous systems to describe population dynamics more accurately reflects the dynamic effects of time-varying environmental drivers on population behavior [24–30].

Furthermore, in ecosystem dynamics, interspecific competition is a fundamental mechanism governing for conflicts over limited shared resources. Classical theories reveal the possibility of competitive exclusion under steady state conditions [31–34]. However, in non-autonomous, time-varying environments, this static competitive equilibrium may be disrupted: Periodic environmental conditions can dynamically alter species' competitive abilities and resource utilization efficiencies. Competition dynamics become particularly complex and fragile when the system is also subject to sudden, discrete disturbances such as pulsed pollution and human interference.

In this paper, we investigate the dynamics of a non-autonomous competitive system subject to discontinuous human disturbances within an impulsive pollution environment. We develop a time-varying model that integrates population competition, pollutant dynamics, and discrete impulsive disturbances and analyze it using a comprehensive approach that combines impulsive differential equations theory, comparative methods, and Lyapunov function techniques. Our objective is to derive threshold conditions for population persistence or extinction. This study aims to theoretically elucidate how the intricate interplay among environmental fluctuations, human external disturbances, and pulse pollution ultimately determines the coexistence or extinction of competing populations.

2. Model formulation

Based on the biological background introduced above, this section establishes the following non-autonomous competitive impulsive system under discontinuous human disturbances in a polluted environment. Specifically, time-varying parameters are introduced to characterize the non-autonomous nature of the environment, and impulsive terms are used to describe external human interference and periodic impulsive pollution events, building on the classical Lotka-Volterra competition model:

$$\begin{cases} \left.\begin{aligned} \frac{\mathrm{d}x_1(t)}{\mathrm{d}t} &= x_1(t)(r_1(t) - a_1(t)x_1(t) - b_1(t)x_2(t) - d_1(t)c_0(t)),\\ \frac{\mathrm{d}x_2(t)}{\mathrm{d}t} &= x_2(t)(r_2(t) - a_2(t)x_2(t) - b_2(t)x_1(t) - d_2(t)c_0(t)),\\ \frac{\mathrm{d}c_0(t)}{\mathrm{d}t} &= k_0c_e(t) - (g+m)c_0(t),\\ \frac{\mathrm{d}c_e(t)}{\mathrm{d}t} &= -hc_e(t), \end{aligned}\right\} t \neq \tau_k,\ t \neq nT,\ n \in \mathbf{N}^+,\ k \in \mathbf{N}^+,\\ \left.\begin{aligned} \Delta x_1(t) &= h_{1k}x_1(t),\\ \Delta x_2(t) &= h_{2k}x_2(t), \end{aligned}\right\} t \in \tau_k,\ k \in \mathbf{N}^+,\\ \left.\begin{aligned} \Delta c_0(t) &= 0,\\ \Delta c_e(t) &= \mu, \end{aligned}\right\} t = nT,\ n \in \mathbf{N}^+, \end{cases} \tag{2.1}$$

where $\mathbf{N}^+ = \{1, 2, \cdots\}$, $\Delta x_i(t) = x_i(t^+) - x_i(t)$ $(i = 1, 2)$, $\Delta c_0(t) = c_0(t^+) - c_0(t)$, $\Delta c_e(t) = c_e(t^+) - c_e(t)$, and $f(t^+) = \lim_{\bar{t}\to t^+} f(\bar{t})$. $x_1(t)$ and $x_2(t)$ represent the densities of the two competing populations at time t, respectively; $r_i(t)$ $(i = 1, 2)$ denotes the time-varying intrinsic growth rates of the population

$x_i(t)$; $a_i(t)$, $b_i(t)$ $(i = 1, 2)$ represent the time-varying intraspecific and interspecific competition coefficients, respectively; $c_0(t)$ and $c_e(t)$ denote the pollutant concentrations within organisms and in the environment at time t, respectively; $d_i(t)$ $(i = 1, 2)$ is the toxicity coefficients of the pollutant to the population $x_i(t)$; $k_0 c_e(t)$ represents the absorption rate of pollutants from the environment by the organism; $(g + m)c_0(t)$ denotes the excretion and purification rates of pollutants within the organism at time t; $hc_e(t)$ represents the reduction in toxin quantity caused by factors such as biological transfer, volatilization, bacterial degradation, and death, as well as photosynthesis; h_{ik} $(i = 1, 2)$ are the human disturbance activities (hunting or stocking) on the population at $x_i(t)$ at $t = \tau_k$; and $\mu < 1$ represents the impulsive increase in pollutant concentration in the environment at $t = nT$. We assume that $a_i(t)$, $b_i(t)$, $r_i(t)$, and $d_i(t)$ $(i = 1, 2)$ are all positive bounded functions defined on $\mathbf{R}^+ = [0, +\infty)$. Furthermore, it assumes that within one pollution emission period T, human disturbance activities on competing populations occur q times, meaning there exists an integer q such that $h_{1k+q} = h_{1k}$, $h_{2k+q} = h_{2k}$, and $\tau_{k+q} = \tau_k + T$. Considering specific biological significance, constraints are imposed such that for all $k \in \mathbf{N}^+$, $1 + h_{1k} > 0$ and $1 + h_{2k} > 0$.

Remark 2.1. *([9]) Since $c_0(t)$ and $c_e(t)$ represent pollutant concentrations, it must be required that $0 \leq c_0(t) \leq 1$ and $0 \leq c_e(t) \leq 1$. A sufficient condition for ensuring the inequalities $0 \leq c_0(t), c_e(t) \leq 1$ hold is that their coefficients satisfy*

$$k_0 \leq g + m, \ \mu \leq 1 - \mathrm{e}^{-hT}. \tag{2.2}$$

In the following, we assume that (2.2) always holds.

Considering the subsystem of system (2.1),

$$\begin{cases} \left.\begin{aligned} \frac{\mathrm{d}c_0(t)}{\mathrm{d}t} &= k_0 c_e(t) - (g + m)c_0(t), \\ \frac{\mathrm{d}c_e(t)}{\mathrm{d}t} &= -\, hc_e(t), \end{aligned}\right\} t \neq nT, \ n \in \mathbf{N}^+, \\ \left.\begin{aligned} \Delta c_0(t) &= 0, \\ \Delta c_e(t) &= \mu, \end{aligned}\right\} t = nT, \ n \in \mathbf{N}^+, \end{cases} \tag{2.3}$$

we have the following lemma.

Lemma 2.1. *([9]) System (2.3) possesses a unique T-periodic solution $(\tilde{c}_0(t), \tilde{c}_e(t))$, and for any solution $(c_0(t), c_e(t))$ of system (2.3), we have $c_0(t) \to \tilde{c}_0(t)$, $c_e(t) \to \tilde{c}_e(t)$ as $t \to +\infty$, where*

$$\begin{cases} \tilde{c}_0(t) = \tilde{c}_0(0)\mathrm{e}^{-(g+m)(t-(n-1)T)} + \dfrac{k_0\mu(\mathrm{e}^{-(g+m)(t-(n-1)T)} - \mathrm{e}^{-h(t-(n-1)T)})}{(h - g - m)(1 - \mathrm{e}^{-hT})}, \\ \tilde{c}_e(t) = \dfrac{\mu \mathrm{e}^{-h(t-(n-1)T)}}{1 - \mathrm{e}^{-hT}}, \\ \tilde{c}_0(0) = \dfrac{k_0\mu(\mathrm{e}^{-(g+m)T} - \mathrm{e}^{-hT})}{(h - g - m)(1 - \mathrm{e}^{-(g+m)T})(1 - \mathrm{e}^{-hT})}, \\ \tilde{c}_e(0) = \dfrac{\mu}{1 - \mathrm{e}^{-hT}}, \end{cases} \tag{2.4}$$

for $t \in (nT, (n + 1)T]$, $n \in \mathbf{N}^+$.

From the meaning of the parameters in system (2.1), the following lemma can be easily derived.

Lemma 2.2. *Suppose $(x_1(t), x_2(t), c_0(t), c_e(t))$ is a solution of system (2.1) with initial value $x_1(0) \geq 0$, $x_2(0) \geq 0$, $c_0(0) \geq 0$, $c_e(0) \geq 0$, then we have $x_1(t) \geq 0$, $x_2(t) \geq 0$, $c_0(t) \geq 0$, $c_e(t) \geq 0$ for all $t \geq 0$.*

3. Persistence and extinction of populations in system (2.1)

3.1. Positivity and boundedness

We use the comparison theorem for impulsive differential equations to study the conditions under which populations x_1 and x_2 can persist. First, we establish the ultimate boundedness of both populations x_1 and x_2.

Theorem 3.1. *If*

$$\liminf_{t\to\infty}\int_t^{t+T} a_i(s)ds > 0, \tag{3.1}$$

and for any $t\in\mathbf{R}^+$, *when* $\theta\in[0,T]$,

$$h_i(t,\theta)=\sum_{t\le\tau_k<t+\theta}\ln(1+h_{ik}) \tag{3.2}$$

is bounded, then there exists an $M>0$ *such that* $\lim_{t\to\infty}\sup x_i(t)\le M$ $(i=1,2)$.

Proof. By the boundedness of $h_i(t,\theta)=\sum_{t\le\tau_k<t+\theta}\ln(1+h_{ik})$, there exists an $H>0$ such that for any $t\in\mathbf{R}^+$, $\theta\in[0,T]$, we have

$$|h_i(t,\theta)|=\left|\sum_{t\le\tau_k<t+\theta}\ln(1+h_{ik})\right|<H. \tag{3.3}$$

In particular, $h_i(t,\theta)=\sum_{t\le\tau_k<t+\theta}\ln(1+h_{ik})$ is bounded. From condition (3.1) and the boundedness of $r_i(t)$, there exist positive numbers m, δ_1, and T_0 such that when $t\ge T_0$, we have

$$\int_t^{t+T}(r_i(s)-ma_i(s))\mathrm{d}s+\sum_{t\le\tau_k<t+\theta}\ln(1+h_{ik})<-\delta_1<0. \tag{3.4}$$

From system (2.1), we obtain

$$\begin{cases}\dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t}\le x_i(t)(r_i(t)-a_i(t)x_i(t)),\\ x_i(\tau_k^+)=(1+h_{ik})x_i(\tau_k).\end{cases} \tag{3.5}$$

Consider the impulsive comparison equation

$$\begin{cases}\dfrac{\mathrm{d}u(t)}{\mathrm{d}t}=u(t)(r_i(t)-a_i(t)u(t)),\\ u(\tau_k^+)=(1+h_{ik})u(\tau_k),\end{cases} \tag{3.6}$$

satisfying the initial conditions $x_i(T_0)\le u(T_0)$.

Now we prove that there exists an $M>0$ such that $\lim_{t\to\infty}\sup u(t)\le M$.

For any solution $u(t)$ of system (3.6), only the following three cases can exist:

(i) There exists a $t_0\ge T_0$ such that for any $t\ge t_0$, $u(t)\ge m$.

(ii) There exists a $t_0\ge T_0$ such that for any $t\ge t_0$, $u(t)\le m$.

(iii) For any $t\ge T_0$, $u(t)$ oscillates with respect to m.

Assume case (i) holds. Let $t = t_0 + T_0$ and $l \in \mathbf{Z}^+$. Integrate the first equation (3.6) from t_0 to t. From (3.4), we obtain

$$\begin{aligned}
u(t) =& u(t_0)\exp\Big[\int_{t_0}^{t}(r_i(s) - a_i(s)u(s))\mathrm{d}s + \sum_{t_0\le\tau_k<t}\ln(1+h_{ik})\Big]\\
\le& u(t_0)\exp\Big[\int_{t_0}^{t_0+T}(r_i(s) - ma_i(s))\mathrm{d}s + \sum_{t_0\le\tau_k<t_0+T}\ln(1+h_{ik}) + \cdots\\
&+\int_{t_0+(l-1)T}^{t_0+lT}(r_i(s) - ma_i(s))\mathrm{d}s + \sum_{t_0+(l-1)T\le\tau_k<t_0+lT}\ln(1+h_{ik})\Big]\\
<& u(t_0)\exp(-l\delta_1).
\end{aligned}$$

If $l \to \infty$, then $u(t) \to 0$. This contradicts the fact that $u(t) \ge m$, so case (i) does not hold.

Assume case (iii) holds. Since $u(t)$ oscillates about m, we can select two sequences $\{\rho_n\}$ and $\{\rho_n^*\}$ such that

$$T_0 < \rho_1 < \rho_1^* < \rho_2 < \rho_2^* < \cdots, \ \lim_{n\to\infty}\rho_n = \lim_{n\to\infty}\rho_n^* = +\infty,$$

satisfying $u(\rho_n) \le m$, $u(\rho_n^+) \ge m$, $u(\rho_n^*) \ge m$, $u(\rho_n^{*+}) \le m$, and $u(t) \ge m$ for $t \in (\rho_n, \rho_n^*]$, as well as $u(t) \le m$ for $t \in (\rho_n^*, \rho_{n+1}]$. For any $t \ge \rho_1$, if $t \in (\rho_n, \rho_n^*]$, there exist $l \ge 0$ and $0 \le v < T$ such that $t = \rho_n + lT + v$, and we have

$$\begin{cases}
\dfrac{\mathrm{d}u(t)}{\mathrm{d}t} \le u(t)(r_i(t) - ma_i(t)),\\
u(\tau_k^+) = (1+h_{ik})u(\tau_k).
\end{cases}$$

Integrating the above inequality from ρ_n to t and using (3.3) and (3.4), we obtain

$$\begin{aligned}
u(t) =& u(\rho_n)\exp\Big[\int_{\rho_n}^{t}(r_i(s) - ma_i(s))\mathrm{d}s + \sum_{\rho_n\le\tau_k<t}\ln(1+h_{ik})\Big]\\
\le& m\exp\Big[\int_{\rho_n}^{\rho_n+T}(r_i(s) - ma_i(s))\mathrm{d}s + \sum_{\rho_n\le\tau_k<\rho_n+T}\ln(1+h_{ik}) + \cdots\\
&+\int_{\rho_n+lT}^{\rho_n+lT+v}(r_i(s) - ma_i(s))\mathrm{d}s + \sum_{\rho_n+lT\le\tau_k<\rho_n+lT+v}\ln(1+h_{ik})\Big]\\
\le& m\exp(-l\delta_1 + AT + H)\\
\le& m\exp(AT + H),
\end{aligned}$$

where $A = \sup_{t\in\mathbf{R}^+}\{|r_i(t)| + m|a_i(t)|\}$. For $t \in (\rho_n^*, \rho_{n+1}]$, we have $u(t) \le m \le m\exp(AT + H)$. Therefore, for all $t \ge \rho_1$, $u(t) \le m\exp(AT + H)$ holds.

Assume case (ii) holds. Then we can directly obtain $u(t) \le m\exp(AT + H)$.

Let $M = m\exp(AT + H)$. So, we have $\lim_{t\to\infty}\sup u(t) \le M$.

By the comparison theorem for impulse differential equations, we obtain $\lim_{t\to\infty}\sup x_i(t) \le M$. This completes the proof.

Remark 3.1. *Theorem 3.1 ensures that, under the combined effects of impulsive pollution and anthropogenic disturbances (such as impulsive harvesting or stocking), the densities of the two* *competing populations have an upper bound. The conditions require that the intraspecific competition coefficients* $a_i(t)$ *are positive on average over a time period (indicating persistent competitive pressure) and that the cumulative effect of anthropogenic disturbances,* $\sum_{t\le\tau_k<t+\theta}\ln(1+h_{ik})$*, is bounded. Biologically, this implies that as long as intraspecific competition is sufficiently strong and the intensity of external disturbances–whether positive (stocking) or negative (harvesting)–does not increase indefinitely, population numbers will not grow without limit. Thereby, resource depletion or system collapse is avoided, providing a mathematical guarantee for the sustainability of the ecosystem.*

Theorem 3.2. *If the conditions of Theorem 3.1 hold and*

$$\int_t^{t+T}[r_i(s)-Mb_i(s)-\tilde{c}_0(s)d_i(s)]ds+\sum_{t\le\tau_k<t+T}\ln(1+h_{ik})>0,\ i=1,2, \tag{3.7}$$

then there exists a $\eta\in\mathbf{R}^+$ *such that* $\lim_{t\to\infty}\inf x_i(t)\ge\eta$.

Proof. Take a sufficiently small $\varepsilon>0$. Then from (3.7), we have

$$\int_t^{t+T}[r_i(s)-Mb_i(s)-(\tilde{c}_0(s)+\varepsilon)d_i(s)]\mathrm{d}s+\sum_{t\le\tau_k<t+T}\ln(1+h_{ik})>0.$$

By conditions (3.2) and (3.7) and the boundedness of $a_i(t)$, there exist positive numbers $\bar{m}$, $\bar{\delta}_1$, and T_2 such that when $t\ge T_2\ge T_1$, we have

$$\int_t^{t+T}[r_i(s)-Mb_i(s)-(\tilde{c}_0(s)+\varepsilon)d_i(s)-\bar{m}a_i(s)]\mathrm{d}s+\sum_{t\le\tau_k<t+T}\ln(1+h_{ik})>\bar{\delta}_1. \tag{3.8}$$

For the given ε above, by Lemma 2.1, there exists a $T_3>T_2$ such that when $t>T_3$, we have

$$\tilde{c}_0(t)-\varepsilon<c_0(t)<\tilde{c}_0(t)+\varepsilon.$$

From system (2.1) and Theorem 3.1, we obtain

$$\begin{cases}\dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t} \ge x_i(t)[r_i(t)-Mb_i(t)-(\tilde{c}_0(t)+\varepsilon)d_i(t)-a_i(t)x_i(t)],\\ x_i(\tau_k^+)=(1+h_{ik})x_i(\tau_k).\end{cases}$$

Consider the impulsive comparison equation

$$\begin{cases}\dfrac{\mathrm{d}z(t)}{\mathrm{d}t} =z(t)[r_i(t)-Mb_i(t)-(\tilde{c}_0(t)+\varepsilon)d_i(t)-a_i(t)z(t)],\\ z(\tau_k^+)=(1+h_{ik})z(\tau_k),\end{cases} \tag{3.9}$$

satisfying the initial conditions $z(T_2)\le x_i(T_2)$. We now prove that there exists an $\eta>0$ such that $\lim_{t\to\infty}\inf z(t)\ge\eta$.

For any solution $z(t)$ of system (3.9), only the following three cases can exist:

(i) There exists a $t_1\ge T_2$ such that for any $t\ge t_1$, $z(t)\le\bar{m}$.

(ii) There exists a $t_1\ge T_2$ such that for any $t\ge t_1$, $z(t)\ge\bar{m}$.

(iii) For any $t \geq T_2$, $z(t)$ oscillates with respect to $\bar{m}$.

Assuming that case (i) holds, let $t = t_1 + lT$ and $l \in \mathbf{N}^+$. Integrate the first equation (3.9) from t_1 to t. From (3.8), we obtain

$$\begin{aligned} z(t) =& z(t_1)\exp\Big[\int_{t_1}^{t}[r_i(s) - Mb_i(s) - (\tilde{c}_0(s)+\varepsilon)d_i(s) - a_i(s)z(s)]\mathrm{d}s + \sum_{t_1\leq\tau_k<t}\ln(1+h_{ik})\Big] \\ \geq& z(t_1)\exp\Big[\int_{t_1}^{t}[r_i(s) - Mb_i(s) - (\tilde{c}_0(s)+\varepsilon)d_i(s) - \bar{m}a_i(s)]\mathrm{d}s + \sum_{t_1\leq\tau_k<t_1+T}\ln(1+h_{ik}) + \cdots \\ &+ \int_{t_1+(l-1)T}^{t_1+lT}[r_i(s) - Mb_i(s) - (\tilde{c}_0(s)+\varepsilon)d_i(s) - \bar{m}a_i(s)]\mathrm{d}s + \sum_{t_1+(l-1)T\leq\tau_k<t_1+lT}\ln(1+h_{ik})\Big] \\ \geq& z(t_1)\exp(l\bar{\delta}_1). \end{aligned}$$

If $l \to +\infty$, then $z(t) \to +\infty$, which contradicts the fact that $z(t) \leq \bar{m}$. Therefore, case (i) does not hold.

Assume that condition (iii) holds. Since $z(t)$ oscillates about $\bar{m}$, we can select two sequences $\{\bar{\rho}_n\}$ and $\{\bar{\rho}_n^*\}$ such that

$$T_2 < \bar{\rho}_1 < \bar{\rho}_1^* < \bar{\rho}_2 < \bar{\rho}_2^* < \cdots, \ \lim_{n\to\infty}\bar{\rho}_n = \lim_{n\to\infty}\bar{\rho}_n^* = +\infty,$$

satisfying $z(\bar{\rho}_n) \leq \bar{m}$, $z(\bar{\rho}_n^+) \geq \bar{m}$, $z(\bar{\rho}_n^*) \geq \bar{m}$, $z(\bar{\rho}_n^{*+}) \leq \bar{m}$, and $z(t) \geq \bar{m}$ for $t \in (\bar{\rho}_n, \bar{\rho}_n^*]$ as well as $z(t) \leq \bar{m}$ for $t \in (\bar{\rho}_n^*, \bar{\rho}_{n+1}]$. For any $t \geq \bar{\rho}_1$, if $t \in (\bar{\rho}_n, \bar{\rho}_n^*]$, there exist $l' \geq 0$ and $0 \leq \nu < T$ such that $t = \bar{\rho}_n + l'T + \nu$, and we have

$$\begin{cases} \dfrac{\mathrm{d}z(t)}{\mathrm{d}t} \geq z(t)[r_i(t) - Mb_i(t) - (\tilde{c}_0(t)+\varepsilon)d_i(t) - \bar{m}a_i(t)], \\ z(\tau_k^+) = (1+h_{ik})z(\tau_k). \end{cases}$$

Integrating the above inequality from $\bar{\rho}_n$ to t and using (3.3) and (3.8), we obtain

$$\begin{aligned} z(t) =& z(\bar{\rho}_n)\exp\Big[\int_{\bar{\rho}_n}^{t}[r_i(s) - Mb_i(s) - (\tilde{c}_0(s)+\varepsilon)d_i(s) - \bar{m}a_i(s)]\mathrm{d}s + \sum_{\bar{\rho}_n\leq\tau_k<t}\ln(1+h_{ik})\Big] \\ \geq& \bar{m}\exp\Big[\int_{\bar{\rho}_n}^{\bar{\rho}_n+T}[r_i(s) - Mb_i(s) - (\tilde{c}_0(s)+\varepsilon)d_i(s) - \bar{m}a_i(s)]\mathrm{d}s + \sum_{\bar{\rho}_n\leq\tau_k<\bar{\rho}_n+T}\ln(1+h_{ik}) + \cdots \\ &+ \int_{\bar{\rho}_n+l'T}^{\bar{\rho}_n+l'T+\nu}[r_i(s) - Mb_i(s) - (\tilde{c}_0(s)+\varepsilon)d_i(s) - \bar{m}a_i(s)]\mathrm{d}s + \sum_{\bar{\rho}_n+l'T\leq\tau_k<\bar{\rho}_n+l'T+\nu}\ln(1+h_{ik})\Big] \\ \geq& \bar{m}\exp(l'\bar{\delta}_1 - \bar{A}T - H) \\ \geq& \bar{m}\exp(-\bar{A}T - H), \end{aligned}$$

where $\bar{A} = \sup_{t\in\mathbf{R}^+}\{|r_i(t)| + M|b_i(t)| + |(\tilde{c}_0(t)+\varepsilon)d_i(t)| + |\bar{m}a_i(t)|\}$. For $t \in (\bar{\rho}_n^*, \bar{\rho}_{n+1}]$, we have $z(t) \geq \bar{m} \geq \bar{m}\exp(-\bar{A}T - H)$. Therefore, for all $t \geq \bar{\rho}_1$, $z(t) \geq \bar{m}\exp(-\bar{A}T - H)$ holds.

Assume case (ii) holds. Then we can directly obtain $z(t) \geq \bar{m}\exp(-\bar{A}T - H)$.

Let $\eta = \bar{m}\exp(-\bar{A}T - H)$. So, we have $\lim_{t\to\infty}\inf z(t) \geq \eta$.

By the comparison theorem for impulse differential equations, we have $\lim_{t\to\infty}\inf x_i(t) \geq \eta$. This completes the proof.

Remark 3.2. *Theorem 3.2 provides threshold conditions for the persistent survival of a population, meaning that its density remains consistently above a positive minimum value. In addition to the conditions of Theorem 3.1, it requires that the average net growth rate of the population exceeds the combined loss rate caused by competition, pollution, and disturbances. Specifically, the time-averaged growth rate* $r_i(t)$ *must overcome the competitive effects from the other species* $Mb_i(s)$*, the toxic effects of pollution* $\tilde{c}_0(s)d_i(s)$*, and the net impact of anthropogenic disturbances (for example, positive pulses such as stocking can enhance persistence). This reveals that in a time-varying environment, the long-term survival of a population depends on whether its growth potential can withstand the synergistic effects of competitive pressure, cumulative pollution toxicity, and impulsive disturbances. If the conditions are satisfied, even under severe environmental fluctuations, the population can maintain a minimum sustainable density.*

Theorem 3.2 provides the conditions for the persistence of solutions to system (2.1). Next, we use the method of constructing a Lyapunov function to investigate the conditions for the extinction of competing populations.

Denote

$$\alpha(t) = \min\{a_1(t), a_2(t)\}, \ h_k = \max_{t\in\mathbf{R}^+}\{h_{1k}, h_{2k}\},$$
$$\gamma(t) = \min\{r_1(t) - d_1(t)c_0(t), r_2(t) - d_2(t)c_0(t)\}.$$

Theorem 3.3. *If*

$$\liminf_{t\to\infty} \int_t^{t+T} \alpha(s)ds > 0, \ \limsup_{t\to\infty} \left[\int_t^{t+T} \gamma(s)ds + \sum_{t\le\tau_k<t+T} \ln(1 + h_k)\right] \le 0, \tag{3.10}$$

and $|h(t,\theta)| = \left|\sum_{t\le\tau_k<t+T} \ln(1 + h_k)\right| \le H$*, then for any solution* $(x_1(t), x_2(t))$ *with a positive initial value of system (2.1), we have* $\lim_{t\to\infty} x_i(t) = 0$.

Proof. By (3.10), for any $\varepsilon > 0$, we have

$$\limsup_{t\to\infty} \left[\int_t^{t+T} \left(\gamma(s) - \varepsilon\frac{\alpha(s)}{2}\right) \mathrm{d}s + \sum_{t\le\tau_k<t+T} \ln(1 + h_k)\right] < 0.$$

Therefore, there exist positive number $\tilde{\delta}$ and $\tilde{T}$. For $t \ge \tilde{T}$, we have

$$\int_t^{t+T} \left(\gamma(s) - \varepsilon\frac{\alpha(s)}{2}\right) \mathrm{d}s + \sum_{t\le\tau_k<t+T} \ln(1 + h_k) < -\tilde{\delta}. \tag{3.11}$$

Construct the Lyapunov function by defining $V(t) = x_1(t) + x_2(t)$. Then, when $t \ne \tau_k$,

$$\begin{aligned}
D^+V(t) =&\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} + \frac{\mathrm{d}x_2(t)}{\mathrm{d}t}\\
=&x_1(t)[r_1(t) - c_0(t)d_1(t) - a_1(t)x_1(t)] - b_1(t)x_1(t)x_2(t)\\
&+ x_2(t)[r_2(t) - c_0(t)d_2(t) - a_2(t)x_2(t)] - b_2(t)x_1(t)x_2(t)\\
\le&\gamma(t)V(t) - \alpha(t)(x_1^2(t) + x_2^2(t))\\
\le&V(t)\left(\gamma(t) - \frac{\alpha(t)}{2}V(t)\right).
\end{aligned}$$

When $t = \tau_k$, we have

$$V(\tau_k^+) = x_1(\tau_k^+) + x_2(\tau_k^+) = (1 + h_{1k})x_1(\tau_k) + (1 + h_{2k})x_2(\tau_k) \leq (1 + h_k)V(\tau_k).$$

From (3.11), using a method similar to that in Theorem 3.1, we can obtain $V(t) \leq \varepsilon \exp(CT + H)$ for $t \geq \tilde{T}$, where $C = \sup_{t\in\mathbf{R}^+}\{|\gamma(t)| + \frac{\varepsilon\alpha(t)}{2}\}$. By the arbitrariness of ε, we have $V(t) \to 0$ as $t \to \infty$. From the definition of $V(t)$, we know that $\lim_{t\to\infty} x_1(t) = 0$ and $\lim_{t\to\infty} x_2(t) = 0$. This completes the proof.

Remark 3.3. *Theorem 3.3 describes the conditions leading to population extinction. Its core idea is this: When the average net growth potential of a population is accounted for as the maximum growth rate adjusted for pollution, $\gamma(t)$ is non-positive in the time-averaged sense, and the intraspecific competition intensity $\alpha(t)$ is sufficiently high, the population density will tend toward zero. If anthropogenic disturbances take the form of negative pulses (e.g., harvesting), they further reduce the net growth rate and accelerate extinction. From a biological perspective, this emphasizes that in polluted environments, if the intrinsic growth capacity of a population is insufficient (e.g., due to chronic toxic effects of pollutants inhibiting reproduction or survival) or the combined pressures of competition and disturbances are too severe, the species will be unable to sustain itself, eventually leading to local extinction. This provides a theoretical basis for assessing the ecological risks posed by pollution events and human activities.*

4. Global attractivity of system (2.1)

Having established the threshold conditions for population persistence in Section 3, we now focus on the case where both competing populations survive in the long term (i.e., the conditions of Theorem 3.2 are satisfied). Under this premise, we investigate the global attractivity of the solutions to system (2.1), which concerns whether different initial population densities converge to the same dynamic trajectory over time.

Assuming that system (2.1) satisfies the conditions of Theorem 3.2, for any two positive solutions $(x_1(t), x_2(t))$ and $(x_1'(t), x_2'(t))$ of system (2.1), by Theorems 3.1 and 3.2, there exists $M > 0$ and $\eta > 0$ such that

$$\eta \leq \lim_{t\to\infty} \inf\{x_i(t), x_i'(t)\} \leq \lim_{t\to\infty} \sup\{x_i(t), x_i'(t)\} \leq M, \ i = 1, 2. \tag{4.1}$$

Theorem 4.1. *Suppose the conditions of Theorem 3.2 hold,* $\beta(t) = \min\{a_1(t) - b_2(t), a_2(t) - b_1(t)\} \geq 0$ *for* $t \in \mathbf{R}^+$, *and there exists a* $\lambda > 0$ *such that* $\lim_{t\to\infty} \inf \int_t^{t+\lambda} \beta(s)ds > 0$. *Then the solutions of system (2.1) are globally attractive. That is, for any two positive solutions* $(x_1(t), x_2(t))$ *and* $(x_1'(t), x_2'(t))$ *of system (2.1), we have*

$$\lim_{t\to\infty}(x_1(t) - x_1'(t)) = 0, \ \lim_{t\to\infty}(x_2(t) - x_2'(t)) = 0.$$

Proof. Construct the Lyapunov function $V(t) = |\ln x_1(t) - \ln x_1'(t)| + |\ln x_2(t) - \ln x_2'(t)|$. On one hand, $V(\tau_k^+) = |\ln x_1(\tau_k^+) - \ln x_1'(\tau_k^+)| + |\ln x_2(\tau_k^+) - \ln x_2'(\tau_k^+)| = V(\tau_k)$, i.e., $V(t)$ is continuous at $t \geq 0$. On the other hand, when t is large enough, from (4.1), we obtain

$$\frac{1}{M}|x_1(t) - x_1'(t)| \leq |\ln x_1(t) - \ln x_1'(t)| \leq \frac{1}{\eta}|x_1(t) - x_1'(t)|, \tag{4.2}$$

$$\frac{1}{M}|x_2(t) - x_2'(t)| \leq |\ln x_2(t) - \ln x_2'(t)| \leq \frac{1}{\eta}|x_2(t) - x_2'(t)|, \tag{4.3}$$

$$\begin{aligned}
D^+V(t) =& \operatorname{sgn}(x_1(t) - x_1'(t)) \left(\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} \frac{1}{x_1(t)} + \frac{\mathrm{d}x_1'(t)}{\mathrm{d}t} \frac{1}{x_1'(t)} \right) \\
&+ \operatorname{sgn}(x_2(t) - x_2'(t)) \left(\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} \frac{1}{x_2(t)} + \frac{\mathrm{d}x_2'(t)}{\mathrm{d}t} \frac{1}{x_2'(t)} \right) \\
\leq& \operatorname{sgn}(x_1(t) - x_1'(t))[-a_1(t)(x_1(t) - x_1'(t)) - b_1(t)(x_2(t) - x_2'(t))] \\
&+ \operatorname{sgn}(x_2(t) - x_2'(t))[b_2(t)(x_1(t) - x_1'(t)) - a_2(t)(x_2(t) - x_2'(t))] \\
\leq& -(a_1(t) - b_2(t))|x_1(t) - x_1'(t)| - (a_2(t) - b_1(t))|x_2(t) - x_2'(t)| \\
\leq& -\beta(t)\eta V(t).
\end{aligned}$$

Integrating the above inequality yields $V(t) \leq V(0)\exp(-\eta \int_0^t \beta(s)\mathrm{d}s)$. So, we have $V(t) \to 0$ as $t \to \infty$. From (4.2) and (4.3), we obtain $\lim_{t\to\infty}(x_1(t) - x_1'(t)) = 0$ and $\lim_{t\to\infty}(x_2(t) - x_2'(t)) = 0$. This completes the proof.

Remark 4.1. *Theorem 4.1 addresses the global attractivity of the system's solutions, meaning that any two positive solutions (representing different initial population densities) will converge to the same dynamic trajectory over time. The condition requires that the intraspecific competition intensity is always no less than the interspecific competition intensity (i.e.,* $\beta(t) = \min\{a_1(t) - b_2(t), a_2(t) - b_1(t)\} \geq 0$*), and that this difference is positive in the time-averaged sense. Biologically, this implies that the self-regulatory ability (intraspecific competition) of each species always dominates or equals the competitive pressure from the other species. Consequently, the system exhibits stable dynamic behavior and is not affected by initial conditions. Such predictability is crucial for ecological management. For example, in pollution remediation or species introduction, we can expect that the population will eventually tend to a unique equilibrium or periodic state, which helps in formulating long-term conservation strategies.*

5. Numerical simulations

Theoretical threshold conditions for population persistence and extinction were established in Section 3. In this section, we perform numerical simulations to verify these theoretical results and to illustrate the system dynamics under different parameter configurations.

We first consider a scenario that leads to the persistence of both populations. The initial values are set as $(x_1(0), x_2(0), c_0(0), c_e(0)) = (0.8, 0.6, 0.2, 0.1)$, with the common period $T = 1$, the impulsive moment $\tau_k = 0.5$, $q = 1$, and the simulation step size to 1. The other parameters are chosen as follows: $r_1(t) = 1.1 + \sin(20\pi t)$, $r_2(t) = 1.1 + \cos(20\pi t)$, $a_1(t) = 0.4$, $a_2(t) = 0.2$, $b_1(t) = 0.19$, $b_2(t) = 0.3$, $d_1(t) = 0.2$, $d_2(t) = 0.3$, $k_0 = 0.4$, $g = 0.2$, $m = 0.3$, and $h = 0.2$. In the scenario where the anthropogenic disturbances are positive pulses (e.g., $h_{1k} = 0.8$, $h_{2k} = 0.7$, which may represent protective stocking) and the pollutant discharge level is low ($\mu = 0.1$), we can calculate

$$\liminf_{t\to\infty} \int_t^{t+T} a_1(s)\mathrm{d}s = 0.4 > 0, \liminf_{t\to\infty} \int_t^{t+T} a_2(s)\mathrm{d}s = 0.2 > 0.$$

Given that $|h_i(t,\theta)| = \left|\sum_{t\le\tau_k<t+\theta}\ln(1+h_{ik})\right| < 0.6$, we take $H = 0.6$. Further, we have $A = \sup_{t\in\mathbf{R}^+}\{|r_i(t)| + m|a_i(t)|\} = 2.22$, and thus $M = m\exp(AT+H) = 2.3$. The key integrals are computed as

$$\int_t^{t+T} [r_1(s) - Mb_1(s) - \tilde{c}_0(s)d_1(s)]\mathrm{d}s + \sum_{t\le\tau_k<t+T} \ln(1+h_{1k}) = 1.1478 > 0,$$

$$\int_t^{t+T} [r_2(s) - Mb_2(s) - \tilde{c}_0(s)d_2(s)]\mathrm{d}s + \sum_{t\le\tau_k<t+T} \ln(1+h_{2k}) = 0.8206 > 0.$$

These results satisfy all the conditions of Theorem 3.2, illustrating the persistence of both populations, as shown in Figure 1. Furthermore, we note that the condition $\beta(t) = \min\{a_1(t) - b_2(t), a_2(t) - b_1(t)\} = \min\{0.4 - 0.3, 0.2 - 0.19\} \ge 0$ holds for all t, and its time average is positive. Therefore, the additional criteria in Theorem 4.1 are also met. This implies that the persistent state is not only sustained but also globally attractive, that is, regardless of the initial population densities, the solution trajectories will converge to the same long-term dynamic behavior over time. The numerical results shown in Figure 1 are consistent with this theoretical prediction, demonstrating convergence to a stable coexistence state from the specified initial values.

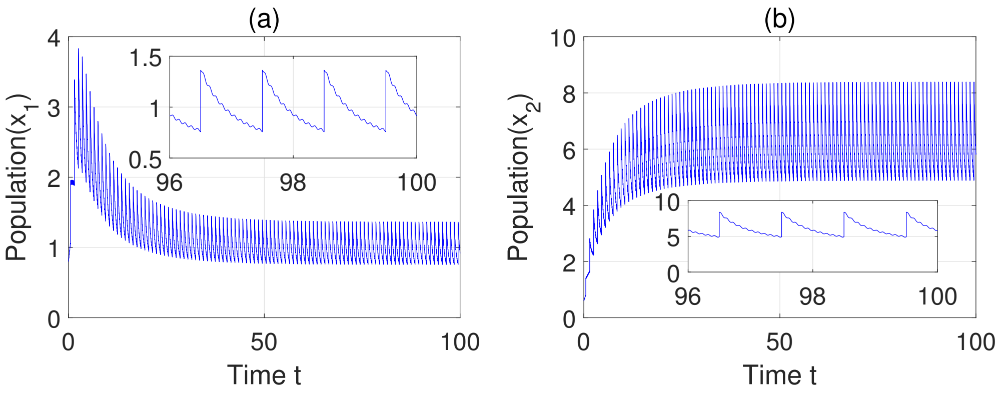

Figure 1. Persistence of system (2.1). (a) Time series plot for population x_1; (b) Time series plot for population x_2, where $h_{1k} = 0.8$, $h_{2k} = 0.7$, and $\mu = 0.1$.

We now consider a scenario designed to validate the extinction threshold stated in Theorem 3.3. Keeping all environmental and pollution parameters identical to the previous case (including $\mu = 0.1$), we alter the nature of the anthropogenic disturbances to strong, sustained negative pulses (e.g., $h_{1k} = -0.7$, $h_{2k} = -0.7$ representing harvesting).

For this configuration, we compute the key composite parameters defined in Theorem 3.3:

$$\alpha(t) = \min\{a_1(t), a_2(t)\} = 0.19,$$

$$h_k = \max_{t\in\mathbf{R}^+}\{h_{1k}, h_{2k}\} = -0.7,$$

$$\gamma(t) = \min\{r_1(t) - d_1(t)c_0(t), r_2(t) - d_2(t)c_0(t)\}.$$

The calculation of the time-averaged quantities yields:

$$\liminf_{t\to\infty} \int_t^{t+T} \alpha(s)\mathrm{d}s = 0.2 > 0,$$

$$\limsup_{t\to\infty}\left[\int_t^{t+T}\gamma(s)\mathrm{d}s+\sum_{t\le\tau_k<t+T}\ln(1+h_k)\right]=-0.6440<0.$$

These results satisfy both inequality conditions of Theorem 3.3. The first condition ensures sufficient intraspecific competitive pressure, while the second confirms that the time-averaged net growth potential, further reduced by the cumulative negative pulses, is non-positive. Therefore, according to the theorem, both populations are predicted to tend toward extinction.

The numerical simulation under this parameter set confirms the theoretical prediction. As shown in Figure 2, the densities of both competing populations decline and converge to zero over time. This example underscores that even with an unchanged pollution level, the introduction of sufficiently strong and frequent negative anthropogenic disturbances (harvesting) can drive the system to collapse, fulfilling the extinction criteria established by Theorem 3.3.

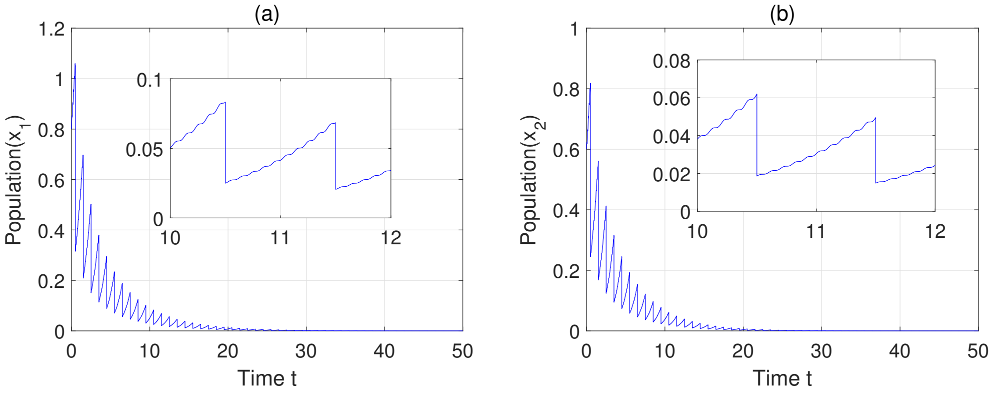

Figure 2. Extinction of system (2.1). (a) Time series plot for population x_1; (b) Time series plot for population x_2, where $h_{1k} = -0.7$, $h_{2k} = -0.7$, and $\mu = 0.1$.

Figure 3 presents a case of competitive exclusion, simulated under parameters identical to the persistence scenario in Figure 1, except for a substantially increased impulsive pollutant input $\mu = 0.8$. The calculated long-term averages yield a critical combination of conditions: The time-averaged intraspecific competition remains positive ($\lim_{t\to\infty}\inf\int_t^{t+T}\alpha(s)\mathrm{d}s = 0.2 > 0$), and the key functionals for the two populations diverge. One satisfies a persistence-like inequality

$$\int_t^{t+T}[r_1(s)-Mb_1(s)-\tilde{c}_0(s)d_1(s)]\mathrm{d}s+\sum_{t\le\tau_k<t+T}\ln(1+h_{1k})=0.5878>0,$$

while the composite growth potential for the more vulnerable population is negative on average:

$$\limsup_{t\to\infty}\left[\int_t^{t+T}\gamma(s)\mathrm{d}s+\sum_{t\le\tau_k<t+T}\ln(1+h_k)\right]=-0.0194<0.$$

This combination of conditions leads to competitive exclusion: Population x_1 (with higher pollution tolerance or growth resilience) survives, while the other (x_2) is driven to extinction, as visually confirmed in Figure 3. The simulation thus demonstrates that a sharp increase in impulsive pollution (μ

from 0.1 to 0.8) can fundamentally alter the outcome, even when all other parameters remain favorable for coexistence.

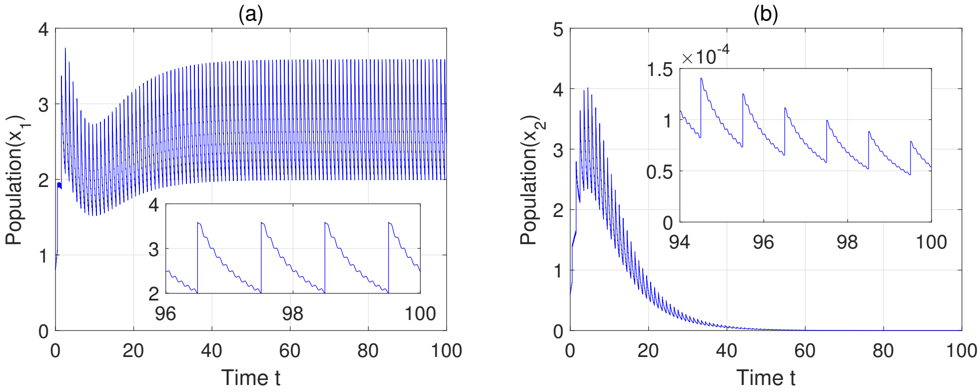

Figure 3. (a) Time series plot for population x_1; (b) Time series plot for population x_2, where $h_{1k} = 0.8$, $h_{2k} = 0.7$, and $\mu = 0.8$.

6. Conclusions and discussion

In this paper, we studied the dynamics of a non-autonomous competitive system subject to simultaneous impulsive pollutant discharge and external anthropogenic discontinuous disturbances, such as impulsive harvesting or stocking. Through theoretical analysis, we derived the conditions for the persistence and extinction of competing populations under such impulsive stress.

The theoretical results show that the solutions of the system are globally positive and bounded. By establishing threshold criteria for ultimate persistence and extinction and constructing appropriate Lyapunov functions, we rigorously proved the conditions for the global attractivity of the positive solutions. The main conclusion indicates that the ultimate fate of the populations is determined not only by traditional intra- and inter-specific competition, but also by the combination of the populations' time-varying growth potential, competitive pressures, the intensity and frequency of impulsive disturbances, and the cumulative toxic effects of pollutants. In particular, Theorems 3.1–3.3 show that when the average "net growth rate" of population exceeds the "combined loss rate" caused by competition, pollution, and impulsive disturbances, the population persists permanently; otherwise, it tends toward extinction.

The numerical simulations presented in Section 5 provide clear visual verification of the theoretical thresholds established in Theorems 3.2 and 3.3. Figure 1 confirms that when the time-averaged net growth rate surpasses the combined losses from competition, pollution, and disturbances, both populations persist, and the solutions converge to a globally attractive state, consistent with Theorem 4.1. In contrast, Figure 2 demonstrates that strong negative anthropogenic pulses can drive both populations to extinction, fulfilling the conditions of Theorem 3.3. Furthermore, Figure 3 reveals a critical scenario of pollution-mediated competitive exclusion: A sharp increase in impulsive pollutant input (μ from 0.1 to 0.8) can alter the competitive balance, leading to the survival of the more tolerant species and the extinction of the more vulnerable one, even under otherwise favorable conditions. These simulations not only validate the theoretical findings but also underscore the practical significance of the model in predicting population fates under complex, coupled impulsive stressors.

This paper extends the theory of non-autonomous competitive systems to more realistic ecological scenarios. The results indicate that, in ecosystems frequently affected by human activities, the effects of discrete and abrupt disturbance events, whether arising from direct population management or indirect pollution inputs, cannot be adequately described by continuous processes. These disturbances interact nonlinearly with the time-varying characteristics of the system and jointly determine population persistence and extinction. Therefore, when formulating ecological conservation or pollution control strategies, it is important not only to consider long-term average intensities, but also to carefully evaluate the timing, frequency, and magnitude of individual impulsive events. Such considerations are essential for preventing cumulative effects, together with environmental fluctuations, from triggering irreversible population extinction or drastic shifts in community structure. Future work may incorporate environmental stochasticity and spatial dispersal to develop more comprehensive models.

To further highlight the novelty of our model, we compare it with two simplified frameworks. First, if the non-autonomous terms are neglected and the system is treated as autonomous, the population dynamics become time-invariant, and the extinction thresholds depend only on constant competition and pollution levels. In this case, the model cannot capture the cumulative effects of time-varying growth rates and disturbance frequencies. Second, if one of the impulsive mechanisms is removed (for example, considering impulses only on population densities but not on pollutant dynamics), the model fails to describe the combined impact of simultaneous perturbations in both biotic and abiotic components. In contrast, the proposed model captures the interaction between non-autonomous dynamics and dual impulsive effects, leading to richer dynamical behaviors, such as species-specific extinction driven by differential toxicities. These features highlight the importance of incorporating both time-varying environments and coupled impulsive disturbances when predicting population persistence in realistic ecological systems.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Nos. 12301616, 12171004), the Doctor Research Project Foundation of Liaoning Province (No. 2023-BS-210), the Liaoning Key Laboratory of Development and Utilization for Natural Products Active Molecules (Nos. LZ202302, LZ202303).

Conflict of interest

The authors declare there are no conflicts of interest.

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