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ERA, 33(3): 1267–1284.
DOI: 10.3934/era.2025056
Received: 09 January 2025
Revised: 09 February 2025
Accepted: 14 February 2025
Published: 04 March 2025

Research article

Dynamic event-triggered H_∞ control for neural networks with sensor saturations and stochastic deception attacks

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Abstract: This paper is devoted to dealing with the dynamic event-triggered H_∞ quantized control for neural networks with sensor saturations and stochastic deception attacks. To save the limited network resources, a dynamic event-triggered scheme is offered, which includes the general one. And a lower trigger frequency can be obtained by appropriately adjusting the triggering error. Then, a new closed-loop quantized control model is established under a dynamic event-triggered scheme, sensor saturations, and stochastic deception attacks, which is described by two independent Bernoulli-distributed variables. Moreover, by Lyapunov-Krasovskii functional theory, a new H_∞ performance criterion is given, and based on the criterion, the controller design approach is derived. Finally, simulations are listed to verify the validity of derived methods.

Keywords: event-triggered scheme; neural networks; sensor saturation; quantization; cyber-attacks

1. Introduction

For several decades, neural networks (NNs) have aroused the research interest of scholars because of their applications in diverse fields, such as pattern recognition, rainfall forecasting, and optimization problems [1]. The stability is the prerequisite for NNs to work, and a large number of stabilization results for NNs have emerged [2]. For example, the dissipative synchronization issue was addressed for semi-Markovian jumping delayed NNs under random deception attacks by a novel event-triggered impulsive control strategy in [3], and the problem of event-triggered synchronization for master-slave NNs was done in [4].

Networked control systems (NCSs) are the closed-loop control systems, which have been successfully applied in smart grids, industrial automation, and mobile communications [5]. In NCSs, data are transmitted via a communication network among the sensor, controller, and actuator. Now, networked control problems have attracted a growing number of researchers. For example, event-triggered sliding-mode control for networked Markov jumping systems with channel fading was done in [6], and the paper [7] investigated event-triggered output feedback H_∞ control for NCSs. Recently, Li et al. [8] researched H_∞ synchronization of semi-Markovian switching complex-valued networks with time-varying delay; Zhao and Wu [9] studied fixed/prescribed stability criteria of stochastic systems with time-delay; Hou et al. [10] addressed observer-based prescribed-time synchronization and topology identification for complex networks of piecewise-smooth systems with hybrid impulses; Tan et al. [11] focused on the dual control for autonomous airborne source search with Nesterov accelerated gradient descent.

To realize the efficient utilization of network resources, an event-triggered scheme (ETS) was introduced. Now, researchers have focused extensively on event-triggered controller design. For example, the output feedback $\mathcal{L}_\infty$ load frequency control of networked power systems was considered in [12] by an adaptive ETS. The leader-following consensus for linear multi-agent systems was studied in [13] by a dynamic ETS. The paper [14] investigated periodic event-triggered dynamic output feedback control for NCSs, and periodic event-triggered control for NCSs subject to input and output delays and external disturbance was studied in [15]. Recently, Wang et al. [16] addressed quantization-dependent dynamic event-triggered control for networked switched systems under Dos attacks; Zhang et al. [17] researched accumulated-state-error-based event-triggered sampling scheme and its application to H_∞ control of sampled-data systems; Zhao et al. [18] investigated prescribed-time synchronization for complex dynamic networks of piecewise smooth systems by a hybrid event-triggering control approach; Hou et al. [19] focused on the practical finite-time synchronization for master-slave Lur'e nonlinear systems with performance constraint and time-varying actuator faults via the memory based quantized dynamic event-triggered control.

Nowadays, the security issue of NCSs has received broad interest. Cyber-attacks are mainly classified into deception attacks and DoS attacks [20]. Now, researchers have focused extensively on cyber-attacks. For example, event-triggered H_∞ load frequency control for multi-area power systems was addressed in [21] with DoS attacks, and co-design of dynamic ETS and resilient observer-based control under aperiodic DoS attacks was done in [22]. The event-triggered control for networked Markovian jump systems was considered in [23] subject to a deception attack. Recently, the paper [24] investigated asynchronous sliding-mode control for discrete-time networked hidden stochastic jump systems with cyber attacks, and Yao et al. [25] solved prescribed-time output feedback control for cyber-physical systems under malicious attacks.

Sensor saturation brings nonlinear characteristics to the system by nonlinear sampled measurements, which can degrade the systems performance or even make it unstable. Therefore, it makes practical sense to take the sensor saturation into account. Now, some efforts have been devoted to researching the sensor saturation. For example, the H_∞ control for time-delay systems was considered in [26] with sensor saturations, and the paper [27] was concerned with the robust non-fragile observer-based dynamic event-triggered sliding mode control for NCSs subject to sensor saturation. The paper [28] has addressed security output feedback control for T-S fuzzy systems with decentralized ETS and multi-sensor saturations. Recently, dual flexible prescribed performance control [29], flexible prescribed performance output feedback control [30], and sliding flexible prescribed performance control [31] are investigated with input saturation.

Based on the above discussion, we found that in the research of deception attacks, the accumulated dynamic cyber-attack is often ignored, and only conventional deception attack, such as those that satisfy the Lipschitz condition are considered. At the same time, with the continuous improvement of ETS, there is still room for further research on how to balance the control performance of the system and the utilization rate of network resources. As for sensor saturation, to our knowledge, few scholars have investigated event-triggered quantized control for neural networks (NNs) with sensor saturations and cyber-attacks. Based on these, the dynamic event-triggered quantized control for NNs with sensor saturations and stochastic deception attacks is studied. The main contributions of this article can be summarized as follows. (1) The dynamic ETS is offered to save limited system resources, which can be reduce to the static one. (2) An improved quantized control model is established for NNs with dynamic ETS, sensor saturations and stochastic deception attacks. (3) A reformative event-triggered H_∞ quantized controller design is derived.

Notations: $Sym\{A\} = A + A^T$; $Pb\{A\}$ is the probability of event A that happens; $*$ denotes the symmetric term; $\mathscr{E}\{\cdot\}$ is the expectation; $B > 0$ shows B is a positive-definite matrix; $diag\{\cdot\}$ means block-diagonal matrix.

2. Problem formulation

A framework of dynamic event-triggered quantized control for NNs with sensor saturations and cyber-attacks is presented in Figure 1.

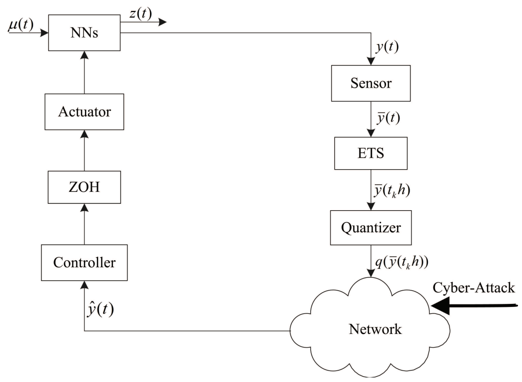

Figure 1. Networked control for NNs.

The plant is described by:

$$\begin{cases} \dot{\chi}(t) = \mathcal{A}\chi(t) + \mathcal{D}u(t) + E\hat{h}(t - \eta(t)) + \mathcal{B}\mu(t) \\ y(t) = C\chi(t) \\ \widetilde{z}(t) = \mathcal{F}\chi(t) \end{cases} \tag{2.1}$$

where $\mathcal{A} \in \mathbb{R}^{n\times n}$, $\mathcal{D} \in \mathbb{R}^{n\times p}$, $C \in \mathbb{R}^{m\times n}$, $\mathcal{B} \in \mathbb{R}^{n\times r}$, $E \in \mathbb{R}^{n\times n}$, and $\mathcal{F} \in \mathbb{R}^{q\times n}$; $\chi(t) = [\,\chi_1(t)\ \chi_2(t)\ \cdots\ \chi_n(t)\,]^T \in \mathbb{R}^n$ is the system state; $u(t) \in \mathbb{R}^p$ is the control input; $\mu(t) \in \mathbb{R}^r$ is the disturbance input; $y(t) = [\,y_1(t)\ y_2(t)\ \cdots\ y_m(t)\,]^T \in \mathbb{R}^m$ is the measured output; $\widetilde{z}(t) \in \mathbb{R}^q$ is the controlled output; $\hat{h}(\chi(t)) = [\,\hat{h}_1(\chi_1(t))\ \hat{h}_2(\chi_2(t))\ \cdots\ \hat{h}_n(\chi_n(t))\,]^T \in \mathbb{R}^n$ denotes the neuron activation function, and satisfies

$$\ell_i^- \leq \frac{\hat{h}_i(l_1) - \hat{h}_i(l_2)}{l_1 - l_2} \leq \ell_i^+, l_1 \neq l_2, i = 1, \cdots, n \tag{2.2}$$

with $\hat{h}(0) = 0$, ℓ_i^- and ℓ_i^+ are known constants.

In Figure 1, sensor saturation is taken into account, which can be described by a saturation function $sat(v) = [sat(v_1), sat(v_2), \cdots, sat(v_m)] \in \mathbb{R}^m$, and it can be decomposed into a linear part and a nonlinear part, that is, $sat(v) = v - \phi(v)$. Then, there exists a real number $\delta \in (0, 1)$ such that

$$\delta v^T v \geq \phi^T(v)\phi(v)$$

Therefore, the real output of the sensor is

$$\bar{y}(t) = sat(y(t)) = y(t) - \phi(y(t))$$

holds for the constraint that for $\delta \in (0, 1)$,

$$\delta y^T(t)y(t) \geq \phi^T(y(t))\phi(y(t)) \tag{2.3}$$

The ETS is given as follows:

$$e_{\bar{y}}^T(lh)\Omega_1 e_{\bar{y}}(lh) \leq \sigma \bar{y}_\rho^T(t_{k_l}h)\Omega_2 \bar{y}_\rho(t_{k_l}h) + \frac{1}{\alpha}\eta(t_{k_l}h) \tag{2.4}$$

where the triggering error

$$e_{\bar{y}}(lh) = \bar{y}(t_k h) - \bar{y}_\rho(t_{k_l}h),\ t_{k_l}h = t_k h + lh$$

and $\bar{y}_\rho(t_{k_l}h) = \rho\bar{y}(t_{k_l}h) + (1-\rho)\bar{y}(t_k h)$ with $0 \leq \rho \leq 1$ and $l \in \mathbb{N}$; $\Omega_k > 0$ $(k = 1, 2)$ are the weighting matrices; $\bar{y}(t_{k_l}h)$ and $\bar{y}(t_k h)$ are the currently sampled signal and the last transmitted signal, respectively; the variable $\eta(t)$ satisfies

$$\dot{\eta}(t) = -\beta\eta(t) + \sigma\bar{y}_\rho^T(t_{k_l}h)\Omega_2\bar{y}_\rho(t_{k_l}h) - e_{\bar{y}}^T(lh)\Omega_1 e_{\bar{y}}(lh),\ t \in [0, \infty) \tag{2.5}$$

where $\beta > 0$ and $\eta\,(0) > 0$.

Remark 1. *Compared with the reported ETSs in [32–34], a new triggering error $e_y(lh) = \bar{y}(t_k h) - \bar{y}_\rho(t_{k_l}h)$ is introduced, and when $\rho = 1$, it can reduce to the general one $e(t) = y(t_k h) - y(t_{k_l}h)$. By this triggering error, when the sampled data have a rapid change arising from the external disturbance, spurious triggering events may decrease. Second, more parameters are used to adjust the ETS (2.4) with (2.5), such as σ, ρ, α, β, Ω_1, and Ω_2. Third, the variable $\eta(t)$ can be adjusted as the system changes instead of a preset constant.*

Remark 2. *When $\rho = 1$, $\Omega_1 = \Omega_2$ and $\alpha \to \infty$ or $\eta(t) \to 0$, the ETS (2.4) degrades into the static ones in [28], [35], and [36]. So, the static ETSs in [28], [35], and [36] are the special case of ETS (2.4). When $\Omega_1 = \Omega_2$, $\rho = 1$, and $y = x$, the ETS (2.4) can reduce to state-based dynamic ETS in [37]. So, the dynamic ETS in [37] can also be the special case of ETS (2.4). Moreover, when $\Omega_1 = \Omega_2$, the ETS (2.4) reduces to the dynamic ETS in [34].*

Remark 3. *Refer to the paper [38]; since the event-triggered condition is only tested at the periodic moment, the minimum value of the time interval of the adjacent event-triggered moment is the sampling period h, which can directly exclude Zeno behavior.*

The logarithmic quantizer is presented as

$$q(\cdot) = [q_1(\cdot), q_2(\cdot), \cdots, q_m(\cdot)]^T$$

where $q_i(\cdot)$ is the i-th subquantizer, and $q_i(\cdot)$ is described by

$$q_i(\bar{y}_i(t_kh)) = \begin{cases} \mu_i^{(l)} & \frac{1}{1+w_i}\mu_i^{(l)} < \bar{y}_i(t_kh) \leq \frac{1}{1-w_i}\mu_i^{(l)} \\ 0 & \bar{y}_i(t_kh) = 0 \\ -g_i(-\bar{y}_i(t_kh)) & \bar{y}_i(t_kh) < 0 \end{cases}$$

with quantized levels set

$$\{\pm\mu_i^{(l)} | \mu_i^{(l)} = (d_i)^l \mu_i^{(0)}, l = 0, \pm1, \pm2, \cdots\} \cup \{0\}$$

where $w_i = \frac{1-d_i}{1+d_i}$, $d_i \in (0, 1)$, $u_i^{(0)} > 0$ denote quantizer parameter, quantizer density, and initial quantization, respectively. From [39], the quantizer is the characteristic of

$$q(\bar{y}(t_kh)) = \bar{y}(t_kh) + h(\bar{y}(t_kh)) \tag{2.6}$$

where

$$h(\bar{y}(t_kh)) = \left[h_1(\bar{y}_1(t_kh)) \;\; h_2(\bar{y}_2(t_kh)) \;\; \cdots h_m(\bar{y}_m(t_kh))\right]^T$$

with

$$-w_i[\bar{y}_i(t_kh)]^2 \leq \bar{y}_i(t_kh)h_i(\bar{y}_i(t_kh)) \leq w_i[\bar{y}_i(t_kh)]^2 \; (i = 1, 2, \cdots, m) \tag{2.7}$$

Assumed that τ_k satisfies $\underline{\tau} \leq \tau_k \leq \overline{\tau}$ $(k = 1, 2, \cdots)$, where $\underline{\tau}$ and $\overline{\tau}$ are two constants. For $t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$, the controller input

$$\widehat{y}(t) = \delta(t_k)q(\bar{y}(t_kh)) + \kappa(t_k)Cf(x(t - \nu(t))) + \lambda(t_k)C \int_{t-\theta(t)}^{t} g(x(q))dq$$

where $f(\cdot)$ and $g(\cdot)$ are the cyber-attacks; $Pb\{\delta(t_k) = 1\} = \bar{\delta}$, $Pb\{\delta(t_k) = 0\} = 1 - \bar{\delta}$, $Pb\{\varphi(t_k) = 1\} = \bar{\varphi}$, $Pb\{\varphi(t_k) = 0\} = 1 - \bar{\varphi}$; $\kappa(t_k) = [1 - \delta(t_k)]\,\varphi(t_k)$, $\lambda(t_k) = [1 - \delta(t_k)]\,[1 - \varphi(t_k)]$ with $\delta(t_k) \in \{0, 1\}$, $\varphi(t_k) \in \{0, 1\}$; $\nu(t) \in (0, \nu_M]$, $\theta(t) \in (0, \theta_M]$.

Assumption 1 [40]: Deception attacks $f(\cdot)$ and $g(\cdot)$ are bounded and satisfy

$$p_j^- \leq \frac{f_j(\ell_1) - f_j(\ell_2)}{\ell_1 - \ell_2} \leq p_j^+, \; q_j^- \leq \frac{g_j(\ell_1) - g_j(\ell_2)}{\ell_1 - \ell_2} \leq q_j^+ \; (\ell_1 \neq \ell_2) \tag{2.8}$$

where p_j^-, p_j^+, q_j^-, q_j^+ $(j = 1, \cdots, n)$ are constants, and $f(0) = g(0) = 0$.

Similar to [41], the interval $[t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$ can be divided as

$$[t_kh + \tau_k, t_{k+1}h + \tau_{k+1}) = \cup_{d=0}^{d_k} \chi_{t_k}^d, \; d_k = t_{k+1} - t_k - 1$$

where $\chi_{t_k}^d = \left[t_kh + dh + \tau_k^d, t_kh + dh + h + \tau_k^{d+1}\right)$ with $\tau_k^0 = \tau_k$ and $\tau_k^{d_k+1} = \tau_{k+1}$. For $t \in \chi_{t_k}^d$, we define $\tau(t) = t - t_kh - dh$ and $e(t) = \rho\bar{y}(t_kh) - \rho\bar{y}(t_kh + dh)$. It is clear that

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M, \quad \bar{y}(t_kh) = \rho^{-1}e(t) + \bar{y}(t - \tau(t))$$

where $\tau_m := \underline{\tau}$ and $\tau_M := \overline{\tau} + h$. So, the dynamic ETS (2.4) can be rewritten as

$$e^T(t)\Omega_1 e(t) \le \sigma\left[\varrho e(t) + \bar{y}(t-\tau(t))\right]^T \Omega_2 \left[\varrho e(t) + \bar{y}(t-\tau(t))\right] + \frac{1}{\alpha}\eta(t-\tau(t)) \tag{2.9}$$

where $\varrho = \rho^{-1} - 1$, and

$$\dot{\eta}(t) = -\beta\eta(t) - e^T(t)\Omega_1 e(t) + \sigma\left[\varrho e(t) + \bar{y}(t-\tau(t))\right]^T \Omega_2 \left[\varrho e(t) + \bar{y}(t-\tau(t))\right] \tag{2.10}$$

The controller is formulated as

$$u(t) = \mathcal{K}\widehat{y}(t) \tag{2.11}$$

where $\mathcal{K}$ is the controller gain.

Substituting (2.11) into (2.1), we can obtain the following system subject to (2.3), (2.7)–(2.9):

$$\begin{cases} \dot{\chi}(t) = \mathcal{A}\chi(t) + E\hat{h}(t-\eta(t)) + \bar{\kappa}\mathcal{D}\mathcal{K}Cf(\chi(t-\nu(t))) \\ \qquad +\bar{\lambda}\mathcal{D}\mathcal{K}C\int_{t-\theta(t)}^{t} g(x(r))dr + \mathcal{B}\mu(t) \\ \qquad +\bar{\delta}\mathcal{D}\mathcal{K}\left[C\chi(t-\tau(t)) + \rho^{-1}e(t) - \phi(C\chi(t-\tau(t))) + h(\bar{y}(t_k h)\right] \\ \qquad +\left[\delta(t_k) - \bar{\delta}\right]\mathcal{D}\mathcal{K}\left[C\chi(t-\tau(t)) + \rho^{-1}e(t) - \phi(C\chi(t-\tau(t))) + h(\bar{y}(t_k h)\right] \\ \qquad +\left[\kappa(t_k) - \bar{\kappa}\right]\mathcal{D}\mathcal{K}Cf(\chi(t-\nu(t))) \\ \qquad +\left[\lambda(t_k) - \bar{\lambda}\right]\mathcal{D}\mathcal{K}C\int_{t-\theta(t)}^{t} g(\chi(r))dr \\ \widetilde{z}(t) = \mathcal{F}\chi(t) \end{cases} \tag{2.12}$$

where $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$, $\bar{\kappa} = (1-\bar{\delta})\bar{\varphi}$ and $\bar{\lambda} = (1-\bar{\delta})(1-\bar{\varphi})$.

Remark 4. *The problem of dynamic event-triggered H_∞ networked control for NNs subject to stochastic deception attacks is studied. Compared to the reported paper [42], a new event-triggered quantized control model for NNs is presented under the dynamic ETS, sensor saturations, and stochastic deception attacks.*

Lemma 1 ([42]). *For a scalar $0 \le \mu(t) \le \mu_M$, any matrices $\mathscr{W} \in \mathbb{S}_n^+$, $\mathscr{U} \in \mathbb{R}^{n\times n}$ satisfying $\left[\begin{smallmatrix} \mathscr{W} & * \\ \mathscr{U} & \mathscr{W} \end{smallmatrix}\right] \ge 0$ and integral function $\{\dot{x}(r) | r \in [-\mu_M, 0]\}$, we have*

$$-\mu_M \int_{t-\mu_M}^{t} \dot{x}^T(r)\mathscr{W}\dot{x}(r)\,dr \le \psi^T(t)\Lambda\psi(t)$$

where

$$\psi(t) = \begin{bmatrix} x(t) \\ x(t-\mu(t)) \\ x(t-\mu_M) \end{bmatrix}, \quad \Lambda = \begin{bmatrix} -\mathscr{W} & * & * \\ \mathscr{W}-\mathscr{U} & -2\mathscr{W}+\mathscr{U}+\mathscr{U}^T & * \\ \mathscr{U} & \mathscr{W}-\mathscr{U} & -\mathscr{W} \end{bmatrix}$$

Lemma 2 ([43]). *$\mathcal{D} = \mathscr{M}\Sigma\mathscr{N}^T \in \mathbb{R}^{n\times m}$ is a matrix with full column rank, where $\Sigma \in \mathbb{R}^{m\times n}$ is a rectangular diagonal matrix, and $\mathscr{M}$ and $\mathscr{N}$ are orthogonal matrices. For $\mathscr{P} \in \mathbb{S}_n$, $\mathscr{X} \in \mathbb{R}^{m\times m}$, the sufficient and necessary condition of $\mathscr{P}\mathcal{D} = \mathcal{D}\mathscr{X}$ is $\mathscr{P} = \mathscr{M}diag\{\mathscr{P}_1, \mathscr{P}_2\}\mathscr{M}^T$ with $\mathscr{P}_1 \in \mathbb{R}^{m\times m}$ and $\mathscr{P}_2 \in \mathbb{R}^{(n-m)\times(n-m)}$.*

3. Main results

We will design a controller such that 1) the system (2.12) is mean-square asymptotically stable; 2) For $\mu(t) \neq 0 \in \mathcal{L}_2[0,\infty)$, $\widetilde{z}(t)$ satisfies

$$\|\widetilde{z}(t)\|_{\mathscr{E}_2} < \gamma \|\mu(t)\|_2$$

under zero initial condition.

Theorem 1. *For parameters γ, $\bar{\delta}$, $\bar{\varphi}$, α, β, ρ, σ, η_M, τ_M, ν_M, θ_M, and matrix $\mathcal{K}$, if matrices $\mathscr{P}$, $\mathscr{U}$, $\mathscr{Q}_\ell$, $\mathscr{R}_\ell$ ($\ell = 1, \cdots, 4$) $\in \mathbb{S}_n^+$, diagonal matrices $\mathscr{F}_{aj} \geq 0$, $\mathscr{G}_{aj} \geq 0$, $\mathscr{F}_{bj} \geq 0$, $\mathscr{G}_{bj} \geq 0$, $\mathscr{F}_{dj} \geq 0$, $\mathscr{G}_{dj} \geq 0$ ($j = 1, 2, 3$), $\Omega_1 \geq 0$, $\Omega_2 \geq 0$, $\mathscr{N}_k$ ($k = 1, \cdots, 4$) and $\mathscr{X}$ such that:*

$$\begin{bmatrix} \Xi & * & * & * \\ B^T \mathscr{P} e_1 & -\gamma^2 I & * & * \\ \widehat{\Upsilon}_1 & \nabla & \Lambda & * \\ \widehat{\Upsilon}_2 & 0 & 0 & \Lambda \end{bmatrix} < 0 \tag{3.1}$$

$$\begin{bmatrix} \mathscr{Q}_i & * \\ \mathscr{N}_i & \mathscr{Q}_i \end{bmatrix} \geq 0 \, (i = 1, \cdots, 4), \quad \begin{bmatrix} \mathscr{U} & \mathscr{X} \\ * & \mathscr{U} \end{bmatrix} \geq 0 \tag{3.2}$$

where

$$\begin{aligned}
\Xi =& \varXi_1 + \varXi_2 + \varXi_3 + e_1^T \mathcal{F}^T \mathcal{F} e_1 \\
\varXi_1 =& Sym\Big\{e_1^T \Big(\mathscr{P}\mathcal{A} e_1 + \bar{\eta} \mathscr{P}\mathcal{D}\mathcal{K}\Big(Ce_4 + \rho^{-1} e_{21} - e_{22} + e_{23}\Big) + \bar{\varphi}\mathscr{P}\mathcal{D}\mathcal{K}Ce_{14} \\
& + \bar{\gamma}\mathscr{P}\mathcal{D}\mathcal{K}Ce_{19} + \mathscr{P}Ee_{11})\Big\} \\
\varXi_2 =& e_1^T (\mathscr{R}_1 + \mathscr{R}_2 + \mathscr{R}_3 + \mathscr{R}_4) e_1 - e_3^T \mathscr{R}_1 e_3 - e_5^T \mathscr{R}_2 e_5 - e_7^T \mathscr{R}_3 e_7 - e_9^T \mathscr{R}_4 e_9 \\
& + \left[\begin{smallmatrix} e_1 \\ e_2 \\ e_3 \end{smallmatrix}\right]^T \Psi_1 \left[\begin{smallmatrix} e_1 \\ e_2 \\ e_3 \end{smallmatrix}\right] + \left[\begin{smallmatrix} e_1 \\ e_4 \\ e_5 \end{smallmatrix}\right]^T \Psi_2 \left[\begin{smallmatrix} e_1 \\ e_4 \\ e_5 \end{smallmatrix}\right] + \left[\begin{smallmatrix} e_1 \\ e_6 \\ e_7 \end{smallmatrix}\right]^T \Psi_3 \left[\begin{smallmatrix} e_1 \\ e_6 \\ e_7 \end{smallmatrix}\right] + \left[\begin{smallmatrix} e_1 \\ e_8 \\ e_9 \end{smallmatrix}\right]^T \Psi_4 \left[\begin{smallmatrix} e_1 \\ e_8 \\ e_9 \end{smallmatrix}\right] \\
& + \nu_M^2 e_{16}^T \mathscr{U} e_{16} - \begin{bmatrix} e_{19} \\ e_{20} \end{bmatrix}^T \begin{bmatrix} \mathscr{U} & \mathscr{X} \\ * & \mathscr{U} \end{bmatrix} \begin{bmatrix} e_{19} \\ e_{20} \end{bmatrix} \\
& + (1+\alpha\beta) \Big[(Ce_4 + \varrho e_{21} - e_{22})^T \sigma \Omega_2 (Ce_4 + \varrho e_{21} - e_{22})\Big] \\
& - (1+\alpha\beta) e_{21}^T \Omega_1 e_{21} + \delta e_4^T C^T C e_4 - e_{22}^T e_{22} - e_{23}^T \widehat{D} e_{23} \\
& + \Big(\rho^{-1} e_{21} + Ce_4 - e22\Big)^T W^T \widehat{D} W \Big(\rho^{-1} e_{21} + Ce_4 - e_{22}\Big) \\
\varXi_3 =& \sum_{i=1}^{3} Sym\{\Big(W_p e_i - e_{i+9}\Big)^T \mathscr{F}_{ai} (e_{i+9} - W_m e_i)\} + \sum_{i=1}^{3} \Big\{e_i^T W \mathscr{G}_{ai} W e_i - e_{i+9}^T \mathscr{G}_{ai} e_{i+9}\Big\} \\
& + Sym\{\Big(K_p e_1 - e_{13}\Big)^T \mathscr{F}_{b1} (e_{13} - K_m e_1)\} \\
& + \sum_{i=2}^{3} Sym\{\Big(K_p e_{i+4} - e_{i+12}\Big)^T \mathscr{F}_{bi} (e_{i+12} - K_m e_{i+4})\} \\
& + e_1^T \widehat{K} \mathscr{G}_{b1} \widehat{K} e_1 - e_{13}^T \mathscr{G}_{b1} e_{13} + \sum_{i=2}^{3} \Big\{e_{i+4}^T \widehat{K} \mathscr{G}_{bi} \widehat{K} e_{i+4} - e_{i+12}^T \mathscr{G}_{bi} e_{i+12}\Big\}
\end{aligned}$$

$$
\begin{aligned}
&+ Sym\{\left(L_p e_1 - e_{16}\right)^T \mathscr{F}_{d1} (e_{16} - L_m e_1)\} \\
&+ \sum_{i=2}^{3} Sym\{\left(L_p e_{i+6} - e_{i+15}\right)^T \mathscr{F}_{di} (e_{i+15} - L_m e_{i+6})\} \\
&+ e_1^T L \mathscr{G}_{d1} L e_1 - e_{16}^T \mathscr{G}_{d1} e_{16} + \sum_{i=2}^{3} \left\{ e_{i+6}^T L \mathscr{G}_{di} L e_{i+6} - e_{i+15}^T \mathscr{G}_{di} e_{i+15} \right\}
\end{aligned}
$$

$$
\begin{aligned}
\Psi_i =& \begin{bmatrix} -\mathscr{Q}_i & * & * \\ \mathscr{Q}_i - N_i & -2\mathscr{Q}_i + \mathscr{N}_i + \mathscr{N}_i^T & * \\ \mathscr{N}_i & \mathscr{Q}_i - \mathscr{N}_i & -\mathscr{Q}_i \end{bmatrix} \quad (i = 1, \cdots, 4) \\
\Upsilon_i =& \begin{bmatrix} \eta_M \Gamma_i^T & \tau_M \Gamma_i^T & d_M \Gamma_i^T & v_M \Gamma_i^T \end{bmatrix}^T (i = 1, 2) \\
\Gamma_1 =& \mathscr{P}\mathcal{A} e_1 + \bar{\eta} \mathscr{P}\mathcal{D}\mathcal{K} \left(C e_4 + \rho^{-1} e_{21} - e_{22} + e_{23} \right) + \bar{\varphi} \mathscr{P}\mathcal{D}\mathcal{K} C e_{14} + \bar{\gamma} \mathscr{P}\mathcal{D}\mathcal{K} C e_{19} + \mathscr{P} E e_{11} \\
\Gamma_2 =& \lambda_1 \mathscr{P}\mathcal{D}\mathcal{K} \left(C e_4 + \rho^{-1} e_{21} - e_{22} + e_{23} \right) + \lambda_2 \mathscr{P}\mathcal{D}\mathcal{K} C e_{14} + \lambda_3 \mathscr{P}\mathcal{D}\mathcal{K} C e_{19} \\
\nabla =& \begin{bmatrix} \eta_M (\mathscr{P}\mathcal{D})^T & \tau_M (\mathscr{P}\mathcal{D})^T & v_M (\mathscr{P}\mathcal{D})^T & \theta_M (\mathscr{P}\mathcal{D})^T \end{bmatrix}^T \\
\Lambda =& diag \left\{ -\mathscr{P}\mathscr{Q}_1^{-1}\mathscr{P}, -\mathscr{P}\mathscr{Q}_2^{-1}\mathscr{P}, -\mathscr{P}\mathscr{Q}_3^{-1}\mathscr{P}, -\mathscr{P}\mathscr{Q}_4^{-1}\mathscr{P} \right\} \\
e_i =& \begin{bmatrix} 0_{n\times(i-1)n} & I_{n\times n} & 0_{n\times(23-i)n} \end{bmatrix} (i = 1, \cdots, 23) \\
\mu_1 =& \sqrt{\bar{\delta}(1-\bar{\delta})}, \quad \mu_2 = \sqrt{\bar{\kappa}(1-\bar{\kappa})}, \quad \mu_3 = \sqrt{\bar{\lambda}(1-\bar{\lambda})}
\end{aligned}
$$

Then the system (2.12) is asymptotically stable.

Proof. The LKF candidate is chosen as

$$
\mathcal{V}(t) = \mathcal{V}_1(t) + \mathcal{V}_2(t) + \mathcal{V}_3(t) + \mathcal{V}_4(t)
$$

where

$$
\begin{aligned}
\mathcal{V}_1(t) =& \chi^T(t) \mathscr{P} \chi(t) \\
\mathcal{V}_2(t) =& \int_{t-\eta_M}^{t} \chi^T(s) \mathscr{R}_1 \chi(s) ds + \int_{t-\tau_M}^{t} \chi^T(s) \mathscr{R}_2 \chi(s) ds \\
&+ \int_{t-d_M}^{t} \chi^T(s) \mathscr{R}_3 \chi(s) ds + \int_{t-v_M}^{t} \chi^T(s) \mathscr{R}_4 \chi(s) ds \\
\mathcal{V}_3(t) =& \eta_M \int_{t-\eta_M}^{t} \int_{s}^{t} \dot{\chi}^T(u) \mathscr{Q}_1 \dot{\chi}(u) du ds + \tau_M \int_{t-\tau_M}^{t} \int_{s}^{t} \dot{\chi}^T(u) \mathscr{Q}_2 \dot{\chi}(u) du ds \\
&+ v_M \int_{t-v_M}^{t} \int_{s}^{t} \dot{\chi}^T(u) \mathscr{Q}_3 \dot{\chi}(u) du ds + \theta_M \int_{t-\theta_M}^{t} \int_{s}^{t} \dot{\chi}^T(u) \mathscr{Q}_4 \dot{\chi}(u) du ds \\
\mathcal{V}_4(t) =& \theta_M \int_{t-\theta_M}^{t} \int_{s}^{t} h^T(\chi(v)) \mathscr{U} h(\chi(v)) dv ds
\end{aligned}
$$

For convenience, we define:

$$
\begin{aligned}
\xi_1(t) &= col\{\chi(t) \;\; \chi(t-\eta(t)) \;\; \chi(t-\eta_M) \;\; \chi(t-\tau(t))\} \\
\xi_2(t) &= col\{\chi(t-\tau_M) \;\; \chi(t-v(t)) \;\; \chi(t-v_M) \;\; \chi(t-\theta(t))\}
\end{aligned}
$$

$$\xi_3(t) = col\left\{\chi(t-\theta_M)\ \hat{h}(\chi(t))\ \hat{h}(\chi(t-\eta(t)))\ \hat{h}(\chi(t-\eta_M))\right\}$$
$$\xi_4(t) = col\{g(\chi(t))\ \ g(\chi(t-v(t)))\ \ g(\chi(t-v_M))\}$$
$$\xi_5(t) = col\{h(\chi(t))\ \ h(\chi(t-\theta(t)))\ \ h(\chi(t-\theta_M))\}$$
$$\xi_6(t) = col\{\int_{t-\theta(t)}^{t} h(\chi(u))du \quad \int_{t-\theta_M}^{t-\theta(t)} h(\chi(u))du\}$$
$$\xi_7(t) = col\{e(t)\quad \phi(\bar{y}(t-\tau(t)))\quad h(\bar{y}(t_k h))\}$$
$$\xi(t) = col\{\xi_1(t)\ \ \xi_2(t)\ \ \xi_3(t)\ \ \xi_4(t)\ \ \xi_5(t)\ \ \xi_6(t)\ \ \xi_7(t)\}$$

From $\mathscr{P} > 0$, $\mathscr{U} > 0$, $\mathscr{Q}_\ell > 0$, $\mathscr{R}_\ell > 0$ $(\ell = 1, \cdots, 4)$, we have $\mathcal{V}(t) > 0$. Computing $\frac{d\mathcal{V}(t)}{dt}$, we have

$$\dot{\mathcal{V}}_1(t) = 2\chi^T(t)\mathscr{P}\dot{\chi}(t) \tag{3.3}$$
$$\begin{aligned}\dot{\mathcal{V}}_2(t) =& \chi^T(t)\left(\mathscr{R}_1+\mathscr{R}_2+\mathscr{R}_3+\mathscr{R}_4\right)\chi(t)\\ &-\chi^T(t-\eta_M)\mathscr{R}_1\chi(t-\eta_M)-\chi^T(t-\tau_M)\mathscr{R}_2\chi(t-\tau_M)\\ &-\chi^T(t-v_M)\mathscr{R}_3\chi(t-v_M)-\chi^T(t-\theta_M)\mathscr{R}_4\chi(t-\theta_M)\end{aligned} \tag{3.4}$$
$$\begin{aligned}\dot{\mathcal{V}}_3(t) =& \dot{\chi}^T(t)\left(\eta_M^2\mathscr{Q}_1+\tau_M^2\mathscr{Q}_2+d_M^2\mathscr{Q}_3+v_M^2\mathscr{Q}_4\right)\dot{\chi}(t)\\ &-\eta_M\int_{t-\eta_M}^{t}\dot{\chi}^T(s)\mathscr{Q}_1\dot{\chi}(s)ds-\tau_M\int_{t-\tau_M}^{t}\dot{\chi}^T(s)\mathscr{Q}_2\dot{\chi}(s)ds\\ &-v_M\int_{t-v_M}^{t}\dot{\chi}^T(s)\mathscr{Q}_3\dot{\chi}(s)ds-\theta_M\int_{t-\theta_M}^{t}\dot{\chi}^T(s)\mathscr{Q}_4\dot{\chi}(s)ds\\ \dot{\mathcal{V}}_4(t) =& \theta_M^2 h^T(\chi(t))\mathscr{U}h(\chi(t))-\theta_M\int_{t-\theta_M}^{t}h^T(\chi(v))\mathscr{U}h(\chi(v))dv\end{aligned}$$

From Lemma 1, there exist $\mathscr{N}_i$ $(i = 1, \cdots, 4)$ satisfying (3.2) such that

$$\begin{aligned}-\eta_M\int_{t-\eta_M}^{t}\dot{\chi}^T(s)\mathscr{Q}_1\dot{\chi}(s)ds &\leq \begin{bmatrix}\chi(t)\\ \chi(t-\eta(t))\\ \chi(t-\eta_M)\end{bmatrix}^T\Psi_1\begin{bmatrix}\chi(t)\\ \chi(t-\eta(t))\\ \chi(t-\eta_M)\end{bmatrix}\\ -\tau_M\int_{t-\tau_M}^{t}\dot{\chi}^T(s)\mathscr{Q}_2\dot{\chi}(s)ds &\leq \begin{bmatrix}\chi(t)\\ \chi(t-\tau(t))\\ \chi(t-\tau_M)\end{bmatrix}^T\Psi_2\begin{bmatrix}\chi(t)\\ \chi(t-\tau(t))\\ \chi(t-\tau_M)\end{bmatrix}\\ -v_M\int_{t-v_M}^{t}\dot{\chi}^T(s)\mathscr{Q}_3\dot{\chi}(s)ds &\leq \begin{bmatrix}\chi(t)\\ \chi(t-v(t))\\ \chi(t-v_M)\end{bmatrix}^T\Psi_3\begin{bmatrix}\chi(t)\\ \chi(t-v(t))\\ \chi(t-v_M)\end{bmatrix}\\ -\theta_M\int_{t-\theta_M}^{t}\dot{\chi}^T(s)\mathscr{Q}_4\dot{\chi}(s)ds &\leq \begin{bmatrix}\chi(t)\\ \chi(t-\theta(t))\\ \chi(t-\theta_M)\end{bmatrix}^T\Psi_4\begin{bmatrix}\chi(t)\\ \chi(t-\theta(t))\\ \chi(t-\theta_M)\end{bmatrix}\end{aligned}$$

By the reciprocally convex inequality, we have

$$\begin{aligned}&-\theta_M\int_{t-\theta_M}^{t}h^T(\chi(v))\mathscr{U}h(\chi(v))dv\\ \leq& -\frac{\theta_M}{\theta(t)}\int_{t-\theta(t)}^{t}h^T(\chi(v))dv\mathscr{U}\int_{t-\theta(t)}^{t}h(\chi(v))dv\\ &-\frac{\theta_M}{\theta_M-\theta(t)}\int_{t-\theta_M}^{t-\theta(t)}h^T(\chi(v))dv\mathscr{U}\int_{t-\theta_M}^{t-\theta(t)}h(\chi(v))dv\end{aligned}$$

$$\leq -\begin{bmatrix} \int_{t-\theta(t)}^{t} h(\chi(v))dv \\ \int_{t-\theta_M}^{t-v(t)} h(\chi(v))dv \end{bmatrix}^T \begin{bmatrix} \mathscr{U} & \mathscr{X} \\ * & \mathscr{U} \end{bmatrix} \begin{bmatrix} \int_{t-\theta(t)}^{t} h(\chi(v))dv \\ \int_{t-\theta_M}^{t-\theta(t)} h(\chi(v))dv \end{bmatrix}$$

Based on the inequality (2.3), we have

$$\begin{aligned} 0 \leq& \delta\bar{y}^T(t-\tau(t))\bar{y}(t-\tau(t)) - \phi^T(\bar{y}(t-\tau(t)))\phi(\bar{y}(t-\tau(t))) \\ =& \delta\chi^T(t-\tau(t))C^T C\chi(t-\tau(t)) - \phi^T(\bar{y}(t-\tau(t)))\phi(\bar{y}(t-\tau(t))) \end{aligned} \tag{3.5}$$

From (2.7), we obtain

$$\begin{aligned} 0 \leq& -[h(\bar{y}(t_k h)) + W\bar{y}(t_k h)]^T \widehat{D}[h(\bar{y}(t_k h)) - W\bar{y}(t_k h)] \\ =& \left[\rho^{-1}e(t) + C\chi(t-\tau(t)) - \phi(\bar{y}(t-\tau(t)))\right]^T W^T \widehat{D} W \\ &\times \left[\rho^{-1}e(t) + C\chi(t-\tau(t)) - \phi(\bar{y}(t-\tau(t)))\right] - h^T(y(t_k h))\widehat{D}h(y(t_k h)) \end{aligned} \tag{3.6}$$

From (2.9), we have

$$\begin{aligned} 0 =& -\beta\eta(t-\tau(t)) + [\varrho e(t) + \bar{y}(t-\tau(t))]^T \sigma\Omega_2 [\varrho e(t) + \bar{y}(t-\tau(t))] - e^T(t)\Omega_1 e(t) \\ \leq& (1+\alpha\beta)[(\varrho e(t) + C\chi(t-\tau(t)) - \phi(\bar{y}(t-\tau(t))))^T \sigma\Omega_2 \\ &\times (\varrho e(t) + C\chi(t-\tau(t)) - \phi(\bar{y}(t-\tau(t)))) - e^T(t)\Omega_1 e(t)] \end{aligned} \tag{3.7}$$

Note that

$$\begin{aligned} \widehat{f_{1i}}(\ell) &:= 2\Big(W_p\chi(\ell) - \hat{h}(\chi(\ell))\Big)^T \mathscr{F}_{ai}\Big(\hat{h}(\chi(\ell)) - W_m\chi(\ell)\Big) \geq 0 \\ \widehat{g_{1i}}(\ell) &:= \chi^T(\ell) W\mathscr{G}_{ai} W\chi(\ell) - \hat{h}^T(\chi(\ell))\mathscr{G}_{ai}\hat{h}(\chi(\ell)) \geq 0 \\ \widehat{f_{2i}}(\ell) &:= 2\Big(K_p\chi(\ell) - g(\chi(\ell))\Big)^T \mathscr{F}_{bi}(g(\chi(\ell)) - K_m\chi(\ell)) \geq 0 \\ \widehat{g_{2i}}(\ell) &:= \chi^T(\ell)\widehat{K}\mathscr{G}_{bi}\widehat{K}\chi(\ell) - g^T(\chi(\ell))\mathscr{G}_{bi} g(\chi(\ell)) \geq 0 \\ \widehat{f_{3i}}(\ell) &:= 2\Big(L_p\chi(\ell) - h(\chi(\ell))\Big)^T \mathscr{F}_{di}(h(\chi(\ell)) - L_m\chi(\ell)) \geq 0 \\ \widehat{g_{3i}}(\ell) &:= \chi^T(\ell) L\mathscr{G}_{di} L\chi(\ell) - h^T(\chi(\ell))\mathscr{G}_{di} h(\chi(\ell)) \geq 0 \end{aligned}$$

where $\mathscr{F}_{ai}$, $\mathscr{F}_{bi}$, $\mathscr{F}_{di}$, $\mathscr{G}_{ai}$, $\mathscr{G}_{bi}$, $\mathscr{G}_{di}$ are n-dimensional diagonal matrices, and

$$\begin{aligned} W_m =& diag\{\ell_1^-, \cdots, \ell_n^-\}, & W_p &= diag\{\ell_1^+, \cdots, \ell_n^+\} \\ W =& diag\{\ell_1, \cdots, \ell_n\}, & \ell_k &= \max\left\{\left|\ell_k^+\right|, \left|\ell_k^-\right|\right\}(k = 1, \cdots, n) \\ K_m =& diag\{p_1^-, \cdots, p_n^-\}, & K_p &= diag\{p_1^+, \cdots, p_n^+\} \\ \widehat{K} =& diag\{p_1, \cdots, p_n\}, & p_j &= \max\left\{\left|p_j^+\right|, \left|p_j^-\right|\right\}(j = 1, \cdots, n) \\ L_m =& diag\{q_1^-, \cdots, q_n^-\}, & L_p &= diag\{q_1^+, \cdots, q_n^+\} \\ L =& diag\{q_1, \cdots, q_n\}, & q_m &= \max\left\{\left|q_m^+\right|, \left|q_m^-\right|\right\}(m = 1, \cdots, n) \end{aligned}$$

Thus, we have

$$\widehat{f_{11}}(t) + \widehat{f_{12}}(t-\eta(t)) + \widehat{f_{13}}(t-\eta_M) + \widehat{g_{11}}(t) + \widehat{g_{12}}(t-\eta(t)) + \widehat{g_{13}}(t-\eta_M)$$

$$+\widehat{f_{21}}(t)+\widehat{f_{22}}(t-d(t))+\widehat{f_{23}}(t-d_M)+\widehat{g_{21}}(t)+\widehat{g_{22}}(t-v(t))+\widehat{g_{23}}(t-v_M)$$
$$+\widehat{f_{31}}(t)+\widehat{f_{32}}(t-\theta(t))+\widehat{f_{33}}(t-\theta_M)+\widehat{g_{31}}(t)+\widehat{g_{32}}(t-\theta(t))+\widehat{g_{33}}(t-\theta_M)$$
$$=\xi^T(t)\Xi_3\xi(t)\geq 0 \tag{3.8}$$

Now, using (3.3)–(3.8) yields

$$\dot{\mathcal{V}}(t)\leq 2\chi^T(t)\mathscr{P}\dot{\chi}(t)+\dot{\chi}^T(t)\widetilde{\mathcal{Q}}\dot{\chi}(t)+\xi^T(t)(\Xi_2+\Xi_3)\xi(t)$$

where $\widetilde{\mathcal{Q}}=\eta_M^2\mathscr{Q}_1+\tau_M^2\mathscr{Q}_2+d_M^2\mathscr{Q}_3+\varphi_M^2\mathscr{Q}_4$.

When $\mu(t)\neq 0$, we have

$$\mathcal{E}\left\{2\chi^T(t)\mathscr{P}\dot{\chi}(t)\right\}=2\xi^T(t)e_1^T\mathscr{P}(\Pi_0\xi(t)+E\omega(t))=\xi^T(t)\Xi_1\xi(t)+2\xi^T(t)e_1^T\mathscr{P}E\omega(t) \tag{3.9}$$

and

$$\mathcal{E}\left\{\dot{\chi}^T(t)\widetilde{\mathcal{Q}}\dot{\chi}(t)\right\}=\mathcal{E}\{(\Pi_0\xi(t)+E\omega(t))^T\widetilde{\mathcal{Q}}(\Pi_0\xi(t)+E\omega(t))+\xi^T(t)\Pi_1^T\widetilde{\mathcal{Q}}\Pi_1\xi(t)\} \tag{3.10}$$

where

$$\Pi_0=Ae_1+\bar{\eta}D\mathcal{K}\left(Ce_4+\rho^{-1}e_{21}-e_{22}+e_{23}\right)+\bar{\varphi}D\mathcal{K}Ce_{14}+\bar{\gamma}D\mathcal{K}Ce_{19}+Ee_{11}$$
$$\Pi_1=\lambda_1D\mathcal{K}\left(Ce_4+\rho^{-1}e_{21}-e_{22}+e_{23}\right)+\lambda_2D\mathcal{K}Ce_{14}+\lambda_3BKCe_{19}$$

Furthermore, from (2.1), we have

$$\widetilde{z}^T(t)\widetilde{z}(t)-\gamma^2\mu^T(t)\mu(t)=\xi^T(t)e_1^T\mathcal{F}^T\mathcal{F}e_1\xi(t)-\gamma^2\mu^T(t)\mu(t) \tag{3.11}$$

We can obtain

$$\begin{aligned}&\mathcal{E}\left\{\dot{\mathcal{V}}(t)+\widetilde{z}^T(t)\widetilde{z}(t)-\gamma^2\mu^T(t)\mu(t)\right\}\\ \leq&\mathcal{E}\Big\{\widehat{\xi}^T(t)\begin{bmatrix}\Xi & *\\ \mathcal{B}^T\mathscr{P}e_1 & -\gamma^2I\end{bmatrix}\widehat{\xi}(t)\\ &-\widehat{\xi}^T(t)(\begin{bmatrix}\Upsilon_1 & F\end{bmatrix}^T\Lambda^{-1}\begin{bmatrix}\Upsilon_1 & F\end{bmatrix}+\begin{bmatrix}\Upsilon_2 & 0\end{bmatrix}^T\Lambda^{-1}\begin{bmatrix}\Upsilon_2 & 0\end{bmatrix})\widehat{\xi}(t)\Big\}\end{aligned} \tag{3.12}$$

where $\widehat{\xi}(t)=col\{\xi(t)\quad\mu(t)\}$. Since the system (2.12) is asymptotically stable, under zero initial conditions, we have

$$\mathcal{E}\left\{\int_0^\infty\dot{\mathcal{V}}(s)ds\right\}=\mathcal{E}\{\mathcal{V}(\infty)-\mathcal{V}(0)\}=0$$

Then by (3.12) and (3.1), we can obtain

$$\mathcal{E}\left\{\int_0^\infty\left[\widetilde{z}^T(s)\widetilde{z}(s)-\gamma^2\mu^T(s)\mu(s)\right]ds\right\}=\mathcal{E}\left\{\int_0^\infty\left[\dot{\mathcal{V}}(s)+\widetilde{z}^T(s)\widetilde{z}(s)-\gamma^2\mu^T(s)\mu(s)\right]ds\right\}<0$$

which means $\|\widetilde{z}(t)\|_{\mathcal{E}_2}<\gamma\|\mu(t)\|_2$. □

Remark 5. *Compared with recently reported works [32], [37], and [42], more system information was used. First, the information of sensor saturation and quantization error $h(y(t_k h))$ was used, as seen from (3.5) and (3.6). Second, the cyber-attack functions $f(x(t))$, $g(x(t))$, and their ramifications*

$$f(x(t-d(t))), f(x(t-d_M)), g(x(t-v(t))), g(x(t-v_M))$$
$$\int_{t-v(t)}^{t} g(x(s))ds, \int_{t-v_M}^{t-v(t)} g(x(s))ds$$

are all employed. Finally, based on the ETS (2.4), the dynamic variable $\eta(t)$ is utilized, as seen from (3.7).

Remark 6. *Theorem 1 is derived based on a Jensen integral inequality. If we employ the Bessel-Legendre inequality [44], it is expected to derive some less conservative results.*

Based on Theorem 1, we will give an event-triggered quantized controller design for NNs with sensor saturation and stochastic deception attacks.

Theorem 2. *For parameters γ, $\bar{\delta}$, $\bar{\varphi}$, α, β, σ, ρ, η_M, τ_M, θ_M, v_M, ϵ_i $(i = 1, \cdots, 4)$, if matrices $\mathscr{P}$, $\mathscr{U}$, $\mathscr{Q}_\ell$, $\mathscr{R}_\ell$ $(\ell = 1, \cdots, 4) \in \mathbb{S}_n^+$, diagonal matrices $\mathscr{F}_{aj} \geq 0$, $\mathscr{G}_{aj} \geq 0$, $\mathscr{F}_{bj} \geq 0$, $\mathscr{G}_{bj} \geq 0$, $\mathscr{F}_{dj} \geq 0$, $\mathscr{G}_{dj} \geq 0$ $(j = 1, 2, 3)$, $\Omega_1 \geq 0$, $\Omega_2 \geq 0$, $\mathscr{N}_k$ $(k = 1, \cdots, 4)$, $\mathscr{X}$ and $\mathscr{Y}$ such that (3.2) and the following LMI hold:*

$$\begin{bmatrix} \widetilde{\Pi} & * & * & * \\ \mathcal{B}^T \mathscr{P} e_1 & -\gamma^2 I & * & * \\ \widetilde{\Upsilon}_1 & \nabla & \widetilde{\Lambda} & * \\ \widetilde{\Upsilon}_2 & 0 & 0 & \widetilde{\Lambda} \end{bmatrix} < 0 \tag{3.13}$$

where

$$\begin{aligned}
\widetilde{\Pi} &= \widetilde{\Xi}_1 + \Xi_2 + \Xi_3 + e_1^T \mathcal{F}^T \mathcal{F} e_1 \\
\widetilde{\Xi}_1 &= Sym\Big\{ e_1^T \Big(\mathscr{P}\mathcal{A} e_1 + \bar{\eta}\mathcal{D}\mathscr{Y} \big(Ce_4 + \rho^{-1} e_{21} - e_{22} + e_{23} \big) + \bar{\varphi}\mathcal{D}\mathscr{Y} Ce_{14} + \bar{\gamma}\mathcal{D}\mathscr{Y} Ce_{19} + \mathscr{P} E e_{11} \Big) \Big\} \\
\widetilde{\Upsilon}_i &= \begin{bmatrix} \eta_M \widetilde{\Gamma}_i^T & \tau_M \widetilde{\Gamma}_i^T & v_M \widetilde{\Gamma}_i^T & \theta_M \widetilde{\Gamma}_i^T \end{bmatrix}^T \ (i = 1, 2) \\
\widetilde{\Gamma}_1 &= \mathscr{P}\mathcal{A} e_1 + \bar{\eta}\mathcal{D}\mathscr{Y} \big(Ce_4 + \rho^{-1} e_{21} - e_{22} + e_{23} \big) + \bar{\varphi}\mathcal{D}\mathscr{Y} Ce_{14} + \bar{\gamma}\mathcal{D}\mathscr{Y} Ce_{19} + \mathscr{P} E e_{11} \\
\widetilde{\Gamma}_2 &= \lambda_1 \mathcal{D}\mathscr{Y} \big(Ce_4 + \rho^{-1} e_{21} - e_{22} + e_{23} \big) + \lambda_2 \mathcal{D}\mathscr{Y} Ce_{14} + \lambda_3 \mathcal{D}\mathscr{Y} Ce_{19} \\
\widetilde{\Lambda} &= diag\{ -2\epsilon_1 \mathscr{P} + \epsilon_1^2 \mathscr{Q}_1, -2\epsilon_2 \mathscr{P} + \epsilon_2^2 \mathscr{Q}_2, -2\epsilon_3 \mathscr{P} + \epsilon_3^2 \mathscr{Q}_3, -2\epsilon_4 \mathscr{P} + \epsilon_4^2 \mathscr{Q}_4 \}
\end{aligned}$$

Then the system (2.12) with $\mathcal{K} = (\mathcal{D}^T \mathscr{P} \mathcal{D})^{-1} \mathcal{D}^T \mathcal{D} \mathscr{Y}$ is asymptotically stable.

Proof. Because of $\mathcal{D}$ is a matrix with full column rank, we have $\mathcal{D} = \mathcal{M} \left[\begin{smallmatrix} D_0 \\ 0 \end{smallmatrix} \right] \mathcal{N}^T$, where $D_0 \in \mathbb{R}^{m \times m}$ is a diagonal matrix. Let $\mathscr{P} = \mathcal{M} diag\{\mathscr{P}_1, \mathscr{P}_2\} \mathcal{M}^T$; by Lemma 2, we can find a matrix $\mathcal{S} \in \mathbb{R}^{m \times m}$ such that $\mathscr{P}\mathcal{D}\mathcal{K} = \mathcal{D}\mathcal{S}\mathcal{K}$. Defining $\mathscr{Y} = S\mathcal{K}$, we have $\mathscr{P}\mathcal{D}\mathcal{K} = \mathcal{D}\mathscr{Y}$ and $\mathcal{K} = (\mathcal{D}^T \mathscr{P} \mathcal{D})^{-1} \mathcal{D}^T \mathcal{D} \mathscr{Y}$, which implies

$$\begin{bmatrix} \widetilde{\Pi} & * & * & * \\ \mathcal{B}^T \mathscr{P} e_1 & -\gamma^2 I & * & * \\ \widetilde{\Upsilon}_1 & \nabla & \Lambda & * \\ \widetilde{\Upsilon}_2 & 0 & 0 & \Lambda \end{bmatrix} < 0 \tag{3.14}$$

There exist $\epsilon_\ell > 0$ ($\ell = 1, \cdots, 4$) satisfy

$$-\mathscr{P}\mathscr{Q}_i^{-1}\mathscr{P} \leq -2\epsilon_i\mathscr{P} + \epsilon_i^2\mathscr{Q}_i$$

Replace $-\mathscr{P}\mathscr{Q}_i^{-1}\mathscr{P}$ with $-2\epsilon_i\mathscr{P} + \epsilon_i^2\mathscr{Q}_i$ in (3.14), and we can obtain (3.13). □

Remark 7. *By introducing dynamic ETS, quantization, sensor saturations, and stochastic deception attacks, we derived a new stability criterion. Based on the criterion, a reformative event-triggered quantized controller design was derived.*

4. Numerical examples

Here, we list a three-order system example to verify the validity of derived results.

Example 1. Considering an NN (2.1) with

$$\mathcal{A} = \begin{bmatrix} -2.1 & 0.1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \mathcal{D} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}, E = \begin{bmatrix} 0.1 & 0.2 & 0 \\ -0.1 & 0.2 & 0.1 \\ -0.2 & 0 & -0.3 \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, C = \mathcal{F} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \hat{h}(x) = \begin{bmatrix} tanh(0.03x_1) \\ tanh(0.06x_2) \\ tanh(0.03x_3) \end{bmatrix}$$

and $\mu(t) = e^{-0.08t}$, the cyber-attacks are

$$f(x) = \begin{bmatrix} tanh(0.04x_1) \\ tanh(0.04x_2) \\ tanh(0.04x_3) \end{bmatrix}, g(x) = \begin{bmatrix} tanh(0.03x_1) \\ tanh(0.06x_2) \\ tanh(0.03x_3) \end{bmatrix}$$

Thus, $W_p = diag\{0.03, 0.06, 0.03\}$, $K_p = diag\{0.04, 0.04, 0.04\}$, $L_p = diag\{0.03, 0.06, 0.03\}$, $K_m = diag\{0, 0, 0\}$, $W_m = diag\{0, 0, 0\}$, $L_m = diag\{0, 0, 0\}$.

For $\gamma = 1$, $h = 0.1$, $\bar{\delta} = 0.5$, $\bar{\varphi} = 0.5$, $\eta_M = 0.5$, $\tau_M = 0.5$, $\theta_M = 0.5$, $\nu_M = 0.5$, $\epsilon_\ell = 1 (\ell = 1, \cdots, 4)$, $\alpha = 5$, $\beta = 0.1$, $\sigma = 1$ and $\rho = 0.3$, by Theorem 2, we have

$$\mathscr{P} = \begin{bmatrix} 3.7741 & -0.1598 & -0.0006 \\ -0.1598 & 2.9349 & -0.0041 \\ -0.0006 & -0.0041 & 3.0832 \end{bmatrix}, \quad \mathscr{Y} = \begin{bmatrix} -0.1512 \\ 0.1667 \\ -0.0239 \end{bmatrix}^T$$

$$\Omega_1 = \begin{bmatrix} 5.3130 & -0.0024 & 0.0008 \\ -0.0024 & 5.8848 & -0.0008 \\ 0.0008 & -0.0008 & 5.3116 \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} 0.2312 & -0.0002 & 0.0001 \\ -0.0002 & 0.2213 & -0.0001 \\ 0.0001 & -0.0001 & 0.2312 \end{bmatrix}$$

Then we can obtain

$$\mathcal{K} = \begin{bmatrix} -0.0506 & 0.0558 & -0.0080 \end{bmatrix}$$

For $x(0) = \begin{bmatrix} -0.5 & -0.3 & 0.4 \end{bmatrix}^T$, Figure 2 displays the state trajectory of system (2.12). It is not difficult to see that the state trajectory of system (2.12) is convergent to zero, which shows the system (2.12) is stochastically asymptotically stable.

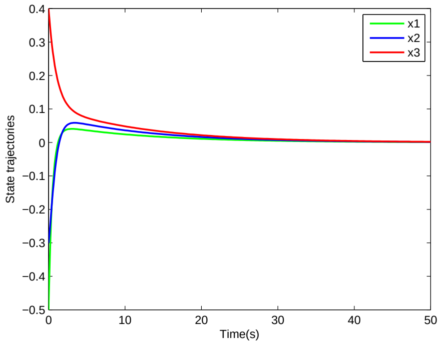

Figure 2. The state trajectory of NN (2.1).

Choose $\eta(0) = 0.0001$. Figure 3 shows event-based signal transmission instants and release intervals. It is not difficult to see that only 38 times are triggered during the period [0, 50s], and we can work out the average trigger rate is 7.6%. Thus, our ETS can save more system resources. Moreover, by adjusting the parameters $\eta(0)$ and ρ, we can obtain the desired result.

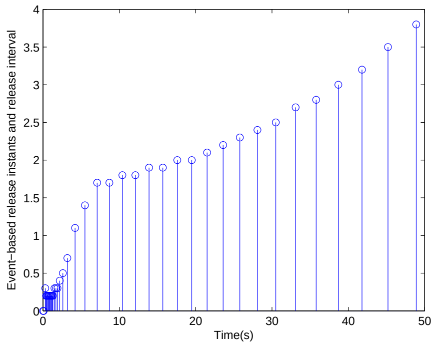

Figure 3. Signal transmission instants and intervals.

Set $\gamma = 1$, $\bar{\delta} = 0.5$, $\bar{\varphi} = 0.5$, $\tau_M = 0.3$, $\nu_M = 0.4$, $\theta_M = 0.4$, $\alpha = 5$, $\beta = 0.1$, $\delta = 0.5$, $\rho = 0.5$, $\sigma = 0.01$, and $\eta_0 = 0.0001$. For different values of h, the number of triggered data during [0,50s] are listed in Table 1.

Table 1. The number of triggered data.

h	0.1	0.2	0.5
ETS in [36]	75	61	52
ETS in [37]	38	31	24
This paper	21	20	15

It is clear that the number of triggered data for various h are far less than those in [36, 37], which means our ETS is more advanced.

5. Conclusions

Dynamic event-triggered quantized control for neural networks with sensor saturations and stochastic deception attacks has been studied. With the dynamic ETS and quantization being introduced, an improved networked control model was formulated with stochastic deception attacks and sensor saturations. By the LKF approach, the event-triggered quantized controller design under sensor saturations and deception attacks was solved. Finally, a numerical example was listed to display the availability of derived methods. In the future, we will study distributed event-triggered H_∞ filtering for NCSs.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

The paper was supported by the National Natural Science Foundation of China (62073188) and Natural Science Foundation of Rizhao City, China (RZ2022ZR65).

Conflict of interest

The authors declare there are no conflicts of interest.

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