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Research article

Synchronization of a class of nonlinear multiple neural networks with delays via a dynamic event-triggered impulsive control strategy

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Abstract: In this paper, the impulsive synchronization of a class of nonlinear multiple neural networks (MNNs) with multi-delays was considered under a dynamic event-based mechanism. To achieve a more comprehensive synchronization outcome and mitigate the conservativeness of impulsive control due to predetermined time sequences, we integrated a dynamic event-triggered strategy. This approach formed a novel control framework for generalized MNNs, where impulsive inputs were applied only under specific conditions governed by event-triggering rules. Towards the above objectives, the impulsive jumping system, resulting from dynamic component, and matrix measure method were invoked to directly increase the computational simplicity and extensibility of the study. As the outcome, the synchronization criteria for the MNNs could be achieved, and the exponential convergence rate is resolved by considering both the generalized comparison principle regarding impulsive systems and the variable parameter formula. Moreover, Zeno-freeness of the achieved triggering regulation is ensured. Finally, two numerical examples confirmed the validity of the designed approach.

Keywords: synchronization; delayed neural networks; impulsive control; dynamic event-triggered

1. Introduction

The growing requirements for synchronization of MNNs has inevitably raised higher precision in various areas of applications, such as biological systems, secure communications, chemical reaction, information processing, etc. (e.g., see [1–4]). Along with the outcome, the performance of network evolution and control immensely depends on the accuracy of the underlying model. However, the occurrence of time-varying delays is widely acknowledged as the major cause of chaos, oscillation, inadequate functionality, and occasional instability of complex networks. Such delays are prevalent in practical plants because of multiple factors like information transmission, controller update, communication bandwidth, and sampling. Neural networks suffering from delays, as shown by [5–9], could depict the practical networks more minutely and shows great superiority, leading to numerous efforts being dedicated to solving the synchronization issue of such networks [10–12].

Despite the unprecedented opportunities presented by networked systems, the restrictions in network facilities and communication channels give rise to emerging challenges that coexist with the applications. The network devices in practice are constrained in processing data, particularly for battery-powered systems. Therefore, more endeavors should be focused on the design of energy-saving control strategies to avoid continuous communication and decrease the frequency of controller updates, achieving a lower communication and computation load of the proposed control strategy in actual application. Several effective strategies for conserving resources have been introduced in the existing literature so far, such as sampled-date feedback control [7, 13], variable structure control [14, 15] and sliding mode control [16–18], and so forth. However, the existing control methods often lead to a periodic transmission of packets, including some data that may be hardly necessary. Along with this consideration, alternative control strategies, such as event-triggered schemes, have been developed and garnered a wealth of research interest [18–21]. It utilizes an appropriate event generator to determine when signals should be delivered to neighboring nodes, resulting in a remarkable reduction in wastage of network resources. Specifically, sampling in event-based control only occurs as the state-dependent system error reaches a certain level. The pioneering work of Dimarogonas was based on an event-triggered control strategy that is associated with the error between the current and previous sampling instants [22]. The information of the controller is updated using the sampling data when the measurable error surpasses the allowable level. Initially, implementing the triggering conditions with static thresholds can be beneficial in minimizing communication costs since the conditions are difficult to fulfill. Nevertheless, these conditions may become easier to meet over time, resulting in frequent triggering and unnecessary updates. As a result, there is an urgent need for scholars to explore more flexible event-triggered schemes that can not only decrease communication expense and update the frequency of controller but also ensure efficient network coordination [23, 24]. With the dynamic event-triggered scheme designed, fortunately, it provides us a direction to devote more effort for further optimization. Notice that only a limited number of research are available for the improved control strategy [25–27]. Li et al. [25] designed the synchronization controllers by incorporating a dynamic event-triggering approach involving absolute errors between system input updates, and a sufficient condition was achieved with synchronization error being exponentially ultimately bounded. In [26], a distributed dynamic event-triggered approach was initially presented utilizing an auxiliary parameter to dynamically regulate the threshold. Following this, certain criteria were developed to ensure the consensus of general linear multi-agent systems with or without a leader. The dynamic function was found to play a crucial role in regulating the triggering threshold dynamically and avoiding the Zeno behaviors in the time sequence.

In addition, it is pertinent to note that the convergence rate of systems can be affected when their state undergoes changes instantaneously at some impulsive moments, resulting in decelerated, accelerated, or non-convergence scenarios. Therefore, investigating the impact of impulsive control upon the synchronization or stability of controlled systems is of great necessity. Numerous studies have concentrated on impulsive control via a time-triggered mechanism for neural networks. However, since the impulsive controller should be designed with fixed impulsive instants, its operating time cannot be altered [28–31]. This may cause constant changes in the actuator state, leading to performance degradation and wastage of resources. To optimize performance, the control instants can be computed by dynamically event-triggered conditions. The combined approach, which inherits the advantages of both single control strategies, also compensates for the drawbacks of event-triggered control by significantly reducing control expense and the resource usage of transmitted information. That is to say, the two approaches complement each other. Therefore, some results about impulsive control based on event-triggered schemes have been proposed recently [32–35]. As an example, Lu and Li [33] examined the consensus of a simple linearity system employing a distributed event-triggered impulsive (ETI) control scheme, in which impulse is implemented in a distributed framework according to the local information. In [34], Chen et al. designed the event-triggering impulsive control scheme, where impulsive instants and event-triggered sequences are separated and independent, and the synchronization of MNNs was discussed based on this.

However, either those studies are based only on some simple systems that the delays, nonlinear terms, and characterizing the dynamical behavior are neglected, or the proposed controlling mechanism remains to be improved. All these matters above notwithstanding, the ETI control of complex systems subject to multi-delays is not yet considered and remains challenging, and a more generalized and utility-controlling theoretical method needs to be given.

Thus, the innovations and contributions of this paper can be summarized as:

1) Compared with the event-triggered control based on mathematical induction applying only to single delay linear system [33], the novel mechanism, introducing the impulsive-jumping system to improve the traditional comparison principle for the impulsive scheme, can solve the event-triggered control problem of system with multi-delays and nonlinear terms for the more practical situation.

2) In comparison with the static event-triggered control in [32], the dynamic one benefiting from the import of dynamic function $\eta(t)$ takes advantage of the resource-efficiency of the networks while guaranteeing the synchronization. Furthermore, the constraint resulting from the upper bound T^* requirement for consecutive impulse intervals can be optimized by the proposed event-triggering regulation.

3) Different from the event-triggered control based on Lyapunov function [34, 36, 37], the designed event-triggered control in terms of observation error and synchronization error shows superior feasibility, which makes the algorithms easy to implement during the realization of synchronization.

4) The p-norm and matrix measure approach is employed to handle the Lyapunov function rather than using cumbersome linear matrix inequalities, leading to the reduction of computational complexity and less conservativeness in the results.

The remainder of this paper is structured as follows: Section 2 introduces the network model and basic knowledge. The main outcomes are demonstrated in Section 3. Section 4 extends the application to memristive neural networks and Section 5 presents the numerical simulations. Finally, the conclusion is made in Section 6.

Notations: $\mathbb{R}$ is the set of real number and $\mathbb{R}^{p\times p}$ denotes the p-dimensional Euclidean space. For a vector or a matrix, the symbol $\|\cdot\|_q(q = 1, 2, \infty)$ stands for its q- norm. Let $\text{diag}\{\xi_1, \cdots, \xi_p\}$ represent the diagonal matrix with diagonal elements ξ_1 to ξ_p. The transpose of matrix Q is given as Q^T and the identity matrix in $\mathbb{R}^{p\times p}$ is denoted as I_p. For a given function $f : \mathbb{R} \to \mathbb{R}$, $f(\mathrm{t}^+) = \lim_{\mathrm{h}\to 0^+} f(\mathrm{t}+\mathrm{h})$ (or $f(\mathrm{t}^-) = \lim_{\mathrm{h}\to 0^-} f(\mathrm{t}+\mathrm{h})$) indicates the upper-left Dini derivative $D^- f(\mathrm{t}) = \lim_{\mathrm{h}\to 0^-} \sup \frac{f(\mathrm{t}+\mathrm{h})-f(\mathrm{t})}{\mathrm{h}}$ (or the upper-right Dini derivative $D^+ f(\mathrm{t}) = \lim_{\mathrm{h}\to 0^+} \sup \frac{f(\mathrm{t}+\mathrm{h})-f(\mathrm{t})}{\mathrm{h}}$).

2. Model description and preliminaries

Consider an array of coupled neural networks with nonlinear coupling and multi-delays, of which the i th neural network model can be represented as

$$\begin{aligned}\dot{\xi}_i(\mathrm{t}) =& -\mathrm{D}\xi_i(\mathrm{t}) + \mathrm{A}f(\mathrm{t}) + \mathrm{B}f(\xi_i(\mathrm{t}-\tau_1(\mathrm{t}))) + c_1\sum_{j=1}^{N} h_{ij}^1\Gamma_1 g_1(\xi_j(\mathrm{t})) \\ &+ c_2\sum_{j=1}^{N} h_{ij}^2\Gamma_2 g_2(\xi_j(\mathrm{t}-\tau_2(\mathrm{t}))) + J + u_i(\mathrm{t}),\ i = 1,2,\cdots,N,\end{aligned} \tag{2.1}$$

with $u = 1,2,\cdots,N$. $\xi_i(\mathrm{t}) = (\xi_{i1}(\mathrm{t}),\xi_{i2}(\mathrm{t}),\cdots,\xi_{in}(\mathrm{t}))^T \in \mathbb{R}^n$ represents the state variable of the i neural network; $\mathrm{D} = \mathrm{diag}\{d_1,d_2,\cdots,d_n\} > 0$ refers to the rate at which the cell returns to its resting state potential when separated from other cells and inputs; A and B respectively stand for the connection weighting matrix with and without delay; $f : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ denotes the activation function referring to a continuous vector-valued function; c_1 and c_2 are positive constants to describe the coupling strengths; The current-state and time-delay inner coupling matrices are denoted by positive defined matrices Γ_1 and Γ_2, respectively; $\tau_s(\mathrm{t}), s = 1,2$ represent the time-varying delays with $0 \le \tau_s(\mathrm{t}) \le \bar{\tau}$. $H^1 = [h_{ij}^1]_{N\times N}$ and $H^2 = [h_{ij}^2]_{N\times N}$ are the current-state and time-delay coupling matrices, respectively, which are defined as follows: $h_{ij}^s > 0$, for $s = 1,2$, if the ith node is capable of receiving the signal from the jth node, otherwise $h_{ij}^s = 0$. Moreover, the diagonal entries are redefined as $h_{ii}^s = -\sum_{j=1}^{N} h_{ij}^s$. $J = (J_1,J_2,\cdots,J_n)$ is an external input constant vector. In particular, $u_i(\mathrm{t})$ denotes an appropriate controller.

The desired trajectory $s(\mathrm{t})$ is given as follows:

$$\dot{s}(\mathrm{t}) = -\mathrm{D}s(\mathrm{t}) + \mathrm{A}f(s(\mathrm{t})) + \mathrm{B}f(s(\mathrm{t}-\tau_1(\mathrm{t}))) + J. \tag{2.2}$$

Then, the synchronization error is further represented by $\epsilon_i(\mathrm{t}) = \xi_i(\mathrm{t}) - s(\mathrm{t})$ with initial condition $\epsilon_i(\mathrm{t}) = \phi_i(\mathrm{t})-\varphi_i(\mathrm{t}) \in \mathrm{C}([-\bar{\tau},0],\mathbb{R}^n)$. Thus, we are situated to explore the stability of the ith error systems:

$$\begin{aligned}\dot{\epsilon}_i(\mathrm{t}) =& -\mathrm{D}\epsilon_i(\mathrm{t}) + \mathrm{A}\tilde{f}(\epsilon_i(\mathrm{t})) + \mathrm{B}\tilde{f}(\epsilon_i(\mathrm{t}-\tau_1(\mathrm{t}))) \\ &+ c_1\sum_{j=1}^{N} h_{ij}^1\Gamma_1\tilde{g}_1(\epsilon_j(\mathrm{t})) + c_2\sum_{j=1}^{N} h_{ij}^2\Gamma_2\tilde{g}_2(\epsilon_j(\mathrm{t}-\tau_2(\mathrm{t}))) + u_i(\mathrm{t}),\end{aligned} \tag{2.3}$$

in which $\mathrm{t} \ge 0, i = 1,2,\cdots,N$, $\tilde{f}(\epsilon_i(\mathrm{t})) = f(\xi_i(\mathrm{t}))-f(s(\mathrm{t}))$, $\tilde{f}(\epsilon_i(\mathrm{t}-\tau_1(\mathrm{t}))) = f(\xi_i(\mathrm{t}-\tau_1(\mathrm{t})))-f(s(\mathrm{t}-\tau_1(\mathrm{t})))$, $\tilde{g}_1(\epsilon_j(\mathrm{t})) = g_1(\xi_j(\mathrm{t})) - g_1(s(\mathrm{t}))$, and $\tilde{g}_2(\epsilon_j(\mathrm{t}-\tau_2(\mathrm{t}))) = g_2(\xi_j(\mathrm{t}-\tau_2(\mathrm{t}))) - g_2(s(\mathrm{t}-\tau_2(\mathrm{t})))$. The event-based impulsive controller $u_i(\mathrm{t})$ for delayed neural networks (2.1) is designed as

$$u_i(\mathrm{t}) = -K_i\epsilon_i(\mathrm{t}_{k-1}) + \sum_{k=1}^{\infty}[\mu_k\epsilon_i(\mathrm{t}) - \epsilon_i(\mathrm{t})]\delta(\mathrm{t}-\mathrm{t}_k),\ k \in N, \tag{2.4}$$

where $t_0 = 0$ is the initial instant and $\{t_1, t_2, \cdots\}$ denotes the sequence of impulsive instants, which will be decided by developed event generator later; $K_i \in \mathbb{R}^{n\times n}$ is the state feedback gain matrix, and $\delta(\cdot)$ denotes the Dirac delta function. Note that the impulsive mechanism demands the control strength $\mu_k \in (0, 1)$ which will be determined later.

Remark 1. *Noticing that the novel controller* (2.4) *consists impulsive term* $\sum_{k=1}^{\infty} [\mu_k \epsilon_i(t) - \epsilon_i(t)]\delta(t - t_k)$ *and event-triggered state feedback term* $-K_i\epsilon_i(t_{k-1})$*, the specific event-triggered condition will be employed to generate the impulsive instants for avoiding the unnecessary energy cost resulting from the preset of time series, and it determines the updating of state feedback controlling term. Previous studies on impulsive control [10, 29, 38, 39] have shown that a sufficiently high impulse frequency is required to achieve a desired convergence rate, in which case the unreasonable controlling time may result in unnecessary resource waste and poor performance. Obviously, in contrast with the traditional impulsive controllers or event-triggered controllers, in practical applications, the novel control mechanism not only makes up the above deficiency but also gives an instantaneous impulsive jump to further enhance the transient performance of the proposed controller.*

Supposing $\epsilon_i(t)$ is right-continuous, we have $\epsilon_i(t_k) = \epsilon_i(t_k^+)$ at $t = t_k$, which further means the error system' solutions exhibit jump discontinuities at $t = t_k$ in the presence of impulsive scheme. In particular, the desired trajectory (2.2) is transmitted as a system input to system (2.1), causing an instantaneous change of corresponding state variable at $t = t_k$. As a result, we obtain the following ETI error system with state feedback control input:

$$\begin{cases} \dot{\epsilon}_i(t) = -D\epsilon_i(t) + A\tilde{f}(\epsilon_i(t)) + B\tilde{f}(\epsilon_i(t - \tau_1(t))) + c_1 \sum_{j=1}^{N} h_{ij}^1 \Gamma_1 \tilde{g}_1(\epsilon_j(t)) \\ \qquad\qquad + c_2 \sum_{j=1}^{N} h_{ij}^2 \Gamma_2 \tilde{g}_2(\epsilon_j(t - \tau_2(t))) - K_i\epsilon_i(t_{k-1}), t \in [t_{k-1}, t_k), \\ \epsilon_i(t_k^+) = \mu_k \epsilon_i(t_k^-), k \in \mathbb{N}. \end{cases}$$

Letting $\tilde{D} = I_N \otimes D$, $\tilde{A} = I_N \otimes A$, $\tilde{B} = I_N \otimes B$, $\tilde{H}^1 = (H^1 \otimes \Gamma_1)$, $\tilde{H}^2 = (H^2 \otimes \Gamma_2)$, $K = \text{diag}\{K_1, K_2, \cdots, K_N\} > 0$, $\epsilon(t) = (\epsilon_1(t), \epsilon_2(t), \cdots, \epsilon_N(t))^T$, $\tilde{g}_1(\epsilon(t)) = (\tilde{g}_1(\epsilon_1(t)), \tilde{g}_2(\epsilon_2(t)), \cdots, \tilde{g}_1(\epsilon_N(t)))^T$, and $\tilde{g}_2(\epsilon(t - \tau_2(t))) = (\tilde{g}_2(\epsilon_1(t - \tau_2(t))), \tilde{g}_2(\epsilon_2(t - \tau_2(t))), \cdots, g_2(\epsilon_N(t - \tau_2(t))))^T$, the compact form of error systems can be expressed as

$$\begin{cases} \dot{\epsilon}(t) = -\tilde{D}\epsilon(t) + \tilde{A}\tilde{f}(\epsilon(t)) + \tilde{B}\tilde{f}(\epsilon(t - \tau_1(t))) + c_1\tilde{H}^1\tilde{g}_1(\epsilon(t)) \\ \qquad + c_2\tilde{H}^2\tilde{g}_2(\epsilon(t - \tau_2(t))) - K\epsilon(t_{k-1}), t \in [t_{k-1}, t_k), \\ \epsilon(t_k^+) = \mu_k\epsilon(t_k^-), k \in N. \end{cases} \tag{2.5}$$

Remark 2. *In this research, the goal is to devise the ETI control for solving the synchronization for general network systems with multi-delays. Toward this objective, several problems must be taken into account during the investigation. First, how to develop a control framework that combines the existing achievements on event-triggered control [20, 40] and impulsive control [28, 29, 39] while retaining the main advantages of each strategy. Second, due to the unpredictability of controlling instants, the extraordinary assumption of the impulsive interval is no longer valid, and it becomes impossible and unreasonable to verify the theorem conditions in terms of the generated infinite controlling time. Third, for the distributed event-triggered strategy proposed in [41, 42], every node is being updated based on the local information. Nevertheless, the ETI, with distributed form [33], may result in unsuitable* *impulses and raise great challenges for deriving the correlation between $V(t_k^-)$ and $V(t_k^+)$. Instead, in accordance with the impulsive scheme, the centralized event-triggered scheme [37, 43] should be a superior alternative to the candidate for synthesizing the innovative approach. Besides, the distributed one can be reviewed in future studies.*

Definition 1. *[44] The matrix measure of a real square matrix $Z = (z_{ij})_{N\times N}$ is defined as*

$$m_p(Z) = \lim_{h\to 0^+} \frac{\|I_N + hZ\|_p - 1}{h}, \tag{2.6}$$

where $p = 1, 2, \infty$.

Given a constant matrix Z, recall the matrix norms by $\|Z\|_1 = \max_j \sum_{i=1}^N |z_{ij}|$, $\|Z\|_2 = \sqrt{\lambda_{\max}(Z^TZ)}$, $\|Z\|_\infty = \max_j \sum_{j=1}^N$. Then, the corresponding matrix measures are derived as: $m_1(Z) = \max_j\{z_{ij} + \sum_{i=1,i\neq j}^n |z_{ij}|\}$, $m_2(Z) = (\frac{1}{2}\lambda_{\max}(Z^T + Z))$, $m_\infty = \max_i\{z_{ii} + \sum_{j=1,i\neq j}^n |z_{ij}|\}$.

Assumption 1. *The function $f(\cdot)$, $\tilde{g}_1(\cdot)$, and $\tilde{g}_2(\cdot)$ are Lipschitz continuous, i.e., there exist constants $L_1, L_2, L_3 > 0$, such that*

$$\begin{aligned} \|f(u) - f(v)\|_p &\leq L_1\|u - v\|_p, \\ \|\tilde{g}_1(u) - \tilde{g}_1(v)\|_p &\leq L_2\|u - v\|_p, \\ \|\tilde{g}_2(u) - \tilde{g}_2(v)\|_p &\leq L_3\|u - v\|_p, \end{aligned}$$

for all $u, v \in \mathbb{R}$, and $p = 1, 2, \infty$.

Lemma 1. *[28] Let $0 \leq \tau_i(t) \leq \tau$, $F(t, u, u_1, \cdots, u_m) : \mathbb{R}^+ \times \overbrace{\mathbb{R}\times\cdots\times\mathbb{R}}^{m+1} \to \mathbb{R}$ be nondecreasing in u_i for each fixed $(t, u, u_1, \cdots, u_{i-1}, u_{i+1}, \cdots, u_m)$, $i = 1, 2, \cdots, m$, and $I_k(u) : \mathbb{R} \to \mathbb{R}$ be nondecreasing in u. Suppose that $u(t), v(t) \in PC([-\tau, +\infty), \mathbb{R})$ satisfy*

$$\begin{cases} D^+u(t) \leq F(t, u(t), u(t - \tau_1(t)), \ldots, u(t - \tau_m(t))), & t \neq t_k, t \geq 0, \\ u(t_k^+) \leq I_k(u(t_k^-)), & k \in \mathbb{N}, \end{cases}$$

and

$$\begin{cases} D^+v(t) > F(t, v(t), v(t - \tau_1(t)), \ldots, v(t - \tau_m(t))), & t \neq t_k, t \geq 0 \\ v(t_k^+) \geq I_k(v(t_k^-)), & k \in \mathbb{N}. \end{cases}$$

Then $u(t) \leq v(t)$ for $-\tau \leq t < 0$ implies $u(t) \leq v(t)$ for $t \geq 0$. where $D^+u(t) = \overline{\lim}_{h\to 0^+} \frac{u(t+h)-u(t)}{h}$.

Recalling the theory of differential equations, $W(t, s) = \prod_{s<t_k<t} \mu_k e^{\int_s^t \mathfrak{f}(u)du}$ is denoted as the Cauchy matrix of linear equations:

$$\begin{cases} \dot{v}(t) = \mathfrak{f}(t)v(t), t \geq t_0, \\ v(t_k^+) = \mu_k v(t_k), k \in Z_+. \end{cases} \tag{2.7}$$

Then, the following useful lemma is presented.

Remark 3. *For Cauchy matrix* $W(t,s)=\prod_{s<t_k<t}\mu_k e^{\int_s^t \mathfrak{f}(u)du}$*, in analyzing impulsive systems, it is often necessary to solve differential or difference equations with impulsive effects. The Cauchy matrix is frequently used to solve these equations, especially in pulse systems with periodic or quasi-periodic characteristics. The Cauchy matrix provides an effective tool in the convolution operations within pulse systems, aiding in the analysis of the response of the system to impulsive inputs.*

Lemma 2. *Let* $\{t_k\}$ *be a sequence satisfying* $0\le t_0<t_1<t_2<\cdots$ *and* $\lim_{t\to\infty} t_k=\infty$*. The function* $v(\cdot)\in PC^1(R_+,R)$ *is right continuous in each* t_k*. Suppose that the following inequalities hold*

$$\begin{cases}\dot v(t)\le \mathfrak{f}(t)v(t)+\mathfrak{g}(t),\ t\ge t_0,\\ v(t_k^+)\le \mu_k v(t_k^-)+c_k,\ k\in Z_+,\end{cases} \tag{2.8}$$

where $\mathfrak{f}(\cdot),\ \mathfrak{g}(\cdot)\in C(R_+,R)$*,* $\mu_k\ge 0$ *and* c_k *are constants. Then, we have*

$$v(t)\le v(t_0)W(t,t_0)+\int_{t_0}^{t} W(t,s)\mathfrak{g}(s)ds+\sum_{t_0\le t_k<t} W(t,t_k)\mu_k,\ t\ge t_0. \tag{2.9}$$

Proof. Letting $t\in[t_0,t_1)$, according to (2.8), one gets

$$\left(v(t)e^{-\int_{t_0}^{t}\mathfrak{f}(u)du}\right)'\le \mathfrak{g}(t)e^{-\int_{t_0}^{t}f(u)du}.$$

Taking integral from t_0 to t, it has

$$v(t)\le v(t_0)e^{\int_{t_0}^{t}\mathfrak{f}(u)du}+\int_{t_0}^{t}\mathfrak{g}(s)e^{\int_s^t f(u)du}ds,\ t_0\le t<t_1,$$

therefore, (2.9) holds over the time interval $t\in[t_0,t_1]$. Assume that there exists a $n>1$, when $t\in[t_0,t_n]$, the Eq (2.9) is satisfied. Then, it can be verified for $t\in[t_n,t_{n+1})$ that

$$v(t)\le v(t_n^+)e^{\int_{t_n}^{t}\mathfrak{f}(u)du}+\int_{t_n}^{t}\mathfrak{g}(s)e^{\int_s^t \mathfrak{f}(u)du}ds,$$

recalling the inequalities (2.8), it gets

$$v(t_n^+)\le \mu_n v(t_n^-)+c_n,$$

then

$$v(t)\le (\mu_n v(t_n)+c_n)e^{\int_{t_n}^{t}\mathfrak{f}(u)du}+\int_{t_n}^{t}\mathfrak{g}(s)e^{\int_s^t \mathfrak{f}(u)du}ds.$$

By applying mathematical induction, we have

$$\begin{aligned}v(t)\le&\mu_n e^{\int_{t_n}^{t}\mathfrak{f}(u)du}\Big[v(t_0)\prod_{t_0<t_k<t_n}\mu_k e^{\int_{t_0}^{t_n}\mathfrak{f}(v)dv}\\&+\sum_{t_0<t_k<t_n}\Big(\prod_{t_k<t_j<t_n}\mu_j e^{\int_{t_k}^{t_n}\mathfrak{f}(u)du}\Big)c_k+\int_{t_0}^{t_n}\Big(\prod_{s<t_k<t_n}\mu_k e^{\int_s^{t_n}\mathfrak{f}(u)du}\mathfrak{g}(s)\Big)ds\Big]\\&+c_n e^{\int_{t_n}^{t}\mathfrak{f}(u)du}+\int_{t_n}^{t}\mathfrak{g}(s)e^{\int_s^t \mathfrak{f}(u)du}ds.\end{aligned} \tag{2.10}$$

By means of certain algebraic manipulations, it can be verified that (2.10) is the (2.9) for $t\in[t_0,t_{n+1}]$. Hence, the proof is complete.

Remark 4. *Lemma 2 proposes the form of a generalized impulsive system with $c_k \neq 0$. Obviously, it is able to characterize more complicated impulsive mechanisms than classical homogeneous impulses with $c_k = 0$ [28, 39, 43], which is much less conservative than many other reported impulsive strategy results involved. In this paper, the generalization impulsive system is first introduced to address the synchronization problem of complex networks, which may greatly expand the application of the impulsive control method such that more intricate impulsive mechanisms can be employed in future works.*

3. Main results

In this section, the ETI control schemes will be constructed to study the synchronization of general delayed MNNs with time-varying delays for the first time.

Define the measurement error of the error system as

$$\hat{\epsilon}_i(\mathrm{t}) = \epsilon_i(\mathrm{t}_{k-1}) - \epsilon_i(\mathrm{t}),\ \mathrm{t} \in [\mathrm{t}_{k-1}, \mathrm{t}_k),$$

and $\hat{\epsilon}(\mathrm{t}) = (\hat{\epsilon}_1(\mathrm{t}), \hat{\epsilon}_2(\mathrm{t}), \cdots, \hat{\epsilon}_N(\mathrm{t}))^T$.

Based on Assumption 2, the following dynamic variable $\eta(\mathrm{t})$ is proposed to satisfy the differential equation as follows:

$$\dot{\eta}(\mathrm{t}) = -\beta\eta(\mathrm{t}) - \varepsilon(\|K\|_p\|\hat{\epsilon}(\mathrm{t})\|_p - \sigma\|\epsilon(\mathrm{t})\|_p), \tag{3.1}$$

with $\eta(0) > 0$, $\beta > 0$, and appropriate parameters $\varepsilon \in [0, 1]$, $\sigma \in [0, 1]$ guaranteeing $\dot{\eta}(\mathrm{t}) < 0$. These dynamic variables are associated with the triggering laws, which will be developed in Theorem 1.

Given θ and T^* are positive constants, the triggering time sequence is determined by

$$\mathrm{t}_k = \inf\Big\{\mathrm{t} > \mathrm{t}_{k-1} : \mathrm{t} - \mathrm{t}_{k-1} > T^* \text{ or } \|\hat{\epsilon}(\mathrm{t})\|_p \geq \frac{1}{\theta(\|K\|_p + \sigma)}\eta(\mathrm{t}) + \frac{\sigma}{\|K\|_p + \sigma}\|\epsilon(\mathrm{t}_{k-1})\|_p\Big\}. \tag{3.2}$$

Remark 5. *The internal dynamic variable $\eta(\mathrm{t})$ in (3.2) is involved, of which the updating law involves the measurement error, negative self-feedback, and state error. Different from the static event-triggered scheme, the adoption of $\eta(\mathrm{t})$ in (3.2) is a vital element to adjust the triggering level adaptively. If $\eta(\mathrm{t})$ is devised to be zero, the event-triggered condition degenerates into the static one, which is a special case for (3.2). Moreover, it is not hard to prove that the triggering interval of two consecutive instants is lengthened due to the injection of $\eta(\mathrm{t})$. Moreover, it results in the lessening of triggering frequency.*

Remark 6. *In almost all impulsive schemes, the average impulsive interval T is introduced as a definition or assumption for the preset impulsive instants such that the synchronization of the complex system can be ensured. However, for event-triggered control, the triggering times are generated by the designed condition, it is unreasonable to propose the criteria involves the controlling instant t_k owing to the uncertainty, infinity, and unpredictability of event-triggering instants. The incorporation of impulsive control and event-triggering schemes inevitably possesses additional limitations. Fortunately, benefiting from the event-triggered machine, it can provide great flexibility during the construction of an appropriate triggering strategy such that some specific objectives can be achieved. For example, in [37], the triggering condition provided the upper bound of the involved time-vary delay during the controlling. Consequently, as for the unpredictable impulsive instants, the former average impulsive interval may be weakened into the triggering condition characterized as the constant T^* here.*

In what follows, the criteria of the delayed MNNs under the dynamic ETI strategy are formulated and the elimination of the Zeno phenomenon is discussed as well.

Theorem 1. *Suppose that Assumption 1 holds. Then, the delayed neural networks are synchronized via the controller* (2.4) *with dynamic event-triggered conditions* (3.2) *if there exist constants* $\gamma_1, \gamma_2, \omega > 0$ *such that*

$$(\Lambda + \frac{\ln\tilde{\mu}}{T^*}) + \gamma_1 + \gamma_2 < 0, \tag{3.3}$$

where the control gains satisfy

$$(1 - e^{-\omega T^*}) + e^{-\omega T^*}\mu_k < \mu_{k+1}, \tag{3.4}$$

where $\Lambda = \max\{(m_p(-K) + L_p\|\tilde{\mathrm{A}}\|_p + \|cH - \tilde{\mathrm{D}}\|_p),\ \frac{1-\varepsilon}{\theta} - \beta\}$, $\tilde{\mu} = \max_k\{\mu_k\}$, $\gamma_1 = L_1\|\mathrm{B}\|_p$, $\gamma_2 = L_3c_2\|\tilde{H}^2\|_p$, $\omega \geq -(\Lambda + \frac{ln\tilde{\mu}}{T^*})$, *and* $\lambda > 0$ *is a unique positive solution for* $\lambda + (\Lambda + \frac{ln\tilde{\mu}}{T^*}) + (\gamma_1 + \gamma_2)e^{\lambda\bar{\tau}} = 0$.

Proof. In accordance with the triggering condition, one has

$$\|\hat{\epsilon}(\mathrm{t})\|_p \leq \frac{1}{\theta(\|K\|_p + \sigma)}\eta(\mathrm{t}) + \frac{\sigma}{\|K\|_p + \sigma}\|\epsilon(\mathrm{t}_{k-1})\|_p,\ \mathrm{t} \in [\mathrm{t}_{k-1}, \mathrm{t}_k). \tag{3.5}$$

Moreover, recalling the Eq (3.1), we have $\dot{\eta}(\mathrm{t}) \geq -\beta\eta(\mathrm{t}) - \frac{\varepsilon}{\theta}\eta(\mathrm{t})$. Thus

$$\eta(\mathrm{t}) \geq \eta(0)e^{-(\beta+\frac{\varepsilon}{\theta})\mathrm{t}} > 0. \tag{3.6}$$

Design the Lyapunov functional as follows:

$$\tilde{V}(\mathrm{t}) = V(\mathrm{t}) + \eta(\mathrm{t}), \tag{3.7}$$

where $V(\mathrm{t}) = \|\epsilon(\mathrm{t})\|_p$, $p = 1, 2, \infty$. Computing the upper Dini derivative of (3.7) for $[\mathrm{t}_{k-1}, \mathrm{t}_k)$ along the trajectory of error system (2.5) and control input (2.4), there exists

$$\begin{aligned}
D^+\tilde{V}(\mathrm{t}) &= \lim_{\mathrm{h}\to 0^+} \frac{\|\epsilon(\mathrm{t}+\mathrm{h})\|_p - \|\epsilon(\mathrm{t})\|_p}{\mathrm{h}} + \dot{\eta}(\mathrm{t}) \\
&= \lim_{\mathrm{h}\to 0^+} \frac{\|\epsilon(\mathrm{t}) + \mathrm{h}\dot{\epsilon}(\mathrm{t}) + o(\mathrm{h})\|_p - \|\epsilon(\mathrm{t})\|_p}{\mathrm{h}} + \dot{\eta}(\mathrm{t}) \\
&= \lim_{\mathrm{h}\to 0^+} \frac{1}{\mathrm{h}}\Big\{\|\epsilon(\mathrm{t}) + \mathrm{h}(-\tilde{\mathrm{D}}\epsilon(\mathrm{t}) + \tilde{\mathrm{A}}\tilde{f}(\epsilon(\mathrm{t})) + \tilde{\mathrm{B}}\tilde{f}(\epsilon(\mathrm{t}-\tau_1(\mathrm{t}))) + c_1\tilde{H}^1\tilde{g}_1(\epsilon(\mathrm{t})) \\
&\quad + c_2\tilde{H}^2\tilde{g}_2(\epsilon(\mathrm{t}-\tau_2(\mathrm{t}))) - K\epsilon(\mathrm{t}_{k-1})) + o(\mathrm{h})\|_p - \|\epsilon(\mathrm{t})\|_p\Big\} + \dot{\eta}(\mathrm{t}) \\
&\leq \lim_{\mathrm{h}\to 0^+} \frac{\|I_n - \mathrm{h}K\|_p - 1}{\mathrm{h}}\|\epsilon(\mathrm{t})\|_p + \|\tilde{\mathrm{A}}\|_p\|f(\epsilon(\mathrm{t}))\|_p + \|\tilde{\mathrm{B}}\|_p\|f(\epsilon(\mathrm{t}-\tau_1(\mathrm{t})))\|_p \\
&\quad + \|\tilde{\mathrm{D}}\|_p\|\epsilon(\mathrm{t})\|_p + c_1\|\tilde{H}^1\|_p\|\tilde{g}_1(\epsilon(\mathrm{t}))\|_p + c_2\|\tilde{H}^2\|_p\|\tilde{g}_2(\epsilon(\mathrm{t}-\tau_2(\mathrm{t})))\|_p + \|K\|_p\|\hat{\epsilon}(\mathrm{t})\|_p + \dot{\eta}(\mathrm{t}) \\
&\leq (m_p(-K) + L_1\|\tilde{\mathrm{A}}\|_P + \|\tilde{\mathrm{D}}\|_p + c_1L_2\|\tilde{H}^1\|_p)\|\epsilon(\mathrm{t})\|_p + L_1\|\mathrm{B}\|_p\|\epsilon(\mathrm{t}-\tau_1(\mathrm{t}))\|_p \\
&\quad + L_3c_2\|\tilde{H}^2\|_p\|\epsilon(\mathrm{t}-\tau_2(\mathrm{t}))\|_p + \|K\|_p\|\hat{\epsilon}(\mathrm{t})\|_p - \beta\eta(\mathrm{t}) + \varepsilon(\sigma\|\epsilon(\mathrm{t})\|_p - \|K\|_p\|\hat{\epsilon}(\mathrm{t})\|_p) \\
&\leq (m_p(-K) + L_1\|\tilde{\mathrm{A}}\|_P + \|\tilde{\mathrm{D}}\|_p + c_1L_2\|\tilde{H}^1\|_p)\|\epsilon(\mathrm{t})\|_p + (\frac{1-\varepsilon}{\theta} - \beta)\eta(\mathrm{t})
\end{aligned}$$

$$+ L_1\|\mathrm{B}\|_p\|\epsilon(\mathrm{t}-\tau_1(\mathrm{t}))\|_p + L_3c_2\|\tilde{H}^2\|_p\|\epsilon(\mathrm{t}-\tau_2(\mathrm{t}))\|_p$$
$$\leq \Lambda\tilde{V}(\mathrm{t}) + \gamma_1\tilde{V}(\mathrm{t}-\tau_1(\mathrm{t})) + \gamma_2\tilde{V}(\mathrm{t}-\tau_2(\mathrm{t})), \tag{3.8}$$

where $\Lambda = \max\{(m_p(-K) + L_1\|\tilde{\mathrm{A}}\|_P + \|\tilde{\mathrm{D}}\|_p + c_1L_2\|\tilde{H}^1\|_p),\ \frac{1-\varepsilon}{\theta} - \beta\}$, and $\gamma_1 = L_1\|\mathrm{B}\|_p,\ \gamma_2 = L_3c_2\|\tilde{H}^2\|_p$.

Hereafter, we are devoted to developing the correlation between $V(\mathrm{t}_k^+)$ and $V(\mathrm{t}_k^-)$ which allows the outcomes to match the framework in Lemma 2. If $\mathrm{t} = \mathrm{t}_k$, there exists

$$\tilde{V}(\mathrm{t}_k^+) = V(\mathrm{t}_k^+) + \eta(\mathrm{t}_k^+) = \mu_k V(\mathrm{t}_k^-) + \eta(\mathrm{t}_k^-) = \mu_k\tilde{V}(\mathrm{t}_k^-) + (1-\mu_k)\eta(\mathrm{t}_k^-). \tag{3.9}$$

Denoting $c_k = (1-\mu_k)\eta(\mathrm{t}_k^-)$, $\forall\kappa > 0$, $v(\mathrm{t})$ is a unique solution of the novel impulsive system

$$\begin{cases} \dot{v}(\mathrm{t}) = \Lambda v(\mathrm{t}) + \gamma_1 v(\mathrm{t}-\tau_1(\mathrm{t})) + \gamma_2 v(\mathrm{t}-\tau_2(\mathrm{t})) + \kappa, & \mathrm{t} \neq \mathrm{t}_k, \\ v(\mathrm{t}_k) = \mu_k v(\mathrm{t}_k^-) + c_k, & \mathrm{t} = \mathrm{t}_k, \\ v(s) = \|\epsilon(s)\|_p, & -\tau \leq s \leq 0. \end{cases}$$

From Lemma 1, one gets $v(\mathrm{t}) \geq \tilde{V}(\mathrm{t}) \geq 0$, $\forall\mathrm{t} \geq 0$. Applying the result of Lemma 2, it can be obtained that

$$v(\mathrm{t}) = W(\mathrm{t},0)v(0) + \int_0^{\mathrm{t}} W(\mathrm{t},s)[\gamma_1 v(s-\tau_1(s)) + \gamma_2 v(s-\tau_2(s)) + \kappa]ds + \sum_{\mathrm{t}_0\leq\mathrm{t}_k<\mathrm{t}} W(\mathrm{t},\mathrm{t}_k)c_k, \tag{3.10}$$

in which $W(\mathrm{t},s)$ ($\mathrm{t}, s \geq 0$) refer to the Cauchy matrix of the corresponding linear impulsive system.

In light of the formulation of the Cauchy matrix, we have the below inequality:

$$W(\mathrm{t},s) = \prod_{s<\mathrm{t}_k<\mathrm{t}} \bar{\mu}e^{\int_s^{\mathrm{t}}\Lambda dv} \leq e^{\Lambda(\mathrm{t}-s)}\cdot\bar{\mu}^{\frac{\mathrm{t}-s}{T^*}} = e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(\mathrm{t}-s)}.$$

Let $q = \sup_{-\bar{\tau}\leq s\leq 0}\{\|\epsilon(s)\|_p\}$. From (3.10), it gets

$$v(\mathrm{t}) \leq qe^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})\mathrm{t}} + \int_0^{\mathrm{t}} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(\mathrm{t}-s)}\Big[\gamma_1 v(s-\tau_1(s)) + \gamma_2 v(s-\tau_2(\mathrm{t})) + \kappa\Big]ds + \sum_{\mathrm{t}_0\leq\mathrm{t}_k<\mathrm{t}} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(\mathrm{t}-\mathrm{t}_k)}c_k. \tag{3.11}$$

Let $h(\lambda) = \lambda + (\Lambda + \frac{ln\tilde{\mu}}{T^*}) + (\gamma_1+\gamma_2)e^{\lambda\bar{\tau}}$. Utilizing the condition in Theorem 1, we have

$$h(0^+) = (\Lambda + \frac{ln\tilde{\mu}}{T^*}) + \gamma_1 + \gamma_2 < 0. \tag{3.12}$$

Obviously, we have $h(+\infty) > 0$ and $h'(\lambda) = 1 + \bar{\tau}(\gamma_1+\gamma_2)e^{\lambda\bar{\tau}} > 0$. Therefore, there is a only positive solution $\lambda > 0$ of the equation $h(\lambda) = 0$.

Thereafter, we will prove that the following inequality is satisfied for $\forall\mathrm{t} \geq 0$

$$v(\mathrm{t}) < qe^{-\lambda\mathrm{t}} + \sum_{\mathrm{t}_0\leq\mathrm{t}_k<\mathrm{t}} e^{-\lambda\mathrm{t}+\omega\mathrm{t}_k}c_k + \frac{\kappa}{-(\Lambda+\frac{ln\tilde{\mu}}{T^*}) - \gamma_1 - \gamma_2}, \tag{3.13}$$

where $(\Lambda + \frac{ln\tilde{\mu}}{T^*}) \geq -\omega$.

If it does not hold, a positive constant t^* can be found with

$$v(t^*) \geq qe^{-\lambda t^*} + \sum_{t_0 \leq t_k < t^*} e^{-\lambda t+\omega t_k} c_k + \frac{\kappa}{-(\Lambda + \frac{ln\tilde{\mu}}{T^*}) - \gamma_1 - \gamma_2}. \tag{3.14}$$

$$v(t) < qe^{-\lambda t} + \sum_{t_0 \leq t_k < t} e^{-\lambda t+\omega t_k} c_k + \frac{\kappa}{-(\Lambda + \frac{ln\tilde{\mu}}{T^*}) - \gamma_1 - \gamma_2}, \; t < t^*. \tag{3.15}$$

From (3.11) and (3.15), it holds that

$$\begin{aligned}
v(t^*) \leq & qe^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})t^*} + \int_0^{t^*} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(t^*-s)}\Big[\gamma_1 v(s-\tau_1(s)) + \gamma_2 v(s-\tau_2(s)) + \kappa\Big]ds \\
& + \sum_{t_0 \leq t_k < t^*} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(t^*-t_k)} c_k \\
< & qe^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})t^*} + e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})t^*} \int_0^{t^*} e^{-(\lambda+(\Lambda+\frac{ln\tilde{\mu}}{T^*}))s}(\gamma_1+\gamma_2)[qe^{\lambda\bar{\tau}} + \sum_{t_0 \leq t_k < t^*} e^{-\lambda(s-\bar{\tau})+\omega t_k} c_k]ds \\
& + \frac{\kappa}{-(\Lambda + \frac{ln\tilde{\mu}}{T^*}) - \gamma_1 - \gamma_2} \int_0^{t^*} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(t^*-s)}(\gamma_1+\gamma_2)ds + \kappa \int_0^{t^*} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(t^*-s)}ds \\
& + \sum_{t_0 \leq t_k < t^*} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(t^*-t_k)} c_k \\
\leq & qe^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})t^*} + qe^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})t^*} \int_0^{t^*} e^{-(\lambda+(\Lambda+\frac{ln\tilde{\mu}}{T^*}))s}[-(\lambda + (\Lambda + \frac{ln\tilde{\mu}}{T^*}))]ds + \sum_{t_0 \leq t_k < t^*} e^{-\lambda t^*+\omega t_k} c_k \\
& - \sum_{t_0 \leq t_k < t^*} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})t^*+\omega t_k} c_k + \sum_{t_0 \leq t_k < t^*} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(t^*-t_k)} c_k \\
& + \frac{-\kappa(\Lambda + \frac{ln\tilde{\mu}}{T^*})}{-(\Lambda + \frac{ln\tilde{\mu}}{T^*}) - \gamma_1 - \gamma_2} \int_0^{t^*} e^{(\Lambda+\frac{ln\tilde{\mu}}{T^*})(t^*-s)}ds \\
< & qe^{-\lambda t^*} + \sum_{t_0 \leq t_k < t^*} e^{-\lambda t^*+\omega t_k} c_k + \frac{\kappa}{-(\Lambda + \frac{ln\tilde{\mu}}{T^*}) - \gamma_1 - \gamma_2},
\end{aligned}$$

which results in a contradiction with (3.14). Thus, the inequality (3.13) is satisfied. Letting $\kappa \to 0$, it holds for $t \geq 0$ that

$$\|e(t)\|_p \leq \tilde{V}(t) \leq v(t) \leq qe^{-\lambda t} + \sum_{t_0 \leq t_k < t} e^{-\lambda t+\omega t_k} c_k, \quad t \geq 0,$$

which indicates

$$\|e(t)\|_p \leq \sup_{-\bar{\tau} \leq s \leq 0} \|\epsilon(s)\|_p e^{-\lambda t} + \sum_{t_0 \leq t_k < t} e^{-\lambda t+\omega t_k} c_k.$$

Recalling the differential Eq (3.1), it has

$$\eta(t) = e^{-\beta t}\Big(\eta(0) + \int_0^t e^{\beta s}(\varepsilon(\|K\|_p\|\hat{\epsilon}(s)\|_p - \sigma\|\epsilon(s)\|_p))ds\Big).$$

Noticing the noincreasing of dynamic function $\eta(\cdot)$, the impulsive gain μ_{k+1} for the event-triggered instant $\mathrm{t} = \mathrm{t}_{k+1}$ can be designed such that

$$\frac{(1-\mu_{k+1})\eta(\mathrm{t}_{k+1})}{(1-\mu_k)\eta(\mathrm{t}_k)} < e^{-\omega T^*} < 1,$$

then, we can have

$$(1-e^{-\omega T^*}) + e^{-\omega T^*}\mu_k < \mu_{k+1}. \tag{3.16}$$

Finally, for the constant $M > 0$, one has

$$\|e(\mathrm{t})\|_p \le (\sup_{-\bar{\tau}\le s\le 0} \|\epsilon(s)\|_p + M)e^{-\lambda \mathrm{t}}. \tag{3.17}$$

The theorem is proved.

Remark 7. *Note from the condition* (3.4) *that the sequential impulsive gain* μ_{k+1} *is closely related to the system parameter term* Λ*, former impulsive strength* μ_k *and the length of* T^**. Obviously, each impulsive instant gives us a guideline for balancing the selection of impulsive gain* μ_k *to obtain a suitable hybrid impulsive controller. Just event-triggering rules may not guarantee the realization of synchronization for underlying networks, some restricted conditions should be additionally put forward for certain objectives. For example, as a compromise, there exists a constant* ρ_2 *which satisfies* $\rho_2 \ge \sup_{k\in\mathbb{N}}\{\mathrm{t}_k - \mathrm{t}_{k-1}\}$ *for triggering instants in [32], and the authors choose a suitable parameter* β *such that* $\max_{\forall k}\{e^{\ln(\beta\gamma)+\lambda(\mathrm{t}_{k+1}-\mathrm{t}_k)}\} < \alpha < 1$ *in [36].*

Remark 8. *Different from previous works, the existence of dynamic function* $\eta(\mathrm{t})$ *greatly improves the challenges of the considered problem, which not only induces the constants* c_k*, in each impulsive instants, turning the traditional impulsive system into impulsive jumping systems but also should promise the monotone decreasing of dynamic triggering function that* $\dot{\eta} < 0$ *at the same time.*

The following theorem will demonstrate that the proposed control scheme gets rid of the Zeno-phenomenon, which means $\mathrm{t}_{k+1} - \mathrm{t}_k > 0$ can be guaranteed with a lower bound of interval times.

Theorem 2. *Supposed that criteria* (3.3) *and* (3.4) *hold, then the time interval* T_k *exists and satisfies the following*

$$\mathrm{t}_k = \mathrm{t}_{k+1} - \mathrm{t}_k \ge \sigma > 0. \tag{3.18}$$

That is to say, the Zeno phenomenon of the proposed control scheme is excluded.

Proof. According to (3.17), we have for $\mathrm{t} \in [\mathrm{t}_{k-1}, \mathrm{t}_k)$ that

$$\begin{aligned}
&\|\epsilon(\mathrm{t}_{k-1})\|_p \le (\sup_{-\bar{\tau}\le s\le 0} \|\epsilon(s)\|_p + M)e^{-\lambda \mathrm{t}_{k-1}},\\
&\|\epsilon(\mathrm{t}-\tau_1(\mathrm{t}))\|_p = \|\epsilon(\mathrm{t})\|_p + \xi_1(\mathrm{t}) \le (\sup_{-\bar{\tau}\le s\le 0} \|\epsilon(s)\|_p + M)e^{-\lambda \mathrm{t}_{k-1}},
\end{aligned}$$

where $\xi(\mathrm{t}) \ge 0$ is associated with $\tau(\mathrm{t})$.

Taking derivation of $\hat{\epsilon}(\mathrm{t})$ for $[\mathrm{t}_{k-1}, \mathrm{t}_k), k \in \mathbb{N}$, yields

$$\begin{aligned}
D^+\|\epsilon(\mathrm{t})\| &\leq \|\dot{\epsilon}(\mathrm{t})\| \\
&= \| - \tilde{\mathrm{D}}\epsilon(\mathrm{t}) + \tilde{\mathrm{A}}\tilde{f}(\epsilon(\mathrm{t})) + \tilde{\mathrm{B}}\tilde{f}(\epsilon(\mathrm{t} - \tau_1(\mathrm{t}))) + c_1\tilde{H}^1\tilde{g}_1(\epsilon(\mathrm{t})) + c_2\tilde{H}^2\tilde{g}_2(\epsilon(\mathrm{t} - \tau_2(\mathrm{t}))) - K\epsilon(\mathrm{t}_{k-1})\| \\
&\leq (\|\tilde{\mathrm{D}}\| + c_1L_2\tilde{H}^1)\|\epsilon(\mathrm{t})\| + L_1\|\tilde{\mathrm{A}}\|\|\epsilon(\mathrm{t})\| + L_1\|\tilde{\mathrm{B}}\|\|\epsilon(\mathrm{t} - \tau_1(\mathrm{t}))\| + c_2L_3\tilde{H}^2\|\epsilon(\mathrm{t} - \tau_2(\mathrm{t}))\| + \|K\|\|\epsilon(\mathrm{t}_{k-1})\| \\
&\leq (\|cH - \tilde{\mathrm{D}}\| + L_f\|\tilde{\mathrm{A}}\| + L_f\|\tilde{\mathrm{B}}\|)\|\hat{\epsilon(\mathrm{t})}\| + (\|cH - \tilde{\mathrm{D}}\| + L_f\|\tilde{\mathrm{A}}\| + L_f\|\tilde{\mathrm{B}}\| + \|K\|)\|\epsilon(\mathrm{t}_{k-1})\| + L_f\|\tilde{\mathrm{B}}\|\xi(\mathrm{t}) \\
&= \Upsilon\|\hat{\epsilon(\mathrm{t})}\| + \tilde{C}(\mathrm{t}),
\end{aligned}$$

where $\Upsilon = \|cH - \tilde{\mathrm{D}}\| + L_f\|\tilde{\mathrm{A}}\| + L_f\|\tilde{\mathrm{B}}\|$, $\tilde{C}(\mathrm{t}) = (\|cH - \tilde{\mathrm{D}}\| + L_f\|\tilde{\mathrm{A}}\| + L_f\|\tilde{\mathrm{B}}\| + \|K\|)\|\epsilon(\mathrm{t}_{k-1})\| + L_f\|\tilde{\mathrm{B}}\|\xi(\mathrm{t})$.

Therefore, we can solve the equality with the initial condition $\hat{\epsilon}(\mathrm{t}_k) = 0$ for $\mathrm{t} \in [\mathrm{t}_{k-1}, \mathrm{t}_k)$ as follows:

$$\|\hat{\epsilon}(\mathrm{t})\| \leq \frac{\tilde{C}(\mathrm{t})}{\Upsilon}[e^{\Upsilon(\mathrm{t}-\mathrm{t}_{k-1})} - 1].$$

By virtue of the dynamic triggering condition, when $\mathrm{t} = \mathrm{t}_k$ we have

$$\frac{1}{\theta(\|K\|_p + \sigma)}\eta(\mathrm{t}) + \frac{\sigma}{\|K\|_p + \sigma}\|\epsilon(\mathrm{t}_{k-1})\| \leq \frac{\tilde{C}(\mathrm{t})}{\Upsilon}[e^{\Upsilon(\mathrm{t}_k-\mathrm{t}_{k-1})} - 1],$$

which means that

$$\mathrm{t}_k = \mathrm{t}_k - \mathrm{t}_{k-1} \geq \frac{1}{\Upsilon}\ln[1 + \frac{\Upsilon(\eta(\mathrm{t}) + \theta\sigma\|\epsilon(\mathrm{t}_{k-1})\|)}{\theta(\|K\| + \sigma)\tilde{C}(\mathrm{t})}].$$

Thus $\tilde{C}(\mathrm{t}) \leq (\sup_{-\bar{\tau}\leq s\leq 0} \|\epsilon(s)\|_p + M)(\|cH - \tilde{\mathrm{D}}\| + L_f\|\tilde{\mathrm{A}}\| + L_f\|\tilde{\mathrm{B}}\| + \|K\| + L_f\|\tilde{\mathrm{B}}\|) = \tilde{C}$. It follows that

$$\mathrm{t}_k \geq \frac{1}{\Upsilon}\ln[1 + \frac{\Upsilon(\eta(\mathrm{t}) + \theta\sigma\|\epsilon(\mathrm{t}_{k-1})\|)}{\theta(\|K\| + \sigma)\tilde{C}}].$$

Hence, for $\mathrm{t} \in [\mathrm{t}_{k-1}, \mathrm{t}_k), k \in \mathbb{N}$,

$$\mathrm{t}_k \geq \frac{1}{\Upsilon}\ln[1 + \frac{\Upsilon(\eta(\mathrm{t}_k) + \theta\sigma\|\epsilon(\mathrm{t}_{k-1})\|)}{\theta(\|K\| + \sigma)\tilde{C}}] > 0.$$

Thus, the lower bound of inter-execution time exists and satisfies $T_k = \mathrm{t}_k - \mathrm{t}_{k-1} \geq \sigma > 0$. The completes the proof.

Remark 9. *It can be found that according to property of dynamic function* $\eta(\mathrm{t})$*, only when the synchronization comes true with error state* $\epsilon(\mathrm{t}) = 0$*,* $\epsilon(\mathrm{t}_{k-1}) = 0$ *and the dynamic function decreasing to* $\eta(\mathrm{t}) = 0$*, the Zeno phenomenon can emerge. That is to say, the Zeno-behavior is ruled out for error system* (2.5).

The proposed control scheme is capable of improving the traditional impulsive. When the interval T^* refers to the average impulsive interval T, with event-triggered strategy removing, it is easy to obtain the following result.

Corollary 1. *Suppose that the Assumption 1 holds. Then, the delayed MNNs are synchronized via the controller* (2.4) *with the dynamic event-triggering rule* (3.2)*, if*

$$(\Lambda + \frac{\ln\tilde{\mu}}{T^*}) + \gamma_1 + \gamma_2 < 0, \tag{3.19}$$

and selecting

$$-(\Lambda + \frac{\ln\tilde{\mu}}{T^*}) < \omega < \lambda,$$

where $\Lambda = m_p(-K) + L_p\|\tilde{\mathrm{A}}\|_p + \|cH - \tilde{\mathrm{D}}\|_p$, $\tilde{\mu} = \max_k\{\mu_k\}$, $\gamma_1 = L_1\|\mathrm{B}\|_p, \gamma_2 = L_3 c_2\|\tilde{H}^2\|_p$ *and* $\lambda > 0$ *is a unique positive solution of* $\lambda + (\Lambda + \frac{ln\tilde{\mu}}{T^*}) + (\gamma_1 + \gamma_2)e^{\lambda\bar{\tau}} = 0$.

Remark 10. *If the dynamic event-triggered mechanism is separated, the investigation becomes the traditional impulsive synchronization problem. The corollary not only contains some correlational previous problems as a special case but also, with the introduction of p-norm, the simpler and easier verifiable criterion is achieved compared with some former achievement [28, 39].*

4. Numerical simulation

In this section, we present two numerical simulations to illustrate the efficiency of the theoretical results obtained in this paper.

Example 1. The effectiveness of Theorem 1 is verified in this section. To illustrate the efficiency of the proposed theoretical results, the following cellular neural network is considered the leader

$$\begin{bmatrix}\dot{s}_1(\mathrm{t})\\ \dot{s}_2(\mathrm{t})\end{bmatrix} = -\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}s_1(\mathrm{t})\\ s_2(\mathrm{t})\end{bmatrix} + \begin{bmatrix}1+\pi/4 & 20\\ 0.1 & 1+\pi/4\end{bmatrix}\begin{bmatrix}f_1(s_1(\mathrm{t}))\\ f_2(s_2(\mathrm{t}))\end{bmatrix} + \begin{bmatrix}-\sqrt{2}\pi\frac{13}{40} & 0.1\\ 0.1 & -\sqrt{2}\pi\frac{13}{40}\end{bmatrix}\begin{bmatrix}f_1(s_1(\mathrm{t}-\tau_1(\mathrm{t})))\\ f_2(s_2(\mathrm{t}-\tau_1(\mathrm{t})))\end{bmatrix}$$

where $f_i(s_i) = (|s_i + 1| - |s_i - 1|)/2$, $i = 1, 2$. It is clearly that $\Pi = \mathrm{diag}\{1, 1\}$ [39]. Then, to validate the feasibility of Theorem 1, we consider a networked multi-agent system of six agents, and the coupled cellular neural network satisfies (2) with

$$\mathrm{D} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \quad \mathrm{A} = \begin{bmatrix}1+\pi/4 & 20\\ 0.1 & 1+\pi/4\end{bmatrix}, \quad \mathrm{B} = \begin{bmatrix}-\sqrt{2}\pi\frac{13}{40} & 0.1\\ 0.1 & -\sqrt{2}\pi\frac{13}{40}\end{bmatrix}.$$

It is obviously the Lipschitz condition in Assumption 1 is computed as $L = 0.6I_3$. Assume that $\Gamma_1 = \Gamma_2 = \Gamma_3 = diag\{1, 1\}$ and $\tau_1(\mathrm{t}) = \tau_2(\mathrm{t}) = 0.1\frac{e^{\mathrm{t}}}{e^{\mathrm{t}}+1}$. The current-state coupling and time-delay can be respectively chosen as

$$H1 = \begin{pmatrix}-2 & 1 & 0 & 1 & 0 & 0\\ 0 & -1 & 1 & 0 & 0 & 0\\ 1 & 0 & -3 & 1 & 0 & 1\\ 0 & 0 & 0 & -2 & 1 & 0\\ 0 & 0 & 0 & 0 & -1 & 1\\ 1 & 0 & 0 & 0 & 0 & -3\end{pmatrix}, \; H2 = \begin{pmatrix}-1 & 0 & 0 & 0 & 10 & 0\\ 0 & -2 & 1 & 1 & 0 & 0\\ 0 & 0 & -2 & 0 & 0 & 1\\ 1 & 0 & 1 & -2 & 0 & 0\\ 0 & 0 & 0 & 0 & -1 & 1\\ 0 & 0 & 1 & 0 & 0 & -1\end{pmatrix}.$$

Without the lost of general, the nonlinear coupling function $g_1(x) = g_2(x) = 0.32x - 0.295(|x+1| - |x-1|)$, coupling strength $c_1 = c_2 = 0.5$, and the state feedback control gain matrix $K = 10 * \text{diag}\{1, 1\}$. Considering the 2-norm of the vector or matrix in the simulations, therefore, we have $m_p(-K) = -10$. Noticing that the upper impulsive bound T^*, $\gamma_1 = 1.539L1$ and $\gamma_2 = 1.5432L_3$ can be calculated to satisfy the condition (13) in Theorem 1. By solving the inequality, we can set the controlling interval $T^* = 20$ and maximum impulsive gain $\tilde{\mu} = 0.943$. Therefore, conditions (13) and (14) can be satisfied. To clearly demonstrate the complexity of our algorithm, the method of choosing the parameters in the controller can refer to Algorithm 1.

Algorithm 1 Dynamic event-triggered impulsive control implementation

Require: Give the initial values $\xi_i(t)$, $s(t)$, $\eta(t)$ for all $i = 1, 2, \cdots, N$, $t_0 = 0$, $k = 0$. Set t, sampling time h, and simulation time T;

1: Chose the parameters β, ε, σ, θ, T^*, control feedback parameters K, μ_k;

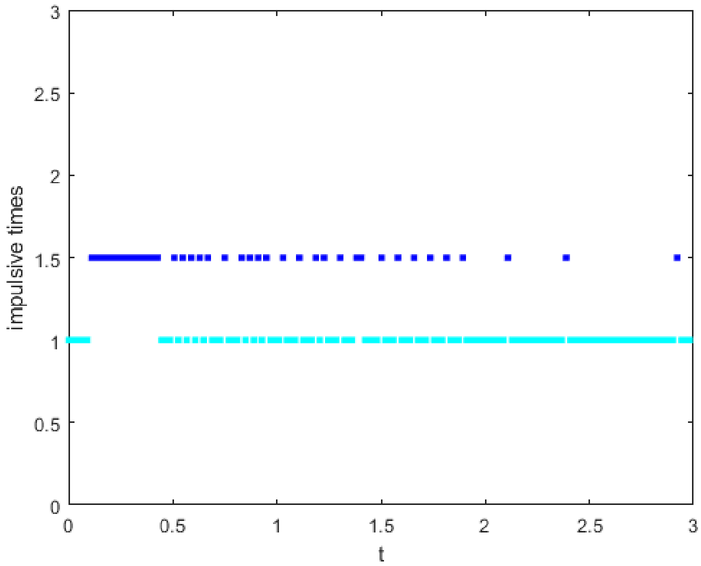
Ensure:

2: **if** Assumption 1 holds simultaneously, **then**
3:     go to Line 7
4: **else**
5:     go to Line 1
6: **end if**
7: **for** $0 < t < T$ **do**
8:     **if** $t = 0$; **then**
9:         Compute the value of $\Lambda = \max\{(m_p(-K) + L_p\|\tilde{\mathrm{A}}\|_p + \|cH - \tilde{\mathrm{D}}\|_p), \frac{1-\varepsilon}{\theta} - \beta\}$, $\tilde{\mu} = \max_k\{\mu_k\}$, $\gamma_1 = L_1\|\mathrm{B}\|_p, \gamma_2 = L_3 c_2\|\tilde{H}^2\|_p$, $\omega \geq -(\Lambda + \frac{\ln\tilde{\mu}}{T^*})$, and $\lambda > 0$ is a unique positive solution for $\lambda + (\Lambda + \frac{ln\tilde{\mu}}{T^*}) + (\gamma_1 + \gamma_2)e^{\lambda\bar{\tau}} = 0$, and control input u_i(t) in (2.4) using the initial value ϵ_i(t);
10:     **else**
11:         **if** event-triggered condition (3.2) holds; **then**
12:             $k = k + 1$;
13:             t_k = t;
14:         **else**
15:             t_k keeps its value of k-triggering;
16:         **end if**
17:         Compute the impulsive gain μ_k in (2.4);
18:         Compute the control input in (2.4);
19:     **end if**
20:     t = t + h;
21: **end for**

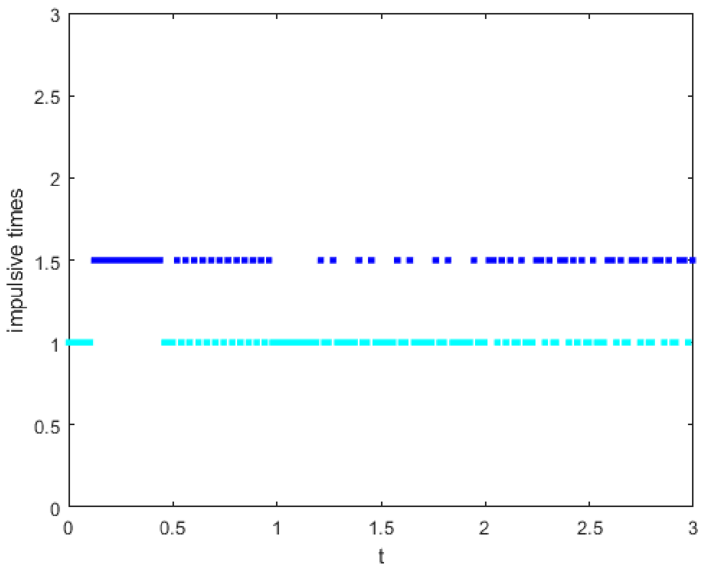
Next, we will show the dynamic state-triggering rule (3.2) revolution with $\theta = 10$, $\sigma = 1$ and dynamic function $\eta_i(0) = 10$. The dynamic ETI instants are given in Figure 1(a) and static ETI one [33, 37, 43] in Figure 1(b), in which the value equal to 1.5 refers to the triggering generated from the upper triggering bound T^* in blue and 1 resulting from the error triggering condition in pale green. Obviously, compared with the static triggering condition with the same parameters as the dynamic one, it is shown that the dynamic one can significantly decrease the triggering number, especially behind t = 2. Figure 2(a) shows the state trajectories of the leader and the following coupled neural networks. With the event-based impulsive controller is replaced by

$$u_i(\mathrm{t}) = -K\epsilon_i(\mathrm{t}_{k-1}) + \sum_{k=1}^{\infty}[\mu\epsilon_i(\mathrm{t}) - \epsilon_i(\mathrm{t})]\delta(\mathrm{t}-\mathrm{t}_k),\ k \in N, \tag{4.1}$$

the error system of (2.5) under the designed control is shown in Figure 2(b). Obviously, the exponential synchronization of the coupled neural networks and the state leader is achieved with multi-delays and nonlinear terms.

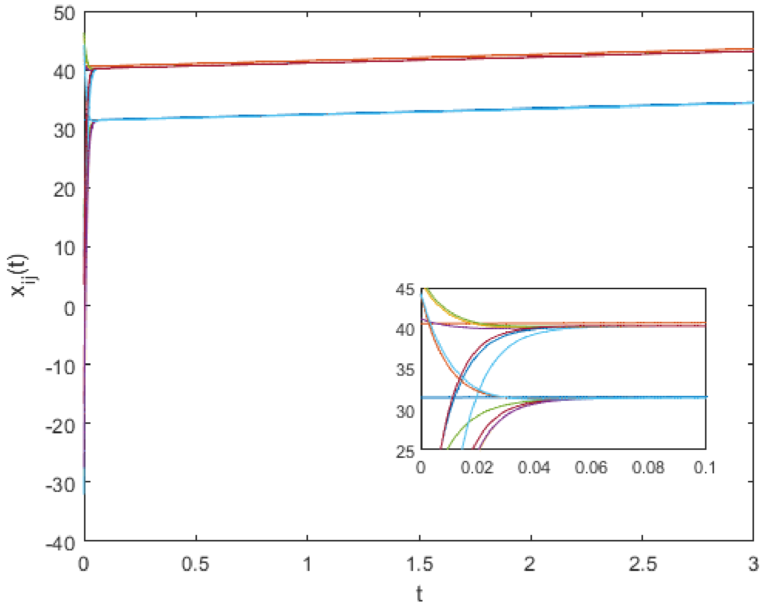


(a) Triggering time sequences of dynamic event-triggered strategy.

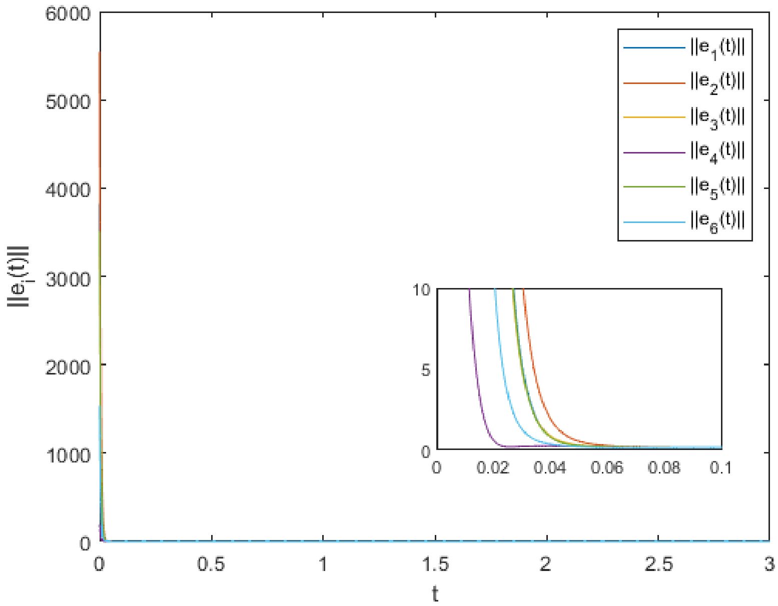


(b) Triggering time sequences of static event-triggered strategy.

Figure 1. Triggering time sequences of Example 1 with 6 coupled neural networks.

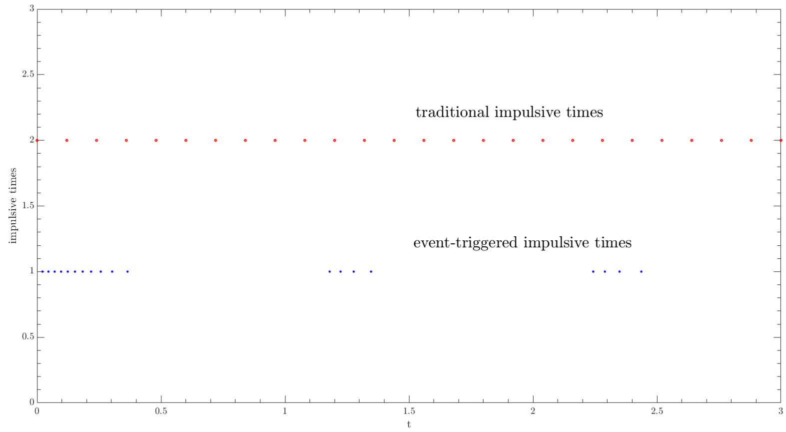


(a) State trajectories of six coupled neural networks in Example 1.

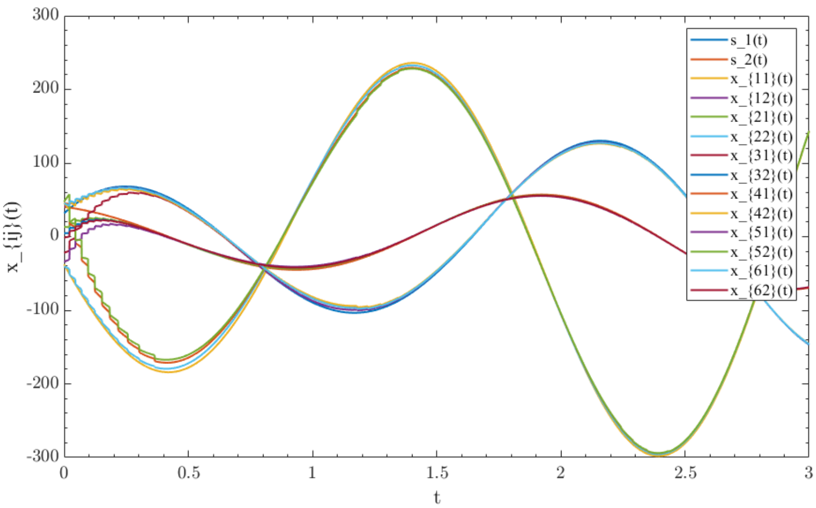


(b) State Trajectories of the error system of the neural networks with the leader.

Figure 2. State Trajectories of Example 1 with 6 coupled neural networks.



(a) Impulsive instants for event-triggered impulsive control and single impulsive control.



(b) Each node's and the leader's evolution of Chua's circuits model.

Figure 3. The simulations of Example 2 with Chua's circuits model.

Example 2. To demonstrate the effectiveness of the dynamic EIC method in practical applications. Consider the Chua's circuits model, of which the dynamic is described by

$$\begin{cases} \dot{s}_1(t) = a(s_2(t) - G(s_1(t))), \\ \dot{s}_2(t) = s_1(t) - s_2(t) + s_3(t), \\ \dot{s}_3(t) = -bs_2(t), \end{cases} \tag{4.2}$$

where $G(s_1(t)) = 0.32x - 0.295(|s_1(t)+1| - |s_1(t)-1|)$, $a = 10$, $b = 14.87$. Without loss of generality, we consider the system (4.2) as the leader of the networked system. Therefore, we can obtain the system parameters in the following

$$A = \begin{pmatrix} -0.32a & -a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{pmatrix}, B = \begin{pmatrix} -a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The nonlinear function $f(s_1(t)) = -0.295(|s_1(t) + 1| - |s_1(t) - 1|)$. Thus the Lipschitz condition in Assumption 1 can be calculated as $L = 0.6I_3$. To verify the feasibility of Theorem 1, we consider a networked system of six agents, where the dynamic of each agent is depicted by the Chua's circuits (4.2), the coupling delay $\tau_0 = 0.2$, coupling strength $d = 0.5$, and the coupling matrix can be chosen as

$$H = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}.$$

Basing simulation of example 1 and taking the same parameters of controller and event-triggered method, some similar results can be obtained. Obviously, the exponential synchronization of Chua's circuits model is achieved under the event-triggered impulsive control, and the state trajectories of the leader and following agents are shown in Figure 3(a). In addition, the event-triggered impulsive instants and traditional impulsive instants are given in Figure 3(b). It should be noted that if impulsive instants are set before, some impulsive instants are unnecessary, and the instants sequence has to be unrelenting owing to the unknown terminal time.

5. Conclusions

Basing the traditional impulsive control theory, this paper combines the dynamic event-triggered rule to further improve control performance and reduce impulse consumption, of which the impulsive instants are generated according to the predesigned event-triggering condition. Moreover, the synchronization problem of coupled neural networks with nonlinear and multi-delays is investigated to enhance the applicability and universality of the outcome. By achieving the solution and impulsive jumping system and introducing measure measure method, the dynamic event-based impulsive mechanism is successfully constructed and applied to receive some sufficient conditions of the considered synchronization problem, and the Zeno behavior has been excluded in the proposed strategy. Finally, two numerical simulations are conducted to verify the correctness of the proposed algorithm. Although our network model can describe certain real-world phenomena, it still has some limitations. It should be noted that state-dependent networks, of which the communication structure varies related to the state of nodes, can better characterize the practice network. We will extend the proposed dynamic ETI scheme to the synchronization problem of state-dependent networks in our future work.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there are no conflicts of interest.

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