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Research article

Non-fragile sampled-data control for synchronizing Markov jump Lur'e systems with time-variant delay

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Abstract: The issue of non-fragile sampled-data control for synchronizing Markov jump Lur'e systems (MJLSs) with time-variant delay was investigated. The time-variant delay allowed for uncertainty that was constrained to an interval with defined upper and lower boundaries. The components of the nonlinear function within the MJLSs were considered to satisfy either Lipschitz continuity or non-decreasing monotonicity. Numerically tractable conditions that ensured stochastic synchronization with a predefined $\mathcal{L}_2 - \mathcal{L}_\infty$ disturbance attenuation level for the drive-response MJLSs were established, utilizing time-dependent two-sided loop Lyapunov-Krasovskii functionals, together with integral and matrix inequalities. An exact mathematical expression of the desired controller gains can be obtained based on these conditions. Finally, an example with numerical simulation was employed to demonstrate the effectiveness of the proposed control strategies.

Keywords: synchronization; Lur'e system; time delay; Markov jump process; sampled-data control; non-fragile control

1. Introduction

Synchronization of chaotic systems stands as a prominent research area within nonlinear system science. Its potential utility spans diverse domains, including the transmission of digital signals, secure communication, and information processing [1–3]. Numerous chaotic systems, such as Chua's circuit and Hopfield network, can be effectively represented as Lur'e systems [4]. Consequently, the synchronization of Lur'e systems (LSs) has garnered substantial attention. Within the master-slave framework established by Pecora and Carroll [5], a wealth of research results addressing various synchronization issues, including quasi-synchronization [6], cluster synchronization [7], prespecified-time synchronization [8], and bipartite synchronization [9], have been reported.

In actual engineering systems, abrupt changes can occur in the structure or parameters due to component failures, sudden environmental disturbances, and changes in connections between subsystems. Markov chain often serves as a suitable candidate to model these abrupt change behaviors [10–13]. Synchronization of Markov jump Lur'e systems (MJLSs) was initially investigated by [14], where a delay feedback control scheme was introduced. Following that, reference [15] considered singular perturbations and developed a mode-dependent control strategy to ensure stochastic synchronization. Reference [16] took into account both time delays and external disturbances, presenting a design for state-feedback controllers to achieve finite-time $\mathcal{H}_\infty$ synchronization.

Recent years have witnessed the introduction of various networked control strategies, including quantized control [17], event-triggered control [18], and sampled-data control (SDC) [19], in the study of synchronization for MJLSs [20–22]. Under SDC strategies, the system's state or output is sampled at specific time instants, and the control action is updated and applied at those discrete time points, thereby effectively decreasing the amount of transmitted information and conserving communication bandwidth [23, 24]. In the study of [21], MJLSs with a single time delay were examined. Building on a novel Lyapunov-Krasovskii functional (LKF), two SDC control approaches, formulated in terms of linear matrix inequalities (LMIs), were established to guarantee stochastic synchronization between the master and slave MJLSs. This research was extended to encompass multiple time delays in [22], where a design approach for mean-square exponential synchronization was developed.

Despite theoretical progress in the research of SDC for delayed Markov jump systems, there are still concerns that need to be addressed further. In particular, the time delays are assumed to be time-invariant in [21, 22], whereas, in practical application circumstances, they often display time-variant behavior [25, 26]. Incorporating time-variant delays is more challenging but may result in more generic solutions. Furthermore, the control designs in [21, 22] do not account for the influence of gain fluctuations. In engineering implementations, controllers/filters frequently exhibit a degree of parameter inaccuracies due to digital rounding errors, memory constraints, and analog-digital conversion imprecision [27]. As indicated by [28], even a minor gain fluctuation can impair the effectiveness of control/filtering.

Motivated by the preceding discussion, this paper focuses on the issue of non-fragile sampled-data control for synchronizing delayed MJLSs. In contrast to the studies conducted in [21, 22], the present research incorporates considerations for both time-variant delay and gain fluctuations. The main contributions of this paper can be summarized as follows: 1) The components of the nonlinear function within the MJLSs are assumed to be either Lipschitz continuous or monotonic nondecreasing. This enables the construction of two distinct two-sided loop LKFs; 2) Two sufficient conditions concerning the non-fragile sampled-data controller design are derived to ensure that the drive-response MJLSs are stochastically synchronized with a prescribed $\mathcal{L}_2 - \mathcal{L}_\infty$ disturbance attenuation level (DAL). An exact mathematical expression of the desired controller gains can be obtained based on these conditions.

Notations: Throughout, the notation $\mathcal{E}\{\cdot\}$ represents the mathematical expectation. $col\{\cdots\}$ and $diag\{\cdots\}$ stand for a column vector and a block-diagonal matrix, respectively. Represent by $\mathbb{R}^n$ the n-dimensional Euclidean space, by $\mathbb{R}^{l\times n}$ the set of all $l \times n$ real matrices, by $\mathbb{S}^n$ the $n \times n$ symmetric matrices, and by $\mathbb{S}^n_+$ the $n \times n$ symmetric and positive-define matrices. The superscripts "-1" and "T" stand for the inverse and transpose of a matrix, respectively. $\mathscr{S}(P)$ is the sum of matrix P and its transpose (i.e., $\mathscr{S}(P) = P + P^T$). $\lambda_{\max}(P)$ is used to denote the largest eigenvalue of P. I_n and $0_{l\times n}$ represent the $n \times n$ identity matrix and $l \times n$ zero matrix, respectively. The symbol "$*$" denotes a symmetric block.

2. Preliminaries

Consider the following MJLSs with time-variant delay:

$$\begin{cases} \dot{x}(t) = A(\delta(t))x(t) + W(\delta(t))x(t-\sigma(t)) \\ \qquad + H(\delta(t))f(Dx(t)), \\ \hat{z}(t) = C(\delta(t))x(t), \end{cases} \tag{2.1}$$

$$\begin{cases} \dot{y}(t) = A(\delta(t))y(t) + W(\delta(t))y(t-\sigma(t)) \\ \qquad + H(\delta(t))f(Dy(t)) + u(t) \\ \qquad + G(\delta(t))\omega(t), \\ \check{z}(t) = C(\delta(t))y(t), \end{cases} \tag{2.2}$$

where $x(t) = col\{x_1(t), x_2(t), \ldots, x_n(t)\}$ and $y(t) = col\{y_1(t), y_2(t), \ldots, y_n(t)\}$ denote the state vectors of the drive and response system, respectively; $\hat{z}(t) = col\{\hat{z}_1(t), \hat{z}_2(t), \ldots, \hat{z}_m(t)\}$ and $\check{z}(t) = col\{\check{z}_1(t), \check{z}_2(t), \ldots, \check{z}_m(t)\}$ are the measurement output of the drive and response system, respectively; $u(t) \in \mathbb{R}^n$ is the control input; $f(Dx(t)) = \left[f_1(d_1^T x_1(t)), \ldots, f_n(d_n^T x_n(t))\right]$ and $f(Dy(t)) = \left[f_1(d_1^T y_1(t)), \ldots, f_n(d_n^T y_n(t))\right]$ are nonlinear function vectors, where d_i^T is the ith row of matrix D; $\omega(t)$ denotes the exterior disturbance; and $\sigma(t)$ is the time delay, which, as in [29–31], is considered to be continuous and satisfies the following constraints:

$$0 \le \sigma_1 \le \sigma(t) \le \sigma_2, \ \sigma_{12} = \sigma_2 - \sigma_1,$$

where σ_1 and σ_2 are the lower and upper bounds of the variable time delay, respectively. System matrices $A(\delta(t))$, $W(\delta(t))$, $H(\delta(t))$, $G(\delta(t))$, and $C(\delta(t))$ are known matrices with appropriate dimensions. $x(s) = \hat{\rho}(s)$ and $y(s) = \check{\rho}(s)$, $s \in [-\sigma_2, 0]$ denote the initial condition with σ_2 being the upper bounds of delay function $\sigma(t)$. The time-homogenous Markov jump process with right continuous trajectories is represented by $\{\delta(t)\}$ that takes values in $\Gamma = \{1, 2, \ldots, \tilde{\Gamma}\}$. The transition probability matrix (TPM) is characterized by $\tilde{\pi} = \pi_{mn}$, which is defined as follows:

$$\Pr\{\delta(\mathrm{t}+\varphi) = \mathrm{n} | \delta(\mathrm{t}) = \mathrm{m}\} = \begin{cases} \pi_{mn}\varphi + o(\varphi), \ m \neq n, \\ 1 + \pi_{mm}\varphi + o(\varphi), \ m = n, \end{cases}$$

where $\varphi > 0$, $\lim_{\varphi\to 0}(\frac{o(\varphi)}{\varphi}) = 0$, and π_{mn}, for all $m, n \in \Gamma$, denotes the switching rate from m to n with $\pi_{mn} \ge 0$, $m \neq n$, and $\pi_{mm} = -\sum_{n=1, n\neq m}^{\tilde{\Gamma}} \pi_{mn} < 0$ [32,33]. The sampled-data controller considered in this study differs from the state-feedback controllers discussed in [34–36]. It is based on output feedback and takes into account gain fluctuations. The controller structure is given by:

$$u(t) = (K(\delta(t)) + \Delta K(\delta(t)))(\hat{z}(t_k) - \check{z}(t_k)), \tag{2.3}$$

where $K(\delta(t))$ denotes the controller gain matrix to be determined and t_k ($k = 0, 1, 2, \ldots$) is the updated instant time of the zero-order-hold [37]. Throughout this work, the sampling periods are considered to be bounded and time-variant, satisfying

$$t_{k+1} - t_k = h_k \in [h_1, h_2],$$

where h_k denotes the variable sampling interval, and scalars h_1 and h_2 denote the lower and upper bounds of h_k, respectively. $\Delta K(\delta(t))$ stands for the gain uncertainty with the following form:

$$\Delta K(\delta(t)) = E(\delta(t))\Theta N(\delta(t)), \tag{2.4}$$

where $E(\delta(t))$ and $N(\delta(t))$ are certain real constant matrices, and Θ stands for an uncertain matrix meeting $\Theta^T\Theta \leq I$ [38–40]. Thus, by (2.4), (2.3) can be restructured as

$$u(t) = (K(\delta(t)) + E(\delta(t))\Theta N(\delta(t)))(\hat{z}(t_k) - \check{z}(t_k)). \tag{2.5}$$

For $\delta(t) = m$, $m \in \Gamma$, from (2.1), (2.2), and (2.5), we can obtain the following synchronization error system:

$$\dot{\eta}(t) = A_m\eta(t) + W_m\eta(t - \sigma(t)) + H_m f(D\eta(t)) - (K_m + E_m\Theta N_m)C_m\eta(t_k) - G_m\omega(t),\ t \in [t_k,\ t_{k+1}), \tag{2.6}$$

where $f(D\eta(t)) = f(D\eta(t) + Dy(t)) - f(Dy(t))$. The nonlinear functions are supposed to adhere to one of the two distinct assumptions outlined below:

Assumption 1. *There exists a matrix $L = diag\{L_i, \ldots, L_n\} > 0$ that ensures that*

$$|f_i(\eta_1) - f_i(\eta_2)| \leq L_i\,|\eta_1 - \eta_2)|\,,\ i \in \{1, \ldots, n\},$$

for any two different scalars η_1, $\eta_2 \in \mathbb{R}$.

Assumption 2. *There exists a matrix $L = diag\{L_i, \ldots, L_n\} > 0$ that ensures that*

$$0 \leq \frac{f_i(\eta_1) - f_i(\eta_2)}{\eta_1 - \eta_2} \leq L_i,\ i \in \{1, \ldots, n\},$$

for any two different scalars η_1, $\eta_2 \in \mathbb{R}$.

Remark 1. *In most extant works discussing systems with nonlinear functions, Assumptions 1 or 2 are two extensively employed hypotheses (see, e.g., [41–43]). Assumption 1 imposes a condition on the nonlinear function vector, requiring only Lipschitz continuity of its components. Assumption 2 strengthens this requirement by demanding not only Lipschitz continuity but also nondecreasing monotonicity. Examples of functions that satisfy Assumption 1 but violate Assumption 2 include the cosine function $cos(t)$ and the exponential function $exp(-t^2)$.*

Definition 1. *[44] Error system (2.6) is said to be stochastically stable if, when $\omega(t) \equiv 0$,*

$$\int_0^\infty \mathcal{E}\left\{\|\eta(s)\|^2\,|\eta_0, \delta_0\right\} ds < \infty,$$

holds true.

Definition 2. *Given a scalar $\gamma > 0$, drive-response MJLSs (2.1) and (2.2) are said to be stochastically synchronized with a prescribed $\mathcal{L}_2 - \mathcal{L}_\infty$ DAL γ if error system (2.6) is stochastically stable and*

$$\sup_{t\geq 0} \mathcal{E}\{z^T(t)z(t)\} \leq \gamma^2 \int_0^\infty \omega^T(\beta)\omega(\beta)d\beta, \tag{2.7}$$

holds for all nonzero $\omega(t) \in \mathcal{L}_2[0, \infty]$ and the zero initial condition.

The aim of this paper is to figure out a non-fragile sampled-data feedback controller in the compact form of (2.5) to ensure that drive-response MJLSs (2.1) and (2.2) are stochastically synchronized with a prescribed $\mathcal{L}_2 - \mathcal{L}_\infty$ DAL γ.

The $\mathcal{L}_2 - \mathcal{L}_\infty$ DAL, also called the energy-to-peak DAL, was proposed by Wilson in [45]. The level is a metric that quantifies the controller's performance to limit the impact of energy-bounded disturbance on the peak of the system's output.

In order to address such an issue, the following four lemmas should be employed:

Lemma 1. *[46, 47] For a given matrix $R \in \mathbb{S}_+^n$ and any differentiable function μ in $[\lambda_1, \lambda_2] \to \mathbb{R}^n$, we have*

$$\int_{\lambda_1}^{\lambda_2} \dot{\mu}^T(s) R \dot{\mu}(s) ds \geq \frac{1}{\lambda_2 - \lambda_1} \bar{\Theta}^T diag\{R,\ 3R,\ 5R\} \bar{\Theta},$$

where

$$\bar{\Theta} = \begin{bmatrix} \mu(\lambda_2) - \mu(\lambda_1) \\ \mu(\lambda_2) + \mu(\lambda_1) - \frac{2}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} \mu(s) ds \\ \mu(\lambda_2) - \mu(\lambda_1) - \frac{6}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} \upsilon_{\lambda_1, \lambda_2}(s) \mu(s) ds \end{bmatrix},$$

$$\upsilon_{\lambda_1, \lambda_2}(s) = 2\left(\frac{s - \lambda_1}{\lambda_2 - \lambda_1}\right) - 1.$$

Lemma 2. *[48] For a scalar $\beta \in (0,\ 1)$, matrices θ_1 and $\theta_2 \in \mathbb{S}_+^n$, and θ_3 and $\theta_4 \in \mathbb{R}^{n \times n}$, the following inequality holds true:*

$$\begin{bmatrix} \frac{\theta_1}{\beta} & 0 \\ 0 & \frac{\theta_2}{1-\beta} \end{bmatrix} \geq \begin{bmatrix} \theta_1 + (1-\beta)\tilde{\theta}_1 & (1-\beta)\theta_3 + \beta\theta_4 \\ * & \theta_2 + \beta\tilde{\theta}_2 \end{bmatrix},$$

where $\tilde{\theta}_1 = (\theta_1 - \theta_4 \theta_2^{-1} \theta_4^T)$ and $\tilde{\theta}_2 = (\theta_2 - \theta_3^T \theta_1^{-1} \theta_3)$.

Lemma 3. *[49] For real matrices R and S of suitable dimensions and a scalar $\alpha > 0$, one has*

$$RS^T + SR^T \leq \alpha^{-1} RR^T + \alpha SS^T.$$

Lemma 4. *[50] The inequality*

$$\begin{bmatrix} R & U \\ U^T & S \end{bmatrix} > 0,$$

is equivalent to

$$S > 0 \text{ and } R - US^{-1}U^T > 0.$$

3. Main results

This section focuses on the non-fragile sampled-data synchronization problem for MJLSs with time-variant delay. Sufficient conditions are provided to ensure stochastic synchronization with a predefined $\mathcal{L}_2 - \mathcal{L}_\infty$ DAL for the drive-response MJLSs and the corresponding desired controller gains will be given.

To go further, we need to introduce some notations as follows:

$$
\begin{aligned}
p_\iota &= \begin{bmatrix} 0_{n\times(\iota-1)n} & I_n & 0_{n\times(19-\iota)n} \end{bmatrix}, \iota = 1, \cdots, 19,\\
\bar{\Omega}_0 &= col\{p_{17}, p_1 - p_2, p_1 + p_2 - 2p_5, p_2 - p_4, \hat{\Omega}_0\},\\
\hat{\Omega}_0 &= \sigma_{12}(p_2 + p_4) - 2(p_{11} + p_{13}),\\
\bar{\Omega}_1(\chi) &= col\{p_1, \sigma_1 p_5, \sigma_1 p_6, p_{11} + p_{13}, \hat{\Omega}_1(\chi)\},\\
\hat{\Omega}_1(\chi) &= (\sigma_2 - \chi)(p_{11} + p_{14}) + (\chi - \sigma_1)(p_{12} - p_{13}),\\
\Omega_2 &= col\{p_1 - p_2, p_1 + p_2 - 2p_5, p_1 - p_2 - 6p_6\},\\
\Omega_3 &= col\{p_2 - p_3, p_2 + p_3 - 2p_7, p_2 - p_3 - 6p_8\},\\
\Omega_4 &= col\{p_3 - p_4, p_3 + p_4 - 2p_9, p_3 - p_4 - 6p_{10}\},\\
\Omega_{34} &= col\{\Omega_3, \Omega_4\}, \Omega_5 = col\{p_1 - p_{18}, p_{19} - p_1\},\\
\Omega_6 &= col\{p_{18}, p_{19}\},\\
\upsilon_0(t) &= col\{\eta(t), \eta(t-\sigma_1), \eta(t-\sigma(t)), \eta(t-\sigma_2)\},\\
\upsilon_1(t) &= \frac{1}{\sigma_1}\begin{bmatrix} \int_{-\sigma_1}^0 \eta_t^T(s)ds & \int_{-\sigma_1}^0 e_1(s)\eta_t^T(s)ds \end{bmatrix}^T,\\
\upsilon_2(t) &= \frac{1}{\sigma(t)-\sigma_1}\begin{bmatrix} \int_{-\sigma(t)}^{-\sigma_1} \eta_t^T(s)ds & \int_{-\sigma(t)}^{-\sigma_1} e_2(s)\eta_t^T(s)ds \end{bmatrix}^T,\\
\upsilon_3(t) &= \frac{1}{\sigma_2-\sigma(t)}\begin{bmatrix} \int_{-\sigma_2}^{-\sigma(t)} \eta_t^T(s)ds & \int_{-\sigma_2}^{-\sigma(t)} e_3(s)\eta_t^T(s)ds \end{bmatrix}^T,\\
\upsilon_4(t) &= (\sigma(t)-\sigma_1)\upsilon_2(t), \upsilon_5(t) = (\sigma_2 - \sigma(t))\upsilon_3(t),\\
\xi(t) &= col\{\xi_0(t), \xi_1(t), \xi_2(t)\},\\
\xi_0(t) &= col\{\upsilon_0(t), \ldots, \upsilon_5(t)\},\\
\xi_1(t) &= col\{f(D\eta(t)), \omega(t), \dot{\eta}(t)\},\\
\xi_2(t) &= col\{\eta(t_k), \eta(t_{k+1})\},
\end{aligned}
$$

where $\eta_t^T(s) = \eta^T(t+s)$ and $e_j(s)\,(j = 1, \cdots, 4)$ are given by

$$
\begin{aligned}
e_1(s) &= 2\frac{s+\sigma_1}{\sigma_1} - 1, e_2(s) = 2\frac{s+\sigma(t)}{\sigma(t)-\sigma_1} - 1,\\
e_3(s) &= 2\frac{s+\sigma_2}{\sigma_2-\sigma(t)} - 1, e_4(s) = 2\frac{s+\sigma_2}{\sigma_{12}} - 1.
\end{aligned}
$$

For the nonlinear function of error system (2.6), we consider the following two different assumptions. Under Assumption 1, the following inequality holds true

$$f_i^2(d_i^T\eta_i(\cdot)) \le (L_i d_i^T\eta_i(\cdot))^2, i = 1, 2, \ldots n. \tag{3.1}$$

In this case, we can propose the following condition:

Theorem 1. *Under Assumption 1, for given scalars $\gamma > 0$, $h_2 \ge h_1 > 0$, $u > 0$, suppose that there exist scalars $\epsilon_m > 0$, matrices $\tilde{P}$ in $\mathbb{S}_+^{5n}$, P_m, S_1, S_2, R_1, R_2, S_5, S_6 in $\mathbb{S}_+^n$, S_3 in $\mathbb{S}^{2n}$, S_4 in $\mathbb{S}^n$, diagonal matrix*

T_5 in $\mathbb{S}_+^n$, arbitrary matrices M_1, M_2 in $\mathbb{R}^{3n\times 3n}$, M_3, M_4, T_1, S_7, S_8, X_m in $\mathbb{R}^{n\times n}$, and T_3, T_4 in $\mathbb{R}^{19n\times 2n}$, such that

$$\begin{bmatrix} \tilde{\Pi}_1^{h_k}(\sigma_1) + \epsilon_m \bar{N}_m \bar{N}_m^T & \Pi_1^{12} & \bar{E}_m \\ * & \Pi_1^{22} & 0 \\ * & * & -\epsilon_m I \end{bmatrix} < 0, \tag{3.2}$$

$$\begin{bmatrix} \tilde{\Pi}_1^{h_k}(\sigma_2) + \epsilon_m \bar{N}_m \bar{N}_m^T & \Pi_1^{13} & \bar{E}_m \\ * & \Pi_1^{22} & 0 \\ * & * & -\epsilon_m I \end{bmatrix} < 0, \tag{3.3}$$

$$\begin{bmatrix} \tilde{\Pi}_2^{h_k}(\sigma_1) + \epsilon_m \bar{N}_m \bar{N}_m^T & \Pi_2^{12} & \bar{E}_m \\ * & \Pi_2^{22} & 0 \\ * & 0 & -\epsilon_m I \end{bmatrix} < 0, \tag{3.4}$$

$$\begin{bmatrix} \tilde{\Pi}_2^{h_k}(\sigma_2) + \epsilon_m \bar{N}_m \bar{N}_m^T & \Pi_2^{13} & \bar{E}_m \\ * & \Pi_2^{22} & 0 \\ * & * & -\epsilon_m I \end{bmatrix} < 0, \tag{3.5}$$

$$\begin{bmatrix} P_m & C_m^T \\ * & \gamma^2 I \end{bmatrix} > 0, \tag{3.6}$$

hold for $m \in \Gamma$, $h_k \in \{h_1, h_2\}$, and $\chi \in \{\sigma_1, \sigma_2\}$, where

$$\begin{aligned}
\tilde{\Pi}_1^{h_k}(\chi) &= \Pi_0(\chi) + \bar{T} + \check{T}_1 + h_k \bar{\Lambda}_2 - \Omega_5^T \bar{\Lambda}_s^1 \Omega_5, \\
\tilde{\Pi}_2^{h_k}(\chi) &= \Pi_0(\chi) + \bar{T} + \check{T}_1 + h_k \bar{\Lambda}_3 - \Omega_5^T \bar{\Lambda}_s^2 \Omega_5, \\
\Pi_1^{12} &= \begin{bmatrix} (p_{19} - p_1)^T M_3^T & \Omega_3^T M_2 \end{bmatrix}, \\
\Pi_1^{13} &= \begin{bmatrix} (p_{19} - p_1)^T M_3^T & \Omega_4^T M_1^T \end{bmatrix}, \\
\Pi_2^{12} &= \begin{bmatrix} (p_1 - p_{18})^T M_4 & \Omega_3^T M_2 \end{bmatrix}, \\
\Pi_2^{13} &= \begin{bmatrix} (p_1 - p_{18})^T M_4 & \Omega_4^T M_1^T \end{bmatrix}, \\
\Pi_1^{22} &= diag\{-S_5, -\check{R}_2\}, \\
\Pi_2^{22} &= diag\{-S_6, -\check{R}_2\}, \\
\Pi_0(\chi) &= \mathscr{S}(p_1^T P_m p_{17} + \bar{\Omega}_1^T(\chi) \tilde{P} \bar{\Omega}_0) + \sum_{n=1}^{\tilde{\Gamma}} \pi_{mn} p_1^T P_n p_1 + \bar{S} + p_{17}^T (\sigma_1^2 R_1 + \sigma_{12}^2 R_2) p_{17} \\
&\quad - \Omega_2^T \check{R}_1 \Omega_2 - \Omega_{34}^T \check{R}_{TM} \Omega_{34} + \bar{\Lambda}_1 + T(\chi) - p_{16}^T p_{16}, \\
\bar{S} &= diag\{S_1, -S_1 + S_2, 0_{n\times n}, -S_2, 0_{15n\times 15n}\}, \\
\check{R}_i &= diag\{R_i, 3R_i, 5R_i\}, \ i = \{1, 2\}, \\
\check{R}_M &= \begin{bmatrix} \check{R}_2 + \frac{\sigma_2 - \chi}{\sigma_{12}} \check{R}_2 & \frac{\sigma_2 - \chi}{\sigma_{12}} M_1 + \frac{\chi - \sigma_1}{\sigma_{12}} M_2 \\ * & \check{R}_2 + \frac{\chi - \sigma_1}{\sigma_{12}} \check{R}_2 \end{bmatrix}, \\
\bar{\Lambda}_s^1 &= \begin{bmatrix} S_5 & M_4 \\ * & 2S_6 \end{bmatrix}, \ \bar{\Lambda}_s^2 = \begin{bmatrix} 2S_5 & M_3 \\ * & S_6 \end{bmatrix}, \\
\bar{\Lambda}_1 &= -(p_1^T - p_{18}^T) S_4 (p_1 - p_{18}) + \mathscr{S}[(p_{19}^T - p_1^T) S_7 p_{19} + (p_{19}^T - p_1^T) S_8 p_{18}], \\
\check{T}_1 &= \mathscr{S}[(p_1^T T_1 + p_{17}^T u T_1)(A_m p_1 + W_m p_3 + H_m p_{15} - G_m p_{16} - p_{17})]
\end{aligned}$$

$$
\begin{aligned}
&- \mathscr{S}[(p_1^T + up_{17}^T)X_mC_mp_{18}],\\
T(\chi) &= \mathscr{S}(T_3\alpha_1(\chi) + T_4\alpha_2(\chi)),\\
\alpha_1(\chi) &= (\chi - \sigma_1)\begin{bmatrix} p_7 \\ p_8 \end{bmatrix} - \begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix},\\
\alpha_2(\chi) &= (\sigma_2 - \chi)\begin{bmatrix} p_9 \\ p_{10} \end{bmatrix} - \begin{bmatrix} p_{13} \\ p_{14} \end{bmatrix},\\
\bar{T} &= (p_1^T D^T LT_5LDp_1 - p_{15}^T T_5 p_{15}),\\
\bar{\Lambda}_2 &= -\Omega_6^T S_3\Omega_6 + h_k p_{17}^T S_6 p_{17} - \mathscr{S}(p_{17}^T S_7 p_{19} + p_{17}^T S_8 p_{18}),\\
\bar{\Lambda}_3 &= \mathscr{S}((p_1^T - p_{18}^T)S_4 p_{17}) + h_k p_{17}^T S_5 p_{17} + \Omega_6^T S_3\Omega_6,\\
\bar{E}_m &= -(p_1^T T_1 + p_{17}^T uT_1)E_m,\\
\bar{N}_m &= N_mC_mp_{18}.
\end{aligned}
$$

Then, drive-response MJLSs (2.1) and (2.2) are stochastically synchronized with a predefined $\mathcal{L}_2-\mathcal{L}_\infty$ DAL if the SDC gains in (2.3) are given by

$$K_m = T_1^{-1}X_m,\ m \in \Gamma. \tag{3.7}$$

Proof. For $\delta(t) = m \in \Gamma$, choose the following LKF:

$$\mathscr{V}(\eta(t),\, \delta(t),\, t) = \mathscr{V}_1(\eta(t),\, \delta(t),\, t) + \mathscr{V}_2(t) + \mathscr{V}_3(t) + \mathscr{V}_4(t) + \mathscr{V}_5(t),\ [t_k,\, t_{k+1}), \tag{3.8}$$

with

$$
\begin{aligned}
\mathscr{V}_1(\eta(t),\, \delta(t),\, t) &= \eta^T(t)P(\delta(t))\eta(t),\\
\mathscr{V}_2(t) &= \zeta_1^T(t)\tilde{P}\zeta_1(t),\\
\mathscr{V}_3(t) &= \int_{t-\sigma_1}^{t} \eta^T(s)S_1\eta(s)ds + \int_{t-\sigma_2}^{t-\sigma_1} \eta^T(s)S_2\eta(s)ds,\\
\mathscr{V}_4(t) &= \sigma_1\int_{-\sigma_1}^{0}\int_{t+\theta}^{t} \dot{\eta}^T(s)R_1\dot{\eta}(s)dsd\theta + \sigma_{12}\int_{-\sigma_2}^{-\sigma_1}\int_{t+\theta}^{t} \dot{\eta}^T(s)R_2\dot{\eta}(s)dsd\theta,\\
\mathscr{V}_5(t) &= (t_{k+1} - t)(t - t_k)\xi_2^T S_3\xi_2 + (t_{k+1} - t) \times (\eta(t) - \eta(t_k))^T S_4(\eta(t) - \eta(t_k))\\
&\quad + (t_{k+1} - t)h_k\int_{t_k}^{t} \dot{\eta}^T(s)S_5\dot{\eta}(s)ds - (t - t_k)h_k\int_{t}^{t_{k+1}} \dot{\eta}^T(s)S_6\dot{\eta}(s)ds\\
&\quad + 2(t - t_k)(\eta(t_{k+1}) - \eta(t))^T \times (S_7\eta(t_{k+1}) + S_8\eta(t_k)),
\end{aligned}
$$

where

$$\zeta_1(t) = col\{\eta(t),\, \int_{-\sigma_1}^{0}\eta_t(s)ds,\, \int_{-\sigma_1}^{0} e_1(s)\eta_t(s)ds,\, \int_{-\sigma_2}^{-\sigma_1}\eta_t(s)ds,\, \sigma_{12}\int_{-\sigma_2}^{-\sigma_1} e_4(s)\eta_t(s)ds\}.$$

In consideration of

$$
\begin{aligned}
\sigma_{12}e_4(s) &= (\sigma(t) - \sigma_1)e_2(s) + (\sigma_2 - \sigma(t))\\
&= (\sigma_2 - \sigma(t))e_3(s) - (\sigma(t) - \sigma_1),
\end{aligned}
$$

one has

$$
\begin{aligned}
&\sigma_{12}\int_{-\sigma_2}^{-\sigma_1} e_4(s)\eta_t(s)ds\\
=&(\sigma(t)-\sigma_1)\Big(\int_{-\sigma(t)}^{-\sigma_1} e_2(s)\eta_t(s)ds\Big)-(\sigma(t)-\sigma_1)\Big(\int_{-\sigma_2}^{-\sigma(t)}\eta_t(s)ds\Big)\\
&+(\sigma_2-\sigma(t))\Big(\int_{-\sigma(t)}^{-\sigma_1}\eta_t(s)ds\Big)+(\sigma_2-\sigma(t))\Big(\int_{-\sigma_2}^{-\sigma(t)} e_3(s)\eta_t(s)\Big)\\
=&\hat{\Omega}_1(\sigma(t))\xi(t). \qquad (3.9)
\end{aligned}
$$

In light of

$$
\begin{aligned}
\eta(t)&=p_1\xi(t),\\
\int_{-\sigma_1}^{0}\eta_t(s)ds&=\sigma_1 p_5\xi(t),\\
\int_{-\sigma_1}^{0}e_1(s)\eta_t(s)ds&=\sigma_1 p_6\xi(t),\\
\int_{-\sigma_2}^{-\sigma_1}\eta_t(s)ds&=(p_{11}+p_{13})\xi(t),
\end{aligned}
$$

and (3.9), we can write

$$\zeta_1^T(t)=\xi^T(t)\bar{\Omega}_1^T(\sigma(t)).$$

Define $\mathfrak{L}$ as the infinitesimal generator of random process $\{\eta(t),\ \delta(t)\}$. For each $\delta(t)=m$,

$$
\begin{aligned}
&\mathfrak{L}\mathscr{V}(\eta(t),\ \delta(t),\ t)\\
=&\lim_{\varphi\to 0^+}\frac{1}{\varphi}\Big[\mathcal{E}\{\mathscr{V}(\eta(t+\varphi),\ \delta(t+\varphi),\ t+\varphi\,|\eta(t),\ \delta(t),\ t)\}-\mathscr{V}(\eta(t),\ \delta(t),\ t)\Big].
\end{aligned}
$$

Subsequently, we can deduce that

$$
\begin{aligned}
&\mathfrak{L}\mathscr{V}_1(\eta(t),\ \delta(t),\ t)\\
=&\lim_{\varphi\to 0^+}\frac{1}{\varphi}\Big[\sum_{n=1,n\neq m}^{\tilde{\Gamma}}(\pi_{mn}\varphi+o(\varphi))\times\eta^T(t+\varphi)P_n\eta(t+\varphi)+\big(1+\pi_{mm}\varphi+o(\varphi)\big)\\
&\times\eta^T(t+\varphi)P_m\eta(t+\varphi)-\eta^T(t)P_m\eta(t)\Big]\\
=&\lim_{\varphi\to 0^+}\frac{1}{\varphi}\Big[\sum_{n=1}^{\tilde{\Gamma}}(\pi_{mn}\varphi+o(\varphi))\eta^T(t+\varphi)P_n\eta(t+\varphi)+\eta^T(t+\varphi)P_m\eta(t+\varphi)-\eta^T(t)P_m\eta(t)\Big]\\
=&\eta^T(t)(\sum_{n=1}^{\tilde{\Gamma}}\pi_{mn}P_n)\eta(t)+\lim_{\varphi\to 0^+}\frac{1}{\varphi}[\eta^T(t+\varphi)P_m\eta(t+\varphi)-\eta^T(t)P_m\eta(t)]\\
=&\eta^T(t)(\sum_{n=1}^{\tilde{\Gamma}}\pi_{mn}P_n)\eta(t)+2\eta^T(t)P_m\dot{\eta}(t). \qquad (3.10)
\end{aligned}
$$

It follows that

$$\mathfrak{L}\mathscr{V}_2(t) = \xi^T(t)\mathscr{S}(\bar{\Omega}_1^T(\chi)\tilde{P}\bar{\Omega}_0)\xi(t), \tag{3.11}$$

$$\mathfrak{L}\mathscr{V}_3(t) = \xi^T(t)\bar{S}\xi(t), \tag{3.12}$$

$$\begin{aligned}\mathfrak{L}\mathscr{V}_4(t) = &\dot{\eta}^T(t)(\sigma_1^2 R_1 + \sigma_{12}^2 R_2)\dot{\eta}(t) - \sigma_1 \int_{t-\sigma_1}^{t} \dot{\eta}^T(s)R_1\dot{\eta}(s)ds \\ &- \sigma_{12}\int_{t-\sigma_2}^{t-\sigma_1} \dot{\eta}^T(s)R_2\dot{\eta}(s)ds,\end{aligned} \tag{3.13}$$

$$\begin{aligned}\mathfrak{L}\mathscr{V}_5(t) = &\xi^T(t)\bar{\Lambda}_1\xi(t) + \xi^T(t)((t-t_k)\bar{\Lambda}_2 + (t_{k+1}-t)\bar{\Lambda}_3)\xi(t) - h_k\int_{t_k}^{t}\dot{\eta}^T(s)S_5\dot{\eta}(s)ds \\ &- h_k\int_t^{t_{k+1}}\dot{\eta}^T(s)S_6\dot{\eta}(s)ds.\end{aligned} \tag{3.14}$$

For the integral in (3.13), by Lemma 1, one has

$$-\sigma_1\int_{t-\sigma_1}^{t}\dot{\eta}^T(s)R_1\dot{\eta}(s)ds \le -\xi^T(t)\Omega_2^T\check{R}_1\Omega_2\xi(t), \tag{3.15}$$

$$-\sigma_{12}\int_{t-\sigma_2}^{t-\sigma_1}\dot{\eta}^T(s)R_2\dot{\eta}(s)ds \le -\xi^T(t)\begin{bmatrix}\Omega_3\\ \Omega_4\end{bmatrix}^T\begin{bmatrix}\frac{\sigma_{12}\check{R}_2}{\sigma(t)-\sigma_1} & 0\\ * & \frac{\sigma_{12}\check{R}_2}{\sigma_2-\sigma(t)}\end{bmatrix}\begin{bmatrix}\Omega_3\\ \Omega_4\end{bmatrix}\xi(t). \tag{3.16}$$

Employing Jensen's inequality, for the integral items in (3.14), we have

$$\begin{aligned}&- h_k\int_{t_k}^{t}\dot{\eta}^T(s)S_5\dot{\eta}(s)ds \\ \le &-\frac{h_k}{t-t_k}(\eta(t)-\eta(t_k))^T S_5(\eta(t)-\eta(t_k)),\end{aligned} \tag{3.17}$$

$$\begin{aligned}&- h_k\int_{t}^{t_{k+1}}\dot{\eta}^T(s)S_6\dot{\eta}(s)ds \\ \le &-\frac{h_k}{t_{k+1}-t}(\eta(t_{k+1})-\eta(t))^T S_6(\eta(t_{k+1})-\eta(t)).\end{aligned} \tag{3.18}$$

For any matrices M_1, M_2 in $\mathbb{R}^{3n\times 3n}$ and M_3, M_4 in $\mathbb{R}^{n\times n}$, from Lemma 2 we can obtain the following inequalities:

$$-\xi^T(t)\begin{bmatrix}\Omega_3\\ \Omega_4\end{bmatrix}^T\begin{bmatrix}\frac{\sigma_{12}\check{R}_2}{\sigma(t)-\sigma_1} & 0\\ * & \frac{\sigma_{12}\check{R}_2}{\sigma_2-\sigma(t)}\end{bmatrix}\begin{bmatrix}\Omega_3\\ \Omega_4\end{bmatrix}\xi(t) \le -\xi^T(t)\Omega_{34}^T\check{R}_M\Omega_{34}\xi(t), \tag{3.19}$$

$$\begin{aligned}&-\frac{h_k}{t-t_k}(\eta(t)-\eta(t_k))^T S_5(\eta(t)-\eta(t_k)) \\ &-\frac{h_k}{t_{k+1}-t}(\eta(t_{k+1})-\eta(t))^T S_6(\eta(t_{k+1})-\eta(t)) \\ \le &-\xi^T(t)\Omega_5^T\begin{bmatrix}\frac{h_k}{t-t_k}S_5 & 0\\ 0 & \frac{h_k}{t_{k+1}-t}S_6\end{bmatrix}\Omega_5\xi(t)\end{aligned}$$

$$
\begin{aligned}
&\le -\xi^T(t)\Omega_5^T \breve{S}_M \Omega_5 \xi(t) \\
&\le -\xi^T(t)\Omega_5^T \left[\frac{t-t_k}{h_k}\bar{\Lambda}_s^1 + \frac{t_{k+1}-t}{h_k}\bar{\Lambda}_s^2\right]\Omega_5\xi(t),
\end{aligned} \tag{3.20}
$$

with

$$
\begin{aligned}
\breve{S}_M &= \begin{bmatrix} S_5 + (\frac{t_{k+1}-t}{h_k})\breve{S}_{M_4} & (\frac{t_{k+1}-t}{h_k})M_3 + \frac{t-t_k}{h_k}M_4 \\ * & S_6 + \frac{t-t_k}{h_k}\breve{S}_{M_3} \end{bmatrix}, \\
\breve{S}_{M_4} &= (S_5 - M_4 S_6^{-1} M_4^T), \\
\breve{S}_{M_3} &= (S_6 - M_3^T S_5^{-1} M_3).
\end{aligned}
$$

Furthermore, along (2.6), using the free-weighting-matrix approach [51], for any matrices T_1, T_2 in $\mathbb{R}^{n\times n}$, the following is satisfied:

$$
\begin{aligned}
0 =&2[\eta^T(t)T_1 + \dot{\eta}^T(t)T_2][A_m\eta(t) + W_m\eta(t-\sigma(t)) \\
&+ H_m f(D\eta(t)) - (K_m + E_m\Theta N_m)C_m\eta(t_k) - G_m\omega(t) - \dot{\eta}^T(t)].
\end{aligned} \tag{3.21}
$$

On the other hand, by the definitions of $\alpha_1(\sigma(t))$ and $\alpha_2(\sigma(t))$, for any matrices T_3, T_4 in $\mathbb{R}^{19n\times 2n}$, one has

$$
2\xi^T(t)(T_3\alpha_1(\sigma(t)) + T_4\alpha_2(\sigma(t)))\xi(t) = 0. \tag{3.22}
$$

For any diagonal matrix T_5 in $\mathbb{S}_+^n$, from (3.1), the following holds:

$$
f^T(D\eta(t))T_5 f(D\eta(t)) \le \eta^T(t)D^T L T_5 L D\eta(t). \tag{3.23}
$$

For $t \in [t_k,\ t_{k+1})$, combining the above inequalities, one obtains

$$
\mathcal{E}\{\mathfrak{L}\mathscr{V}(\eta(t),\ \delta(t),\ t)\} \le \mathcal{E}\{\xi^T(t)\Big[\frac{t-t_k}{h_k}\Pi_1^{h_k}(\sigma(t)) + \frac{t_{k+1}-t}{h_k}\Pi_2^{h_k}(\sigma(t))\Big]\xi(t) + \omega^T(t)\omega(t)\},
$$

where

$$
\begin{aligned}
\Pi_1^{h_k}(\sigma(t)) &= \Pi_0(\sigma(t)) + \bar{T} + \breve{T} + h_k\bar{\Lambda}_2 - \Omega_5^T\bar{\Lambda}_s^1\Omega_5, \\
\Pi_2^{h_k}(\sigma(t)) &= \Pi_0(\sigma(t)) + \bar{T} + \breve{T} + h_k\bar{\Lambda}_3 - \Omega_5^T\bar{\Lambda}_s^2\Omega_5, \\
\breve{T} &= \mathscr{S}[(p_1^T T_1 + p_{17}^T T_2)(A_m p_1 + W_m p_3 + H_m p_{15} - (K_m + E_m\Theta N_m)C_m p_{18} - G_m p_{16} - p_{17})].
\end{aligned}
$$

By setting $T_2 = uT_1$ and using (3.7), $\Pi_1^{h_k}(\sigma(t))$ can be rearranged as $\tilde{\Pi}_1^{h_k}(\sigma(t)) + \mathscr{S}(\bar{E}_m\Theta\bar{N}_m^T)$. Using Lemma 3, one has

$$
\mathscr{S}(\bar{E}_m\Theta\bar{N}_m^T) \le \epsilon_m^{-1}\bar{E}_m\bar{E}_m^T + \epsilon_m\bar{N}_m\bar{N}_m^T,
$$

and, thus, it can be concluded that $\tilde{\Pi}_1^{h_k}(\sigma(t)) + \mathscr{S}(\bar{E}_m\Theta\bar{N}_m^T)$ can be guaranteed by

$$
\begin{bmatrix} \tilde{\Pi}_1^{h_k}(\sigma_1) + \epsilon_m^{-1}\bar{E}_m\bar{E}_m^T + \epsilon_m\bar{N}_m\bar{N}_m^T & \Pi_1^{12} \\ * & \Pi_1^{22} \end{bmatrix} < 0,
$$

which is equivalent to (3.2) by Lemma 4. Since $\Pi_1^{h_k}(\cdot)$ and $\Pi_2^{h_k}(\cdot)$ are convex functions, it is easy to get $\xi^T(t)\Pi_1^{h_k}(\sigma(t))\xi(t) < 0$ from (3.2) and (3.3). Similarly, we can show that $\xi^T(t)\Pi_2^{h_k}(\sigma(t))\xi(t) < 0$ can be assured by (3.4) and (3.5). Thus, for $t \in [t_k, t_{k+1})$, one has

$$\mathcal{E}\{\mathfrak{L}\mathscr{V}(\eta(t),\ \delta(t),\ t)\} \le \mathcal{E}\left\{\omega^T(t)\omega(t)\right\}. \tag{3.24}$$

When $\omega(t) \equiv 0$, from (3.24), there exists a scalar $\alpha > 0$ such that

$$\mathcal{E}\{\mathfrak{L}\mathscr{V}(\eta(t),\ \delta(t),\ t)\} \le -\alpha \left\|\eta(t)\right\|^2,\ t_k \le t < t_{k+1}. \tag{3.25}$$

Using Dynkin's formula, one has

$$\begin{aligned}&\mathcal{E}\{\mathscr{V}(\eta(t_{k+1}^-),\ \delta(t_{k+1}^-),\ t_{k+1})\} - \mathcal{E}\{\mathscr{V}(\eta(t_k),\ \delta(t_k),\ t_k)\}\\ &\le -\alpha\mathcal{E}\left\{\int_{t_k}^{t_{k+1}^-} \|\eta(s)\|^2\, ds\right\}.\end{aligned} \tag{3.26}$$

Since $\mathscr{V}_5(t_k) = 0$ and $\lim_{t\to t_k} \mathscr{V}(\eta(t),\ \delta(t),\ t) = \mathscr{V}(\eta(t_k),\ \delta(t_k),\ t_k)$, $\mathscr{V}(\eta(t),\ \delta(t),\ t)$ is continuous. Thus, from (3.26), one obtains

$$\sum_{k=0}^{\infty}\mathcal{E}\left\{\int_{t_k}^{t_{k+1}^-} \|\eta(s)\|^2\, ds\right\} \le \alpha^{-1}\mathcal{E}\{\mathscr{V}(\eta(0),\ \delta(0),\ 0)\} < \infty.$$

Therefore, from Definition 1, error system (2.6) is stochastically stable.

Next, we consider the case that $\omega(t) \neq 0$ and introduce a $\mathcal{L}_2 - \mathcal{L}_\infty$ performance index function of error system (2.6) as follows:

$$I(t) = \mathcal{E}\{\mathscr{V}(\eta(t),\ \delta(t),\ t)\} - \int_0^t \omega^T(s)\omega(s)ds.$$

Then, under the zero initial condition, using Dynkin's formula for (3.24) gives

$$\begin{aligned}I(t) &= \mathcal{E}\{\mathscr{V}(\eta(0),\ \delta(0),\ 0)\} + \mathcal{E}\left\{\int_0^t \mathfrak{L}\mathscr{V}((\eta(s),\ \delta(s),\ s)ds\right\} - \int_0^t \omega^T(s)\omega(s)ds\\ &= \mathcal{E}\left\{\int_0^t \mathfrak{L}\mathscr{V}(\eta(s),\ \delta(s),\ s)ds - \omega^T(s)\omega(s)ds\right\}.\end{aligned}$$

From (3.24), one gets $I(t) \le 0$. Thus,

$$\mathcal{E}\{\mathscr{V}(\eta(t),\ \delta(t),\ t)\} \le \int_0^t \omega^T(s)\omega(s)ds. \tag{3.27}$$

Applying Lemma 4, (3.6) is equivalent to

$$P_m - \frac{1}{\gamma^2}C_m^T C_m > 0. \tag{3.28}$$

In virtue of (3.8), (3.27), and (3.28), one has

$$\begin{aligned}\mathcal{E}\left\{z^T(t)z(t)\right\} =& \mathcal{E}\left\{\eta^T(t)C_m^TC_m\eta(t)\right\}\\ &< \mathcal{E}\left\{\gamma^2\eta^T(t)P_m\eta(t)\right\}\\ &< \mathcal{E}\left\{\gamma^2\mathscr{V}(\eta(t),\, \delta(t),\, t)\right\}\\ &< \gamma^2\int_0^t \omega^T(s)\omega(s)ds\\ &< \gamma^2\int_0^\infty \omega^T(s)\omega(s)ds.\end{aligned}$$

Hence, system (2.6) has a $\mathcal{L}_2 - \mathcal{L}_\infty$ performance. In this way, the proof is completed.

On the basis of Assumption 2, the following inequalities

$$f_i^2(d_i^T\eta_i(\cdot)) \le d_i^T\eta_i(\cdot)L_if_i(d_i^T\eta_i(\cdot)), \tag{3.29}$$

$$f_i^2(d_i^T\eta_i(\cdot)) \le (L_id_i^T\eta_i(\cdot))^2], \tag{3.30}$$

hold true for $i = 1, \ldots, n$.

Based on LKF:

$$\bar{\mathscr{V}}(\eta(t), \delta(t), t) = \mathscr{V}(\eta(t), \delta(t), t) + 2\sum_{i=1}^{n}\int_0^{d_i^T\eta_i(t)} r_if_i(s)ds, \tag{3.31}$$

and we can easily derive the following theorem:

Theorem 2. *Under Assumption 2, for given scalars $\gamma > 0$, $h_2 \ge h_1 > 0$, $u > 0$, suppose that there are scalar $\epsilon_m > 0$, matrices $\tilde{P}$ in $\mathbb{S}_+^{5n}$, P_m, S_1, S_2, R_1, R_2, S_5, S_6 in $\mathbb{S}_+^n$, S_3 in $\mathbb{S}^{2n}$, S_4 in $\mathbb{S}^n$, diagonal matrices T_5, T_6, $\Lambda = diag\{r_1, \ldots, r_n\}$ in $\mathbb{S}_+^n$, arbitrary matrices M_1, M_2 in $\mathbb{R}^{3n\times 3n}$, M_3, M_4, T_1, S_7, S_8, X_m in $\mathbb{R}^{n\times n}$, and T_3, T_4 in $\mathbb{R}^{19n\times 2n}$, such that the LMIs in (3.6) and*

$$\begin{bmatrix}\tilde{\Pi}_{11}^{h_k}(\sigma_1) + \epsilon_m\bar{N}_m\bar{N}_m^T & \Pi_1^{12} & \bar{E}_m\\ * & \Pi_1^{22} & 0\\ * & * & -\epsilon_mI\end{bmatrix} < 0, \tag{3.32}$$

$$\begin{bmatrix}\tilde{\Pi}_{11}^{h_k}(\sigma_2) + \epsilon_m\bar{N}_m\bar{N}_m^T & \Pi_1^{13} & \bar{E}_m\\ * & \Pi_1^{22} & 0\\ * & * & -\epsilon_mI\end{bmatrix} < 0, \tag{3.33}$$

$$\begin{bmatrix}\tilde{\Pi}_{21}^{h_k}(\sigma_1) + \epsilon_m\bar{N}_m\bar{N}_m^T & \Pi_2^{12} & \bar{E}_m\\ * & \Pi_2^{22} & 0\\ * & 0 & -\epsilon_mI\end{bmatrix} < 0, \tag{3.34}$$

$$\begin{bmatrix}\tilde{\Pi}_{21}^{h_k}(\sigma_2) + \epsilon_m\bar{N}_m\bar{N}_m^T & \Pi_2^{13} & \bar{E}_m\\ * & \Pi_2^{22} & 0\\ * & * & -\epsilon_mI\end{bmatrix} < 0, \tag{3.35}$$

hold for $m \in \Gamma$, $h_k \in \{h_1,\, h_2\}$, and $\chi \in \{\sigma_1,\, \sigma_2\}$, where

$$
\begin{aligned}
\tilde{\Pi}_{11}^{h_k}(\chi) &= \Pi_0(\chi) + \bar{T}_1 + \breve{T}_1 + h_k\bar{\Lambda}_2 - \Omega_5^T\bar{\Lambda}_s^1\Omega_5, \\
\tilde{\Pi}_{21}^{h_k}(\chi) &= \Pi_0(\chi) + \bar{T}_1 + \breve{T}_1 + h_k\bar{\Lambda}_3 - \Omega_5^T\bar{\Lambda}_s^2\Omega_5, \\
\Pi_1^{12} &= \begin{bmatrix} (p_{19} - p_1)^T M_3^T & \Omega_3^T M_2 \end{bmatrix}, \\
\Pi_1^{13} &= \begin{bmatrix} (p_{19} - p_1)^T M_3^T & \Omega_4^T M_1^T \end{bmatrix}, \\
\Pi_2^{12} &= \begin{bmatrix} (p_1 - p_{18})^T M_4 & \Omega_3^T M_2 \end{bmatrix}, \\
\Pi_2^{13} &= \begin{bmatrix} (p_1 - p_{18})^T M_4 & \Omega_4^T M_1^T \end{bmatrix}, \\
\Pi_1^{22} &= diag\{-S_5, -\breve{R}_2\}, \\
\Pi_2^{22} &= diag\{-S_6, -\breve{R}_2\}, \\
\Pi_0(\chi) &= \mathscr{S}(p_1^T P_m p_{17} + \bar{\Omega}_1^T(\chi)\tilde{P}\bar{\Omega}_0) + \sum_{n=1}^{\tilde{\Gamma}} \pi_{mn} p_1^T P_n p_1 + \bar{S} - \Omega_2^T \breve{R}_1 \Omega_2 \\
&\quad + p_{17}^T(\sigma_1^2 R_1 + \sigma_{12}^2 R_2) p_{17} + T(\chi) - \Omega_{34}^T \breve{R}_{TM} \Omega_{34} + \bar{\Lambda}_1 - p_{16}^T p_{16}, \\
\bar{S} &= diag\{S_1, -S_1 + S_2, 0_{n\times n}, -S_2, 0_{15n\times 15n}\}, \\
\breve{R}_i &= diag\{R_i, 3R_i, 5R_i\}, \ i = \{1, 2\}, \\
\breve{R}_M &= \begin{bmatrix} \breve{R}_2 + \frac{\sigma_2-\chi}{\sigma_{12}}\breve{R}_2 & \frac{\sigma_2-\chi}{\sigma_{12}}M_1 + \frac{\chi-\sigma_1}{\sigma_{12}}M_2 \\ * & \breve{R}_2 + \frac{\chi-\sigma_1}{\sigma_{12}}\breve{R}_2 \end{bmatrix}, \\
\bar{\Lambda}_s^1 &= \begin{bmatrix} S_5 & M_4 \\ * & 2S_6 \end{bmatrix}, \ \bar{\Lambda}_s^2 = \begin{bmatrix} 2S_5 & M_3 \\ * & S_6 \end{bmatrix}, \\
\bar{\Lambda}_1 &= -(p_1^T - p_{18}^T)S_4(p_1 - p_{18}) + \mathscr{S}[(p_{19}^T - p_1^T)S_7 p_{19} + (p_{19}^T - p_1^T)S_8 p_{18}], \\
\breve{T}_1 &= \mathscr{S}[(p_1^T T_1 + p_{17}^T u T_1)(A_m p_1 + W_m p_3 + H_m p_{15} - G_m p_{16} - p_{17})] \\
&\quad - \mathscr{S}[(p_1^T + u p_{17}^T) X_m C_m p_{18}], \\
T(\chi) &= \mathscr{S}(T_3\alpha_1(\chi) + T_4\alpha_2(\chi)), \\
\alpha_1(\chi) &= (\chi - \sigma_1)\begin{bmatrix} p_7 \\ p_8 \end{bmatrix} - \begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix}, \\
\alpha_2(\chi) &= (\sigma_2 - \chi)\begin{bmatrix} p_9 \\ p_{10} \end{bmatrix} - \begin{bmatrix} p_{13} \\ p_{14} \end{bmatrix}, \\
\bar{T}_1 &= \mathscr{S}[p_{15}^T \Lambda D p_{17} + (p_1^T D^T L - p_{15}^T) T_5 p_{15}] \\
&\quad + p_1^T D^T L T_6 L D p_1 - p_{15}^T T_6 p_{15}, \\
\bar{\Lambda}_2 &= -\Omega_6^T S_3 \Omega_6 + h_k p_{17}^T S_6 p_{17} - \mathscr{S}(p_{17}^T S_7 p_{19} + p_{17}^T S_8 p_{18}), \\
\bar{\Lambda}_3 &= \mathscr{S}((p_1^T - p_{18}^T) S_4 p_{17}) + \Omega_6^T S_3 \Omega_6 + h_k p_{17}^T S_5 p_{17}, \\
\bar{E}_m &= -(p_1^T T_1 + p_{17}^T u T_1) E_m, \\
\bar{N}_m &= N_m C_m p_{18}.
\end{aligned}
$$

Then, drive-response MJLSs (2.1) and (2.2) are stochastically synchronized with a predefined $\mathcal{L}_2-\mathcal{L}_\infty$ DAL if the SDC gains in (2.3) are given by (3.7).

Proof. For any $t \in [t_k, t_{k+1})$, calculate that

$$\mathfrak{L}\bar{\mathscr{V}}(\eta(t), \delta(t), t) = \mathfrak{L}\mathscr{V}(\eta(t), \delta(t), t) + 2f^T(D\eta(t))\Lambda D\dot{\eta}(t).$$

Further, by (3.29) and (3.30), for any diagonal matrices T_5, T_6 in $\mathbb{S}_+^n$, the following hold true:

$$2[\eta^T(t)D^T L - f^T(D\eta(t))]T_5 f^T(D\eta(t)) \geq 0,$$
$$\eta^T(t)D^T LT_6 LD\eta(t) - f^T(D\eta(t))T_6 f(D\eta(t)) \geq 0.$$

The remainder of the proof is consistent with Theorem 1, which is omitted here.

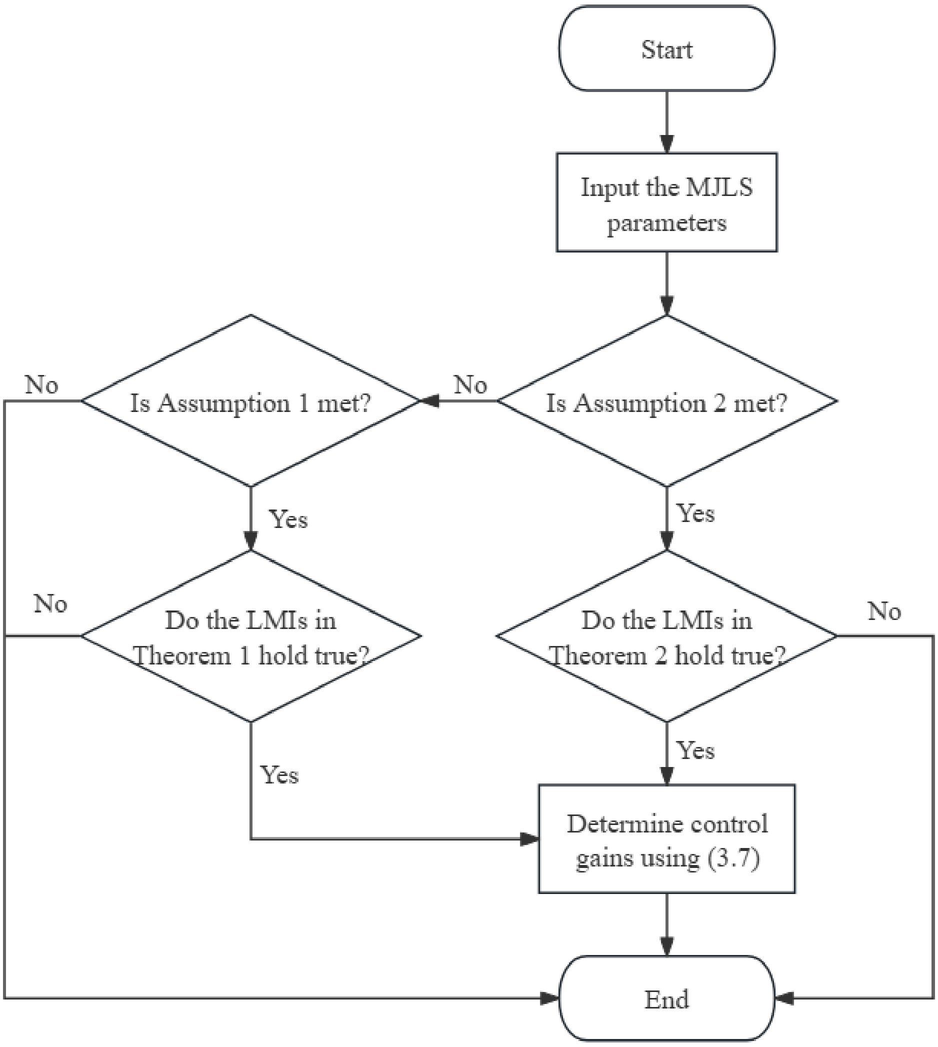

Figure 1. Flow chart of the proposed controller design.

Remark 2. *On the basis of the LMI feasible solutions, Theorems 1 and 2 establish two different conditions for the desired SDC gains, which are capable of being easily verified by publicly accessible MATLAB toolboxes. For a clearer understanding of the proposed controller design, a flowchart is* *presented in Figure 1. The LKF term* $2\sum_{i=1}^{n}\int_{0}^{d_i^T\eta_i(t)} r_i f_i(s)ds$ *is introduced based on Assumption 2, which has the potential to effectively mitigate conservatism. This will be confirmed by comparing the maximum allowable upper bound (MAUB)* h_2 *in the following section.*

When there is no gain perturbation (i.e., $\Delta K(\delta(t)) = 0$) and no mode switching (i.e., $m = 1$), (2.3) can be rewritten as $u(t) = KC\eta(t_k)$. Correspondingly, error system (2.6) is simplified to

$$\dot{\eta}(t) = A\eta(t) + W\eta(t-\sigma(t)) + Hf(D\eta(t)) - KC\eta(t_k) - G\omega(t),\ t \in [t_k,\ t_{k+1})\,. \tag{3.36}$$

The following criterion can be obtained.

Corollary 1. *Under Assumption 1, for given scalars* $\gamma > 0$, $h_2 \geq h_1 > 0$, $u > 0$, *suppose that there are matrices* $\tilde{P}$ *in* $\mathbb{S}_+^{5n}$, P, S_1, S_2, R_1, R_2, S_5, S_6 *in* $\mathbb{S}_+^n$, S_3 *in* $\mathbb{S}^{2n}$, S_4 *in* $\mathbb{S}^n$, *diagonal matrix* T_5 *in* $\mathbb{S}_+^n$, *arbitrary matrices* M_1, M_2 *in* $\mathbb{R}^{3n\times 3n}$, M_3, M_4, T_1, S_7, S_8, X *in* $\mathbb{R}^{n\times n}$, *and* T_3, T_4 *in* $\mathbb{R}^{19n\times 2n}$, *such that*

$$\begin{bmatrix} \bar{\Pi}_1^{h_k}(\sigma_1) & \Pi_1^{12} \\ * & \Pi_1^{22} \end{bmatrix} < 0, \tag{3.37}$$

$$\begin{bmatrix} \bar{\Pi}_1^{h_k}(\sigma_2) & \Pi_1^{13} \\ * & \Pi_1^{22} \end{bmatrix} < 0, \tag{3.38}$$

$$\begin{bmatrix} \bar{\Pi}_2^{h_k}(\sigma_1) & \Pi_2^{12} \\ * & \Pi_2^{22} \end{bmatrix} < 0, \tag{3.39}$$

$$\begin{bmatrix} \bar{\Pi}_2^{h_k}(\sigma_2) & \Pi_2^{13} \\ * & \Pi_2^{22} \end{bmatrix} < 0, \tag{3.40}$$

$$\begin{bmatrix} P & C^T \\ * & \gamma^2 I \end{bmatrix} > 0, \tag{3.41}$$

hold for $h_k \in \{h_1,\ h_2\}$ *and* $\chi \in \{\sigma_1,\ \sigma_2\}$, *where*

$$\begin{aligned}
\bar{\Pi}_1^{h_k}(\chi) &= \bar{\Pi}_0(\chi) + \bar{T} + \check{T}_2 + h_k\bar{\Lambda}_2 - \Omega_5^T\bar{\Lambda}_s^1\Omega_5,\\
\bar{\Pi}_2^{h_k}(\chi) &= \bar{\Pi}_0(\chi) + \bar{T} + \check{T}_2 + h_k\bar{\Lambda}_3 - \Omega_5^T\bar{\Lambda}_s^2\Omega_5,\\
\bar{\Pi}_0(\chi) &= \mathscr{S}(p_1^T P p_{17} + \bar{\Omega}_1^T(\chi)\tilde{P}\bar{\Omega}_0) + \bar{S} + p_{17}^T(\sigma_1^2 R_1 + \sigma_{12}^2 R_2)p_{17}\\
&\quad - \Omega_2^T\check{R}_1\Omega_2 - \Omega_{34}^T\check{R}_{TM}\Omega_{34} + \bar{\Lambda}_1 + T(\chi) - p_{16}^T p_{16},\\
\check{T}_2 &= \mathscr{S}[(p_1^T T_1 + p_{17}^T u T_1)(Ap_1 + Wp_3 + Hp_{15} - Gp_{16} - p_{17})]\\
&\quad - \mathscr{S}[(p_1^T + up_{17}^T)XCp_{18}],\\
\Pi_1^{12} &= \begin{bmatrix} (p_{19}-p_1)^T M_3^T & \Omega_3^T M_2 \end{bmatrix},\\
\Pi_1^{13} &= \begin{bmatrix} (p_{19}-p_1)^T M_3^T & \Omega_4^T M_1^T \end{bmatrix},\\
\Pi_2^{12} &= \begin{bmatrix} (p_1-p_{18})^T M_4 & \Omega_3^T M_2 \end{bmatrix},\\
\Pi_2^{13} &= \begin{bmatrix} (p_1-p_{18})^T M_4 & \Omega_4^T M_1^T \end{bmatrix},\\
\Pi_1^{22} &= diag\{-S_5,\ -\check{R}_2\},\\
\Pi_2^{22} &= diag\{-S_6,\ -\check{R}_2\},\\
\bar{S} &= diag\{S_1,\ -S_1 + S_2,\ 0_{n\times n},\ -S_2,\ 0_{15n\times 15n}\},
\end{aligned}$$

$$
\begin{aligned}
\breve{R}_i &= diag\{R_i,\ 3R_i,\ 5R_i\},\ i = \{1,\ 2\},\\
\breve{R}_M &= \begin{bmatrix} \breve{R}_2 + \frac{\sigma_2-\chi}{\sigma_{12}}\breve{R}_2 & \frac{\sigma_2-\chi}{\sigma_{12}}M_1 + \frac{\chi-\sigma_1}{\sigma_{12}}M_2 \\ * & \breve{R}_2 + \frac{\chi-\sigma_1}{\sigma_{12}}\breve{R}_2 \end{bmatrix},\\
\bar{\Lambda}_s^1 &= \begin{bmatrix} S_5 & M_4 \\ * & 2S_6 \end{bmatrix},\ \bar{\Lambda}_s^2 = \begin{bmatrix} 2S_5 & M_3 \\ * & S_6 \end{bmatrix},\\
\bar{\Lambda}_1 &= -(p_1^T - p_{18}^T)S_4(p_1 - p_{18}) + \mathscr{S}[(p_{19}^T - p_1^T)S_7 p_{19}\\
&\quad + (p_{19}^T - p_1^T)S_8 p_{18} - (p_1^T + up_{17}^T)X_m C_m p_{18}],\\
T(\chi) &= \mathscr{S}(T_3\alpha_1(\chi) + T_4\alpha_2(\chi)),\\
\alpha_1(\chi) &= (\chi - \sigma_1)\begin{bmatrix} p_7 \\ p_8 \end{bmatrix} - \begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix},\\
\alpha_2(\chi) &= (\sigma_2 - \chi)\begin{bmatrix} p_9 \\ p_{10} \end{bmatrix} - \begin{bmatrix} p_{13} \\ p_{14} \end{bmatrix},\\
\bar{T} &= (p_1^T D^T L T_5 L D p_1 - p_{15}^T T_5 p_{15}),\\
\bar{\Lambda}_2 &= -\Omega_6^T S_3 \Omega_6 + h_k p_{17}^T S_6 p_{17} - \mathscr{S}(p_{17}^T S_7 p_{19} + p_{17}^T S_8 p_{18}),\\
\bar{\Lambda}_3 &= \mathscr{S}((p_1^T - p_{18}^T)S_4 p_{17}) + \Omega_6^T S_3 \Omega_6 + h_k p_{17}^T S_5 p_{17},\\
\bar{E}_m &= -(p_1^T T_1 + p_{17}^T u T_1)E_m,\\
\bar{N}_m &= N_m C_m p_{18}.
\end{aligned}
$$

Then, error system (3.36) is stochastically stable and has a predefined $\mathcal{L}_2 - \mathcal{L}_\infty$ DAL if the SDC gain is determined by $K = T_1^{-1}X$.

Corollary 2. *Under Assumption 2, for given scalars $\gamma > 0$, $h_2 \geq h_1 > 0$, $u > 0$, suppose that there are matrices $\tilde{P}$ in $\mathbb{S}_+^{5n}$, P, S_1, S_2, R_1, R_2, S_5, S_6 in $\mathbb{S}_+^n$, S_3 in $\mathbb{S}^{2n}$, S_4 in $\mathbb{S}^n$, diagonal matrix T_5 in $\mathbb{S}_+^n$, arbitrary matrices M_1, M_2 in $\mathbb{R}^{3n\times 3n}$, M_3, M_4, T_1, S_7, S_8, X, in $\mathbb{R}^{n\times n}$, and T_3, T_4 in $\mathbb{R}^{19n\times 2n}$, such that the LMIs in (3.41) and*

$$\begin{bmatrix} \bar{\Pi}_{11}^{h_k}(\sigma_1) & \Pi_1^{12} \\ * & \Pi_1^{22} \end{bmatrix} < 0, \tag{3.42}$$

$$\begin{bmatrix} \bar{\Pi}_{11}^{h_k}(\sigma_2) & \Pi_1^{13} \\ * & \Pi_1^{22} \end{bmatrix} < 0, \tag{3.43}$$

$$\begin{bmatrix} \bar{\Pi}_{21}^{h_k}(\sigma_1) & \Pi_2^{12} \\ * & \Pi_2^{22} \end{bmatrix} < 0, \tag{3.44}$$

$$\begin{bmatrix} \bar{\Pi}_{21}^{h_k}(\sigma_2) & \Pi_2^{13} \\ * & \Pi_2^{22} \end{bmatrix} < 0, \tag{3.45}$$

hold for $h_k \in \{h_1,\ h_2\}$ and $\chi \in \{\sigma_1,\ \sigma_2\}$, where

$$
\begin{aligned}
\bar{\Pi}_{11}^{h_k}(\chi) &= \bar{\Pi}_0(\chi) + \bar{T}_1 + \breve{T}_2 + h_k\bar{\Lambda}_2 - \Omega_5^T\bar{\Lambda}_s^1\Omega_5,\\
\bar{\Pi}_{21}^{h_k}(\chi) &= \bar{\Pi}_0(\chi) + \bar{T}_1 + \breve{T}_2 + h_k\bar{\Lambda}_3 - \Omega_5^T\bar{\Lambda}_s^2\Omega_5,\\
\bar{\Pi}_0(\chi) &= \mathscr{S}(p_1^T P p_{17} + \bar{\Omega}_1^T(\chi)\tilde{P}\bar{\Omega}_0) + \bar{S} + p_{17}^T(\sigma_1^2 R_1 + \sigma_{12}^2 R_2)p_{17} - \Omega_2^T\breve{R}_1\Omega_2\\
&\quad - \Omega_{34}^T\breve{R}_{TM}\Omega_{34} + \bar{\Lambda}_1 + T(\chi) - p_{16}^T p_{16},
\end{aligned}
$$

$$\begin{aligned}
\breve{T}_2 &= \mathscr{S}[(p_1^T T_1 + p_{17}^T u T_1)(A p_1 + W p_3 + H p_{15} - G p_{16} - p_{17})] \\
&\quad - \mathscr{S}[(p_1^T + u p_{17}^T) X C p_{18}], \\
\Pi_1^{12} &= \begin{bmatrix} (p_{19} - p_1)^T M_3^T & \Omega_3^T M_2 \end{bmatrix}, \\
\Pi_1^{13} &= \begin{bmatrix} (p_{19} - p_1)^T M_3^T & \Omega_4^T M_1^T \end{bmatrix}, \\
\Pi_2^{12} &= \begin{bmatrix} (p_1 - p_{18})^T M_4 & \Omega_3^T M_2 \end{bmatrix}, \\
\Pi_2^{13} &= \begin{bmatrix} (p_1 - p_{18})^T M_4 & \Omega_4^T M_1^T \end{bmatrix}, \\
\Pi_1^{22} &= diag\{-S_5, -\breve{R}_2\}, \\
\Pi_2^{22} &= diag\{-S_6, -\breve{R}_2\}, \\
\bar{S} &= diag\{S_1, -S_1 + S_2, 0_{n\times n}, -S_2, 0_{15n\times 15n}\}, \\
\breve{R}_i &= diag\{R_i, 3R_i, 5R_i\}, \ i = \{1, 2\}, \\
\breve{R}_M &= \begin{bmatrix} \breve{R}_2 + \frac{\sigma_2-\chi}{\sigma_{12}}\breve{R}_2 & \frac{\sigma_2-\chi}{\sigma_{12}}M_1 + \frac{\chi-\sigma_1}{\sigma_{12}}M_2 \\ * & \breve{R}_2 + \frac{\chi-\sigma_1}{\sigma_{12}}\breve{R}_2 \end{bmatrix}, \\
\bar{\Lambda}_s^1 &= \begin{bmatrix} S_5 & M_4 \\ * & 2S_6 \end{bmatrix}, \ \bar{\Lambda}_s^2 = \begin{bmatrix} 2S_5 & M_3 \\ * & S_6 \end{bmatrix}, \\
\bar{\Lambda}_1 &= -(p_1^T - p_{18}^T) S_4 (p_1 - p_{18}) + \mathscr{S}[(p_{19}^T - p_1^T) S_7 p_{19} + (p_{19}^T - p_1^T) S_8 p_{18} \\
&\quad - (p_1^T + u p_{17}^T) X_m C_m p_{18}], \\
T(\chi) &= \mathscr{S}(T_3 \alpha_1(\chi) + T_4 \alpha_2(\chi)), \\
\alpha_1(\chi) &= (\chi - \sigma_1)\begin{bmatrix} p_7 \\ p_8 \end{bmatrix} - \begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix}, \\
\alpha_2(\chi) &= (\sigma_2 - \chi)\begin{bmatrix} p_9 \\ p_{10} \end{bmatrix} - \begin{bmatrix} p_{13} \\ p_{14} \end{bmatrix}, \\
\bar{T}_1 &= \mathscr{S}[p_{15}^T \Lambda D p_{17} + (p_1^T D^T L - p_{15}^T) T_5 p_{15}] \\
&\quad + p_1^T D^T L T_6 L D p_1 - p_{15}^T T_6 p_{15}, \\
\bar{\Lambda}_2 &= -\Omega_6^T S_3 \Omega_6 + h_k p_{17}^T S_6 p_{17} - \mathscr{S}(p_{17}^T S_7 p_{19} + p_{17}^T S_8 p_{18}), \\
\bar{\Lambda}_3 &= \mathscr{S}((p_1^T - p_{18}^T) S_4 p_{17}) + \Omega_6^T S_3 \Omega_6 + h_k p_{17}^T S_5 p_{17}, \\
\bar{E}_m &= -(p_1^T T_1 + p_{17}^T u T_1) E_m, \ \bar{N}_m = N_m C_m p_{18}.
\end{aligned}$$

Then, error system (3.36) is stochastically stable and has a predefined $\mathcal{L}_2 - \mathcal{L}_\infty$ DAL if the SDC gain is determined by $K = T_1^{-1} X$.

4. Numerical example

Consider two-mode ($m = 1, 2$) drive-response MJLSs modeled in (2.1) and (2.2) with parameters (refer to [20]):

$$A_1 = \begin{bmatrix} -\frac{9}{7} & 9 & 0 \\ 1 & -1 & 1 \\ 0 & -14.52 & 0 \end{bmatrix}, \ A_2 = \begin{bmatrix} -\frac{18}{7} & 9 & 0 \\ 1 & -1 & 1 \\ 0 & -14.28 & 0 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -0.1 & 0 & 0 \\ -0.1 & 0 & 0 \\ 0.2 & 0 & -0.1 \end{bmatrix}, W_2 = \begin{bmatrix} -0.09 & 0 & 0 \\ -0.09 & 0 & 0 \\ 0.18 & 0 & -0.09 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} \frac{18}{7} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, H_2 = \begin{bmatrix} \frac{27}{7} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, G_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

Consider the controller gain fluctuation (2.4) with the following parameters:

$$\begin{aligned} E_1 &= diag\{0.5,\ 0.5,\ 0.5\}, \\ E_2 &= diag\{0.7,\ 0.7,\ 0.7\}, \\ \Theta &= col\{\sin(t),\ \cos(t),\ \sin(t)\}, \\ N_1 &= N_2 = 0.1. \end{aligned}$$

The time delay is chosen as $\sigma(t) = 0.4 + \beta|\sin t|$, which implies that σ_1 and σ_2 are 0.4 and $0.4 + \beta$, respectively. In addition, the TPM here is assumed to be

$$\tilde{\pi} = \begin{bmatrix} -0.5 & 0.5 \\ 0.8 & -0.8 \end{bmatrix}.$$

In the following, we set $u = 0.5$, $h_1 = 0.001$, and consider two cases of nonlinear functions:

Case 1: $f(Dx(t)) = col\{\frac{1}{2}(|x_1(t) + 1| - |x_1(t) - 1|),\ 0,\ 0\}$.

In this case, it's easy to see that $f(\cdot)$ satisfies both Assumptions 1 and 2 with $L = diag\{1, 0, 0\}$. The prescribed $\mathcal{L}_2 - \mathcal{L}_\infty$ performance γ is chosen as 0.25. In view of Theorems 1 and 2, Table 1 details MAUB h_2 when σ_1 is fixed and β increases from 0.2 to 1.8. As can be seen from Table 1, the MAUB h_2 depends on the value of β, and h_2 in Theorem 1 is always less than h_2 in Theorem 2. Thus, in comparison to Theorem 1, Theorem 2 yields less conservative results. Furthermore, it demonstrates the effectiveness of the intentionally introduced LKF $2\sum_{i=1}^{n}\int_0^{d_i^T\eta_i(t)} r_i f_i(s)ds$.

Table 1. The MAUB of h_2 for various β.

β	0.2	0.6	1.0	1.4	1.8
Theorem 1	0.145	0.140	0.136	0.131	0.127
Theorem 2	0.305	0.292	0.280	0.270	0.261

Case 2: $f(Dx(t)) = col\{\sin(x_1),\ 0,\ 0\}$.

Obviously, the nonlinear function $f(\cdot)$ satisfies Assumption 1 but does not satisfy Assumption 2 with $L = diag\{1, 0, 0\}$. Therefore, the proposed condition in Theorem 2 is no longer valid, while the one in

Theorem 1 remains applicable. We set $h_2 = 0.121$ and $\gamma = 0.25$. By solving the LMIs in Theorem 1, One has

$$T_1 = \begin{bmatrix} 11.0140 & 1.8636 & 4.3164 \\ 5.2193 & 37.2755 & -1.3122 \\ 3.9106 & 3.1960 & 8.9907 \end{bmatrix},$$

$$X_1 = \begin{bmatrix} 64.3188 & 42.9094 & 22.4470 \\ 26.7319 & 112.4413 & 33.1270 \\ 16.9245 & -74.6275 & 63.2178 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 45.3174 & 35.6183 & 19.1237 \\ 22.7783 & 118.3422 & 31.5949 \\ 7.5621 & -74.0286 & 60.5995 \end{bmatrix}.$$

Then, the control gain matrices can be obtained as

$$K_1 = \begin{bmatrix} 6.1565 & 8.5683 & -0.9106 \\ -0.1708 & 1.3762 & 1.2619 \\ -0.7347 & -12.5167 & 6.9790 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 4.5651 & 7.7285 & -1.1361 \\ -0.0676 & 1.6637 & 1.2458 \\ -1.1205 & -12.1870 & 6.7916 \end{bmatrix}.$$

For the simulation, we set the initial conditions of the drive-response MJLSs to be $x(s) = col\{0.2, 0.3, 0.2\}$, $y(s) = col\{-0.3, -0.1, 0.4\}$, $s \in [-1.8, 0]$, the external disturbance $\omega(t) = col\{\exp(-0.5t), \exp(-0.5t), \exp(-0.5t)\}$, and the above parameters. The chaotic behavior of drive system (2.1) with $u(t) = 0$ is shown in Figure 2. Figures 3 and 4 depict the Markov-jump signal and sampling intervals, respectively. Under the designed SDC method, the synchronization between drive-response MJLSs (2.1) and (2.2) is achieved in Figure 5. Figure 6 depicts the trajectories of the error system (2.6). Define a new function

$$\mathcal{L}(t) = \sqrt{\frac{\mathcal{E}\{z^T(t)z(t)\}}{\int_0^t \omega^T(\beta)\omega(\beta)d\beta}}.$$

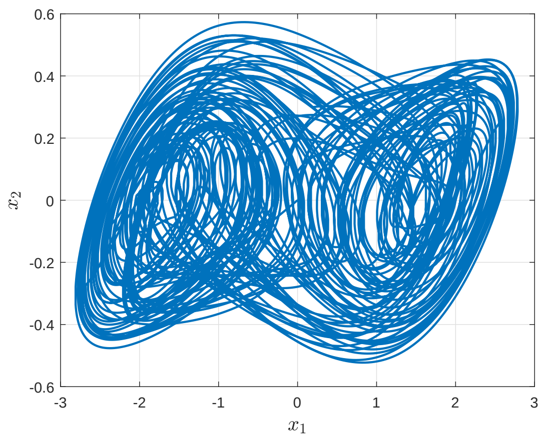

Figure 2. Phase portrait of drive system (2.1) with initial condition $col\{0.2, 0.3, 0.2\}$.

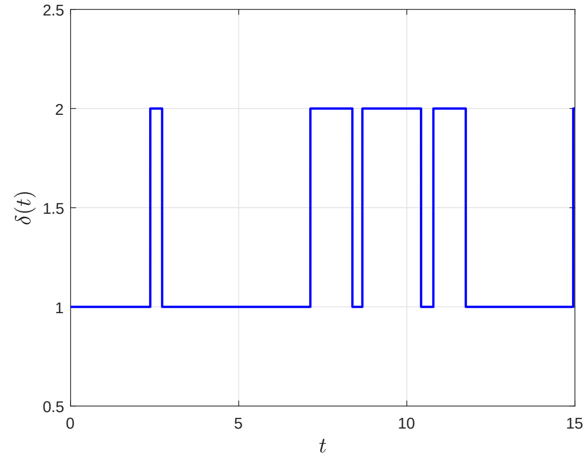

Figure 3. Markov jump signal $\delta(t)$.

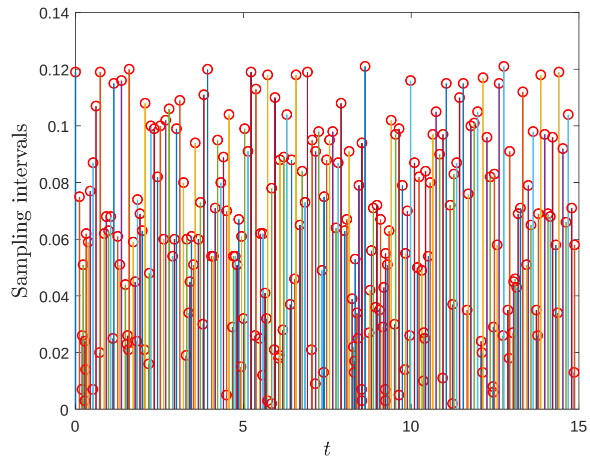

Figure 4. Sampling instants and the corresponding sampling intervals.

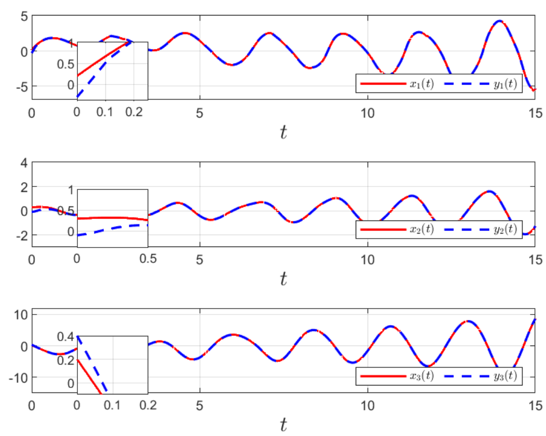

Figure 5. State trajectories of $x(t)$ and $y(t)$ with control.

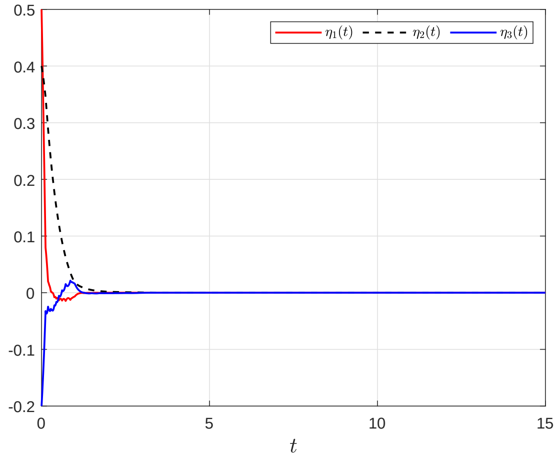

Figure 6. State trajectories of $\eta(t)$ with control.

Its evolution versus time is shown in Figure 7. From the figure, one can observe that $\sup_t \mathcal{L}(t) = 0.0192 < \gamma = 0.21$, which implies that the prescribed $\mathcal{L}_2 - \mathcal{L}_\infty$ performance γ is validity guaranteed.

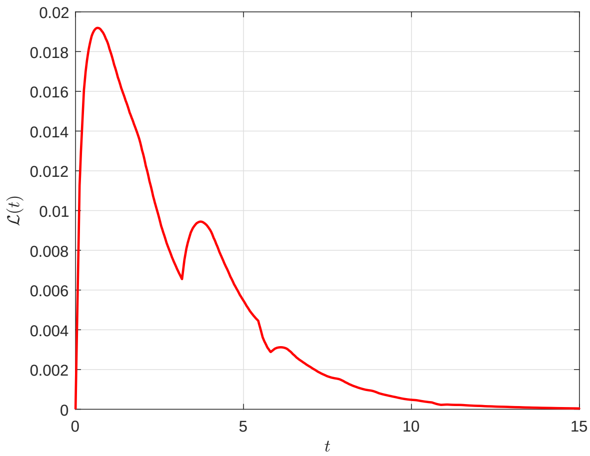

Figure 7. Evolution of $\mathcal{L}(t)$.

5. Conclusions

The non-fragile sampled-data synchronization control issue has been studied for MJLSs with time-variant delay. On the foundation of two different assumptions of the nonlinear function vector, two time-dependent two-sided loop LKFs (see (3.8) and (3.31)) have been constructed. By employing these two LKFs and several inequalities, numerically tractable conditions for the design of a non-fragile sampled-data controller (2.5) have been provided in Theorems 1 and 2 to guarantee the drive and response MJLSs realize stochastic synchronization with a prescribed $\mathcal{L}_2 - \mathcal{L}_\infty$ DAL. Finally, an example has been given to verify the validity of the non-fragile sampled-data controller approaches. In this paper, the transition probabilities of the Markov jump process are assumed to be completely known. In practical applications, however, it is often costly or difficult to obtain all the elements in the TPM. As an extension of the present work, we will further investigate the non-fragile sampled-data synchronization control for Markov jump systems with partially unknown transition probabilities.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this paper.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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