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Research article

An event-based decision and regulation strategy for the production-warehousing-selling model

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Abstract: This paper focused on the decision and regulation for the production-warehousing-selling (P-W-S) model. A novel event-triggered mechanism (ETM) was meticulously developed to determine when to impose control, alongside the development of the corresponding impulsive strategy. By applying the input-to-state stability (ISS) theory, some quantitative relationships between system parameters and ETM were integrated into the estimation of the state of the P-W-S model. It was shown that under the designed ETM and impulsive strategy, the warehouse was able to autonomously adjust inventory levels based on factory production efficiency and market selling trends, and hence the quantity of goods could be maintained within a reasonable range, avoiding excessive inventory while adequately meeting market requirements. At last, an example with numerical simulations was presented to validate our results.

Keywords: production-warehousing-selling; event-triggered mechanism; decision and regulation; impulsive control; input-to-state stability

1. Introduction

With the vigorous development of modern market economy, a growing number of enterprises are inclined to build a complete industrial chain structure to ensure their dominant position in the market. The production-warehousing-selling (P-W-S) model refers to a comprehensive management model and stands as a pivotal framework. It describes the seamless integration of production, warehousing, and selling activities, and provides a comprehensive perspective on the dynamic interactions within supply chains [1, 2], thereby addressing the fundamental challenges encountered in managing the flow of goods from production facilities to end consumers. First, the production phase involves the planning and execution of manufacturing processes. Efficient production scheduling and resource allocation are critical to ensuring optimal utilization of manufacturing capacities and minimizing production costs. Subsequently, the warehousing phase entails the storage, handling, and management of inventory within designated facilities. Effective warehouse management is essential for maintaining inventory accuracy, optimizing storage space utilization, and facilitating timely order fulfillment. Moreover, advanced inventory control techniques, such as just-in-time inventory management [3], can be employed to streamline warehouse operations and reduce holding costs. Finally, the selling phase encompasses the activities involved in marketing, selling, and distributing goods to customers. A robust selling strategy is imperative for driving revenue growth, expanding market reach, and enhancing customer satisfaction. By applying selling forecasting methods and customer relationship management systems [4], enterprises can adjust their selling strategy to meet evolving customer requirements and market trends. Overall, the P-W-S model serves as a foundational framework for supply chain optimization and strategic decision-making, which enables enterprises to enhance operational efficiency, minimize costs, and capitalize on market opportunities in the current business environment [5–8].

Impulsive control has obtained significant attention in recent years as a powerful technique for regulating and stabilizing dynamic systems in numerous fields, including engineering [9], physics [10], biology [11], economics [12], and so on. To be more precise, "impulsive control" refers to the use of impulses for control purposes at the predetermined impulsive instants [13]. This characteristic is advantageous for installation, implementation, and maintenance since it lowers the resource requirements for control updates and information transmission [14–16]. For the decision of an impulsive time sequence, the most advanced one is the so-called event-triggered strategy, whose principle lies in the utilization of an event-triggered mechanism (ETM) to determine when to update the control [17, 18]. These events can be defined based on a variety of criteria, such as changes in system states, deviations from desired trajectories, or the occurrence of specific events in the environment. By monitoring these events and updating impulsive control only when necessary, event-triggered impulsive control (ETIC) minimizes the computational burden and communication cost, while ensuring that control objectives are met with high precision and efficiency. In summary, ETIC represents an effective approach for dynamic systems, and its discrete nature, combined with its adaptability and efficiency, makes it well-suited for addressing the challenges posed by complex and uncertain environments. Through intensive research, ETIC continues to offer innovative schemes to a wide range of control problems and remains a promising avenue in engineering practice [19–22].

Input-to-state stability (ISS) is a fundamental concept in the control field that provides a comprehensive framework for assessing the stability of nonlinear systems subject to external input [23]. It means that for any initial condition, system states will eventually approach a neighborhood of origin, and the state evolution is determined by the size of external input. As a result, ISS is crucial for designing robust control systems, and is a powerful tool for analyzing the dynamic performance of various control systems, especially for impulsive systems [24–30]. The challenge of designing robust model predictive control for linear parameter-varying systems, for example, was tackled by [26]. The triggered condition was derived using the ISS concept, which reduced the burden of information interaction. In the study of nonlinear impulsive systems with external disturbance, [27] searched for dwell-time conditions that ensured the ISS property. In particular, the obtained conditions included the possibility of simultaneous instability of continuous and discontinuous dynamics.

This paper aims to provide an effective decision and regulation strategy for the P-W-S model so as to enhance operational efficiency, minimize costs, and capitalize on market opportunities. For this purpose, we first model the P-W-S as a nonlinear system that presents original hybrid impulses and external input. A novel ETM coupled with impulsive strategy is then designed and the potential Zeno behavior is excluded. Specially, owing to the original hybrid impulses, the triggered cases are more complicated and some novel methods of analysis and induction are developed. Additionally, compared with the existing results [18,31,32], our designed ETM permits the system to autonomously decide the frequency and intensity of the control input according to its current state without any other artificial intervention. By applying the ISS theory, some sufficient estimation conditions for the P-W-S model are derived, under which the quantity of goods can be maintained within a reasonable range, thereby avoiding excessive inventory while adequately meeting market requirements.

In what follows, the fundamental P-W-S model and some preliminaries are introduced in Section 2. Subsequently, Section 3 designs an effective ETM and addresses the estimation problem for the quantity of goods. A numerical example is discussed in Section 4 to validate the results and, finally, Section 5 closes this paper along with some concluding remarks.

Notations: The supremum in the interval $\mathcal{J}$ is labeled as $|\cdot|_{\mathcal{J}}$. Given a symmetric matrix A, its minimum and maximum eigenvalues are respectively expressed as $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$. Given any set of constants $\{h_i\}_{i\in\mathbf{Z}_+}$, $\Sigma_{i=1}^{0} h_i$ is specified as 0.

2. Preliminaries

2.1. Model description

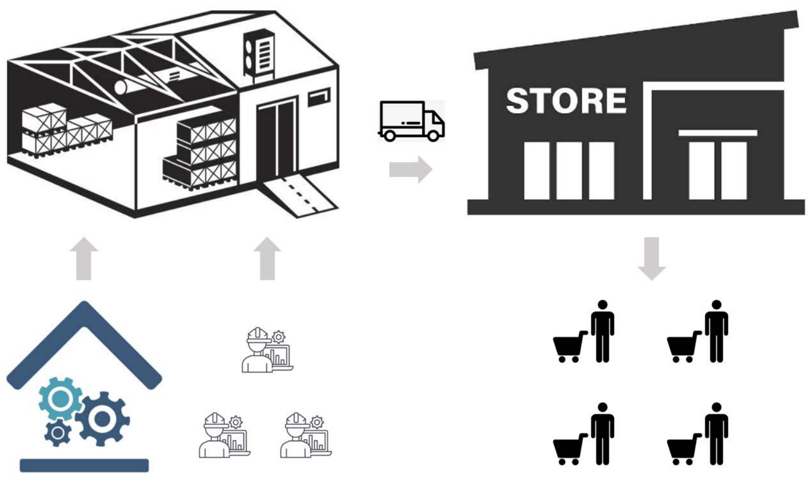

Figure 1. Illustration for the P-W-S model.

In the P-W-S model, our primary goal lies in monitoring the quantity of each type of good in the warehouse and adjusting inventory levels based on factory production efficiency and market selling trends. Specifically, the main factory manufactures goods and stores them in the warehouse. Concurrently, several subsidiary factories also produce goods and periodically deliver them to the warehouse. Due to the simultaneous presence of multiple types of good in the warehouse, the quantity of each type of good is constrained, and this constraint fluctuates as the quantities of other goods vary. Once the quantity of a particular type of good in the warehouse reaches its constraint, it must be dispatched to various stores for selling. According to the market requirement, regular shipments are necessary to ensure adequate supply, even if the inventory levels have not reached their upper constraints. In this process, the rework and wastage of goods are possible, and the unqualified goods will be transformed to be qualified by factories. Besides, due to the changes in factory productivity or transportation capability, a certain nonlinearity should be taken into account in the P-W-S model. See Figure 1 for more details.

Based on the above statement, we consider the following P-W-S model with hybrid impulses:

$$\begin{cases} \dot{x}(t) = Sx(t) + R\mathcal{F}(x(t)) + B\omega(t),\ t \neq \tau_k, \\ x(t) = G_k x(t^-),\ t = \tau_k, \end{cases} \tag{2.1}$$

where $x \in \mathbf{R}^p$ represents the quantities of p types of goods in the warehouse, and $\dot{x}$ denotes its right-hand derivative. $\mathcal{F} \in \mathbf{R}^p$ is a smooth nonlinear vector-valued function and $\mathcal{F}(0) = 0$, which may be caused by the changes in factory productivity or transportation capability. $\omega(t) \in \mathbf{R}^q$ is a measurable external input satisfying $0 < |\omega(t)| \leq \Delta$, which represents the possible rework and wastage of goods. $S \in \mathbf{R}^{p\times p}, R \in \mathbf{R}^{p\times p}$, and $B \in \mathbf{R}^{p\times q}$ are known parameter matrices, which stand for the coupling relationships between different types of goods, such as the binding relationships between camera and film, coffee and milk, and so on. G_k is the hybrid impulsive gain representing the sudden change in the quantities of goods at τ_k. $\{\tau_k\}_{k\in\mathbf{Z}_+}$ is a class of impulsive time sequences satisfying $\tau_k - \tau_{k-1} \geq \tau$ with a positive constant τ. It is assumed that all of the signals are right continuous and have left limits at all instants. The solution to system (2.1) with initial value $x(t_0) = x_0$ uniquely exists in $[t_0, \infty)$.

Assumption 1. *There is a positive constant l_f such that $|\mathcal{F}(x) - \mathcal{F}(y)| \leq l_f|x - y|$, where $x, y \in \mathbf{R}^p$.*

Definition 1. *Given a locally Lipschitz function $V : \mathbf{R}^p \to \mathbf{R}_+$, define the upper right-hand Dini derivative of V with respect to system (2.1) as*

$$D^+V(x) = \limsup_{h\to 0^+} \frac{1}{h}(V(x + hg(x, \omega)) - V(x)).$$

Definition 2. *[33] For the prescribed sequence $\{\tau_k\}_{k\in\mathbb{Z}_+}$, system (2.1) is said to be input-to-state stable (ISS) if there exist functions $\vartheta \in \mathcal{KL}$ and $\chi \in \mathcal{K}_\infty$ such that for every initial condition x_0 and each bounded external input $\omega(t)$, it holds that*

$$|x(t)| \leq \vartheta(|x_0|, t - t_0) + \chi(|\omega|_{[t_0,t]}),\ t \geq t_0.$$

Assumption 2. *[34] For any real vectors x, y and symmetric positive matrix Q with compatible dimension, the following inequality holds:*

$$2x^T y \leq x^T Q^{-1} x + y^T Qy.$$

By constructing a Lyapunov candidate $V(x(t)) = x^T(t)Px(t)$, where $P \in \mathbf{R}^{p\times p}$, the following lemma can be derived to estimate the dynamic behavior of system (2.1). For convenience, we denote $V(t) = V(x(t))$.

Lemma 1. Under Assumptions 1 and 2, if there exist positive matrices P, U, Q, constant μ_k, and positive constants γ, l_f such that

$$-\gamma P + PS + S^T P + PRU^{-1}R^T U + l_f^2 U + PBQ^{-1}B^T P \leq 0, \tag{2.2}$$

$$-\exp(\mu_k)P + G_k^T P G_k \leq 0, \tag{2.3}$$

then we have

$$D^+V(t) \leq cV(t), \text{ whenever } V(t) \geq \varphi(|\omega|_{[t_0,t]}),$$
$$V(\tau_k) \leq \exp(\mu_k)V(\tau_k^-), \ k \in \mathbf{Z}_+,$$

where $c = \gamma + \upsilon, \varphi(s) = \frac{\lambda_{max}(Q)(s^2)}{\upsilon}$, and υ is an arbitrary positive constant.

Proof. Assume that $x(t) = x(t, t_0, x_0)$ is the solution to system (2.1) with initial value (t_0, x_0). Taking the time derivative of $V(t)$ along the trajectory of (2.1) in $[\tau_{k-1}, \tau_k)$ yields

$$\begin{aligned}&D^+V(t)\\=&2x^T(t)P\dot{x}(t)\\=&x^T(t)(PS + S^TP)x(t) + 2x^T(t)PR\mathcal{F}(x(t)) + 2x^T(t)PB\omega(t).\end{aligned}$$

According to Assumptions 1 and 2, one gets $2x^T(t)PR\mathcal{F}(x(t)) \leq x^T(t)PRU^{-1}R^TPx(t) + l_f^2 x^T(t)Ux(t)$ and $2x^T(t)PB\omega(t) \leq x^T(t)PBQ^{-1}B^TPx(t) + \omega^T(t)Q\omega(t)$. Then condition (2.2) derives that

$$\begin{aligned}&D^+V(t)\\\leq&\gamma V(t) + \lambda_{max}(Q)|\omega(t)|^2\\\leq&\gamma V(t) - \upsilon V(t) + \upsilon V(t) + \lambda_{max}(Q)|\omega(t)|^2,\end{aligned}$$

which further implies that

$$D^+V(t) \leq cV(t), \text{ whenever } V(t) \geq \varphi(|\omega|_{[t_0,t]}), \tag{2.4}$$

where $c = \gamma + \upsilon > 0, \varphi(s) = \frac{\lambda_{max}(Q)(s^2)}{\upsilon}$, and υ is an arbitrary positive constant.

When $t = \tau_k$, condition (2.3) means that

$$\begin{aligned}&V(\tau_k)\\=&x^T(\tau_k)Px(\tau_k)\\=&x^T(\tau_k^-)G_k^TPG_kx(\tau_k^-)\\\leq&\exp(\mu_k)V(\tau_k^-).\end{aligned}$$

Assumption 3. *[35] There is a constant $\bar{\mu}$ such that $\bar{\mu} = \max_{k\in\mathbf{Z}_+}\mu_k$. Also, impulsive gain sequence $\{\mu_k\}_{k\in\mathbf{Z}_+}$ satisfies the average impulsive gain condition. That is, there is a constant μ such that*

$$\frac{\mu_m + \mu_{m+1} + ... + \mu_{m+h}}{h+1} \leq \mu.$$

3. Theoretical results

This section will apply the ISS theory into the estimation of the dynamic behavior of the P-W-S model. First of all, we tend to develop an effective decision strategy for determining when to dispatch goods to various stores for selling purposes, thereby enhancing operational efficiency, minimizing costs, and capitalizing on market opportunities.

3.1. An event-based decision and regulation mechanism

In light of the Lyapunov candidate $V(t)$ given previously, we construct the following event:

$$\Xi_{n-1}(t) = \exp(\zeta_n - \lambda(t - t_{n-1}))V(t_{n-1}) + \delta\varphi(|\omega|_{[t_0,t]}), \tag{3.1}$$

where $\lambda > 0, \delta > 1$, and $\zeta_n > 0$ satisfying $\Sigma_{n=1}^{+\infty}\zeta_n = +\infty$. Then we present an event-triggered mechanism (ETM) as

$$t_n = \inf\{t \geq t_{n-1}|V(t) \geq \Xi_{n-1}(t)\}, \tag{3.2}$$

where $\{t_n\}_{n\in\mathbf{Z}_+}$ is the triggered time sequence. Subsequently, impulsive control strategy is designed for regulation purposes and is in the form of $x(t_n) = K_n x(t_n^-)$, where K_n represents the control gain.

Remark 1. ETM (3.2) allows the system to autonomously decide the frequency of impulsive control input according to its current state, and provides a dynamic threshold for the state of the system (2.1). Please refer to Figure 2 for details (the red line represents the original hybrid impulses and the gray line represents the designed stabilized impulse).

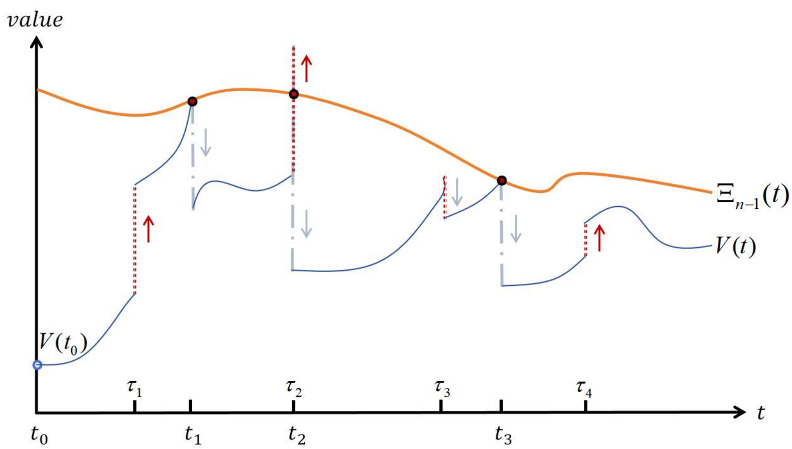

Figure 2. Illustration for the event-based decision and regulation mechanism.

Remark 2. It is possible that the original hybrid impulses and the event-based impulse occur simultaneously at some instants. However, in this scenario, it does not mean that the original impulse is ignored. Without loss of generality, it is assumed that the original impulse is the destabilizing impulse. Within the framework of our studied P-W-S model, this process means that the quantity of goods in the warehouse reaches the constraint and, at the same time, the goods produced by subsidiary factories are delivered to the warehouse. In this scenario, the goods in the warehouse should be dispatched to stores first, so as to make room for the newly produced goods, which are then put into the warehouse. That is, the system state will be reset first according to the event-based impulse and then the original destabilizing impulse updates the system state. Clearly, the occurring order of the event-based impulse and the original destabilizing impulse should be more realistic in the P-W-S model. Besides, it is possible that the original destabilizing impulse triggers the event (3.2), and then the event-based impulse will be input at the next instant. It is shown that the goods produced by subsidiary factories are delivered to the warehouse, which makes the quantity of goods in the warehouse reach the constraint. Then, these goods must be dispatched to various stores for selling.

Under the designed ETM (3.2), the following result is presented to rule out the potential Zeno behavior [36].

Theorem 1. Under Assumptions 1–3 and Lemma 1, there is no Zeno behavior for system (2.1) under ETM (3.2). Moreover, the triggered time sequence $\{t_n\}_{n\in\mathbf{Z}_+}$ satisfies

$$t_n - t_{n-1} \geq \frac{\zeta_n}{\frac{\mu}{\tau} + c + \lambda} \wedge \frac{\ln\delta}{\frac{\mu}{\tau} + c}. \tag{3.3}$$

Proof. Assume that $x(t) = x(t, t_0, x_0)$ is the solution to system (2.1) with initial value (t_0, x_0). Specially, the Zeno behavior can be ruled out naturally if the event is triggered finitely many times. Thereby, we here focus on when the event is triggered infinitely many times. For any interval $[t_{n-1}, t_n), n \in \mathbf{Z}_+$, if $V(t) \geq \varphi(|\omega|_{[t_0,t]})$ holds for all $t \in [t_{n-1}, t_n)$, then Lemma 1 gives $D^+V(t) \leq cV(t)$. Let $N(t, s)$ denote the number of the original hybrid impulses in $[s, t)$, and one then gets

$$V(t) \leq \exp(\Sigma_{k=N(t_{n-1},t_0)+1}^{N(t,t_0)}\mu_k + c(t - t_{n-1}))V(t_{n-1}),\ t \in [t_{n-1}, t_n),$$

which gives

$$V(t_n^-) \leq \exp(\Sigma_{k=N(t_{n-1},t_0)+1}^{N(t_n,t_0)}\mu_k + c(t_n - t_{n-1}))V(t_{n-1}).$$

Due to the existence of the original hybrid impulses, it can be obtained from (3.2) that

$$V(t_n^-) \geq \exp(\zeta_n - \lambda(t_n - t_{n-1}))V(t_{n-1}) + \delta\varphi(|\omega|_{[t_0,t_n]}),$$

which leads to

$$\exp(\zeta_n - \lambda(t_n - t_{n-1}))V(t_{n-1}) + \delta\varphi(|\omega|_{[t_0,t_n]}) \leq \exp(\Sigma_{k=N(t_{n-1},t_0)+1}^{N(t_n,t_0)}\mu_k + c(t_n - t_{n-1}))V(t_{n-1}).$$

Thus, it holds that

$$\zeta_n - \lambda(t_n - t_{n-1}) \leq \Sigma_{k=N(t_{n-1},t_0)+1}^{N(t_n,t_0)}\mu_k + c(t_n - t_{n-1}).$$

Note that Assumption 3 means that $\Sigma_{k=N(t_{n-1},t_0)+1}^{N(t_n,t_0)}\mu_k \leq \frac{t_n - t_{n-1}}{\tau}\mu$, from which we can get that

$$t_n - t_{n-1} \geq \frac{\zeta_n}{\frac{\mu}{\tau} + c + \lambda}.$$

If there exist $t \in [t_{n-1}, t_n)$ such that $V(t) \leq \varphi(|\omega|_{[t_0,t]})$, due to that $\delta > 1$, then it holds that $V(t_n^-) \geq \exp(\zeta_n - \lambda(t_n - t_{n-1}))V(t_{n-1}) + \delta\varphi(|\omega|_{[t_0,t_n]}) \geq \varphi(|\omega|_{[t_0,t_n]})$. Thus, given $t_n^* := \sup\{t \geq t_{n-1} | V(t) \leq \varphi(|\omega|_{[t_0,t]})\}$, one has that $V(t) \geq \varphi(|\omega|_{[t_0,t]}), t \in [t_n^*, t_n)$. Similarly, we have

$$V(t_n^-) \leq \exp(\Sigma_{k=N(t_n^*,t_0)+1}^{N(t_n,t_0)}\mu_k + c(t_n - t_n^*))V(t_n^*).$$

According to the definition of t_n^*, one gets $V(t_n^*) \leq \varphi(|\omega|_{[t_0,t_n^*]})$, under which one has

$$\exp(\zeta_n - \lambda(t_n - t_{n-1}))V(t_{n-1}) + \delta\varphi(|\omega|_{[t_0,t_n]}) \leq \exp(\Sigma_{k=N(t_n^*,t_0)+1}^{N(t_n,t_0)}\mu_k + c(t_n - t_n^*))\varphi(|\omega|_{[t_0,t_n]}).$$

Moreover, it is derived that

$$\delta \leq \exp((\frac{\mu}{\tau} + c)(t_n - t_n^*)),$$

which gives that

$$t_n - t_{n-1} \geq t_n - t_n^* \geq \frac{\ln\delta}{\frac{\mu}{\tau} + c}.$$

Thus in all cases, one finally obtains

$$t_n - t_{n-1} \geq \frac{\zeta_n}{\frac{\mu}{\tau} + c + \lambda} \wedge \frac{\ln\delta}{\frac{\mu}{\tau} + c}.$$

Summarily, the Zeno behavior can be excluded with

$$t_n \geq \sum_{i=1}^{n}(\frac{\zeta_i}{\frac{\mu}{\tau} + c + \lambda} \wedge \frac{\ln\delta}{\frac{\mu}{\tau} + c}) + t_0.$$

Remark 3. Theorem 1 is presented to rule out the possible Zeno behavior in ETM (3.2), thereby preventing the event from being triggered infinitely fast. Otherwise, it will greatly increase the transportation cost. Besides, by analyzing condition (3.3), it is clear that both the larger μ and the lower τ can decrease the triggered intervals, meaning that the high strength and high density of the original impulses will drive the event to be triggered more frequently to achieve the ISS property.

Remark 4. Owing to the fact that $\tau_k - \tau_{k-1} \geq \tau$ holds for all $k \in \mathbf{Z}_+$, the extreme case that Remark 2 discussed will not cause the Zeno behavior.

3.2. ISS analysis

Theorem 2. Under Assumptions 1–3 and Lemma 1, if conditions (2.2), (2.3), and the following hold

$$-\exp(-\varrho_n)P + K_n^T P K_n \leq 0,\ n \in \mathbf{Z}_+, \tag{3.4}$$

$$\inf_{n\in\mathbf{Z}_+}\{\varrho_n - \zeta_{n+1} - \bar{\mu}\} > 0, \tag{3.5}$$

where $\varrho_n > 0$, then system (2.1) is ISS under ETM (3.2).

Proof. Assume that $x(t) = x(t, t_0, x_0)$ is the solution to system (2.1) with initial value (t_0, x_0). When $t = t_n$, it follows from condition (3.4) that

$$
\begin{aligned}
&V(t_n)\\
=&x^T(t_n)Px(t_n)\\
=&x^T(t_n^-)K_n^T PK_n x(t_n^-)\\
\leq&\exp(-\varrho_n)V(t_n^-).
\end{aligned}
\tag{3.6}
$$

In view of the existence of the original hybrid impulses, here we define an auxiliary function as

$$
\sigma(t_n^-)=\begin{cases} 1, \ \text{if } t_n\in\{\tau_k\}\cap\{t_n\},\\ 0, \ \text{if } t_n\notin\{\tau_k\}\cap\{t_n\}.\end{cases}
$$

Given the characteristics of ETM (3.2), we consider three cases:

Case 1: Event (3.2) is never triggered. In this case, one has

$$
V(t)\leq\exp(\zeta_1-\lambda(t-t_0))V(t_0)+\delta\varphi(|\omega|_{[t_0,t]}),\ \forall t\in[t_0,t). \tag{3.7}
$$

Case 2: Event (3.2) is triggered finitely many times. For $t\in[t_0,t_1)$, one gets

$$
\begin{aligned}
&V(t)\\
\leq&\exp(\sigma(t_1^-)\bar{\mu})[\exp(\zeta_1-\lambda(t-t_0))V(t_0)+\delta\varphi(|\omega|_{[t_0,t]})]\\
=&\exp(\sigma(t_1^-)\bar{\mu}+\zeta_1-\lambda(t-t_0))V(t_0)+\exp(\sigma(t_1^-)\bar{\mu})\delta\varphi(|\omega|_{[t_0,t]}),\ \forall t\in[t_0,t_1).
\end{aligned}
\tag{3.8}
$$

For all $[t_1,t_2)$, it is clear that

$$
\begin{aligned}
&V(t)\\
\leq&\exp(\sigma(t_2^-)\bar{\mu})[\exp(\zeta_2-\lambda(t-t_1))V(t_1)+\delta\varphi(|\omega|_{[t_0,t]})]\\
=&\exp(\sigma(t_2^-)\bar{\mu}+\zeta_2-\lambda(t-t_1))V(t_1)+\exp(\sigma(t_2^-)\bar{\mu})\delta\varphi(|\omega|_{[t_0,t]}).
\end{aligned}
$$

According to (3.8), it holds that

$$
V(t_1^-)\leq\exp(\sigma(t_1^-)\bar{\mu}+\zeta_1-\lambda(t_1-t_0))V(t_0)+\exp(\sigma(t_1^-)\bar{\mu})\delta\varphi(|\omega|_{[t_0,t_1]}),
$$

which combining with (3.6) gives

$$
\begin{aligned}
&V(t_1)\\
\leq&\exp(-\varrho_1)V(t_1^-)\\
=&\exp(\sigma(t_1^-)\bar{\mu}+\zeta_1-\varrho_1-\lambda(t_1-t_0))V(t_0)+\exp(\sigma(t_1^-)\bar{\mu}-\varrho_1)\delta\varphi(|\omega|_{[t_0,t_1]}).
\end{aligned}
$$

Thereby, it is deduced that

$$
\begin{aligned}
&V(t)\\
\leq&\exp(\sigma(t_2^-)\bar{\mu}+\sigma(t_1^-)\bar{\mu}+\zeta_2+\zeta_1-\varrho_1-\lambda(t-t_0))V(t_0)\\
&+\exp(\sigma(t_2^-)\bar{\mu}+\sigma(t_1^-)\bar{\mu}+\zeta_2-\varrho_1)\delta\varphi(|\omega|_{[t_0,t]})\\
&+\exp(\sigma(t_2^-)\bar{\mu})\delta\varphi(|\omega|_{[t_0,t]}),\ t\in[t_1,t_2).
\end{aligned}
\tag{3.9}
$$

For all $t \in [t_2, t_3)$, it is clear that

$$\begin{aligned}
&V(t)\\
&\leq \exp(\sigma(t_3^-)\bar{\mu})[\exp(\zeta_3 - \lambda(t - t_2))V(t_2) + \delta\varphi(|\omega|_{[t_0,t]})]\\
&= \exp(\sigma(t_3^-)\bar{\mu} + \zeta_3 - \lambda(t - t_2))V(t_2) + \exp(\sigma(t_3^-)\bar{\mu})\delta\varphi(|\omega|_{[t_0,t]}).
\end{aligned}$$

According to (3.6) and (3.9), one can obtain

$$\begin{aligned}
&V(t_2)\\
&\leq \exp(\sigma(t_2^-)\bar{\mu} + \sigma(t_1^-)\bar{\mu} + \zeta_2 + \zeta_1 - \varrho_2 - \varrho_1 - \lambda(t_2 - t_0))V(t_0)\\
&\quad + \exp(\sigma(t_2^-)\bar{\mu} + \sigma(t_1^-)\bar{\mu} + \zeta_2 - \varrho_2 - \varrho_1)\delta\varphi(|\omega|_{[t_0,t_2]})\\
&\quad + \exp(\sigma(t_2^-)\bar{\mu} - \varrho_2)\delta\varphi(|\omega|_{[t_0,t_2]}),
\end{aligned}$$

which further gives

$$\begin{aligned}
&V(t)\\
&\leq \exp(\sigma(t_3^-)\bar{\mu} + \sigma(t_2^-)\bar{\mu} + \sigma(t_1^-)\bar{\mu} + \zeta_3 + \zeta_2 + \zeta_1 - \varrho_2 - \varrho_1 - \lambda(t - t_0))V(t_0)\\
&\quad + \exp(\sigma(t_3^-)\bar{\mu} + \sigma(t_2^-)\bar{\mu} + \sigma(t_1^-)\bar{\mu} + \zeta_3 + \zeta_2 - \varrho_2 - \varrho_1)\delta\varphi(|\omega|_{[t_0,t]})\\
&\quad + \exp(\sigma(t_3^-)\bar{\mu} + \sigma(t_2^-)\bar{\mu} + \zeta_3 - \varrho_2)\delta\varphi(|\omega|_{[t_0,t]})\\
&\quad + \exp(\sigma(t_3^-)\bar{\mu})\delta\varphi(|\omega|_{[t_0,t]}),\ \forall t \in [t_2, t_3).
\end{aligned}$$

Eventually, after repeating the above steps, we come to a conclusion as follows:

$$\begin{aligned}
&V(t)\\
&\leq \exp[\bar{\mu} + \zeta_1 - \lambda(t - t_0) + \sum_{i=1}^{n-1}(\sigma(t_{i+1}^-)\bar{\mu} + \zeta_{i+1} - \varrho_i)]V(t_0)\\
&\quad + \exp(\bar{\mu})[1 + \sum_{j=1}^{n-1}\exp(\sum_{i=j}^{n-1}(\sigma(t_{i+1}^-)\bar{\mu} + \zeta_{i+1} - \varrho_i))]\delta\varphi(|\omega|_{[t_0,t]}),\ \forall t \in [t_{n-1}, t_n).
\end{aligned} \tag{3.10}$$

Case 3: Event (3.2) is triggered infinitely many times. According to Theorem 1, there is no Zeno behavior. Repeating the similar discussions as in **Case 2**, (3.10) still holds for all $t \in [t_{n-1}, t_n)$.

Summarily, with all three cases, it can be obtained that (3.10) is always true. Let $\varpi = \inf_{n\in\mathbf{Z}_+}\{\varrho_n - \zeta_{n+1} - \bar{\mu}\}$, then according to condition (3.5), one eventually concludes that

$$V(t) \leq \exp(\bar{\mu} + \zeta_1 - \lambda(t - t_0))V(t_0) + \exp(\bar{\mu})\frac{\delta}{1 - e^{-\varpi}}\varphi(|\omega|_{[t_0,t]}),\ \forall t \in [t_{n-1}, t_n). \tag{3.11}$$

Thereby, the ISS property for system (2.1) under ETM (3.2) is achieved.

Remark 5. Recently, using event-triggered impulsive control, extensive research on the ISS issue has been reported, see [18, 31, 32] and the references therein. It should be pointed out that the ETMs designed in the previous results generally involve an additional time-triggered mechanism. In this scene, the event will be triggered to impose unnecessary control input if the event is not triggered for a given period, even if the function $V(t)$ does not reach the event threshold, possibly causing waste of resources. By contrast, ETM (3.2) allows the system to autonomously decide the frequency and intensity of control input according to its current state, without any other artificial intervention.

3.3. Discussions for decision and regulation

In view of the fact that there are some unknown matrices involved in the LMI-based conditions when we guarantee the ISS property of the P-W-S model (2.1), a feasible algorithm deserves to be developed. Through some reasonable transformations, the crucial conditions can be respectively rewritten as

$$\begin{aligned} &\gamma, l_f, \mu_k, \varrho_n, \zeta_n, \\ &s.t. \begin{cases} \begin{bmatrix} -\gamma P + PS + S^T P + l_f^2 U & PR & PB \\ \star & -U & 0 \\ \star & \star & -Q \end{bmatrix} \leq 0, \\ \begin{bmatrix} -\exp(\mu_k) P & G_k^T P \\ \star & -P \end{bmatrix} \leq 0, \ k \in \mathbf{Z}_+, \\ \begin{bmatrix} -\exp(-\varrho_n) P & Z_n \\ \star & -P \end{bmatrix} \leq 0, \ n \in \mathbf{Z}_+, \\ \inf_{n \in \mathbf{Z}_+} \{\varrho_n - \zeta_{n+1} - \bar{\mu}\} > 0. \end{cases} \end{aligned} \tag{3.12}$$

Based on the above translation processes, the feasible solutions P, U, Q, Z_n can be obtained. Moreover, the control gain can be designed by $K_n = P^{-1} Z_n^T$. Next, some crucial discussions will be given for decision and regulation of the P-W-S model (2.1) under ETM (3.2):

- It is shown that the triggered frequency may be adjusted by varying the event parameters ζ_n and λ : *i)* The event threshold for the system state will be small if we select a tiny value for ζ_n, thereby resulting in more frequent triggering. Conversely, it can potentially reduce the frequency of impulsive control with a larger ζ_n; *ii)* Between any two adjacent triggered instants, event $\Xi_{n-1}(t)$ presents the tendency of convergence. Specially, the system state will converge to the origin with decay rate λ for any vanishing external input $\omega(t)$. Therefore, in real-world scenarios, ζ_n and λ can be varied to attain the required performance of the system.

- Condition (3.5) reveals the relationships among ETM, original impulses, and impulsive control. Specially, the larger value of ϱ_n suggests that impulsive control has a stronger effect and performs better when stabilizing the system. The value of ζ_{n+1} is permitted to be larger in this situation. In other words, the event's boundedness increases, permitting the longer triggered interval. Conversely, a lower value of ϱ_n suggests that the role of impulsive control might be weakened, meaning that ζ_{n+1} should have a lower value and the event should occur more frequently so as to stabilize the system. Thereby, our developed ETM (3.2) offers a relatively flexible approach to impulsive control.

- It also deserves to be mentioned that original hybrid impulses may prompt the system state to exceed the threshold of event (3.1), i.e., $V(t) > \Xi_{n-1}(t)$, and hence there are some k and n satisfying that $\tau_k = t_n$. In this case, condition (3.5) means that the control gain must be larger than the disturbance gain, thereby counteracting the negative effect of hybrid impulses on the ISS property.

At the end of this section, the event-based impulsive strategy for decision and regulation of the P-W-S model is illustrated by Algorithm 1.

Algorithm 1 An event-based impulsive strategy for decision and regulation of the P-W-S model

Input: Step size h; initial instant t_0; total time Δ; dimension p; system parameters S, R, B, G_k, event parameters ζ_n, λ, δ; control gains K_n.

Output: The quantities of p types of goods.

Initialization with given x_0;
for t **do** t_0 to $t_0 + \Delta$
  Update the state $x(t)$ using (2.1);
  Compute the threshold by formal (3.1);
  if $V(t) \geq \Xi_{n-1}(t)$ **then**
    The event has been triggered, and record triggered instant $t_n \leftarrow t, n \leftarrow n-1$;
    Update the state $x(t)$ at triggered instant t_n with $x(t_n) = K_n x(t_n^-)$;
  else
    Update the state $x(t)$ at instant t which belongs to interval $[t_{n-1}, t_n)$;
    if $t = \tau_k$ **then**
      Update the state $x(t)$ at instant τ_k with $x(\tau_k) = G_n x(\tau_k^-)$;
    end if
  end if
end for

4. Applications

This section provides a numerical example to validate our results. Specially, consider the P-W-S model with two types of goods that are proportionally bundled, such as cameras and film, coffee and milk, and so on. The corresponding model is given as follows:

$$\begin{bmatrix} \dot{\xi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} \xi \\ \theta \end{bmatrix} + \begin{bmatrix} 0.2 & 0.2 \\ 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} sat(\xi) \\ sat(\theta) \end{bmatrix} + \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} \tanh(t) \\ \sin(t) \end{bmatrix}, \tag{4.1}$$

where ξ and θ are respectively the quantities of good I and good II. Figure 3 shows that the dynamic behavior of (4.1) is non-ISS. Our purpose here is to design ETM (3.2) for model (4.1) such that it is ISS. For the original impulses in (4.1), it is assumed that $\tau_{3k-2} = 2k - 1.5, \tau_{3k-1} = 2k - 1$, and $\tau_{3k} = 2k - 0.5$ with

$$G_{3k-2} = \begin{bmatrix} 1.1 & 0.1 \\ 0.1 & 1.1 \end{bmatrix}, \; G_{3k-1} = \begin{bmatrix} 1 & 0.2 \\ 0.1 & 1 \end{bmatrix}, \; G_{3k} = \begin{bmatrix} 0.5 & 0.2 \\ 0 & 0.5 \end{bmatrix}, \; k \in \mathbf{Z}_+.$$

Without loss of generality, we let $K_n \equiv K$, $\varrho_n \equiv \varrho$, and $\zeta_n \equiv \zeta$ for all $n \in \mathbf{Z}_+$. Given $\gamma = 3.01, l_f = 1$, $\mu_{3k-2} = 0.38$, $\mu_{3k-1} = 0.27$, $\mu_{3k} = -1$, $\varrho = 0.89$, and $\zeta = 0.3$ satisfying $\varrho - \zeta - \bar{\mu} > 0$, by solving LMI (3.12), it is deduced that

$$P = \begin{bmatrix} 3.1821 & -0.7084 \\ -0.7084 & 5.6998 \end{bmatrix}, \; U = \begin{bmatrix} 2.5210 & 1.2720 \\ 1.2720 & 3.2495 \end{bmatrix},$$
$$Q = \begin{bmatrix} 4.0662 & -2.6556 \\ -2.6556 & 4.0327 \end{bmatrix}, \; Z_n \equiv \begin{bmatrix} 1.5783 & 0.7637 \\ -1.0942 & 3.1925 \end{bmatrix},$$

from which we deduce that

$$K = \begin{bmatrix} 0.5408 & -0.0254 \\ 0.0012 & 0.5321 \end{bmatrix}.$$

Consequently, the ISS performance of (4.1) is guaranteed theoretically. Let $\lambda = 2$ and $\delta = 1.1$, the corresponding ETM is designed as

$$t_n = \inf\{t \geq t_{n-1} | V(t) \geq \exp(0.3 - 2(t - t_{n-1}))V(t_{n-1}) + 1.1\varphi(|\omega|_{[t_0,t]})\}. \tag{4.2}$$

For simulation purposes, given $\xi_0 = 2$ and $\theta_0 = 1.5$, then under ETM (4.2), the state trajectories of (4.1) and the corresponding triggered time sequence are displayed by Figure 4. Besides, based on the discussion in Subsection 3.3, Figure 5 illustrates that the triggered time sequence varies as ζ changes. Specially, the event will be triggered in a lower frequency as the value of ζ increases.

Note that it has been pointed out in Remark 5 that the ETMs containing the time-triggered mechanism in [18, 31, 32] may impose unnecessary control input and hence cause waste of resources. To support this statement, here we insert the time-triggered period $T = 1$ into ETM (4.2), and then one has

$$\begin{cases} t_n = t_n^\circ \wedge (t_{n-1} + T), \\ t_n^\circ = \inf\{t \geq t_{n-1} | V(t) \geq \exp(0.3 - 2(t - t_{n-1}))V(t_{n-1}) + 1.1\varphi(|\omega|_{[t_0,t]})\}, \end{cases} \tag{4.3}$$

under which Figure 6 displays the state trajectories of (4.1) and the corresponding triggered time sequence. Comparing Figure 4 with 6, it is clear that the total triggered number under (4.3) is 6 times more than that under (4.2). Thus, the transmission burden and control cost can be reduced significantly under our designed ETM, which corresponds with the discussion in Remark 5. Moreover, as is shown in Figure 7, the total triggered number will increase as the time-triggered period is shortened, indicating that the frequent artificial intervention should be avoided.

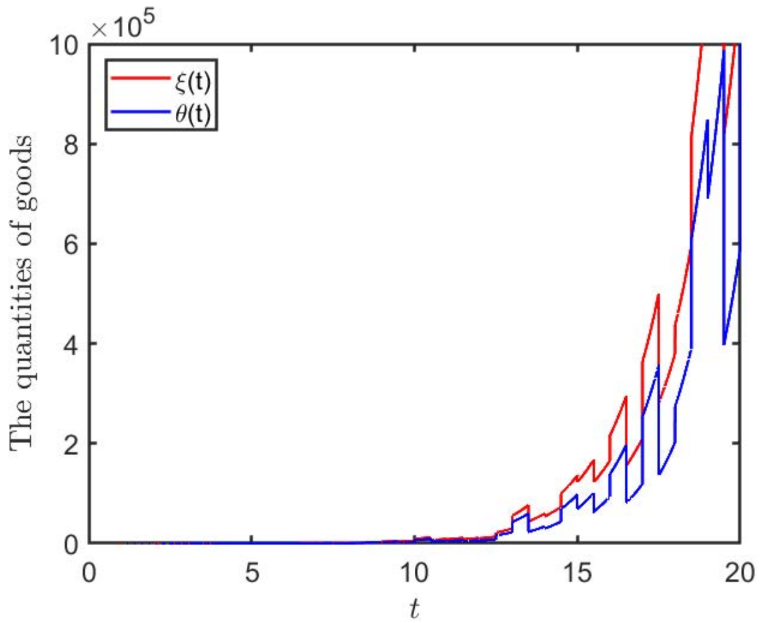

Figure 3. State trajectories of the P-W-S model (4.1) without control input.

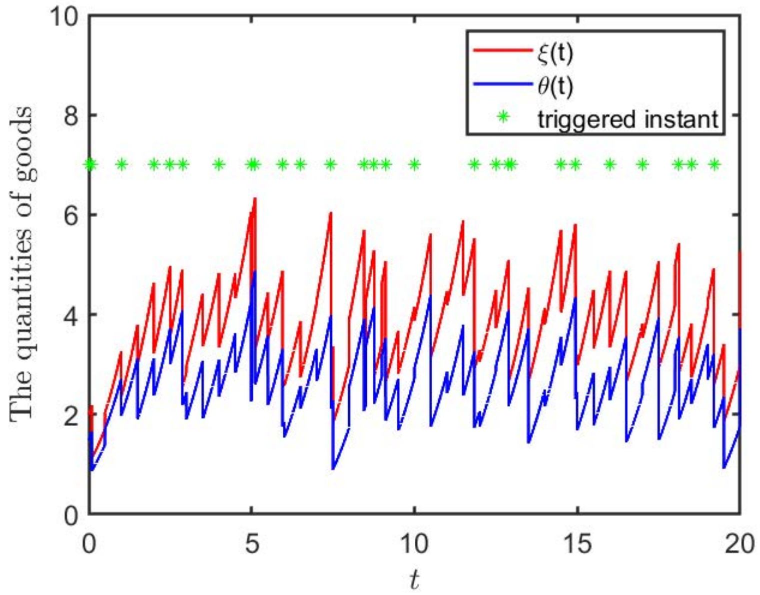

Figure 4. State trajectories of the P-W-S model (4.1) under ETM (4.2).

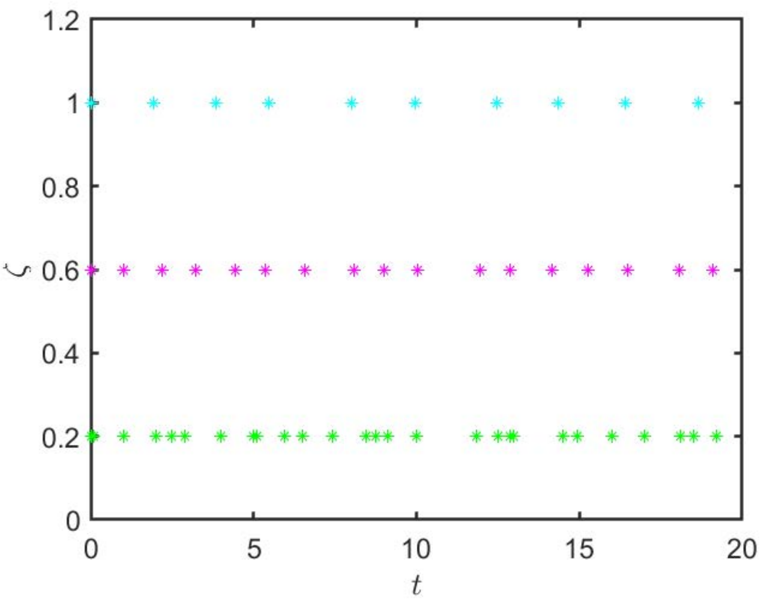

Figure 5. Triggered instants under different values of ζ.

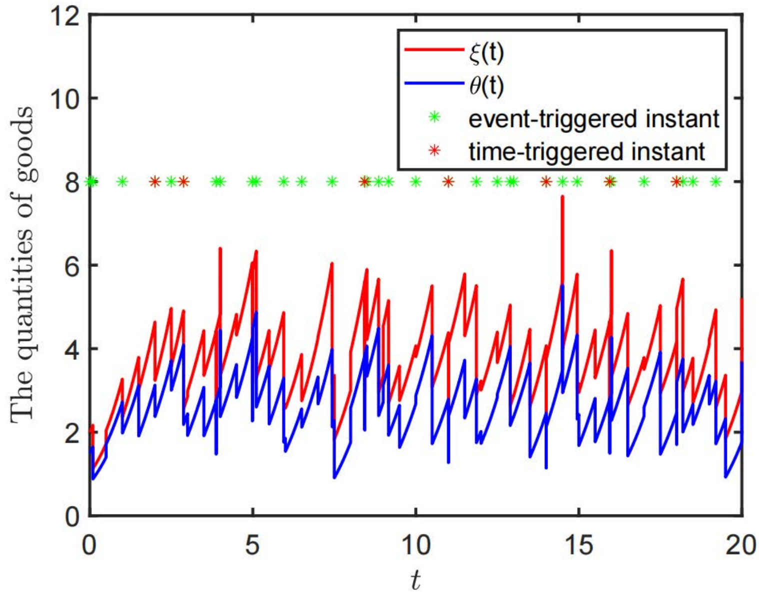

Figure 6. State trajectories of the P-W-S model (4.1) under ETM (4.3).

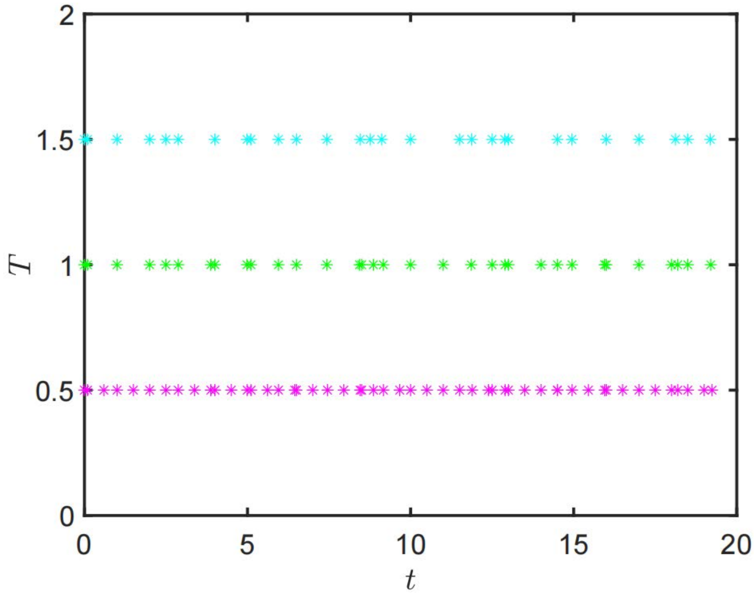

Figure 7. Triggered instants under different values of T.

5. Conclusions

The ETM and impulsive control are respectively designed for the decision and regulation purposes of the P-W-S model. Unlike traditional time-triggered control, ETM can autonomously adjust the frequency of control according to the current system state, thereby avoiding the excessive consumption of resources. By using the ISS theory, some sufficient criteria are derived, which can effectively exclude infinitely fast triggering behavior and guarantee that the quantity of goods can be maintained in a reasonable range. It is shown that our designed strategy of decision and regulation can reasonably allocate resources and dynamically adjust the relationship between production and selling according to market requirements.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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