Electronic Research Archive

https://www.aimspress.com/journal/era

ERA, 33(4): 2032–2060.
DOI: 10.3934/era.2025090
Received: 22 December 2024
Revised: 08 March 2025
Accepted: 21 March 2025
Published: 11 April 2025

Research article

Fixed-time synchronization of mixed-delay fuzzy cellular neural networks with *Lévy* noise

Tianyi Li[1], **Xiaofeng Xu**[2,3,4] **and Ming Liu**[1,*]

[1] Department of Mathematics, Northeast Forestry University, Harbin 150040, China
[2] Engineering Research Center of Agricultural Microbiology Technology, Ministry of Education, Harbin 150080, China
[3] Heilongjiang Provincial Key Laboratory of Ecological Restoration and Resource Utilization for Cold Region, Harbin 150080, China
[4] School of Mathematical Science, Heilongjiang University, Harbin 150080, China

* **Correspondence:** Email: liuming_girl@163.com.

Abstract: In this paper, we investigate the fixed-time synchronization of fuzzy stochastic cellular neural networks (FSCNNs) with mixed delay and *Lévy* noise. We designed the feedback controller and adaptive controller for cellular neural networks with *Lévy* noise to achieve fixed-time synchronization. Using the Lyapunov theory and the *Itô* formula, we established the criterion for fixed-time synchronization of FSCNNs with *Lévy* noise. Additionally, we obtained the resulting settling time, which is independent of the initial values of the system. The practicality and validity of the theoretical conclusions are demonstrated through two examples. The research results show that when the intensity of random interference is not large, FSCNNs can achieve synchronization through appropriate control means.

Keywords: fixed-time synchronization; fuzzy cellular neural networks; *Lévy* noise; feedback control; adaptive control

1. Introduction

Neural networks are widely used in many fields such as biomedical, image processing, automatic control, pattern recognition, signal processing, and secure communication; the synchronization problem of neural networks has been widely studied in recent years [1–7]. In 1988, Chua and Yang first proposed cellular neural networks through parallel processing [8]; subsequently, CNNs attracted the attention of many scholars. In order to deal with some uncertain and incomplete data, Yang et al. added fuzzy logic to the structure of traditional CNNs and first proposed the fuzzy cellular neural network model (FCNNs) [9]. Compared with traditional CNNs, FCNNs have many advantages such as better adaptability, fault tolerance and robustness, and the ability to process information faster [10]. In addition, neural networks in the real environment are inevitably affected by signal transmission delay and external stochastic interference, which make the analysis of the networks more complicated.

Synchronization is a very important dynamic behavior of neural networks and has become a hot topic. In most of the research on neural networks synchronization, scholars have focused on the asymptotic and exponential synchronization of neural networks [11–15]. Asymptotic or exponential synchronization means that the synchronization can only be achieved when time approaches infinity. The applications of these theories are limited, because many systems in practical applications such as secure communications, manufacturing engineering, and power networks require achieving synchronization in finite time. Therefore, the finite-time synchronization of complex networks has gradually been studied by many researchers. In [16], Yang and Cao investigated the finite-time synchronization of complex networks with white noise perturbations by using the finite-time stability theorem and inequality techniques. In [17], Wang investigated the finite-time synchronization of fuzzy cellular neural networks with proportional delays. In [18], Duan et al. researched the finite-time synchronization of delayed fuzzy cellular neural networks with discontinuous activations. In [19], Wu et al. obtained the finite-time synchronization criteria for a chaotic dynamic neural networks with mixed time-varying delays and stochastic disturbance by using the state feedback control and adaptive control. In [20], Abdurahman et al. studied the finite-time synchronization of fuzzy cellular neural networks with time-varying delays. Other literature on finite-time synchronization of neural networks can be found in references [21–25].

Compared with the asymptotic synchronization, the finite-time synchronization can be achieved in a finite time and has faster convergence speed and stronger anti-interference. However, the convergence time of the finite-time synchronization is usually very dependent on the initial state of the systems. That is, different initial states can lead to different convergence times. The initial state of the systems in many practical problems are difficult to know. Therefore, obtaining a fixed convergence time that does not depend on the initial states of the systems has been the focus of scholars in recent years. In [26], Yang et al. investigated the fixed-time synchronization of complex networks with impulsive effects by designing a new Lyapunov function and constructing comparison systems. In [27], Khanzadeh and Pourgholi investigated the fixed-time synchronization of complex dynamical networks with nonidentical nodes in the presence of bounded uncertainties and disturbances by using the sliding mode control technique. In [28], Zhang et al. studied the fixed-time synchronization of stochastic complex networks with white noise and designed a continuous controller to avoid the chattering phenomenon. In [29], Ren et al. investigated the fixed-time synchronization of stochastic memristor-based neural networks with white noise via the state feedback control and the adaptive control. In [30], Kong et al. studied the fixed-time synchronization of a class of discontinuous fuzzy inertial neural networks with time-varying delays by relaxing the conditions of the C-regular Lyapunov function. Other literature on fixed-time synchronization of stochastic neural networks can be found in references [31–35].

It is worth noting that in the above studies on fixed-time synchronization, stochastic neural networks are all driven by white noise. White noise is a continuous effect that can be used to characterize continuous stochastic interference. However, discontinuous stochastic perturbations should also be considered, which can be used to describe sudden external influences. *Lévy* noise combines time, Brownian motion, and random jumps and can better simulate extremely unusual and sudden events such as earthquakes, flash floods, and epidemics, which can not be depicted just by white noise. The introduction of *Lévy* noise brings new difficulties and challenges to the research of neural networks. In [36], Zhu proved several Razumikhin-type theorems for the pth moment exponential stability of stochastic functional differential equations with *Lévy* noise. In [37], Zhou et al. investigated the stabilization of stochastic coupled systems with time-varying delays and *Lévy* noise via the periodically intermittent control. In [38], Zhou et al. studied the cluster synchronization of coupled neural networks with *Lévy* noise by proposing a trigger mechanism and designing a data sampling strategy. In [39], Shi et al. discussed the pth moment exponential synchronization of one-leader multi-follower systems by using the Lyapunov stability theory and the exponential stability criterion. In [40], Zhang et al. researched the semi-globally exponential synchronization of stochastic systems with mixed time delay and *Lévy* noise under the aperiodic intermittent delayed sampled-data control. However, few studies on fixed-time synchronization of stochastic neural networks with *Lévy* noise can be found. In [41], a fixed-time synchronization criterion for stochastic cellular neural networks with *Lévy* noise was established by utilizing the Lyapunov function method and the inequality technique. The research in this area is far from enough and deserves further study.

Based on the above analysis, this article is devoted to the fixed-time synchronization of FSCNNs with white noise, *Lévy* noise, and mixed time delay under feedback control and adaptive control. The following three points reflect the core contributions of this paper.

- A first attempt is made to study the fixed-time synchronization of FSCNNs with *Lévy* noise. Based on the Lyapunov theory, a fixed-time synchronization criterion for FSCNNs under the influence of *Lévy* noise is derived. The obtained synchronization time is independent of the initial value of the systems.
- This paper designs two control methods, feedback control and adaptive control, to achieve the fixed-time synchronization of FSCNNs with *Lévy* noise.
- In the discussion of the fixed-time synchronization of FSCNNs with *Lévy* noise, we use high-order terms to limit the impact of *Lévy* noise.

In fact, the main purpose of this paper is to generalize the fixed-time synchronization method of stochastic neural networks with white noise in reference [29] to stochastic neural networks with *Lévy* noise. Moreover, compared with the *Lévy* noises in existing literature such as [36–41], the assumption about the intensity function of *Lévy* noise in this paper is different.

The remainder of the article is divided into the sections below. Section 2 presents some preliminary descriptions and model characterizations. Section 3 presents the derivation of a fixed-time synchronization criterion with *Lévy* noise based on the Lyapunov theory and the *Itô* formula. In Section 4, some numerical examples are given to validate the proposed criterion.

2. Preliminaries and model descriptions

This paper considers a class of FSCNNs with discrete and distributed delays under white noise and *Lévy* noise

$$
\begin{aligned}
\mathrm{d}\alpha_i(t) =\Big[& - a_i\alpha_i(t) + \sum_{j=1}^{l}\Big(\varsigma_{ij}\widetilde{f}(\alpha_j(t)) + \nu_{ij}\widetilde{f}(\alpha_j(t-h)) + \theta_{ij}\int_{t-h}^{t}\widetilde{f}(\alpha_j(s))\mathrm{d}s\Big) \\
&+ \bigwedge_{j=1}^{l} m_{ij}\widetilde{g}(\alpha_j(t-h)) + \bigvee_{j=1}^{l} \iota_{ij}\widetilde{g}(\alpha_j(t-h)) + \bigwedge_{j=1}^{l} o_{ij}\int_{t-h}^{t}\widetilde{g}(\alpha_j(s))\mathrm{d}s \\
&+ \bigvee_{j=1}^{l} \xi_{ij}\int_{t-h}^{t}\widetilde{g}(\alpha_j(s))\mathrm{d}s + \bigwedge_{j=1}^{l} G_{ij}p_j + \bigvee_{j=1}^{l} H_{ij}p_j + D_i\Big]\mathrm{d}t + w_i(\alpha_i(t), t)\mathrm{d}B(t) \\
&+ \int_{S}\gamma_i(\alpha_i(t), t, u)\tilde{N}(\mathrm{d}t, \mathrm{d}u),
\end{aligned}
\tag{2.1}
$$

where $\alpha_i(t)$ represents the state of the ith node at time t with $i \in \{1, 2, \cdots, l\}$, and l is the total number of neuron nodes. a_i represents the passive decay rate to the ith node; h represents the delay that occurs at the dynamic node; ς_{ij}, ν_{ij}, and θ_{ij} represent the elements of the feedback template; p_i represents the input of the ith neuron; D_i is the deviation of the ith neuron. m_{ij}, o_{ij}, ι_{ij}, ξ_{ij} are the elements of the fuzzy feedback template; G_{ij}, H_{ij} are the elements of the fuzzy feed-forward template. $\widetilde{f}(\cdot) : \mathbb{R} \to \mathbb{R}$ and $\widetilde{g}(\cdot) : \mathbb{R} \to \mathbb{R}$ are the activation functions. $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ is the complete probability of the sample space with the filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). The intensity of white noise is denoted by $w_i : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$, and $B(t)$ is the one-dimensional Brownian motion defined on $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$. $\gamma(\cdot) : \mathbb{R} \times \mathbb{R}^+ \times \mathbb{S} \to \mathbb{R}$ serves as the *Lévy* noise intensity function; the compensated Poisson random measure is represented as $\tilde{N}(\mathrm{d}t, \mathrm{d}u) = N(\mathrm{d}t, \mathrm{d}u) - \lambda(du)$, where $N(\mathrm{d}t, \mathrm{d}u)$ is the Poisson counting measure on $[0, +\infty] \times \mathbb{S}$. λ is the intensity measure and $\mathbb{S}$ is the measurable subset of $\mathbb{R}^+$ such that $\lambda(\mathbb{S}) < \infty$ and $\lambda(\mathbb{S}) = \bar{\lambda}$. Write $\mathcal{L}^2_{\mathcal{F}_0}\left([-h, 0]; \mathbb{R}^l\right)$ for the family of all $\mathcal{F}_0$-measurable $C\left([-h, 0]; \mathbb{R}^l\right)$-value random variables x such that $E(\|x\|^2) < \infty$, where $E(\cdot)$ denotes the mathematical expectation with respect to the given probability measure $\mathbb{P}$.

The system (2.1) is used as a drive system and the following is the response system

$$
\begin{aligned}
\mathrm{d}\beta_i(t) =\Big[& - a_i\beta_i(t) + \sum_{j=1}^{l}\Big(\varsigma_{ij}\widetilde{f}(\beta_j(t)) + \nu_{ij}\widetilde{f}(\beta_j(t-h)) + \theta_{ij}\int_{t-h}^{t}\widetilde{f}(\beta_j(s))\mathrm{d}s\Big) \\
&+ \bigwedge_{j=1}^{l} m_{ij}\widetilde{g}(\beta_j(t-h)) + \bigvee_{j=1}^{l} \iota_{ij}\widetilde{g}(\beta_j(t-h)) + \bigwedge_{j=1}^{l} o_{ij}\int_{t-h}^{t}\widetilde{g}(\beta_j(s))\mathrm{d}s \\
&+ \bigvee_{j=1}^{l} \xi_{ij}\int_{t-h}^{t}\widetilde{g}(\beta_j(s))\mathrm{d}s + \bigwedge_{j=1}^{l} G_{ij}p_j + \bigvee_{j=1}^{l} H_{ij}p_j + D_i + u_i(t)\Big]\mathrm{d}t + w_i(\beta_i(t), t)\mathrm{d}B(t) \\
&+ \int_{S}\gamma_i(\beta_i(t), t, u)\tilde{N}(\mathrm{d}t, \mathrm{d}u),
\end{aligned}
\tag{2.2}
$$

where $u_i(t)$ is a suitable controller and will be designed to ensure that systems (2.1) and (2.2) are synchronized at a fixed time. The initial conditions of systems (2.1) and (2.2) are

$$
\phi(s) = (\phi_1(s), \phi_2(s), \cdots, \phi_l(s)) \in \mathcal{L}^2_{\mathcal{F}_0}\left([-h, 0]; \mathbb{R}^l\right),\ \psi(s) = (\psi_1(s), \psi_2(s), \cdots, \psi_l(s)) \in \mathcal{L}^2_{\mathcal{F}_0}\left([-h, 0]; \mathbb{R}^l\right),
$$

respectively. Define the error as $\zeta_i(t) = \beta_i(t) - \alpha_i(t)$. From (2.1) and (2.2), the error system is as follows

$$\begin{aligned}
\mathrm{d}\zeta_i(t) =& \Bigg[- a_i\zeta_i(t) + \sum_{j=1}^{l} \Big(\varsigma_{ij}\widetilde{F}(\zeta_j(t)) + \nu_{ij}\widetilde{F}(\zeta_j(t-h)) + \theta_{ij}\int_{t-h}^{t}\widetilde{F}(\zeta_j(s))\mathrm{d}s \Big) \\
&+ \bigwedge_{j=1}^{l} m_{ij}\widetilde{G}(\zeta_j(t-h)) + \bigvee_{j=1}^{l} \iota_{ij}\widetilde{G}(\zeta_j(t-h)) + \bigwedge_{j=1}^{l} o_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s \\
&+ \bigvee_{j=1}^{l} \xi_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s + u_i(t) \Bigg]\mathrm{d}t + W_i\,\mathrm{d}B(t) + \int_{S}\Gamma_i\,\tilde{N}(\mathrm{d}t,\mathrm{d}u),
\end{aligned}$$

where

$$\begin{aligned}
&\widetilde{F}(\zeta_j(\cdot)) = \widetilde{f}(\beta_j(\cdot)) - \widetilde{f}(\alpha_j(\cdot)), \quad \bigwedge_{j=1}^{l} m_{ij}\widetilde{G}(\zeta_j(\cdot)) = \bigwedge_{j=1}^{l} m_{ij}\widetilde{g}(\beta_j(\cdot)) - \bigwedge_{j=1}^{l} m_{ij}\widetilde{g}(\alpha_j(\cdot)), \\
&W_i = w_i(\beta_i(t),t) - w_i(\alpha_i(t),t), \ \ \Gamma_i \ = \gamma_i(\beta_i(t),t,u) - \gamma_i(\alpha_i(t),t,u).
\end{aligned}$$

Consider a stochastic system as follows

$$\mathrm{d}\rho(t) = \widetilde{f}(t,\rho(t))\mathrm{d}(t) + w(t,\rho(t))\mathrm{d}B(t) + \int_{S}\gamma(\rho(t),t,u)\tilde{N}(\mathrm{d}t,\mathrm{d}u), \tag{2.3}$$

where $\rho(0) = \rho_0 \in \mathbb{R}^l$. The system (2.3) is a *Lévy* process-driven stochastic differential equation, and the *Itô* formulation for the generalization of such equations is given in the monograph [42]. The first hitting time is a function about the settling time expressed as $T(\rho_0,\epsilon) = \inf\{t|\rho(t) = 0, t \geq 0\}$, where $\epsilon \in \Omega$ is a fundamental event.

Definition 2.1. *[29] For any initial state $\rho_0 \in \mathbb{R}^l$, if the following are satisfied,*

1) The origin is globally stochastic finite-time stable in probability.

2) The mathematical expectation of $T(\rho_0,\epsilon)$ has a upper bound $M > 0$, which is not dependent on the selection of initial values,

$$T_\epsilon = E(T(\rho_0,\epsilon)) \leq M, \ \ \forall\rho_0 \in \mathbb{R}^l,$$

then the trivial solution of system (2.3) is considered to be stochastic fixed-time stable in probability.

Assumption 2.2. *There exists a positive number $\varkappa_i$, such that*

$$\mathrm{Tr}[W_i^T W_i] \leq \varkappa_i\zeta_i^T(t)\zeta_i(t).$$

Assumption 2.3. *For any $s_1, s_2 \in \mathbb{R}$, there exist positive constants L_1 and L_2 satisfying*

$$\begin{aligned}
|\widetilde{f}(s_1) - \widetilde{f}(s_2)| &\leq L_1|s_1 - s_2|, \\
|\widetilde{g}(s_1) - \widetilde{g}(s_2)| &\leq L_2|s_1 - s_2|.
\end{aligned}$$

Assumption 2.4. *For any $\zeta(t) \in \mathbb{R}^l$, $t \in \mathbb{R}^+$, $u \in \mathbb{S}$ and $m_1, m_2, \ldots, m_n > 1$, there exists a positive number φ_i, such that*

$$|\Gamma_i| \leq \varphi_i|\zeta_i(t)|^{m_i}.$$

Lemma 2.5. *[9] Assuming ρ_1 and ρ_2 represent two states of system* (2.1)*, then*

$$\left|\bigwedge_{j=1}^{l} m_{ij}\widetilde{g}(\rho_1) - \bigwedge_{j=1}^{l} m_{ij}\widetilde{g}(\rho_2)\right| \leq \sum_{j=1}^{l} |m_{ij}||\widetilde{g}(\rho_1) - \widetilde{g}(\rho_2)|,$$
$$\left|\bigvee_{j=1}^{l} \iota_{ij}\widetilde{g}(\rho_1) - \bigvee_{j=1}^{l} \iota_{ij}\widetilde{g}(\rho_2)\right| \leq \sum_{j=1}^{l} |\iota_{ij}||\widetilde{g}(\rho_1) - \widetilde{g}(\rho_2)|.$$

Lemma 2.6. *[43] If $S_1, S_2, \ldots, S_l \geq 0$ and $0 < m < 1$, $M > 1$, then*

$$\textstyle\sum_{i=1}^{l} S_i^m \geq \left(\sum_{i=1}^{l} S_i\right)^m, \quad \sum_{i=1}^{l} S_i^M \geq l^{1-M}\left(\sum_{i=1}^{l} S_i\right)^M.$$

This section establishes an adequate criterion for fixed-time synchronization of FSCNNs based on the Lyapunov method.

2.1. Feedback control

State feedback control aims to generate a control input that drives a system to a desired state by utilizing the state variables of the system. By monitoring the current state of the system, the controller will adjust the control input so that the error between the current state and the expected state of the system is constantly reduced and the two states are finally in agreement. Because the control rules are fixed and the feedback gain does not change over time, the design process of the feedback controller is relatively simple and easy to implement. In addition, the synchronization criterion of a system under feedback control is also easier to derive. The advantages of feedback controller are its simple design, small amount of computation, easy derivation of synchronization criteria, and convenient realization. These advantages make the state feedback control a most commonly used technique in modern control theory. First, we design a state feedback controller $u_i(t)$ to synchronize FSCNNs at a fixed time as follows:

$$u_i(t) = \begin{cases} -\aleph_i\zeta_i(t) - \sum_{j=1}^{l}\Big(\omega_{ij}|\zeta_j(t-h)|\text{sign}(\zeta_i(t)) - \frac{1}{2}\varpi_{ij}\int_{t-h}^{t}|\zeta_i(s)|^2 ds\frac{\zeta_i(t)}{|\zeta_i(t)|^2}\Big) & \\ -\mu_1|\zeta_i(t)|^q\text{sign}(\zeta_i(t)) - \eta_1[|\zeta_i(t)|^{2M-1} + |\zeta_i(t)|^{2m-1}]\text{sign}(\zeta_i(t)), & \text{if } \zeta_i(t) \neq 0, \\ 0, & \text{if } \zeta_i(t) = 0, \end{cases} \tag{2.4}$$

where $\aleph_i$, ω_{ij}, μ_1, η_1, ϖ_{ij} are all nonnegative numbers, $0 < q < 1$, and $M > m > 1$. The block diagram of feedback control model is shown in Figure 1.

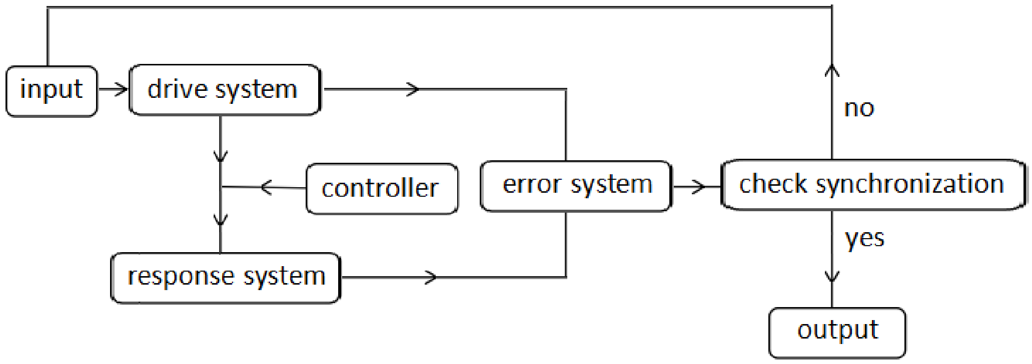

Figure 1. Block diagram of feedback control model.

Theorem 2.7. *Under the Assumptions 2.2–2.4, the systems* (2.1) *and* (2.2) *are stochastic fixed-time synchronization under the feedback control* (2.4)*, if the following four conditions hold*

$$
\begin{aligned}
&-2a_i - 2\aleph_i + \varkappa_i + \sum_{j=1}^{l}\left(L_1\varsigma_{ij} + L_1\varsigma_{ji} + \theta_{ij} + o_{ij} + \xi_{ij}\right) \le 0,\\
&2\sum_{j=1}^{l}\left(L_1\nu_{ij} + |m_{ij}|L_2 + |\iota_{ij}|L_2 - \omega_{ij}\right) \le 0,\\
&\sum_{j=1}^{l}\left(h\theta_{ij}(L_1)^2 + hs_{ji}(L_2)^2 + hc_{ji}(L_2)^2 - \varpi_{ij}\right) \le 0,\\
&2\eta_1 l^{1-M} - \bar{\lambda} l\bar{\varphi} > 0.
\end{aligned}
$$

And the settling time is

$$
T_\epsilon = \frac{1}{\hat{\mu}_1}\frac{1}{1-Q} + \frac{1}{\hat{\eta}_1}\frac{1}{M-1},
$$

where $\bar{\varphi} = \max[\varphi_i^2]$*,* $M = \max[m_i]$*,* $m = \min[m_i]$*,* $\hat{\mu}_1 = 2\mu_1$*,* $Q = \frac{1+q}{2}$*,* $\hat{\eta}_1 = 2\eta_1 l^{1-M} - \bar{\lambda} l\bar{\varphi}$*.*

Proof. We structure the Lyapunov function $V(\zeta(t)) = \sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)$. According to the Itô formula, we have

$$\begin{aligned}
\mathcal{L}V =& 2\sum_{i=1}^{l}\zeta_i^T(t)\bigg[-a_i\zeta_i(t)+\sum_{j=1}^{l}\Big(\varsigma_{ij}\widetilde{F}(\zeta_j(t))+\nu_{ij}\widetilde{F}(\zeta_j(t-h))+\theta_{ij}\int_{t-h}^{t}\widetilde{F}(\zeta_j(s))\mathrm{d}s\Big)\\
&+\bigwedge_{j=1}^{l}m_{ij}\widetilde{G}(\zeta_j(t-h))+\bigvee_{j=1}^{l}\iota_{ij}\widetilde{G}(\zeta_j(t-h))+\bigwedge_{j=1}^{l}o_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s\\
&+\bigvee_{j=1}^{l}\xi_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s+u_i(t)\bigg]+\sum_{i=1}^{l}\mathrm{Tr}(W_i^TW_i)\\
&+\sum_{i=1}^{l}\int_S\Big[(\zeta_i(t)+\Gamma_i)^T(\zeta_i(t)+\Gamma_i)-\zeta_i^T(t)\zeta_i(t)-2\zeta_i^T(t)\Gamma_i\Big]\lambda(du)\\
=&2\sum_{i=1}^{l}-a_i\zeta_i^T(t)\zeta_i(t)+2\sum_{i=1}^{l}\zeta_i^T(t)\sum_{j=1}^{l}\varsigma_{ij}\widetilde{F}(\zeta_j(t))+2\sum_{i=1}^{l}\zeta_i^T(t)\sum_{j=1}^{l}\nu_{ij}\widetilde{F}(\zeta_j(t-h))\\
&+2\sum_{i=1}^{l}\zeta_i^T(t)\sum_{j=1}^{l}\theta_{ij}\int_{t-h}^{t}\widetilde{F}(\zeta_j(s))\mathrm{d}s+2\sum_{i=1}^{l}\zeta_i^T(t)\bigwedge_{j=1}^{l}m_{ij}\widetilde{G}(\zeta_j(t-h))+2\sum_{i=1}^{l}\zeta_i^T(t)\bigvee_{j=1}^{l}\iota_{ij}\widetilde{G}(\zeta_j(t-h))\\
&+2\sum_{i=1}^{l}\zeta_i^T(t)\bigwedge_{j=1}^{l}o_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s+2\sum_{i=1}^{l}\zeta_i^T(t)\bigvee_{j=1}^{l}\xi_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s+2\sum_{i=1}^{l}\zeta_i^T(t)u_i(t)\\
&+\sum_{i=1}^{l}\mathrm{Tr}(W_i^TW_i)+\sum_{i=1}^{l}\int_S\Big[(\zeta_i(t)+\Gamma_i)^T(\zeta_i(t)+\Gamma_i)-\zeta_i^T(t)\zeta_i(t)-2\zeta_i^T(t)\Gamma_i\Big]\lambda(du).
\end{aligned} \tag{2.5}$$

By calculation, we can write the expression for LV (2.5) as $\mathcal{L}V=\sum_{i=1}^{10}\mathcal{L}V_i$, where

$$\begin{aligned}
LV_1 &= 2\sum_{i=1}^{l}\zeta_i^T(t)\sum_{j=1}^{l}\varsigma_{ij}\widetilde{F}(\zeta_j(t)),\quad LV_2=2\sum_{i=1}^{l}\zeta_i^T(t)\sum_{j=1}^{l}\nu_{ij}\widetilde{F}(\zeta_j(t-h)),\\
LV_3 &= 2\sum_{i=1}^{l}\zeta_i^T(t)\bigwedge_{j=1}^{l}m_{ij}\widetilde{G}(\zeta_j(t-h)),\quad LV_4=2\sum_{i=1}^{l}\zeta_i^T(t)\bigvee_{j=1}^{l}\iota_{ij}\widetilde{G}(\zeta_j(t-h)),\\
LV_5 &= 2\sum_{i=1}^{l}\zeta_i^T(t)\sum_{j=1}^{l}\theta_{ij}\int_{t-h}^{t}\widetilde{F}(\zeta_j(s))\mathrm{d}s,\\
LV_6 &= 2\sum_{i=1}^{l}\zeta_i^T(t)\left(\bigwedge_{j=1}^{l}o_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s+\bigvee_{j=1}^{l}\xi_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s\right),\\
LV_7 &= 2\sum_{i=1}^{l}\zeta_i(t)u_i(t),\quad LV_8=\sum_{i=1}^{l}\int_S\Big[(\zeta_i(t)+\Gamma_i)^T(\zeta_i(t)+\Gamma_i)-\zeta_i^T(t)\zeta_i(t)-2\zeta_i^T(t)\Gamma_i\Big]\lambda(du)\\
LV_9 &= 2\sum_{i=1}^{l}-a_i\zeta_i^T(t)\zeta_i(t),\quad LV_{10}=\sum_{i=1}^{l}\mathrm{Tr}(W_i^TW_i).
\end{aligned}$$

Now, we use the inequality scaling for each part of $\mathcal{L}V$. From Assumption 2.3, we can obtain that

$$\begin{aligned} LV_1 &\leq 2\sum_{i=1}^{l}|\zeta_i(t)|\sum_{j=1}^{l}\varsigma_{ij}L_1|\zeta_j(t)| \leq 2\sum_{i,j=1}^{l}\varsigma_{ij}L_1|\zeta_i(t)||\zeta_j(t)| \leq \sum_{i,j=1}^{l}\left(\varsigma_{ij}L_1|\zeta_i(t)|^2+\varsigma_{ij}L_1|\zeta_j(t)|^2\right) \\ &\leq \sum_{i,j=1}^{l}\varsigma_{ij}L_1|\zeta_i(t)|^2+\sum_{i,j=1}^{l}\varsigma_{ji}L_1|\zeta_i(t)|^2 \leq \sum_{i,j=1}^{l}\left(\varsigma_{ij}L_1+\varsigma_{ji}L_1\right)|\zeta_i(t)|^2, \end{aligned} \tag{2.6}$$

and

$$LV_2 \leq 2\sum_{i=1}^{l}|\zeta_i(t)|\sum_{j=1}^{l}\nu_{ij}L_1|\zeta_j(t-h)| \leq 2\sum_{i,j=1}^{l}\nu_{ij}L_1|\zeta_i(t)||\zeta_j(t-h)|. \tag{2.7}$$

By using Assumption 2.3 and Lemma 2.5, one has

$$LV_3 \leq 2\sum_{i=1}^{l}|\zeta_i(t)|\sum_{j=1}^{l}|m_{ij}||\widetilde{G}(\zeta_j(t-h))| \leq 2\sum_{i,j=1}^{l}|m_{ij}|L_2|\zeta_i(t)||(\zeta_j(t-h))|, \tag{2.8}$$

and

$$LV_4 \leq 2\sum_{i=1}^{l}|\zeta_i(t)|\sum_{j=1}^{l}|\iota_{ij}||\widetilde{G}(\zeta_j(t-h))| \leq 2\sum_{i,j=1}^{l}|\iota_{ij}|L_2|\zeta_i(t)||(\zeta_j(t-h))|. \tag{2.9}$$

In the same way, we have

$$\begin{aligned} LV_5 &\leq 2\sum_{i,j=1}^{l}\theta_{ij}\,|\zeta_i(t)|\int_{t-h}^{t}\widetilde{F}(\zeta_j(s))\mathrm{d}s \leq \sum_{i,j=1}^{l}\theta_{ij}\left(|\zeta_i(t)|^2+\left(\int_{t-h}^{t}\widetilde{F}(\zeta_j(s))\mathrm{d}s\right)^2\right) \\ &\leq \sum_{i,j=1}^{l}\left(\theta_{ij}\,|\zeta_i(t)|^2+\theta_{ij}h\int_{t-h}^{t}(\widetilde{F}(\zeta_j(s)))^2\mathrm{d}s\right) \leq \sum_{i,j=1}^{l}\left(\theta_{ij}\,|\zeta_i(t)|^2+\theta_{ij}h(L_1)^2\int_{t-h}^{t}\left|\zeta_j(s)\right|^2\mathrm{d}s\right), \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} LV_6 \leq& 2\sum_{i=1}^{l}\zeta_i^T(t)\left(\sum_{j=1}^{l}o_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s+\sum_{j=1}^{l}\xi_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s\right) \\ &\leq \sum_{i,j=1}^{l}\left(\left(o_{ij}+\xi_{ij}\right)|\zeta_i(t)|^2+o_{ij}h(L_2)^2\int_{t-h}^{t}\left|\zeta_j(s)\right|^2\mathrm{d}s+\xi_{ij}h(L_2)^2\int_{t-h}^{t}\left|\zeta_j(s)\right|^2\mathrm{d}s\right). \end{aligned} \tag{2.11}$$

In addition, it follows from Lemma 2.6, Assumptions 2.2 and 2.4 that

$$\begin{aligned} LV_7 \leq& -2\sum_{i=1}^{l}\aleph_i|\zeta_i(t)|^2-2\sum_{i,j=1}^{l}\omega_{ij}|\zeta_i(t)||\zeta_j(t-h)|-2\mu_1\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{\frac{1+q}{2}} \\ &-2\eta_1 l^{1-M}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M}-2\eta_1 l^{1-m}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m}-\sum_{i,j=1}^{l}\varpi_{ij}\int_{t-h}^{t}|\zeta_i(s)|^2\mathrm{d}s, \end{aligned} \tag{2.12}$$

$$\begin{aligned} LV_8 &\le \sum_{i=1}^{l}\int_S \Big[|\zeta_i(t)+\Gamma_i|^2-|\zeta_i(t)|^2-2\zeta_i^T(t)\Gamma_i\Big]\lambda(du) \le \sum_{i=1}^{l}\int_S \Gamma_i^2\lambda(du) \\ &\le \sum_{i=1}^{l}\int_S \varphi_i^2|\zeta_i^{m_i}(t)|^2\lambda(du) = \bar{\lambda}\sum_{i=1}^{l}\varphi_i^2|\zeta_i^{m_i}(t)|^2, \end{aligned} \tag{2.13}$$

and

$$LV_{10} \le \sum_{i=1}^{l}\varkappa_i|\zeta_i(t)|^2. \tag{2.14}$$

Substituting (2.6)–(2.14) into $\mathcal{L}V = \sum_{i=1}^{10}\mathcal{L}V_i$, we have

$$\begin{aligned} \mathcal{L}V \le& \sum_{i=1}^{l}\Big(-2a_i-2K_i+\sum_{j=1}^{l}(\varsigma_{ij}L_1+\varsigma_{ji}L_1+\theta_{ij}+o_{ij}+\xi_{ij})+\varkappa_i\Big)|\zeta_i(t)|^2 \\ &+2\sum_{i,j=1}^{l}\Big(\nu_{ij}L_1+|m_{ij}|L_2+|\iota_{ij}|L_2-\omega_{ij}\Big)|\zeta_i(t)||\zeta_j(t-h)| \\ &+\sum_{i,j=1}^{l}\Big(hb_{ji}(L_1)^2+hs_{ji}(L_2)^2+hc_{ji}(L_2)^2-\varpi_{ij}\Big)\int_{t-h}^{t}|\zeta_i(s)|^2\mathrm{d}s \\ &-2\mu_1\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{\frac{1+q}{2}}-2\eta_1 l^{1-M}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M}-2\eta_1 l^{1-m}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m} \\ &+\bar{\lambda}\sum_{i=1}^{l}\varphi_i^2|\zeta_i^{m_i}(t)|^2. \end{aligned}$$

Further, if the conditions in Theorem 2.7 hold, we can obtain that

$$\begin{aligned} \mathcal{L}V \le& -2\mu_1\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{\frac{1+q}{2}}-2\eta_1 l^{1-M}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M}-2\eta_1 l^{1-m}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m} \\ &+\bar{\lambda}\sum_{i=1}^{l}\varphi_i^2|\zeta_i^{m_i}(t)|^2. \end{aligned} \tag{2.15}$$

Let $\bar{\varphi}=\max[\varphi_i^2]$, $M=\max[m_i]$, $m=\min[m_i]$ and recall that $V(\zeta(t))=\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)$. Then

$$\begin{aligned} \bar{\lambda}\sum_{i=1}^{l}\varphi_i^2|\zeta_i^{m_i}(t)|^2 &\le \bar{\lambda}\sum_{i=1}^{l}\bar{\varphi}|\zeta_i^{m_i}(t)|^2 \le \bar{\lambda}\sum_{i=1}^{l}\bar{\varphi}(\zeta_i^T(t)\zeta_i(t))^{m_i} \le \bar{\lambda}\bar{\varphi}\sum_{i=1}^{l}[V(\zeta(t))]^{m_i} \\ &\le \bar{\lambda}\bar{\varphi}\sum_{i=1}^{l}[(V(\zeta(t)))^M+(V(\zeta(t)))^m] \le \bar{\lambda}l\bar{\varphi}[(V(\zeta(t)))^M+(V(\zeta(t)))^m]. \end{aligned} \tag{2.16}$$

Substituting (2.16) into the expression for (2.15), we have

$$\begin{aligned}\mathcal{L}V &\leq -2\mu_1\left(V(\zeta(t))\right)^{\frac{1+q}{2}} - 2\eta_1 l^{1-M}\left(V(\zeta(t))\right)^{M} - 2\eta_1 l^{1-m}\left(V(\zeta(t))\right)^{m} + \bar{\lambda}\sum_{i=1}^{l}\varphi_i^2\left(V(\zeta(t))\right)^{m_i}\\ &\leq -2\mu_1\left(V(\zeta(t))\right)^{\frac{1+q}{2}} - 2\eta_1 l^{1-M}\left(V(\zeta(t))\right)^{M} - 2\eta_1 l^{1-m}\left(V(\zeta(t))\right)^{m}\\ &\quad + \bar{\lambda} l\bar{\varphi}[(V(\zeta(t)))^M + (V(\zeta(t)))^m]\\ &\leq -2\mu_1\left(V(\zeta(t))\right)^{\frac{1+q}{2}} - (2\eta_1 l^{1-M} - \bar{\lambda} l\bar{\varphi})\left(V(\zeta(t))\right)^{M} - (2\eta_1 l^{1-m} - \bar{\lambda} l\bar{\varphi})\left(V(\zeta(t))\right)^{m}\\ &\leq -2\mu_1\left(V(\zeta(t))\right)^{\frac{1+q}{2}} - (2\eta_1 l^{1-M} - \bar{\lambda} l\bar{\varphi})\left(V(\zeta(t))\right)^{M}\\ &\leq -\hat{\mu}_1\left(V(\zeta(t))\right)^{Q} - \hat{\eta}_1\left(V(\zeta(t))\right)^{M},\end{aligned} \tag{2.17}$$

where $\hat{\mu}_1 = 2\mu_1$, $Q = \frac{1+q}{2}$, $\hat{\eta}_1 = 2\eta_1 l^{1-M} - \bar{\lambda} l\bar{\varphi}$.

The next step is to estimate the settling time. According to [44], since the $V(\zeta)$ is a positive and radially unbounded function, there are two K_∞ class functions $\widetilde{\mu_1}$ and $\widetilde{\mu_2}$ such that $\widetilde{\mu_1}(|\zeta|) \leq V(\zeta) \leq \widetilde{\mu_2}(|\zeta|)$ for any $\zeta \in \mathbb{R}^l$.

Let $r > 0$ and $0 < \varepsilon < 1$ be arbitrary, and define $\sigma_r = \inf\{t; |\zeta(t;\zeta_0)| > r\}$, where $\zeta_0 = \beta(0) - \alpha(0)$. By supermartingale inequality,

$$P(\sigma_r \leq t)\widetilde{\mu_1}(r) \leq E[I_{\sigma_r \leq t} V(\zeta(\sigma_r))] \leq E[V(\zeta(t \wedge \sigma_r))] \leq V(\zeta_0) \leq \widetilde{\mu_2}(|\zeta_0|).$$

Taking $\delta = \widetilde{\mu_2}^{-1}(\widetilde{\mu_1}(r)\varepsilon)$, we can obtain that $P(\sigma_r \leq t) \leq \varepsilon$ when $|\zeta_0| \leq \delta$. Let $t \to \infty$, then we have $P(\sigma_r \leq \infty) \leq \varepsilon$, which implies that $P\left(\sup_{t\geq 0}|\zeta(t;\zeta_0)| \leq r\right) \geq 1 - \varepsilon$.

Next, we prove the fixed-time attraction in probability. Structure a function

$$R(V(\zeta)) = \int_0^{V(\zeta)} \frac{1}{\hat{\mu}_1 s^Q + \hat{\eta}_1 s^M} ds.$$

The function $R(V(\zeta))$ is positive definite and twice continuously differentiable in $\mathbb{R}^l \backslash \{\mathbf{0}\}$. For any initial condition $\zeta_0 \in \mathbb{R}^l \backslash \{\mathbf{0}\}$, the stopping time is characterized as $h_k = \inf\{t \geq 0; |\zeta(t;\zeta_0)| \notin (\frac{1}{k}, k)\}$. For $t \leq h_k$,

$$\begin{aligned}R(V(\zeta(t \wedge h_k))) =& R(V(\zeta_0)) + \int_0^{t\wedge h_k} \mathcal{L}R(V(\zeta(s)))ds + \int_0^{t\wedge h_k} \frac{1}{\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M}\frac{\partial V}{\partial \zeta}\widetilde{g}(\zeta(s))dB(s)\\ &+ \int_0^{t\wedge h_k} \mathcal{L}^{(1)} R(V(\zeta(s)))\tilde{N}d(\mathrm{d}t, \mathrm{d}u).\end{aligned} \tag{2.18}$$

Since $\frac{1}{\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M}\frac{\partial V}{\partial \zeta}\widetilde{g}(\zeta(s))$ is bounded in interval $\frac{1}{k} < |\zeta| < k$, we have

$$E\left[\frac{1}{\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M}\frac{\partial V}{\partial \zeta}\widetilde{g}(\zeta(s))\right] = 0.$$

Taking expectations on both sides of (2.18), it follows from $E\left[\int_0^{t\wedge h_k} \mathcal{L}^{(1)} R(V(\zeta(s)))\tilde{N}d(\mathrm{d}t, \mathrm{d}u)\right] = 0$ that

$$E[R(V(\zeta(t \wedge h_k)))] = R(V(\zeta_0)) + \int_0^{t\wedge h_k} \mathcal{L}R(V(\zeta(s)))ds, \tag{2.19}$$

where

$$\begin{aligned}\mathcal{L}R(V(\zeta(t))) =& \frac{1}{\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M} \cdot \mathcal{L}V(\zeta(t)) \\ & - \frac{1}{2} \cdot \frac{\hat{\mu}_1 Q V^{Q-1} + \hat{\eta}_1 M V^{M-1}}{(\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M)^2} \cdot Tr\left[\left(\frac{\partial V}{\partial \zeta}\widetilde{g}(\zeta)\right)^T \left(\frac{\partial V}{\partial \zeta}\widetilde{g}(\zeta)\right)\right] \\ & + \int_S \left[\int_{V(\zeta)}^{V(\zeta+\gamma)} \frac{1}{\hat{\mu}_1 s^Q + \hat{\eta}_1 s^M} ds - \frac{1}{\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M}\left(V(\zeta+\gamma) - V(\zeta)\right)\right] \lambda(du).\end{aligned}$$

Because the function $V(\zeta)$ is positive definite and $\hat{\mu}_1, \hat{\eta}_1, Q, M$ are positive constants, we have

$$\frac{1}{2} \cdot \frac{\hat{\mu}_1 Q V^{Q-1} + \hat{\eta}_1 M V^{M-1}}{(\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M)^2} \cdot Tr\left[\left(\frac{\partial V}{\partial \zeta}\widetilde{g}(\zeta)\right)^T \left(\frac{\partial V}{\partial \zeta}\widetilde{g}(\zeta)\right)\right] > 0.$$

Note that $\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M$ is positive and increasing. Thus

$$\begin{aligned}& \int_S \left[\int_{V(\zeta)}^{V(\zeta+\gamma)} \frac{1}{\hat{\mu}_1 s^Q + \hat{\eta}_1 s^M} ds - \frac{1}{\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M}\left(V(\zeta+\gamma) - V(\zeta)\right)\right] \lambda(du) \\ =& \int_S \left[\left(\frac{1}{\hat{\mu}_1 V(\zeta^*)^Q + \hat{\eta}_1 V(\zeta^*)^M} - \frac{1}{\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M}\right)\left(V(\zeta+\gamma) - V(\zeta)\right)\right] \lambda(du) < 0,\end{aligned}$$

where $\zeta^* \in (\zeta, \zeta+\gamma)$ when $\gamma > 0$ and $\zeta^* \in (\zeta+\gamma, \zeta)$ when $\gamma < 0$. According to (2.17), we can obtain that

$$\mathcal{L}R(V(\zeta(t))) \leq \frac{1}{\hat{\mu}_1 V^Q + \hat{\eta}_1 V^M} \cdot \mathcal{L}V(\zeta(t)) \leq -1.$$

It follows from (2.19) that

$$E[R(V(\zeta(t \wedge h_k)))] \leq R(V(\zeta_0)) - (t \wedge h_k),$$

which implies

$$E[R(V(\zeta(t \wedge h_k)))] \leq R(V(\zeta_0)).$$

Let $k \to \infty$, then we have $t \wedge h_k \to T_\epsilon$ a.s. Thus the settling time could be obtained as

$$T_\epsilon = E\left[T(\zeta, \epsilon)\right] \leq R(V(\zeta_0)) = \int_0^{V(\zeta_0)} \frac{1}{\hat{\mu}_1 s^Q + \hat{\eta}_1 s^M} ds \leq \frac{1}{\hat{\mu}_1}\frac{1}{1-Q} + \frac{1}{\hat{\eta}_1}\frac{1}{M-1}.$$

That is $T_\epsilon < +\infty$ and independent of the initial state.

2.2. Adaptive control

Although feedback control has many advantages, it does not mean that other control methods are meaningless. The fixed structure of feedback controller will not only make the design and implementation simple but also lead to the waste of resources. For example, the feedback gain coefficient is usually so large that it would result in an unreasonable use of resources. In order to avoid such waste of resources, the adaptive controller has been proposed. The adaptive controller constantly measures the states of the system and makes decisions to change the structure and parameters of the controller. Or, it changes the control rules according to the adaptive laws. The main purpose of this controller is to make the system achieve the desired states with less consumption by changing the control rules. Compared with the feedback control, the design of the adaptive control is complicated, its calculation is larger, and its synchronization criterion is more challenging to deduce. However, if the adaptive control scheme is feasible, it can achieve the same goal with a smaller control input. Moreover, the adaptive control generally has better stability and robustness than the feedback control. The comparison of the feedback controller and the adaptive controller can be seen in Table 1 below.

Table 1. Comparison of the feedback controller and the adaptive controller.

Controller	Gains	Calculation	Design	Synchronization time	Consume resources	Stability
feedback controller	fixed	simple	simple	more	more	weak
adaptive controller	dynamic	complex	complex	less	fewer	strong

Next, we design an adaptive controller to synchronize FSCNNs within a fixed time as follows:

$$u_i(t) = \begin{cases} -\aleph_i(t)\zeta_i(t) - \sum_{j=1}^{l}\Big[\omega_{ij}|\zeta_j(t-h)|\mathrm{sign}(\zeta_i(t)) - \frac{1}{2}\varpi_{ij}\int_{t-h}^{t}|\zeta_i(s)|^2\mathrm{d}s\frac{\zeta_i(t)}{|\zeta_i(t)|^2}\Big] & \\ -\mu_2|\zeta_i(t)|^q\mathrm{sign}(\zeta_i(t)) - \eta_2[|\zeta_i(t)|^{2M-1} + |\zeta_i(t)|^{2m-1}]\mathrm{sign}(\zeta_i(t)), & \text{if } \zeta_i(t) \neq 0, \\ 0, & \text{if } \zeta_i(t) = 0, \end{cases} \tag{2.20}$$

where $\aleph_i(t)$ is the adaptive adjustment feedback gain, and the designed adaptive rate is

$$\dot{\aleph}_i(t) = \zeta_i^T(t)\zeta_i(t) - \mu_2\mathrm{sign}(\aleph_i(t)-\aleph_1)(\aleph_i(t)-\aleph_1)^q - \eta_2\mathrm{sign}(\aleph_i(t)-\aleph_1)[(\aleph_i(t)-\aleph_1)^{2M-1} + (\aleph_i(t)-\aleph_1)^{2m-1}], \tag{2.21}$$

where $\aleph_1$ is a constant to be determined.

Theorem 2.8. *Assume that the Assumptions 2.2–2.4 hold. The systems* (2.1) *and* (2.2) *are stochastic fixed-time synchronization under the adaptive control* (2.20) *with adaptive rate* (2.21)*, if the following four conditions hold:*

$$\begin{aligned} & -2a_i - 2\aleph_1 + \varkappa_i + \sum_{j=1}^{l}\Big(L_1\varsigma_{ij} + L_1\varsigma_{ji} + \theta_{ij} + o_{ij} + \xi_{ij}\Big) \leq 0, \\ & 2\sum_{j=1}^{l}\Big(|L_1|\,\nu_{ij} + |m_{ij}|L_2 + |\iota_{ij}|L_2 - \omega_{ij}\Big) \leq 0, \\ & \sum_{j=1}^{l}\Big(hb_{ji}\,(L_1)^2 + hs_{ji}\,(L_2)^2 + hc_{ji}\,(L_2)^2 - \varpi_{ij}\Big) \leq 0, \\ & \Big(2\eta_2 l^{1-M} - \bar{\lambda}l\bar{\varphi}\Big)2^{1-M} > 0. \end{aligned}$$

And the settling time is

$$T_\epsilon = \frac{1}{\hat{\mu}_2}\frac{1}{1-Q} + \frac{1}{\hat{\eta}_2}\frac{1}{M-1},$$

where $\hat{\mu}_2 = 2\mu_2$ *and* $\hat{\eta}_2 = \Big(2\eta_2 l^{1-M} - \bar{\lambda}l\bar{\varphi}\Big)2^{1-M}$.

Proof. We structure the Lyapunov function $V(t,\zeta(t)) = \sum_{i=1}^{l}[\zeta_i^T(t)\zeta_i(t) + (\aleph_i(t) - \aleph_1)^2]$. According to the Itô formula, we have

$$
\begin{aligned}
\mathcal{L}V =& 2\sum_{i=1}^{l}\zeta_i^T(t)\bigg[-a_i\zeta_i(t) + \sum_{j=1}^{l}\Big(\varsigma_{ij}\widetilde{F}(\zeta_j(t)) + \nu_{ij}\widetilde{F}(\zeta_j(t-h)) + \theta_{ij}\int_{t-h}^{t}\widetilde{F}(\zeta_j(s))\mathrm{d}s\Big) \\
&+ u_i(t) + \bigwedge_{j=1}^{l} m_{ij}\widetilde{G}(\zeta_j(t-h)) + \bigvee_{j=1}^{l}\iota_{ij}\widetilde{G}(\zeta_j(t-h)) + \bigwedge_{j=1}^{l} o_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s \\
&+ \bigvee_{j=1}^{l}\xi_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s\bigg] + \sum_{i=1}^{l}\mathrm{Tr}(W_i^T W_i) + 2\sum_{i=1}^{l}(\aleph_i(t) - \aleph_1)\cdot\dot{\aleph}_i(t) \\
&+ \sum_{i=1}^{l}\int_S\Big[(\zeta_i(t)+\gamma_i)^T(\zeta_i(t)+\gamma_i) - \zeta_i^T(t)\zeta_i(t) - 2\zeta_i^T(t)\gamma_i\Big]\lambda(du) \\
=& 2\sum_{i=1}^{l} -a_i\zeta_i^T(t)\zeta_i(t) + 2\sum_{i=1}^{l}\zeta_i^T(t)\sum_{j=1}^{l}\varsigma_{ij}\widetilde{F}(\zeta_j(t)) + 2\sum_{i=1}^{l}\zeta_i^T(t)\sum_{j=1}^{l}\nu_{ij}\widetilde{F}(\zeta_j(t-h)) \\
&+ 2\sum_{i=1}^{l}\zeta_i^T(t)\sum_{j=1}^{l}\theta_{ij}\int_{t-h}^{t}\widetilde{F}(\zeta_j(s))\mathrm{d}s + 2\sum_{i=1}^{l}\zeta_i^T(t)\bigwedge_{j=1}^{l} m_{ij}\widetilde{G}(\zeta_j(t-h)) \\
&+ 2\sum_{i=1}^{l}\zeta_i^T(t)\bigvee_{j=1}^{l}\iota_{ij}\widetilde{G}(\zeta_j(t-h)) + 2\sum_{i=1}^{l}\zeta_i^T(t)\bigwedge_{j=1}^{l} o_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s \\
&+ 2\sum_{i=1}^{l}\zeta_i^T(t)\bigvee_{j=1}^{l}\xi_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s + 2\sum_{i=1}^{l}\zeta_i^T(t)u_i(t) + \sum_{i=1}^{l}\mathrm{Tr}(W_i^T W_i) \\
&+ 2\sum_{i=1}^{l}(\aleph_i(t) - \aleph_1)\cdot\dot{\aleph}_i(t) + \sum_{i=1}^{l}\int_S\Big[(\zeta_i(t)+\Gamma_i)^T(\zeta_i(t)+\Gamma_i) - \zeta_i^T(t)\zeta_i(t) - 2\zeta_i^T(t)\Gamma_i\Big]\lambda(du).
\end{aligned}
\tag{2.22}
$$

Comparing (2.5) and (2.22), we can find that the terms in (2.22) are all scaled in the proof of Theorem 2.7, except for the term

$$
2\sum_{i=1}^{l}\zeta_i^T(t)u_i(t) + 2\sum_{i=1}^{l}(\aleph_i(t) - \aleph_1)\cdot\dot{\aleph}_i(t).
$$

By using (2.21), we obtain that

$$
\begin{aligned}
&2\sum_{i=1}^{l}\zeta_i^T(t)u_i(t)+2\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)\cdot\dot{\aleph}_i(t)\\
\leq&2\sum_{i=1}^{l}\zeta_i^T(t)\bigg(-\aleph_i(t)\zeta_i(t)-\sum_{j=1}^{l}(\omega_{ij}|\zeta_j(t-h)|-\frac{1}{2}\varpi_{ij}\int_{t-h}^{t}|\zeta_i(s)|^2\mathrm{d}s\frac{\zeta_i(t)}{|\zeta_i(t)|^2})\\
&-\mu_2|\zeta_i(t)|^q(\zeta_i(t))-\eta_2[|\zeta_i(t)|^{2M-1}+|\zeta_i(t)|^{2m-1}](\zeta_i(t))\bigg)+2\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)\cdot\bigg(\zeta_i^T(t)\zeta_i(t)\\
&-\mu_2(\aleph_i(t)-\aleph_1)(\aleph_i(t)-\aleph_1)^q-\eta_2(\aleph_i(t)-\aleph_1)[(\aleph_i(t)-\aleph_1)^{2M-1}+(\aleph_i(t)-\aleph_1)^{2m-1}]\bigg)\\
\leq&2\sum_{i=1}^{l}\zeta_i^T(t)\bigg(-(\aleph_i(t)-\aleph_1)\,\zeta_i(t)-\sum_{j=1}^{l}\Big(\omega_{ij}|\zeta_j(t-h)|-\frac{1}{2}\varpi_{ij}\int_{t-h}^{t}|\zeta_i(s)|^2\mathrm{d}s\frac{\zeta_i(t)}{|\zeta_i(t)|^2}\Big)\\
&-\mu_2|\zeta_i(t)|^q(\zeta_i(t))-\eta_2[|\zeta_i(t)|^{2M-1}+|\zeta_i(t)|^{2m-1}](\zeta_i(t))\bigg)+2\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)\cdot\bigg(\zeta_i^T(t)\zeta_i(t)\\
&-\mu_2(\aleph_i(t)-\aleph_1)(\aleph_i(t)-\aleph_1)^q-\eta_2(\aleph_i(t)-\aleph_1)[(\aleph_i(t)-\aleph_1)^{2M-1}+(\aleph_i(t)-\aleph_1)^{2m-1}]\bigg)\\
&-2\sum_{i=1}^{l}\aleph_1\zeta_i^T(t)\zeta_i(t).
\end{aligned}
$$

Therefore, it follows from Lemma 2.6 that

$$
\begin{aligned}
&2\sum_{i=1}^{l}\zeta_i^T(t)u_i(t)+2\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)\cdot\dot{\aleph}_i(t)\\
\leq&-2\sum_{i,j=1}^{l}\omega_{ij}|\zeta_i(t)||\zeta_j(t-h)|-\sum_{i,j=1}^{l}\varpi_{ij}\int_{t-h}^{t}|\zeta_i(s)|^2\mathrm{d}s-2\mu_2\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{\frac{1+q}{2}}\\
&-2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M}-2\eta_2 l^{1-m}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m}-2\mu_2\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^{1+q}\\
&-2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{M}-2\eta_2 l^{1-m}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{m}-2\sum_{i=1}^{l}\aleph_1\zeta_i^T(t)\zeta_i(t).
\end{aligned}
\tag{2.23}
$$

Combined with (2.6)–(2.14), (2.16), and (2.23), we have

$$
\begin{aligned}
\mathcal{L}V \leq & \sum_{i=1}^{l}\left(-2a_i - 2\aleph_1 + \sum_{j=1}^{l}(\varsigma_{ij}L_1 + \varsigma_{ji}L_1 + \theta_{ij} + o_{ij} + \xi_{ij}) + \varkappa_{ij}\right)|\zeta_i(t)|^2 \\
& + 2\sum_{i,j=1}^{l}\left(\nu_{ij}L_1 + |m_{ij}|L_2 + |\iota_{ij}|L_2 - \omega_{ij}\right)|\zeta_i(t)||\zeta_j(t-h)| \\
& + \sum_{i,j=1}^{l}\left(hb_{ji}(L_1)^2 + hs_{ji}(L_2)^2 + hc_{ji}(L_2)^2 - \varpi_{ij}\right)\int_{t-h}^{t}|\zeta_i(s)|^2 \mathrm{d}s \\
& - 2\mu_2\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{\frac{1+q}{2}} - 2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M} - 2\eta_2 l^{1-m}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m} \\
& - 2\mu_2\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{\frac{1+q}{2}} - 2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{M} - 2\eta_2 l^{1-m}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{m} \\
& + \bar{\lambda} l\bar{\varphi}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M} + \bar{\lambda} l\bar{\varphi}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m}.
\end{aligned}
$$

Assume that the conditions in Theorem 2.8 hold, then

$$
\begin{aligned}
\mathcal{L}V \leq & - 2\mu_2\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{\frac{1+q}{2}} - 2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M} - 2\eta_2 l^{1-m}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m} \\
& - 2\mu_2\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{\frac{1+q}{2}} - 2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{M} - 2\eta_2 l^{1-m}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{m} \\
& + \bar{\lambda} l\bar{\varphi}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M} + \bar{\lambda} l\bar{\varphi}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m}.
\end{aligned}
$$

Thus, it follows from Lemma 2.6 that

$$\begin{aligned}
\mathcal{L}V \leq & -2\mu_2\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{\frac{1+q}{2}} - 2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M} - 2\eta_2 l^{1-m}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m} \\
& -2\mu_2\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{\frac{1+q}{2}} - 2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{M} - 2\eta_2 l^{1-m}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{m} \\
& +\bar{\lambda}l\bar{\varphi}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M} + \bar{\lambda}l\bar{\varphi}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{m} + \bar{\lambda}l\bar{\varphi}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{M} \\
& +\bar{\lambda}l\bar{\varphi}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{m} \\
\leq & -2\mu_2\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{\frac{1+q}{2}} - 2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M} \\
& -2\mu_2\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{\frac{1+q}{2}} - 2\eta_2 l^{1-M}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{M} \\
& +\bar{\lambda}l\bar{\varphi}\left(\sum_{i=1}^{l}\zeta_i^T(t)\zeta_i(t)\right)^{M} + \bar{\lambda}l\bar{\varphi}\left(\sum_{i=1}^{l}(\aleph_i(t)-\aleph_1)^2\right)^{M} \\
\leq & -2\mu_2\left(\sum_{i=1}^{l}\left(\zeta_i^T(t)\zeta_i(t)+(\aleph_i(t)-\aleph_1)^2\right)\right)^{\frac{1+q}{2}} \\
& -\left(2\eta_2 l^{1-M}-\bar{\lambda}l\bar{\varphi}\right)2^{1-M}\left(\sum_{i=1}^{l}\left(\zeta_i^T(t)\zeta_i(t)+(\aleph_i(t)-\aleph_1)^2\right)\right)^{M} \\
\leq & -2\mu_2\left(V(\zeta(t))\right)^{\frac{1+q}{2}} - \left(2\eta_2 l^{1-M}-\bar{\lambda}l\bar{\varphi}\right)2^{1-M}\left(V(\zeta(t))\right)^{M} \\
\leq & -\hat{\mu}_2\left(V(\zeta(t))\right)^{Q} - \hat{\eta}_2\left(V(\zeta(t))\right)^{M},
\end{aligned}$$

where $\bar{\varphi} = \max[\varphi_i^2]$, $M = \max[m_i]$, $m = \min[m_i]$, $\hat{\mu}_2 = 2\mu_2$, $Q = \frac{1+q}{2}$, $\hat{\eta}_2 = \left(2\eta_2 l^{1-M} - \bar{\lambda}l\bar{\varphi}\right)2^{1-M}$. By using a similar approach in the proof of Theorem 2.7, we can easily obtain taht the settling time is as follows:

$$T_\epsilon = \frac{1}{\hat{\mu}_2}\frac{1}{1-Q} + \frac{1}{\hat{\eta}_2}\frac{1}{M-1}.$$

3. Numerical examples

In this section, two detailed numerical simulations are provided to demonstrate the validity of our results. First, we give the simulation process:

Step 1. Take the values of the system parameters including the connection weights of coefficient matrix.

Step 2. Define the initial values of the drive and response systems.
Step 3. Generate discrete time nodes. Randomly generate white noise perturbations and the time nodes of *Lévy* jumps. Merge them with the original time series to obtain a new time series.
Step 4. Treatment of fuzzy terms.
Step 5. Establish the iterative format of the drive system.
Step 6. Compute control item and establish the iterative format of the response system.
Step 7. Obtain the error data by iteration.

Let $l = 8$, then the drive system is as follows:

$$\begin{aligned} d\alpha_i(t) =& \Big[-a_i\alpha_i(t) + \sum_{j=1}^{8}\Big(\varsigma_{ij}\widetilde{f}(\alpha_j(t)) + \nu_{ij}\widetilde{f}(\alpha_j(t-h)) + \theta_{ij}\int_{t-h}^{t}\widetilde{f}(\alpha_j(s))ds\Big) \\ &+ \bigwedge_{j=1}^{8} m_{ij}\widetilde{g}(\alpha_j(t-h)) + \bigvee_{j=1}^{8} \iota_{ij}\widetilde{g}(\alpha_j(t-h)) + \bigwedge_{j=1}^{8} o_{ij}\int_{t-h}^{t}\widetilde{g}(\alpha_j(s))ds \\ &+ \bigvee_{j=1}^{8}\xi_{ij}\int_{t-h}^{t}\widetilde{g}(\alpha_j(s))ds + \bigwedge_{j=1}^{8} G_{ij}p_j + \bigvee_{j=1}^{8} H_{ij}p_j + D_i\Big]dt + w_i(\alpha_i(t), t)dB(t) \\ &+ \int_S \gamma_i(\alpha_i(t), t, u)\tilde{N}(dt, du). \end{aligned} \tag{3.1}$$

$a_i = 1.6$, $h = 0.01$, $D_i = 0$, $L_1 = L_2 = 1$, $\widetilde{f}(\cdot) = \widetilde{g}(\cdot) = \tan(\cdot)$, $p_j = 0.1$, $G_{ij} = H_{ij} = 1$, $w_1 = 3\alpha_1(t)$, $w_2 = 3.5\alpha_2(t)$, $w_3 = 3\alpha_3(t)$, $w_4 = 4.5\alpha_4(t)$, $w_5 = -3\alpha_5(t)$, $w_6 = -3.5\alpha_6(t)$, $w_7, t) = -4\alpha_7(t)$, $w_8 = -4.5\alpha_8(t)$, $\gamma_1 = \log(1 + \alpha_1(t))$, $\gamma_2 = \alpha_2(t)\log(1 + \alpha_2(t))$, $\gamma_3 = \alpha_3(t)^{\frac{1}{2}}\log(1 + \alpha_3(t))$, $\gamma_4 = \log(1 + \alpha_4(t))$, $\gamma_5 = \alpha_5(t)\log(1 + \alpha_5(t))$, $\gamma_6 = \alpha_6(t)^{\frac{1}{2}}\log(1 + \alpha_6(t))$, $\gamma_7 = \log(1 + \alpha_7(t))$, $\gamma_8 = \alpha_8(t)\log(1 + \alpha_8(t))$, and

$$(\varsigma_{ij})_{8\times 8} = 0.3 * \begin{bmatrix} 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.6 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 \end{bmatrix},$$

$$(\nu_{ij})_{8\times 8} = 0.2 * \begin{bmatrix} 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.6 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 \end{bmatrix},$$

$$(\theta_{ij})_{8\times 8} = 0.4 * \begin{bmatrix} 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.6 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 \end{bmatrix},$$

$$(m_{ij})_{8\times 8} = 0.4 * \begin{bmatrix} 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.6 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 \end{bmatrix},$$

$$(\iota_{ij})_{8\times 8} = (o_{ij})_{8\times 8} = 0.2 * \begin{bmatrix} 0 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.6 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0.7 & 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0 & 0 & 0.7 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 \end{bmatrix},$$

$$(\xi_{ij})_{8\times 8} = 1.3 * \begin{bmatrix} 0 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0.3 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 & 0.3 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 \end{bmatrix}.$$

The response system is

$$\begin{aligned} \mathrm{d}\beta_i(t) =& \Big[- a_i\beta_i(t) + \sum_{j=1}^{8}\Big(\varsigma_{ij}\widetilde{f}(\beta_j(t)) + \nu_{ij}\widetilde{f}(\beta_j(t-h)) + \theta_{ij}\int_{t-h}^{t}\widetilde{f}(\beta_j(s))\mathrm{d}s\Big) \\ &+ \bigwedge_{j=1}^{8} m_{ij}\widetilde{g}(\beta_j(t-h)) + \bigvee_{j=1}^{8} \iota_{ij}\widetilde{g}(\beta_j(t-h)) + \bigwedge_{j=1}^{8} o_{ij}\int_{t-h}^{t}\widetilde{g}(\beta_j(s))\mathrm{d}s \\ &+ \bigvee_{j=1}^{8} \xi_{ij}\int_{t-h}^{t}\widetilde{g}(\beta_j(s))\mathrm{d}s + \bigwedge_{j=1}^{8} G_{ij}p_j + \bigvee_{j=1}^{8} H_{ij}p_j + D_i + u_i(t)\Big]\mathrm{d}t + w_i(\beta_i(t),t)\mathrm{d}B(t) \\ &+ \int_S \gamma_i(\beta_i(t),t,u)\tilde{N}(\mathrm{d}t,\mathrm{d}u), \end{aligned} \tag{3.2}$$

$w_1 = 3\beta_1(t)$, $w_2 = 3.5\beta_2(t)$, $w_3 = 3\beta_3(t)$, $w_4 = 4.5\beta_4(t)$, $w_5 = -3\beta_5(t)$, $w_6 = -3.5\beta_6(t)$, $w_7 = -4\beta_7(t)$, $w_8 = -4.5\beta_8(t)$, $\gamma_1 = \log(1+\beta_1(t))$, $\gamma_2 = \beta_2(t)\log(1+\beta_2(t))$, $\gamma_3 = \beta_3(t)^{\frac{1}{2}}\log(1+\beta_3(t))$, $\gamma_4 = \log(1+\beta_4(t))$, $\gamma_5 = \beta_5(t)\log(1+\beta_5(t))$, $\gamma_6 = \beta_6(t)^{\frac{1}{2}}\log(1+\beta_6(t))$, $\gamma_7 = \log(1+\beta_7(t))$, $\gamma_8 = \beta_8(t)\log(1+\beta_8(t))$. Thus, we can obtain that the error system is

$$\begin{aligned} \mathrm{d}\zeta_i(t) =& \Big[- a_i\zeta_i(t) + \sum_{j=1}^{8}\Big(\varsigma_{ij}\widetilde{F}(\zeta_j(t)) + \nu_{ij}\widetilde{F}(\zeta_j(t-h)) + \theta_{ij}\int_{t-h}^{t}\widetilde{F}(\zeta_j(s))\mathrm{d}s\Big) \\ &+ \bigwedge_{j=1}^{8} m_{ij}\widetilde{G}(\zeta_j(t-h)) + \bigvee_{j=1}^{8} n_{ij}\widetilde{G}(\zeta_j(t-h)) + \bigwedge_{j=1}^{8} o_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s \\ &+ \bigvee_{j=1}^{8} \xi_{ij}\int_{t-h}^{t}\widetilde{G}(\zeta_j(s))\mathrm{d}s + u_i(t)\Big]\mathrm{d}t + W_i\mathrm{d}B(t) + \int_S \Gamma_i(\zeta_i(t),t,u)\tilde{N}(\mathrm{d}t,\mathrm{d}u). \end{aligned} \tag{3.3}$$

Example 3.1. *Consider the effectiveness of fixed-time synchronization of CNNs with Lévy noise by under the influence of feedback control.*

The feedback control is given by

$$u_i(t) = \begin{cases} -\aleph_i\zeta_i(t) - \sum_{j=1}^{8}\omega_{ij}|\zeta_j(t-h)|\mathrm{sign}(\zeta_i(t)) - \frac{1}{2}\sum_{j=1}^{8}\varpi_{ij}\int_{t-h}^{t}|\zeta_i(s)|^2 ds\frac{\zeta_i(t)}{|\zeta_i(t)|^2} & \\ -\mu_1|\zeta_i(t)|^q\mathrm{sign}(\zeta_i(t)) - \eta_1[|\zeta_i(t)|^{2M-1} + |\zeta_i(t)|^{2m-1}]\mathrm{sign}(\zeta_i(t)), & \text{if } \zeta_i(t) \neq 0, \\ 0, & \text{if } \zeta_i(t) = 0. \end{cases} \tag{3.4}$$

where $\aleph_i = 50$, $\mu_1 = 0.5$, $\eta_1 = 0.6$, $\varpi_{ii} = 4$, $q = 0.1$, $M = 3$, $m = 1.1$, $\omega_{ij} = \xi_{ij}$, and all other values not mentioned are zero.

It is not difficult to verify that all the conditions in Theorem 2.7 can be satisfied. The trajectories of the drive system (3.1) and the response system (3.2) are shown in Figures 2 and 3, respectively.

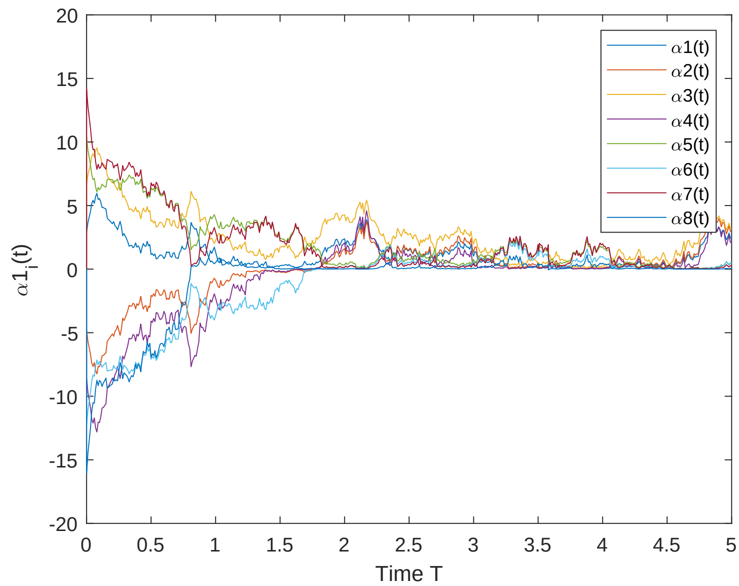

Figure 2. The trajectories of the drive system (3.1).

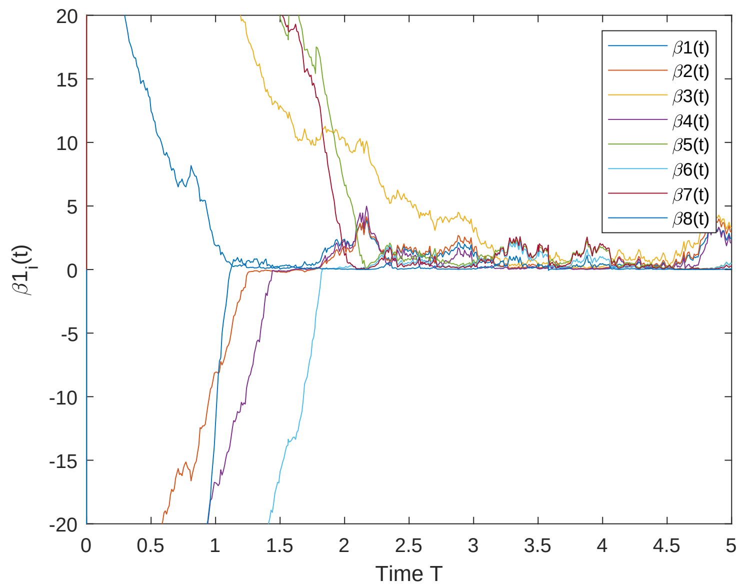

Figure 3. The trajectories of the response system (3.2) under feedback controller (3.4) .

The trajectories of the error system (3.3) with and without feedback control (3.4) are shown in Figures 4 and 5, respectively.

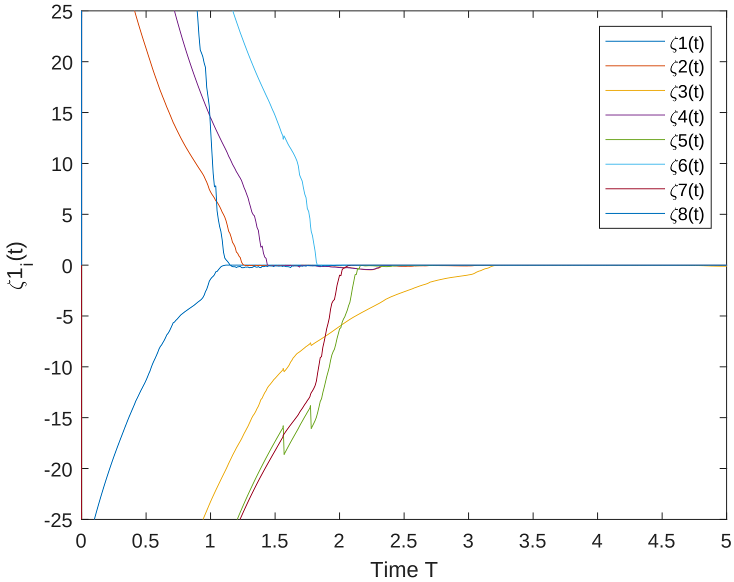

Figure 4. The trajectories of the error system (3.3) with feedback controller (3.4).

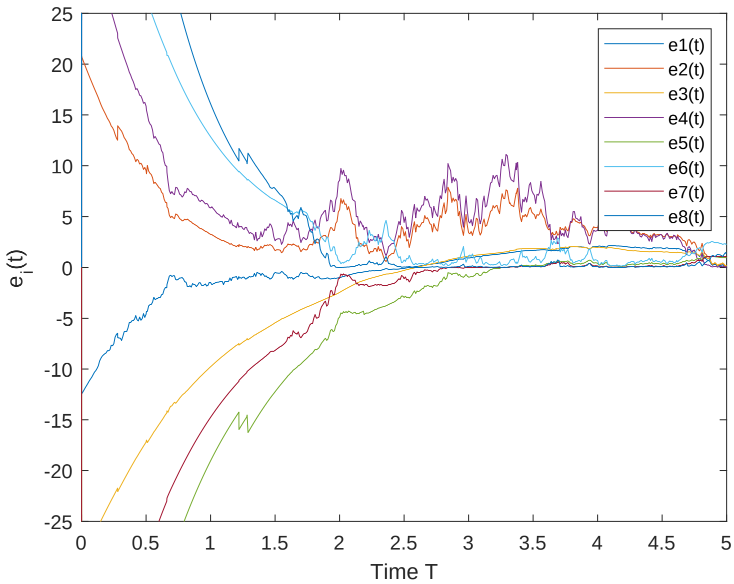

Figure 5. The trajectories of the error system (3.3) without feedback controller (3.4).

In Figure 4, it is obvious that the trajectory of the error system (3.3) tends to zero at a fixed time, which implies that the drive system (3.1) and the response system (3.2) are synchronized at a fixed time. This result shows that Theorem 2.7 is valid. Moreover, Figure 5 shows that the error system (3.3) in the absence of control does not tend to zero, which implies that the drive system (3.1) and the response system (3.2) cannot achieve synchronization.

Example 3.2. *Consider the effectiveness of fixed-time synchronization of CNNs with Lévy noise under adaptive control.*

The adaptive control is

$$u_i(t) = \begin{cases} -\aleph_i(t)\zeta_i(t) - \sum_{j=1}^{8}(\omega_{ij}|\zeta_j(t-h)|\text{sign}(\zeta_i(t)) - \frac{1}{2}\varpi_{ij}\int_{t-h}^{t}|\zeta_i(s)|^2 \mathrm{d}s\frac{\zeta_i(t)}{|\zeta_i(t)|^2}) & \\ -\mu_2|\zeta_i(t)|^q\text{sign}(\zeta_i(t)) - \eta_2[|\zeta_i(t)|^{2M-1} + |\zeta_i(t)|^{2m-1}]\text{sign}(\zeta_i(t)), & \text{if } \zeta_i(t) \neq 0, \\ 0, & \text{if } \zeta_i(t) = 0, \end{cases} \tag{3.5}$$

and the adaptive rate is

$$\dot{\aleph}_i(t) = \zeta_i^T(t)\zeta_i(t) - \mu_2\text{sign}(\aleph_i(t)-\aleph_1)(\aleph_i(t)-\aleph_1)^q - \eta_2\text{sign}(\aleph_i(t)-\aleph_1)[(\aleph_i(t)-\aleph_1)^{2M-1} + (\aleph_i(t)-\aleph_1)^{2m-1}], \tag{3.6}$$

where $\aleph_1 = 90$, $\mu_2 = 0.5$, $\eta_2 = 0.6$, $\varpi_{ii} = 4$, $q = 0.1$, $M = 1.4$, $m = 1.1$, $\omega_{ij} = \xi_{ij}$, and all other values not mentioned are zero.

It is easy to verify that all the conditions in Theorem 2.8 can be satisfied. The trajectories of the drive system (3.1), the response system (3.2), and the error system (3.3) under the adaptive control (3.6) are shown in Figures 6–8, respectively.

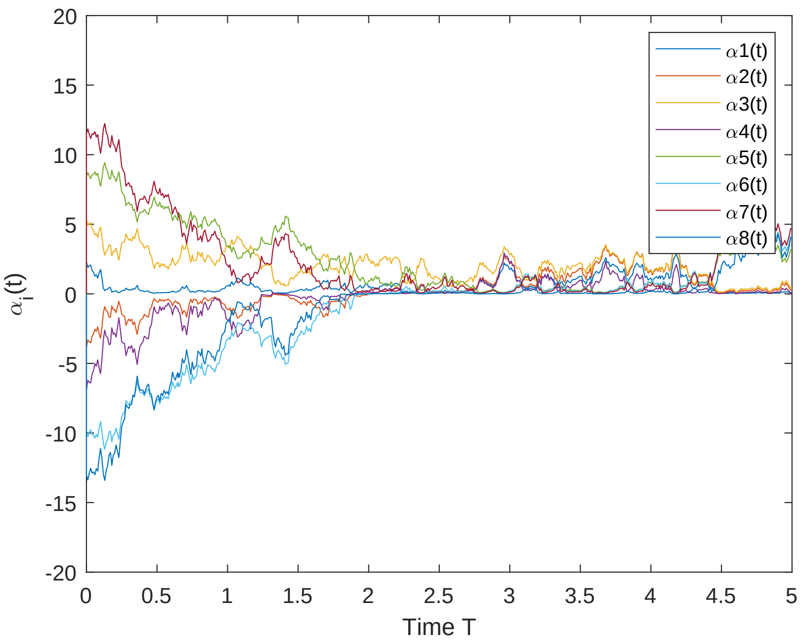

Figure 6. The trajectories of the drive system (3.1).

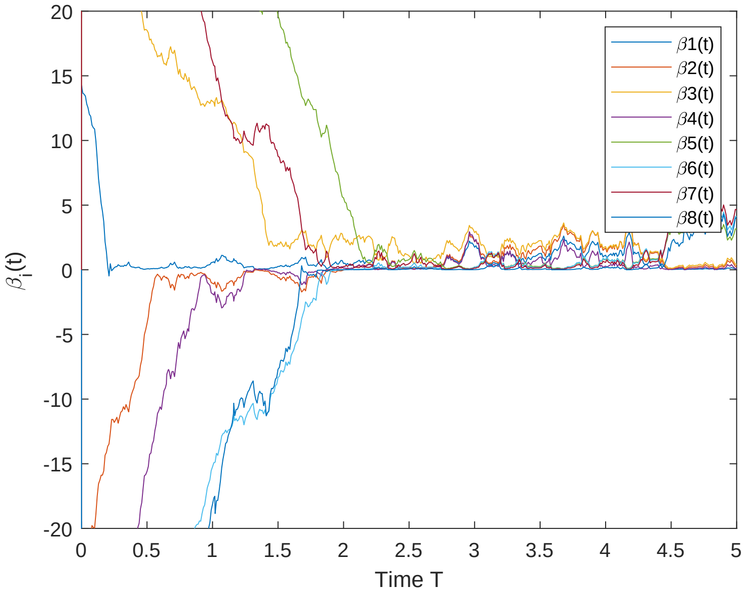

Figure 7. The trajectories of the response system (3.2) under the adaptive controller (3.5).

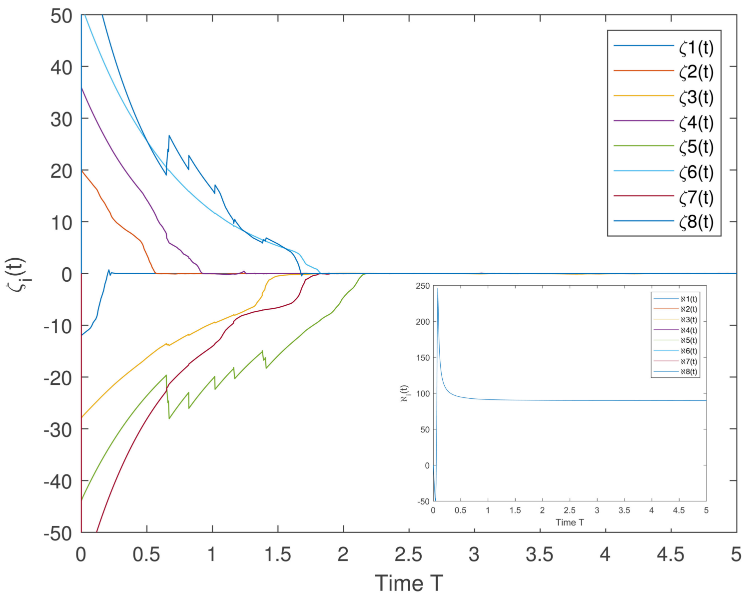

Figure 8. The trajectories of the error system (3.3) under the adaptive controller (3.5). The inset in Figure 8 shows the curve of the adaptive gain.

In Figure 8, it is obvious that the trajectory of the error system (3.3) tends to zero at a fixed time, which implies that the drive system (3.1) and the response system (3.2) are synchronized in a fixed time. This result shows that Theorem 2.8 is valid.

In addition, we can also summarize some rules from the above simulation results. It takes about two units of time for the adaptive controller to synchronize the drive system (3.1) and the response system (3.2), while it takes about three units of time for the feedback controller. This shows that the adaptive controller can synchronize a system faster than the feedback controller.

4. Conclusions

Based on the Lyapunov theory and the Itô formula, we research the fixed-time synchronization of FSCNNs with mixed delays driven by white noise and *Lévy* noise. Theorem 2.7 shows a sufficient criterion for fixed-time synchronization of FSCNNs under a feedback control and a method to compute the stabilization time. In addition, in order to reduce the consumption of resources, we design an adaptive controller. By Theorem 2.8, we obtain a sufficient criterion for fixed-time synchronization of FSCNNs under the adaptive control and the upper bound of the stabilization time. The results show that when the intensities of white noise and *Lévy* noise are small, FSCNNs can achieve synchronization through appropriate control means.

At present, there are many works on the synchronization problem of stochastic neural networks with continuous random perturbations (white noise). However, there are few studies on the synchronization problem of stochastic neural networks with Levy noise. In fact, discontinuous random perturbations (*Lévy* noise) should also be considered, which can be used to describe sudden external influences. Moreover, many works on neural network synchronization focus on asymptotic synchronization. But the infinite synchronization time cannot be used for practical problems in engineering. People are more likely to expect neural networks to be synchronized in a finite time. Since the initial state of a neural network is generally difficult to master, the fixed-time synchronization that does not depend on the initial state of the system is more practical. Our results can be applied to neural networks with *Lévy* noise and obtain a synchronization time independent of the initial state of the system, which complements the synchronization theory of stochastic neural networks.

There are many interesting questions that deserve to be investigated for the fixed-time synchronization of FSCNNs with *Lévy* noise. In this paper, we use two kinds of control methods: the feedback control and the adaptive control. There are many other control methods that are useful and efficient. The effectiveness of other control methods such as pinning control, intermittent control, and state observation control is also worth studying. Moreover, the parameters of the neural networks in this paper are assumed to be constants. In fact, the time-varying parameters and parameters with switching characteristics such as Markov switch and semi-Markov switch should also be considered. These time-varying and switching parameters can make the dynamic behavior of the system more complex. The construction of the Lyapunov function will be more difficult, and the derivation of the synchronization theorem is not known to be feasible. Of course, we have only theoretically demonstrated the synchronization criteria for FSCNNs with *Lévy* noise. How to apply theoretical results to practical problems is also a very challenging problem for us. The difficulties for the practical application of the synchronization theorem mainly focus on the technical realization, environmental factors, and system complexity. In order to overcome these difficulties, it is necessary to use a variety of technical means, and optimize the design according to the specific application scenario.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work is supported by the Natural Science Foundation of Heilongjiang Province (No. LH2022A002) and the National Natural Science Foundation of China (No. 12371160).

Conflict of interest

The authors declare there is no conflict of interest.

References

1. J. L. Wang, L. H. Zhao, PD and PI control for passivity and synchronization of coupled neural networks with multi-weights, *IEEE Trans. Network Sci. Eng.*, **8** (2021), 790–802. https://doi.org/10.1109/TNSE.2021.3052889
2. X. W. Liu, T. P. Chen, Synchronization of nonlinear coupled networks via aperiodically intermittent pinning control, *IEEE Trans. Neural Networks Learn. Syst.*, **26** (2015), 113–126. https://doi.org/10.1109/TNNLS.2014.2311838
3. K. B. Shi, Y. Y. Tang, S. M. Zhong, C. Yin, X. G. Huang, W. Q. Wang, Nonfragile asynchronous control for uncertain chaotic Lurie network systems with Bernoulli stochastic process, *Int. J. Robust Nonlinear Control*, **28** (2018), 1693–1714. https://doi.org/10.1002/rnc.3980
4. J. X. Lv, X. Z. Ju, C. H. Wang, Neural network prescribed-time observer-based output-feedback control for uncertain pure-feedback nonlinear systems, *Expert Syst. Appl.*, **264** (2024). https://doi.org/10.1016/j.eswa.2024.125813
5. D. Y. Li, S. Tong, H. Y. Yang, Q. L. Hu, Time-synchronized control for spacecraft reorientation with time-varying constraints, *IEEE-ASME Trans. Mechatron.*, **2024** (2024), 1–11. https://doi.org/10.1109/TMECH.2024.3430953
6. X. Y. Zhang, Y. H. Liu, C. Y. Su, Adaptive pseudoinverse control for constrained hysteretic nonlinear systems and its application on dielectric elastomer actuator, *IEEE-ASME Trans. Mechatron.*, **28** (2023), 2142–2154. https://doi.org/10.1109/TMECH.2022.3231263
7. C. Q. Guo, J. P. Hu, S. Celikovsky, Non-singular fixed-time tracking control of uncertain nonlinear pure-feedback systems with practical state constraints, *IEEE Trans. Circuits Syst. I Regul. Pap.*, **70** (2023), 3746–3758. https://doi.org/10.1109/TCSI.2023.3291700
8. L. O. Chua, L. Yang, Cellular neural networks theory, *IEEE Trans. Circuits Syst.*, **35** (1988), 1257–1272. https://doi.org/10.1109/31.7600
9. T. Yang, L. B. Yang, C. W. Wu, L. O. Chua, Fuzzy cellular neural networks: Applications, in *1996 Fourth IEEE International Workshop on Cellular Neural Networks and Their Applications Proceedings (CNNA-96)*, (1996), 225–230. https://doi.org/10.1109/cnna.1996.566560

10. K. Ratnavelu, M. Kalpana, P. Balasubramaniam, K. Wong, P. Raveendran, Image encryption method based on chaotic fuzzy cellular neural networks, *Signal Process.*, **140** (2017), 87–96. https://doi.org/10.1016/j.sigpro.2017.05.002

11. J. Karbowski, N. Kopell, Multispikes and synchronization in a large neural network with temporal delays, *Neural Comput.*, **12** (2000), 1573–1606. https://doi.org/10.1162/089976600300015277

12. J. D. Cao, J. Q. Lu, Adaptive synchronization of neural networks with or without time-varying delay, *Chaos*, **16** (2006). https://doi.org/10.1063/1.2178448

13. Y. R. Liu, Z. D. Wang, J. L. Liang, X. H. Liu, Synchronization and state estimation for discrete-time complex networks with distributed delays, *IEEE Tran. Syst. Man Cybern. Part B Cybern.*, **38** (2008), 1314–1325. https://doi.org/10.1109/TSMCB.2008.925745

14. L. L. Hou, P. F. Ma, X. Ma, H. B. Sun, H_∞ Exponential synchronization of switched cellular neural networks based on disturbance observer-based control, *Int. J. Control Autom. Syst.*, **22** (2024), 1430–1441. https://doi.org/10.1007/s12555-022-0917-7

15. T. Y. Jing, T. Y. He, Synchronization of fractional-order neural networks via intermittent quantized control: Optimal algorithm, *Int. J. Theor. Phys.*, **63** (2024). https://doi.org/10.1007/s10773-024-05701-z

16. X. S. Yang, J. D. Cao, Finite-time stochastic synchronization of complex networks, *Appl. Math. Modell.*, **34** (2010), 3631–3641. https://doi.org/10.1016/j.apm.2010.03.012

17. W. T. Wang, Finite-time synchronization for a class of fuzzy cellular neural networks with time-varying coefficients and proportional delays, *Fuzzy Sets Syst.*, **338** (2018), 40–49. https://doi.org/10.1016/j.fss.2017.04.005

18. L. Duan, H. Wei, L. H. Huang, Finite-time synchronization of delayed fuzzy cellular neural networks with discontinuous activations, *Fuzzy Sets Syst.*, **361** (2019), 56–70. https://doi.org/10.1016/j.fss.2018.04.017

19. H. Q. Wu, X. W. Zhang, R. X. Li, R. Yao, Finite-time synchronization of chaotic neural networks with mixed time-varying delays and stochastic disturbance, *Memet. Comput.*, **7** (2015), 231–240. https://doi.org/10.1007/s12293-014-0150-x

20. A. Abdurahman, H. J. Jiang, Z. D. Teng, Finite-time synchronization for fuzzy cellular neural networks with time-varying delays, *Fuzzy Sets Syst.*, **297** (2016), 96–111. https://doi.org/10.1016/j.fss.2015.07.009

21. D. S. Xu, Y. Liu, M. Liu, Finite-time synchronization of multi-coupling stochastic fuzzy neural networks with mixed delays via feedback control, *Fuzzy Sets Syst.*, **411** (2021), 85–104. https://doi.org/10.1016/j.fss.2020.07.015

22. J. Kumar, V. D. Murugan, M. Hymavathi, M. S. Ali, G. K. Thakur, Finite-time synchronization of neutral type fractional-order bidirectional associative memory neural networks with delays via adaptive feedback control strategy, *Discrete Contin. Dyn. Syst. Ser. S*, **2024** (2024). https://doi.org/10.3934/dcdss.2024157

23. H. N. Nie, Y. Zhang, Finite-time cluster synchronization of multi-weighted fractional-order coupled neural networks with and without impulsive effects, *Neural Networks*, **180** (2024). https://doi.org/10.1016/j.neunet.2024.106646

24. J. Zhang, S. Zhu, K. N. Wu, M. Q. Shen, S. P. Wen, Finite-time stabilization of semi-markov reaction-diffusion memristive NNs With unbounded time-varying delays, *IEEE Trans. Circuits Syst. I Regul. Pap.*, **72** (2025), 1832–1842. https://doi.org/10.1109/TCSI.2024.3459913

25. Y. G. Liu, Z. Y. Zhang, X. H. Wang, Z. Wang, C. Lin, Finite-time stabilization for fuzzy complex-valued neural networks with mixed delays via comparison approach, *IEEE Tran. Fuzzy Syst.*, **32** (2024), 621–633. https://doi.org/10.1109/TFUZZ.2023.3304648

26. X. S. Yang, J. Lam, D. W. Ho, Z. G. Feng, Fixed-time synchronization of complex networks with impulsive effects via nonchattering control, *IEEE Trans. Autom. Control*, **62** (2017), 5511–5521. https://doi.org/10.1109/TAC.2017.2691303

27. A. Khanzadeh, M. Pourgholi, Fixed-time sliding mode controller design for synchronization of complex dynamical networks, *Nonlinear Dyn.*, **88** (2017), 2637–2649. https://doi.org/10.1007/s11071-017-3400-x

28. W. L. Zhang, X. S. Yang, C. D. Li, Fixed-time stochastic synchronization of complex networks via continuous control, *IEEE Trans. Cybern.*, **49** (2019), 3099–3104. https://doi.org/10.1109/TCYB.2018.2839109

29. H. W. Ren, Z. P. Peng, Y. Gu, Fixed-time synchronization of stochastic memristor-based neural networks with adaptive control, *Neural Networks*, **130** (2020), 165–175. https://doi.org/10.1016/j.neunet.2020.07.002

30. F. C. Kong, Q. X. Zhu, T. W. Huang, New fixed-time stability lemmas and applications to the discontinuous fuzzy inertial neural networks, *IEEE Tran. Fuzzy Syst.*, **29** (2021), 3711–3722. https://doi.org/10.1109/TFUZZ.2020.3026030

31. C. Hu, J. Yu, Z. H. Chen, H. J. Jiang, T. W. Huang, Fixed-time stability of dynamical systems and fixed-time synchronization of coupled discontinuous neural networks, *Neural Networks*, **89** (2017), 74–83. https://doi.org/10.1016/j.neunet.2017.02.001

32. J. D. Cao, R. X. Li, Fixed-time synchronization of delayed memristor-based recurrent neural networks, *Sci. China Inf. Sci.*, **60** (2017). https://doi.org/10.1007/s11432-016-0555-2

33. Y. Liu, M. Liu, X. F. Xu, Adaptive control design for fixed-time synchronization of fuzzy stochastic cellular neural networks with discrete and distributed delay, *Iran. J. Fuzzy Syst.*, **18** (2021), 13–28.

34. A. Abdurahman, R. Tohti, C. C. Li, New results on fixed-time synchronization of impulsive neural networks via optimized fixed-time stability, *J. Appl. Math. Comput.*, **70** (2024), 2809–2826. https://doi.org/10.1007/s12190-024-02072-w

35. J. A. Wang, R. R. Cai, J. R. Zhang, J. Zhang, M. J. Li, Z. C. Zhao, Fixed-time pinning synchronization of intermittently coupled complex network via economical controller, *Neural Process. Lett.*, **56** (2024). https://doi.org/10.1007/s11063-024-11441-2

36. Q. X. Zhu, Razumikhin-type theorem for stochastic functional differential equations with *Lévy* noise and Markov switching, *Int. J. Control*, **90** (2017), 1703–1712. https://doi.org/10.1080/00207179.2016.1219069

37. H. Zhou, Y. Zhang, W. X. Li, Synchronization of stochastic *Lévy* noise systems on a multi-weights network and its applications of Chua's circuits, *IEEE Trans. Circuits Syst. I Regul. Pap.*, **66** (2019), 2709–2722. https://doi.org/10.1109/TCSI.2019.2899375
38. W. E. Zhou, Y. Q. Sun, P. Shi, Cluster synchronization of coupled neural networks with *Lévy* noise via event-triggered pinning control, *IEEE Trans. Neural Networks Learn. Syst.*, **33** (2022), 6144–6157. https://doi.org/10.1109/TNNLS.2021.3072475
39. M. Shi, D. B. Tong, Q. Y. Chen, W. N. Zhou, pth moment exponential synchronization for delayed multi-agent systems with *Lévy* noise and Markov switching, *IEEE Trans. Circuits Syst. II Express Briefs*, **71** (2024), 697–701. https://doi.org/10.1109/TCSII.2023.3304635
40. N. Zhang, Z. Wang, J. H. Park, W. X. Li, Semi-global synchronization of stochastic mixed time-delay systems with *Lévy* noise under aperiodic intermittent delayed sampled-data control, *Automatica*, **171** (2024). https://doi.org/10.1016/j.automatica.2024.111963
41. C. Li, X. F. Xu, M. Liu, The stochastic fixed-time synchronization of delays neural networks driven by *Lévy* noise, *Syst. Control Lett.*, **190** (2024). https://doi.org/10.1016/j.sysconle.2024.105839
42. R. M. Balan, C. B. Ndongo, Itô formula for integral processes related to space-time *Lévy* white noise, preprint, arXiv:1505.04685.
43. Z. Y. Zuo, L. Tie, Distributed robust finite-time nonlinear consensus protocols for multi-agent systems, *Int. J. Syst. Sci.*, **47** (2016), 1366–1375. https://doi.org/10.1080/00207721.2014.925608
44. J. L. Yin, S. Khoo, Z. H. Man, X. H. Yu, Finite-time stability and instability of stochastic nonlinear systems, *Automatica*, **47** (2011), 2671–2677. https://doi.org/10.1016/j.automatica.2011.08.050