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Research article

A real-time pricing dynamic algorithm for a smart grid with multi-pricing and multiple energy generation

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Abstract: With the proposal of the new power system, power supply from renewable energy sources and traditional power supply have emerged as the future development directions of the power grid, while the traditional pricing mechanisms are facing new challenges. Considering the different characteristics of renewable energy power supply and traditional power supply, such as being clean and sustainable, but unstable, for renewable energy power supply,and being stable and technologically mature, but causing significant pollution, for traditional power supply, a multi-price model with the cost of pollution treatment under the multi-energy electricity generation was established in this paper. A distributed algorithm with the non-dominated sorting genetic algorithm (NSGA-II) was also proposed. In the model, the power supply side includes traditional energy generation, renewable energy generation, and the energy storage device. The proposed algorithm was designed using Lagrangian duality theory, and the multi-price is obtained by solving the different lagrange multipliers. Finally, the numerical results show that the model is reasonable when comparing the obtained price and social welfare with that untreated-pollution model, as well as a single supply model. Also, the proposed algorithm always has better computational efficiency, when compared with PSO, HS, and GA algorithms. The proposed model and algorithm provide a new idea and method for the optimal scheduling of a smart grid.

Keywords: smart grid; real-time pricing; multi-source power supply; multi-energy generation; distributed algorithm; pollution

1. Introduction

In the last two decades, the problem of power shortage has become increasingly severe worldwide. According to the International Energy Agency (IEA) report, global electricity demand grew by 2.4 percent in 2022, while supply capacity increased by only 1.8 percent, which has led to tight power supply in many countries and regions, especially under the dual pressures of frequent extreme weather events and economic growth demands [1]. This imbalance between power supply and demand has prompted countries to accelerate power system reforms and seek more efficient and sustainable power supply solutions.

As is well known, renewable energy could alleviate the problem of power supply shortage. Solar energy, as a typical representative of renewable energy, converts light energy into electrical energy through photovoltaic panels. It is widely used in distributed power generation and large-scale photovoltaic power plants, providing electricity for remote areas and helping to achieve energy self-sufficiency. Wind energy utilizes wind turbines to capture wind energy and convert it into electricity. Large-scale wind farms are booming in areas rich in wind resources such as coastal regions and plateaus. Hydropower, on the other hand, relies on hydropower stations to generate electricity by driving turbines with water flow, which is a stable and efficient source of electricity. These renewable energy sources have significant advantages of being clean and environmentally friendly, producing almost no greenhouse gas emissions, which can greatly alleviate environmental pressure. They are renewable, with inexhaustible resources, avoiding the problem of resource depletion faced by traditional energy sources. However, the intermittency and volatility of renewable energy sources pose huge challenges to the stable supply of electricity, and traditional power supply also faces many limitations in peak shaving and frequency modulation. To address this issue, smart grids have emerged as an effective solution, which could utilize advanced information and intelligent control technologies to perceive the generation status of various power sources and the power consumption status of users in real time.

Demand-side management (DSM) is a crucial aspect in the smart grid, enabling the efficient balance of power supply and demand [2]. As an important part of demand-side management, demand response centers on guiding users to adjust their electricity consumption behavior through price signals, thus achieving a balance between electricity supply and demand. Nowadays, the pricing mechanism of electric power products mainly includes fixed pricing, time-sharing pricing, ladder pricing, and real-time pricing, etc., in which real-time pricing (RTP) is widely regarded as the most ideal pricing mechanism in the electric power market. At present, the pricing of power products has attracted a lot of attention from researchers. It is mainly carried out from two perspectives. On one hand, many studies adopt the game theory method to analyze the supply and demand relationship through the multi-layer game model, and then obtain the equilibrium solution and the electricity supply and use strategy [3–7]. On the other, some studies regard electricity as a public good and use optimization methods to solve the electricity price problem [8–15]. Samadi et al. proposed a power system model of a power supplier with multiple users, and solved the optimal energy consumption and consumption pattern by a distributed Lagrangian dual algorithm [8]. Yuan extended the smart grid real-time pricing model with multiple types of users on this basis [9]. Qu utilized a smoothing function and proposed a smooth Newton's algorithm to improve the accuracy of the model [10]. Zhu improved the risk term in the online real-time pricing risk model by improving the risk term in the online real-time tariff risk model, which is transformed into a dual problem for solving [11]. Song et al. proposed the nonsmooth Levenberg-Marquardt method based on KKT conditions for real-time pricing [12]. Similar research can also be found in [13–15]. However, the above research is mostly based on single power generation, or does not highlight the difference between the traditional energy supply and renewable energy supply. Actually, the cost of power generation from renewable energy sources such as solar energy and wind energy is relatively low, thus the power generation from renewable energy sources is unstable, and there is a possibility of power outages for users. The power generation from traditional energy sources is controllable and has good stability, but the power generation cost is relatively high, and will cause a large degree of environmental pollution. Especially under the background of the “dual carbon” goals, a significant amount of pollution treatment costs need to be paid. Recently, some research improved the smart grid real-time pricing model by considering the power supply side including traditional fossil energy and clean energy generation, and provided some real-time pricing models in a multi-energy generation system [16–21]. Yuan et al. constructed a multi-energy regional bi-level pricing model, introducing regional synergy and a user behavior feedback mechanism [17]. Qu et al. further focused on the interaction between the main grid and the microgrid, and proposed a real-time pricing strategy based on the joint power supply mechanism [18]. Li et al. proposed an optimal pricing method considering generation startup and shutdown costs, emphasizing the role of cost structure in price formation [16]. Luo et al. designed a two-stage response model oriented to user deviation management by incorporating incentive mechanisms [19]. Similar studies can also be found in the literature [20, 21]. Considering that the pollution control cost is typically not addressed, and the differences in power supply between traditional energy supply and renewable energy supply are always from the perspective of a market economy, the social benefits may not necessarily reach the optimal level. A multi-price model with the cost of pollution treatment based on the power supply from multiple sources to maximum the total social benefits is established in this paper.

Actually, the non-dominated sorting genetic algorithm II (NSGA-II) is an effective algorithm for a multi-objective problem, and is always adopted for some models with a smart grid recently [22–25]. Alzahrani et al. constructed a multi-objective dispatch model based on the NSGA-II to achieve simultaneous reductions in operating cost, pollution emission, and load loss expectation while considering renewable energy uncertainty [22]. Mahdi et al. proposed a bi-objective optimization approach for microgrids, integrating the NSGA-II with uncertainty modeling, to significantly reduce operating costs, carbon emissions, and peak-to-average ratios while guaranteeing user comfort [23]. Mundra et al. proposed an independent demand-side management system based on the NSGA-II for future smart grids, which optimizes power consumption scheduling and effectively reduces power peaks and electricity costs while protecting the interests of multiple users [24]. Wang et al. proposed a load resource management method based on the NSGA-II, which improves the frequency and voltage response performance through multi-objective optimization while taking into account the utility loss of users [25]. Considering the effectiveness of the NSGA-II, and it always has significant advantages in the field of multi-objective optimization, we designed an algorithm framework that combines the Lagrangian duality theory with the NSGA-II (non-dominated sorting genetic algorithm II) for solving the proposed algorithm.

The main contributions of this paper are as follows: (1) a multi-price pricing model is constructed with the objective of maximizing social welfare, differentiated electricity prices are implemented for users with different power supply characteristics, and two types of Lagrangian multipliers are introduced through the Lagrangian pairwise algorithm as the corresponding real-time prices. (2) Pollution control costs are introduced into the model to make it closer to the green development requirements of the actual power system. (3) In order to break through the solution bottleneck caused by the non-convex problem, the NSGA-II is adopted to narrow the dyadic gap and improve the accuracy of the solution, so that the obtained results are closer to the optimal solution of the original problem. (4) The numerical results verify the advantages of the proposed method in terms of computational efficiency, and at the same time show that the multiple power supply mode is superior to the single power supply mode, which provides a new method for the optimal scheduling of the smart grid. The comparisons between our work with the related works are listed in Table 1.

Table 1. Comparison of related works.

References	RTP	Welfare	Multi-energy	Multi-pricing	Environmental protection
[16]	✓	✓	✓	×	×
[18]	✓	✓	✓	×	×
[19]	✓	×	✓	✓	×
[20]	✓	✓	✓	×	×
[21]	✓	✓	✓	×	×
This paper	✓	✓	✓	✓	✓

The outline of the paper is as follows. In Section 2, a mathematical formulation of the system model is formulated. In Section 3, a distributed algorithm with the NSGA-II is proposed. In Section 4, the numerical experiment results are shown to illustrate the effectiveness of the model and the algorithm.

2. System model

Consider a smart grid system that includes an energy provider, such as a power plant, solar energy generation, wind power generation, a pollution treatment factory, and a certain number of power users. The interaction relationship of the smart grid is represented by Figure 1. Suppose the power supply and all users are connected with each other through an information communication infrastructure. Divide the whole cycle into k periods, let $N = \{1, 2, ..., n\}$ be the set of users, and $\mathbb{T} = \{1, 2, ..., K\}$ is the set of time periods.

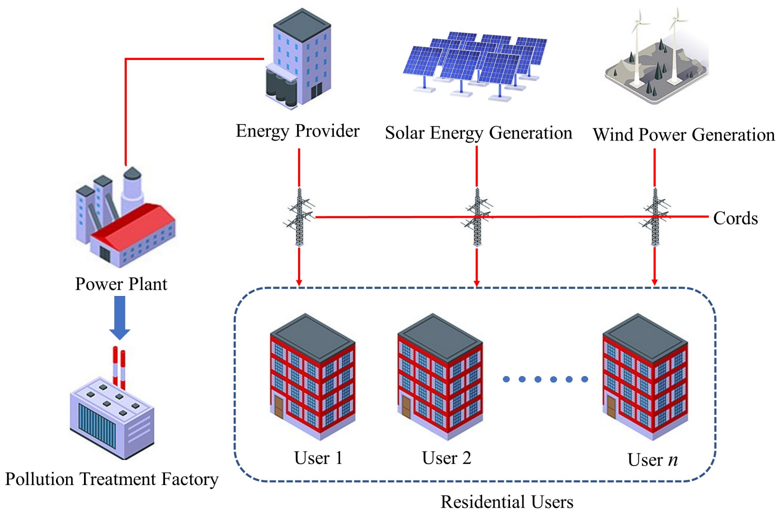


Figure 1. Topology figure of a smart grid system.

2.1. Demand side analysis considering multi-pricing based on different power generation

The power users are the demand main body of the smart grid system. The behavior of electricity users can be expressed in terms of a definite utility function according to microeconomics. Suppose $U(\cdot)$ is a utility function, and it has the properties such that

$$\frac{\partial U(\cdot)}{\partial x} \geq 0, \frac{\partial U^2(\cdot)}{\partial^2 x} \leq 0,$$

where x is the power consumption demand. Utility functions are often used in microeconomics as a measure of the requirements and desires of a customer based on their consumption or leisure activities. These functions represent the relationship between the consumed utility and the amount of a commodity.

In economics, utility functions are often in the form of quadratic, logarithmic, or segmented linear functions. Here, we adopt a commonly used form of a quadratic utility function such that:

$$U(x, \omega) = \begin{cases} \omega x - \frac{\alpha}{2} x^2, & 0 < x < \frac{\omega}{\alpha}, \\ \frac{\omega^2}{2\alpha}, & x \geq \frac{\omega}{\alpha}, \end{cases} \tag{2.1}$$

where x is the electricity consumption of the user, $\alpha > 0$ is a known parameter, and $\omega > 0$ is the elasticity coefficient of electricity consumption, denoting the sensitivity of electricity consumption to changes in electricity prices. Actually, the elasticity coefficients of electricity consumption vary in different time periods, especially in peak and valley times, which may show significant differences in the elasticity of electricity consumption.

Considering that the power generation from renewable energy sources is unstable for users, and the power generation from traditional energy sources is controllable and has good stability, we denote different parameters in this paper, such that $\omega_i^{k,T}$ and $\omega_i^{k,N}$ denote the parameters in (2.1) of the *i-th* user, representing the different satisfactions of users using the electricity from traditional generation and renewable energy generation, respectively. Meanwhile, the single electricity price is not suitable for multi-energy power generation in smart grids. We denote p_T^t, p_N^k as the real-time price of the traditional energy power and the renewable energy power, respectively, $x_i^{k,T}, x_i^{k,N}$ as the traditional and new electricity consumption of the *i-th* user, and then the electricity expenditure of the *i-th* user in the period k can be represented as follows:

$$C_i^k = p_T^k \cdot x_i^{k,T} + p_N^k \cdot x_i^{k,N}. \tag{2.2}$$

To promote the utilization of renewable energy, the government always provides subsidies through economic incentives to provide some subsidies for the users. We denote $R_i^{k,D}$ as the subsidies for the *i-th* user in the period k, and the mechanism of subsidizing the consumption of clean energy is expressed in the form of a subsidy function [26]:

$$R_i^{k,D} = m_1 x_i^{k,N}, \tag{2.3}$$

where $m_1 > 0$ is called the subsidy parameter.

Furthermore, we denote $U(x_i^k, \omega_i^t) = U(x_i^{k,T}, \omega_i^{k,T}) + U(x_i^{k,N}, \omega_i^{k,N})$ the profit R_D^k of the demand supply in the period k as follows:

$$R_D^k = \sum_{i=1}^{n} (U(x_i^k, \omega_i^k) - C_i^k + R_i^{k,D}). \tag{2.4}$$

2.2. Supply side analysis considering the pollution treatment

With its significant advantages of renewability, greenness, and low marginal cost, clean energy has gradually occupied an important position in the modern power grid system. However, as the distributed clean energy is mainly based on wind and solar power generation, it has a small power supply and unstable voltage. Therefore, the introduction of energy storage equipment has become a key link to guarantee the stable output of clean energy. The power supplied by distributed clean energy needs to be stored by energy storage equipment for one cycle before it can be delivered to residential users. Therefore, the amount of electricity supplied by clean energy that can be used stably in the current period is equal to the total amount of electricity supplied by clean energy in the previous period.

The profits of the power suppliers at the time slot k are from two parts. The first one is obtained by selling the electricity to the users, namely,

$$R_S^{k,T} = \sum_{i=1}^{n} p_T^k \cdot x_i^{k,T}, \tag{2.5}$$

and

$$R_S^{k,N} = \sum_{i=1}^{n} p_N^k \cdot x_i^{k,N}. \tag{2.6}$$

The second part is from the subsidies from the government for supplying the renewable energy. We denote $R_i(k, S)$ as the subsidies for the *i-th* user at the slot k. Similar to the subsidies of the demand-supply, the mechanism of subsidizing the *i-th* supply of clean energy is expressed in the form of

$$R^{k,N} = m_2 L_k^N, \tag{2.7}$$

where $m_2 > 0$. This is similar to the subsidies for the users.

The supply cost function is used to describe the total cost to the supplier of producing a given amount of electricity. The cost of supplying electricity from traditional energy sources usually increases with production and it is convex. Here, the power supply cost function for traditional energy sources is adopted in the following from:

$$C_T^k(L_k^T) = a_k(L_k^T)^2 + b_k L_k^T + c_k, \tag{2.8}$$

where L_k^T denotes the total electricity of traditional energy sources at the time slot k, and $a > 0$, $b \geq 0$, and $c \geq 0$ are known parameters.

The wind energy, solar energy, and other natural resources used in renewable energy power generation do not need to pay costs. The raw material cost is ignored. The cost of supplying electricity from clean energy mainly comes from electrical energy storage, which has a low and fixed marginal cost, and we denote it as a linear function such that

$$C_N^{k,1}(L_k^N) = rL_k^N, \tag{2.9}$$

and the maintenance cost of renewable energy power generation equipment is described as [27]

$$C_N^{k,2}(L_k^N) = \theta \cdot (L_k^N)^2 + \eta \cdot L_k^N, \tag{2.10}$$

where $r > 0$ is the marginal cost of clean energy, $\theta > 0, \eta > 0$ are the equipment maintenance cost coefficients, and L_k^N is the total electricity of renewable energy supplied in the k-th time period.

Actually, since the main grid generates electricity from conventional energy sources, it emits pollution into the environment. It is necessary to consider the cost of pollutant treatment and promote the clean development and sustainable operation of the grid by minimizing the cost of pollution treatment. In this paper, we consider the environmental protection cost function:

$$C_P^k = \sum_{i=1}^{m} C_i \gamma_i, \tag{2.11}$$

where m is the type of pollution, γ_i denotes the emission coefficient of the *i-th* pollutant, and C_i is the treatment cost of the corresponding *i-th* pollutant.

In summary, the total profit of the power-supply side at the time slot k is as follows:

$$R_S^k = R_S^{k,T} + R_S^{k,N} + R^{k,N} - C_T^k(L_k^T) - C_N^{k,1}(L_k^N) - C_N^{k,2}(L_k^N) - C_P^k. \tag{2.12}$$

3. Model establishment and analysis

In this section, a multi-price model with the cost of pollution treatment based on the power supply from multiple sources is proposed.

Consider the real-time pricing difference between different power generation and the pollution treatment. We establish the new model in this paper as

$$\begin{aligned} \max \quad & \sum_{k=1}^{K} (R_D^k + R_S^k) \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_i^{k,T} \le L_k^{\mathrm{T}}, k \in \mathcal{T}, \\ & \sum_{i=1}^{n} x_i^{k,N} \le L_k^N, k \in \mathcal{T}. \end{aligned}$$

By straight calculation, the model is in the following form:

$$\begin{aligned} \max \quad & \sum_{k=1}^{K} \{ \sum_{i=1}^{n} (U(x_i^{k,T}, \omega_i^{k,T}) + U(x_i^{k,N}, \omega_i^{k,N})) - C_T^k(L_k^T) - C_N^{k,1}(L_k^N) - C_N^{k,2}(L_k^N) - C_P^k \} \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_i^{k,T} \le L_k^{\mathrm{T}}, k \in \mathcal{T}, \\ & \sum_{i=1}^{n} x_i^{k,N} \le L_k^N, k \in \mathcal{T}, \end{aligned} \tag{3.1}$$

where C_P^k is the treatment cost of pollution.

Obviously, model (3.1) is a convex problem, since the utility function $U(x_i^k, \omega_i^k)$ is concave, the power supply cost function $C_T^k(L_k^T), C_N^{k,2}(L_k^N)$ is convex, the pollution treatment cost $C_N^{k,1}(L_k^N), C_P^k$ is linear, and the constraints are linear. The model is a convex programming problem whose local optimal solution is the global optimal solution.

Consider each period $k \in \mathcal{T}$, and model (3.1) is formulated as follows:

$$\begin{aligned} \max \quad & \sum_{i=1}^{n}(U(x_i^{k,T}, \omega_i^{k,T}) + U(x_i^{k,N}, \omega_i^{k,N})) - C_T^k(L_k^T) - C_N^{k,1}(L_k^N) - C_N^{k,2}(L_k^N) - C_P^k \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_i^{k,T} \le L_k^{\mathrm{T}}, k \in \mathcal{T}, \\ & \sum_{i=1}^{n} x_i^{k,N} \le L_k^N, k \in \mathcal{T}. \end{aligned} \tag{3.2}$$

Both models (3.1) and (3.2) can be used to determine the optimal electricity consumption, electricity supply, and electricity price, but their meanings are different. Model (3.1), in which the specific power consumption and price in each period are usually calculated in a cycle, is called the day-ahead pricing model. Model (3.2) determines the specific electricity consumption and electricity price in each period in real time, so it has more real-time characteristics. At present, the electricity price, which is determined by the two social welfare maximization models, is called real-time pricing.

4. Algorithm based on duality theory

In this section, a distributed algorithm for model (3.1) and the non-dominated sorting genetic algorithm (NSGA-II) for its subproblems are proposed.

Electricity supply and demand in each time period are independent of each other. We adopt the Lagrangian duality algorithm to transform model (3.2) in order to reflect the dynamic process of electricity price changes. The Lagrangian function corresponding to model (3.2) is such that

$$\begin{aligned} & L(x_i^{k,T}, x_i^{k,N}, L_k^T, L_k^N, \lambda_k^T, \lambda_k^N) = \sum_{i=1}^{n} (U(x_i^{k,T}, \omega_i^{k,T}) + U(x_i^{k,N}, \omega_i^{k,N})) - C_T^k(L_k^T) - C_N^{k,1}(L_k^N) \\ & - C_N^{k,2}(L_k^N) - C_P^k + \lambda_k^T \left[L_k^T - \sum_{i=1}^{n} x_i^{k,T} \right] + \lambda_k^N \left[L_k^N - \sum_{i=1}^{n} x_i^{k,N} \right] \\ & = \sum_{i=1}^{n} \left[(U(x_i^{k,T}, \omega_i^{k,T}) + U(x_i^{k,N}, \omega_i^{k,N})) - \lambda_k^T x_i^{k,T} - \lambda_k^N x_i^{k,N} \right] \\ & + \lambda_k^T L_k^T + \lambda_k^N L_k^N - C_T^k(L_k^T) - C_N^{k,1}(L_k^N) - C_N^{k,2}(L_k^N) - C_P^k \end{aligned} \tag{4.1}$$

where $\lambda_k^T, \lambda_k^N \ge 0$ are the Lagrange multipliers. According to the dyadic theory, problem (4.1) can be transformed into:

$$\begin{aligned} DP &= \inf\{\sup L(x_i^{k,T}, x_i^{k,N}, L_k^T, L_k^N, \lambda_k^T, \lambda_k^N)\} \\ &= \min_{\lambda_k^T, \lambda_k^N \ge 0} \{\max L(x_i^{k,T}, x_i^{k,N}, L_k^T, L_k^N, \lambda_k^T, \lambda_k^N)\} \end{aligned}$$

It is worth mentioning that the Lagrange multipliers λ_k^T, λ_k^N mean the shadow prices of electricity in microeconomics, which reflect the value of electricity [28].

Let

$$A_i^k(\lambda_k^T, \lambda_k^N) = \max[(U(x_i^{k,T}, \omega_i^{k,T}) + U(x_i^{k,N}, \omega_i^{k,N})) - \lambda_k^T x_i^{k,T} - \lambda_k^N x_i^{k,N}] \tag{4.2}$$

$$P_k(\lambda_k^T, \lambda_k^N) = \max[\lambda_k^T L_k^T + \lambda_k^N L_k^N - C_T^k(L_k^T) - C_N^{k,1}(L_k^N) - C_N^{k,2}(L_k^N) - C_P^k] \tag{4.3}$$

$$\begin{aligned} D(\lambda_k^T, \lambda_k^N) &= \max L(x_i^{k,T}, x_i^{k,N}, L_k^T, L_k^N, \lambda_k^T, \lambda_k^N) \\ &= \sum_{i=1}^{n} A_i^k(\lambda_k^T, \lambda_k^N) + P_k(\lambda_k^T, \lambda_k^N) \end{aligned}$$

We next present the distributed algorithm for the dual problem as

$$\min_{\lambda_k^T, \lambda_k^N \geq 0} D(\lambda_k^T, \lambda_k^N). \tag{4.4}$$

Algorithm 1—Distributed algorithm

Step 1: Initialize the data: set parameters and initializing residential user data and energy supplier data.

Step 2: Demand side: for each residential user i, the energy consumption $x_i^{k,T}, x_i^{k,N}$ is updated based on the electricity price provided by the supplier λ_k^T, λ_k^N by optimally solving Eq (4.2) with the NSGA-II algorithm. The updated electricity consumption is passed to the energy supplier.

Step 3: Supply side: for each time period $k \in K$, update the electricity prices λ_k^T and λ_k^N using the Eqs (4.5) and (4.6). Optimize the solution of Eq (4.3) by the NSGA-II algorithm to update the electricity supply L_k^T and L_k^N. Receive the electricity consumption $x_i^{k,T}, x_i^{k,N}$ from user $i \in N$ and update the total electricity consumption.

Step 4: Algorithm termination: in each iteration, the demand side and supply side solve the local optimization problem separately and dynamically adjust $x_i^{k,T}, x_i^{k,N}, L_k^T$, and L_k^N. Finally, the optimal solution of the objective function is gradually approximated by genetic manipulation and Lagrange's dual optimization method.

From Algorithm 1, the NSGA-II algorithm plays an important role, which is an effective smart algorithm and always used for multi-objective problems. It could reduce the complexity and has a better distribution of solutions. In Algorithm 1, it was used for solving the subproblems, such as the Eqs (4.2)–(4.4). The specific algorithm steps are shown below and the flow chart of the algorithm can be seen in Figure 2.

The NSGA-II algorithm was proposed for multi-objective optimization [29]. This algorithm has significant advantages in the field of multi-objective optimization. It introduces a fast non-dominated sorting algorithm, which greatly reduces the computational complexity of the algorithm. Compared with traditional algorithms, it can more efficiently process large-scale population data and accelerate the convergence speed of the algorithm, enabling it to find a better solution set in a shorter time. At the same time, the NSGA-II algorithm employs crowding distance and crowding comparison operators. These operators can maintain population diversity while effectively avoiding the clustering of individuals. This ensures that the algorithm can explore a wider solution space during the search process and find Pareto front solutions with a uniform distribution, providing decision-makers with more diverse choices. In order to reduce the dual gap and improve the solution accuracy, we adopt the NSGA-II algorithm in this paper. The concrete steps are as follows.

Subalgorithm: NSGA-II Algorithm

Step 1: Population initialization and objective function calculation: an initial population P_0 is randomly generated, and each individual in the population consists of the user's electricity consumption

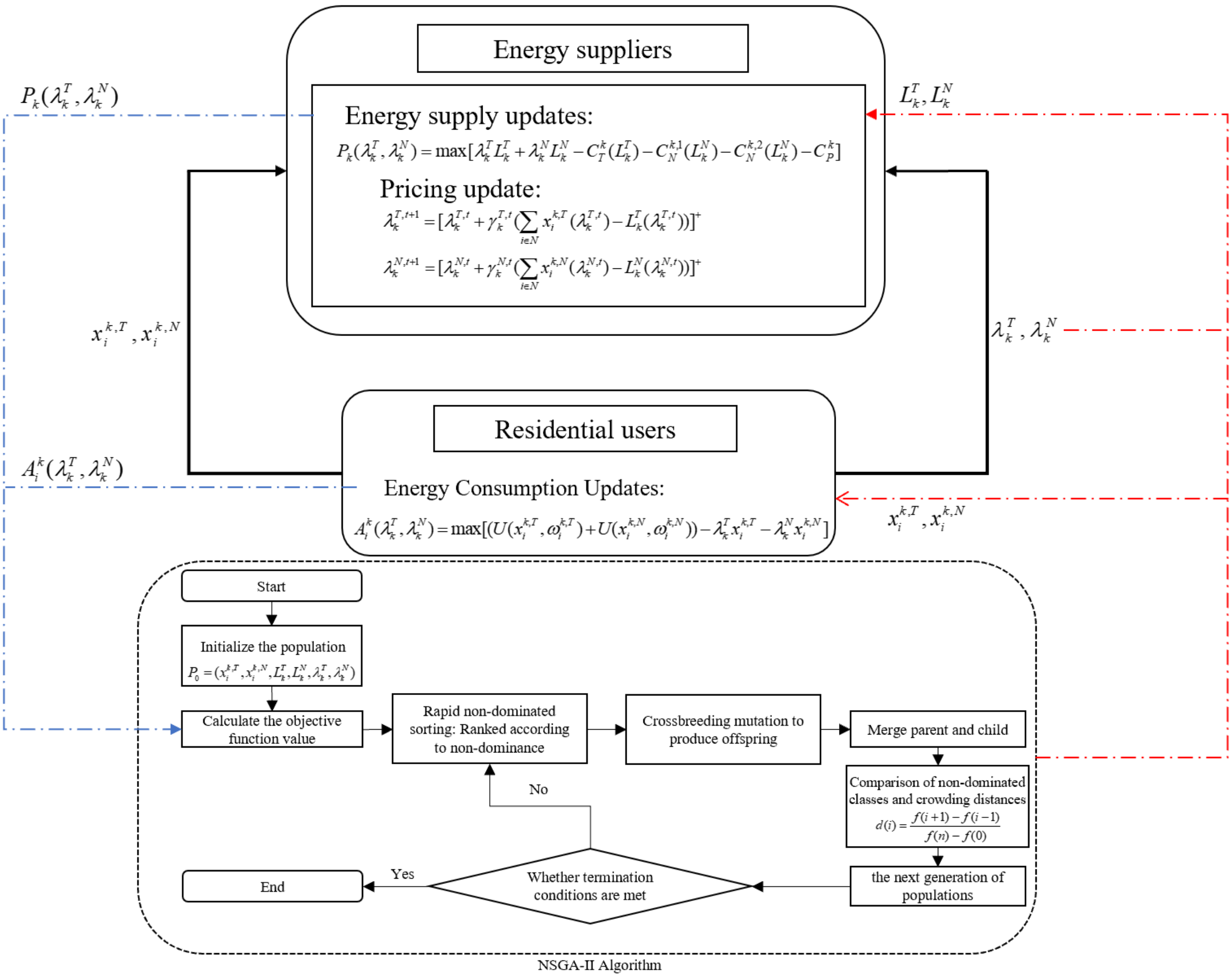


Figure 2. Distributed algorithm for the multi-pricing model.

$x_i^{k,T}, x_i^{k,N}$, the electricity supplier's electricity decision L_k^T, L_k^N, and the k-time electricity price λ_k^T, λ_k^N. The initial population covers the solution space on the user side and the supply provider side. For each individual in the population, Eqs (4.2) and (4.3) are calculated separately to evaluate the fitness of each individual. In order to optimize the dynamic process of tariff allocation, this paper embeds the gradient projection method in the NSGA-II algorithm to iteratively update the Lagrange multipliers λ_k^T, λ_k^N with the update rule:

$$\lambda_k^{T,t+1} = [\lambda_k^{T,t} + \gamma_k^{T,t}(\sum_{i\in N} x_i^{k,T}(\lambda_k^{T,t}) - L_k^T(\lambda_k^{T,t}))]^+ \tag{4.5}$$

$$\lambda_k^{N,t+1} = [\lambda_k^{N,t} + \gamma_k^{N,t}(\sum_{i\in N} x_i^{k,N}(\lambda_k^{N,t}) - L_k^N(\lambda_k^{N,t}))]^+ \tag{4.6}$$

where $\gamma_k^{T,t}, \gamma_k^{N,t}$ is the step parameter, and $[\cdot]^+$ is a non-negative truncation operation to ensure that the price is non-negative.

Step 2: Rapid non-dominated sorting: according to the concept of a non-dominated solution, the population P_0 is stratified into several layers $F_1, F_2, \ldots,$ where the Pareto front F_1 contains the current optimal non-dominated solution. Individuals are ranked according to the non-dominated rank i_{rank}, and the lower the non-dominated rank, the higher the priority of the individual.

Step 3: Crowding distance calculation and selection: in order to maintain the diversity of the solution set, this paper introduces the congestion distance metric $d(i)$. For each individual in the same non-dominated layer F_k, the congestion distance is calculated as

$$d(i) = \frac{f(i+1) - f(i-1)}{f(n) - f(0)}, \quad i = 1, 2, \ldots, n-1,$$

where n is the number of individuals in the dominance layer, $f(\cdot)$ represents the objective function value obtained by solving (4.2) and (4.3), respectively, during the optimization process, $f(i-1)$ and $f(i+1)$ are the values of the objective function of the neighboring individuals before and after individual i, and $f(n)$ and $f(0)$ are the maximum and minimum values of the objective function in the dominance layer, respectively.

To avoid the loss of Pareto optimal solutions, we preferentially select individuals with lower non-dominance rank into the next generation of the population. When the capacity of the population is insufficient, the selection is based on the crowding distance from the largest to the smallest to ensure diversity.

Step 4: Merger of parent and child generations: the child population Q_t is generated by random individual crossover and mutation, and then the parent population P_t and child population Q_t are merged into a temporary population $R_t = P_t \cup Q_t$. Individuals in R_t are sorted by non-dominated rank and crowding distance, and the top N individuals are selected to form the next-generation parent population P_{t+1}.

Step 5: Algorithm termination condition: the algorithm terminates its operation when the value of the objective function defined in (4.2) and (4.3) reaches the maximum number or satisfies the conditions such that

$$|\lambda_k^{N,t+1} - \lambda_k^{N,t}| < \varepsilon, |\lambda_k^{T,t+1} - \lambda_k^{T,t}| < \varepsilon,$$

where ε is small enough.

5. Numerical simulation

In this section, we give some numerical simulations to illustrate the effectiveness of the model and the proposed algorithm.

Consider an intelligent power system containing 6 residential users, including 3 types of energy supply: traditional energy supply, wind energy generation, and solar energy generation. A time period of 24 hours a day is set, and each period is divided into intervals of 1 h. In the demand side, the power elasticity coefficient of residential users is $\omega \in [1, 4]$ [12], and the utility function parameter is selected as $\alpha = 0.5$ [8]. In the supply side, the cost function parameters of traditional energy supply are set as $a = 0.01$ and $b = c = 0$ [8], and the marginal cost of both wind and solar energy is set as $\gamma = 0.01$ [26]. The renewable energy generation maintenance cost coefficient $\theta = 0.01, \eta = 0$ [27]. The power generation of wind and solar energy is randomly distributed in each period at $[2, 8]$, where the solar energy is only operated between 9:00 and 18:00 every day, and the wind energy is set to be operated for the whole day. In addition, this paper introduces pollutant emission and treatment cost

Table 2. Discharge coefficient and cost of pollutants.

Type of pollutant	Treatment costs	Pollutant emission factors		
		PV	WT	Grid
CO_2	0.023	0	0	889
SO_2	6	0	0	1.8
NO_x	8	0	0	1.6

parameters into the model as shown in Table 2. Three types of typical pollutants, carbon dioxide (CO_2), sulfur dioxide (SO_2), and nitrogen oxides (NO_x), are selected as modeling parameters [30]. These three types of pollutants cover the main greenhouse gases and atmospheric pollutants, and can reflect the environmental impact of the traditional thermal power generation process more comprehensively.

To illustrate the effectiveness of the proposed model and algorithm, we first compared the NSGA-II algorithm adopted in this paper to the particle swarm optimization algorithm (PSO), harmonic search algorithm (HS), and genetic algorithm (GA), which are always applied for real-time pricing problems. The comparision of CPU time is shown in Figure 3. We also compare the multi-pricing model with pollution treatment with that not considering pollution treatment, and the result is shown in Figure 4(a),(b). Meanwhile, Figure 5(a) illustrates the comparison of the traditional energy price, renewable energy price, real-time price for single-source power supply, and fixed price under multi-source power supply, while Figure 5(b) presents the comparison of social welfare under multi-source power supply and single-source power supply.

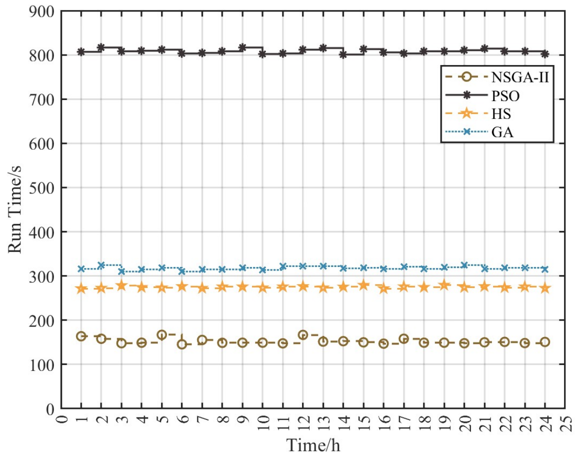
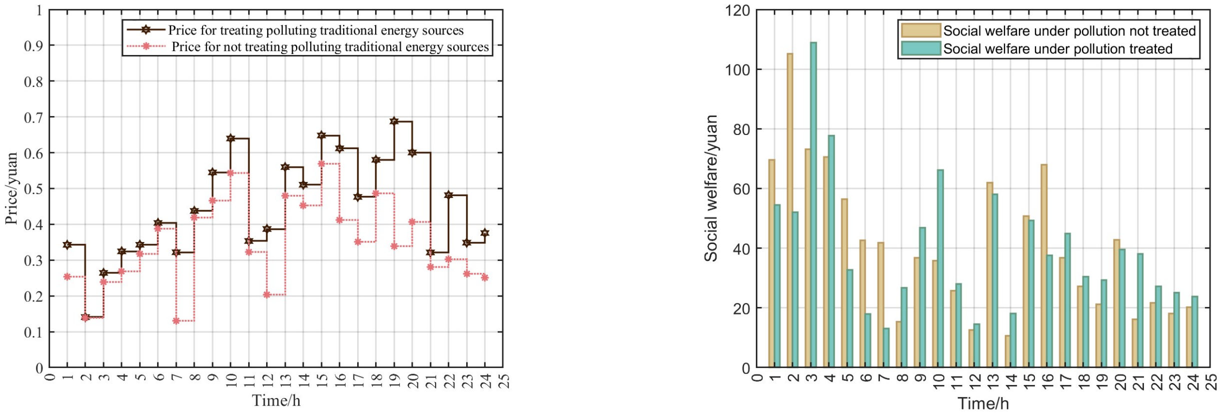


Figure 3. Comparison of the running times of different algorithms.

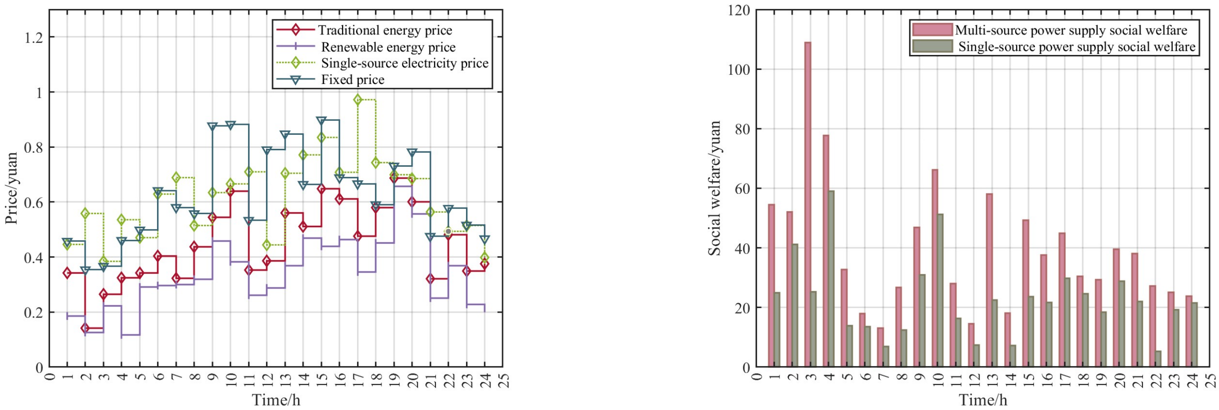
From Figure 3, the NSGA-II has a significantly shorter running time than the other three algorithms. This indicates that the NSGA-II is not only able to meet the demands of solving multi-objective optimization problems, but also has lower time complexity and superior runtime performance compared with PSO, HS, and GA. Therefore, the use of the NSGA-II algorithm as a solution tool in the optimization model of this study has significant advantages and provides effective support for the efficient optimization of smart grid systems.

Figure 4(a),(b) show the results of the comparison of the real-time price and social welfare for the case of considering pollution treatment and the case of not considering pollution treatment,



(a) (b)

Figure 4. Comparison of prices and social welfare under treated and untreated pollution.



(a) (b)

Figure 5. Comparison of prices and social welfare under different energy supply models.

respectively. It can be observed that the real-time prices are slightly higher when pollution treatment is considered than when pollution treatment is not considered. In terms of social welfare, the total social welfare is 980.9158 when pollution is not treated, while it is 960.0440 when pollution is treated. This difference reflects the effect of the cost of pollution treatment on the cost of electricity, suggesting that environmental treatment measures unavoidably increase the economic cost of smart grid operation. However, the difference in total social welfare between the two is relatively small. Starting from the long-term goal of carbon neutrality, the real-time pricing model considering pollution control is more in line with the practical needs, which not only reflects the importance of environmental protection, but also provides theoretical support for the sustainable development of the smart grid. Therefore, the model has important reference value in practical applications and provides a useful theoretical basis for smart grid optimization and policy formulation.

Figure 5(a) illustrates the comparison of traditional energy price, renewable energy price, real-time price for a single-source power supply, and fixed price under multi-source power supply, while Figure 5(b) presents the comparison of social welfare under multi-source power supply and single-source power supply. The results show that renewable energy electricity prices are consistently lower than traditional energy prices, and this difference helps to promote the consumption of renewable energy electricity. In addition, the traditional energy price is generally lower than the fixed price, which significantly reduces the cost of electricity for residential users at all points in time. This is because the real-time price can be dynamically adjusted according to the actual situation of user demand and power supply, achieving the goal of peak shaving and valley filling, thus effectively optimizing the power load. Further analysis shows that the real-time price under single-source power supply is higher than the real-time price under multi-source power supply, and the social welfare of adopting multi-source power supply is always higher than that of single-source power supply at all time points. Therefore, multi-source power supply can not only effectively reduce the electricity cost for users, but also enhance social welfare, showing the obvious advantages of multi-source power supply in smart grids.

In order to further verify the applicability and scalability of the proposed algorithm in large-scale user scenarios, we expand the number of users to 50 and 100, respectively. Figures 6 and 7 show the real-time pricing trends after the expansion of the user scale. Also, we give a comparison of the running time of the PSO, HS, and GA algorithms when the number of resident users is 50. The results are shown in Table 3, where " – " denotes the runtime exceeding 900 seconds.

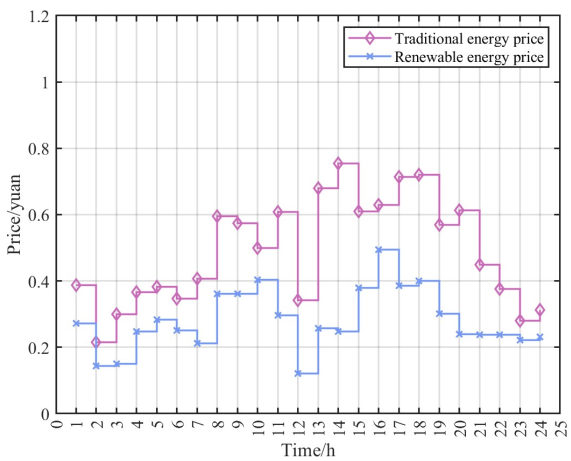


Figure 6. Power supply system with 50 users.

Table 3. Comparison of the running times of different algorithms with 50 users.

	1	2	3	4	5	6	7	8	9	10	11	12
NSGA-II	334.671	336.671	340.133	335.489	337.349	339.430	341.671	345.222	344.869	332.447	344.043	360.417
PSO	–	–	–	–	–	–	–	–	–	–	–	–
HS	574.382	575.671	572.490	577.305	571.689	573.874	575.216	576.458	572.397	579.128	574.663	575.540
GA	716.354	721.320	715.432	723.259	718.452	718.334	721.387	720.856	719.452	724.923	723.526	718.332
	13	14	15	16	17	18	19	20	21	22	23	24
NSGA-II	334.644	342.713	358.814	355.082	343.108	327.430	350.893	348.941	353.756	354.187	354.891	353.637
PSO	–	–	–	–	–	–	–	–	–	–	–	–
HS	573.295	571.842	578.206	574.958	573.781	576.039	572.653	575.789	577.164	573.603	576.497	574.125
GA	724.553	718.674	722.451	717.845	725.332	724.554	728.668	717.372	725.645	721.338	713.528	726.430

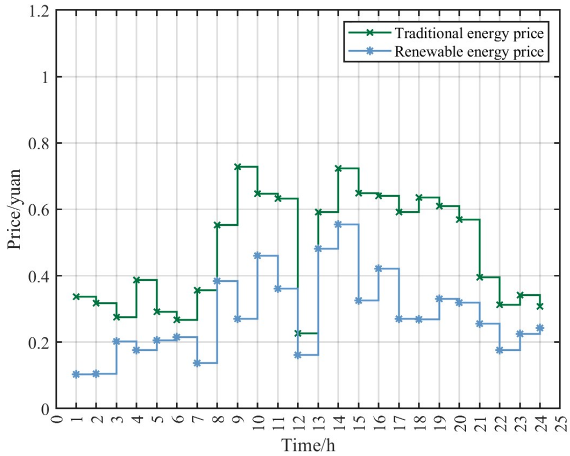


Figure 7. Power supply system with 100 users.

Actually, we also give a comparison of the running times of the PSO, HS, and GA algorithms when the number of resident users is 100. However, the average running time of our proposed algorithm is about 720 seconds, and the other three exceed 900 seconds. We omit listing the results here. From Table 3, it is obvious that the proposed algorithm is effective while dealing with data of different scales. It still has good solution efficiency and stability when the number of users is increased to a medium-to-large scale, which indicates that it has good scalability and practicability. Meanwhile, when the number of users is up to 50 and 100, respectively, the traditional energy price and renewable price obtained are reasonable, as seen in Figures 6 and 7. Also, from the two figures, the renewable energy price is lower than the traditional energy price. Therefore, during the peak electricity consumption periods, the government decision-making departments could guide residents to utilize renewable energy power supply during peak electricity consumption periods, such as at 12:00–15:00 and 18:00–20:00, which could help us level the peak and fill the valley, minimize resource waste, and optimize resource allocation.

6. Conclusions

We consider a multi-price model with the cost of pollution treatment under multi-energy electricity generation in this paper. The model not only considers the cost of pollution treatment, but also integrates the advantages of multi-source power generation. In the solution process, Lagrangian dual theory is used to transform the original problem of convex optimization into a distributed algorithm, and the optimal real-time pricing and optimal power supply strategy are obtained by the NSGA-II algorithm. Numerical simulation results show that the model is reasonable when comparing the obtained price and social welfare in an untreated-pollution model with a single supply model. Also, the proposed algorithm always has better computational efficiency, when compared with the PSO, HS, and GA algorithms. The results of this paper are useful for optimizing the allocation of electric power resources, improving the operation efficiency of the power grid, and reducing the power supply cost. Government departments could guide users to use electricity rationally through price mechanisms, reducing pollution emissions, and achieve a win-win situation in terms of economic, energy, and environmental benefits. This paper provides key theoretical support and practical guidance for the sustainable development of smart grids. However, we only consider a single type of residential user and fail to classify users in detail, such as commercial users, industrial users, and so on. We focused on the multi-energy generation multi-price model under the cost of pollution treatment, while the price fluctuations in the energy market and the cost of energy storage technologies are not discussed in our model, which are the future topics we care about. Moreover, with the rapid development of algorithms such as deep learning, reinforcement learning, and neural networks, we will attempt to use them to solve the real-time pricing model of smart grids that incorporates various factors such as energy storage technologies and uncertainties in the smart grid.

Author Contributions

Linsen Song was responsible for conceptualization, funding acquisition, investigation, methodology, project administration, supervision, validation, and both writing the original draft and reviewing and editing the manuscript. Yichen Du contributed to data curation, formal analysis, methodology, software development, visualization, and also participated in writing the original draft as well as reviewing and editing the manuscript.

Use of AI tools declaration

This article declares that no artificial intelligence (AI) tools were used in the writing of this article.

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Conflict of interest

The authors of this article declare there are no conflicts of interest.

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