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Research article

Synchronization analysis of delayed quaternion-valued memristor-based neural networks by a direct analytical approach

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Abstract: This issue discusses the asymptotic synchronization and the exponential synchronization for memristor-based quaternion-valued neural networks under the time-varying delays. Some criteria for synchronization of the memristor-based quaternion-valued neural networks are given by exploiting the set-valued theory, the differential inclusion theory, some analytic techniques, as well as constructing novel controllers, It is worth noting that the synchronization problem about the memristor-based quaternion-valued neural networks were studied by the direct analysis method in this paper. Finally, the main theoretical results were verified by numerical simulations.

Keywords: exponential synchronization; memristor-based neural networks; memristor; quaternion-valued

1. Introduction

Memristor was proposed by Chua in [1], as the fourth basic electronic component besides resistors, inductors, and capacitors. The memristor value changes with the circuit current, and after the circuit is powered off, its resistance value is the value at the moment of power off. In other words, memristor is a class of nonlinear resistors with memory functions. Due to the fact that nanotechnology was far from mature at that time, the physical realization of memristors was extremely difficult. Therefore, the research of memristors failed to achieve a major breakthrough. Until 2008, the Hewlett-Packard [2] laboratory developed memristors. Since then, memristor and its applications have attracted the attention of numerous scholars [3–18].

Previous research results indicated that the memristors has the function of simulating brain synapses. Due to the above properties of memristors, researchers use memristors as the connection weights of traditional neural networks to obtain memristor-based neural networks [3–6]. Obviously, the model of memristor-based neural networks is a special type of nonlinear system. Recently, the study about the above system's dynamical behavior attracted a lot of research attention and achieved many interesting results. Currently, the research on memristor-based neural networks mainly concentrates in either the real number domain or the complex number domain [7–18]. However, the related research in the field of quaternion is relatively rare.

Quaternion, which is a class of divisible algebras, was proposed by Hamilton [19] in 1843. The exchange law of quaternion multiplication is not, which makes the study of quaternion is slower than the real number or the complex number, and is more difficult. However, due to the development of modern mathematics, applications of quaternion in digital image processing, face recognition, quantum mechanics and other fields have been discovered [20–24]. A series of results have been achieved, especially in three-dimensional Data modeling and processing in space and four-dimensional space [21, 22]. For example, for rotation and affine transformation in three-dimensional space, the quaternion representation is not only more compact and effective, but also can effectively avoid the defects of matrix and Euler representations. Therefore, quaternion is increasingly attracting the attention of scholars.

The quaternion was introduced into the traditional neural network, and quaternion-valued neural networks (QVNNs) have been built. QVNNs show improved performances in color night vision [25], 3D wind forecasting [26, 27], image compression [28], and so on. As the extension of real-valued neural networks(RVNNs) and complex-valued neural networks(CVNNs), QVNNs have some notable advantages such as a low dimensionality and a high efficiency in handling multi-dimensional data. Using the three imaginary parts of the quaternion to denote the three primary colors in the color space, QVNNs not only do not need to deal with the three primary colors separately, but also show the correlation between the three primary colors. Furthermore, QVNNs show an improved performance than RVNNs or CVNNs in handling certain optimization and estimation problems [29].

By introducing the quaternion algebra into memristor-based neural networks(MNNs), the quaternion-value memristor-based neural networks(QVMNNs) is obtained. The connection weights, state variables, and activation function values of QVMNNs are all derived from the quaternion domain. Recently, some interesting conclusions have been shown with the in-depth research of QVMNNs [30–36]. In [31], exponential synchronization for QVMNNs with delayed was studied by quantized intermittent control tactics. In [33], the synchronization for fuzzy QVMNNs was discuss by the Lexicographical order method. Finite-time anti-synchronization about the inconsistent markovian QVMNNs under reaction-diffusion terms was investigated in [34]. The exponential synchronization conditions of the delayed inertial QVMNNs were given in the form of linear matrix inequality(LMIs) in [35]. In [36], Wei and Cao studied fixed-time synchronization for the quaternion-valued memristor-based neural networks by dividing the system into real and imaginary parts.

Inspired by the above research, it is worth discussing the asymptotic synchronization and the exponential synchronization for the QVMNNs under time delays. Different from the above research methods, this issue directly tackles the quaternion memristive neural network, which naturally poses a problem when determining which quaternion is big or small. To solve this problem, this paper will adopt the vector ordering approach, which supplies the theoretical basis to determine the "magnitude" of two different quaternions. On this basis, we propose a direct method to discuss the asymptotic synchronization and the exponential synchronization for memristor-based quaternion-valued neural networks under the time-varying delays, which simplifies the proof process.

The main works about this issue are arranged as follows. In Section 2, the model is built and the basics that will be used later are introduced. Several conditions are obtained for the asymptotic synchronization and the exponential synchronization for the QVMNNs under time-varying delays through the controllers in Section 3. Two numerical examples are used to verify the accuracy for the conclusions in Section 4. The last section draws a conclusion.

Notations: In this article, R is the real field and Q is the quaternion field. $co[a,b]$ expresses the closure for the convex hull Q manufactured by the quaternion a,b. $C^{(1)}([\varsigma,0],R^n)$ shows the class of continuous functions from $[\varsigma,0]$ to R^n.

2. Model and preliminaries

The quaternion is the hypercomplex number consisting of one real part and three imaginary parts. For $u\in Q$, this is written as

$$u=u^R+u^Ii+u^Jj+u^Kk$$

where $u^R,u^I,u^J,u^K\in R$. Moreover, i,j,k are the imaginary parts, which follow the Hamilton principle:

$$i^2=-1,j^2=-1,k^2=-1,ij=-ji=,ki=-ik=jk,jk=-kj=i.$$

The conjugate of u is denoted by$\bar{u}=u^R-u^Ii-u^Jj-u^Kk$. The modulus of u is written as

$$|u|=\sqrt{\bar{u}u}=\sqrt{(u^R)^2+(u^I)^2+(u^I)^2+(u^K)^2}.$$

$\|u\|_1=\sum_{m=1}^{n}|u_m|$ is the norm of x. For two quaternions $u_1=u_1^R+u_1^Ii+u_1^Jj+u_1^Kk$ and $u_2=u_2^R+u_2^Ii+u_2^Jj+u_2^Kk$, the addition between them is defined as

$$u_1+u_2=u_1^R+u_2^R+\left(u_1^I+u_2^I\right)i+\left(u_1^J+u_2^J\right)j+\left(u_1^K+u_2^K\right)k.$$

According to the Hamilton rule, the product of two quaternions is

$$\begin{aligned}u_1u_2 &= (u_1^Ru_2^R-u_1^Iu_2^I-u_1^Ju_2^J-u_1^Ku_2^K)+(u_1^Ru_2^I+u_1^Iu_2^R+u_1^Ju_2^K-u_1^Ku_2^J)i\\ &\quad+(u_1^Ru_2^J+u_1^Ju_2^R+u_1^Ku_2^I-u_1^Iu_2^K)j+(u_1^Ru_2^K+u_1^Ku_2^R+u_1^Iu_2^J-u_1^Ju_2^I)k.\end{aligned}$$

Consider the QVMNNs under time-varying delays as follows:

$$\begin{aligned}\dot{\omega}_p(t) &= -c_p\omega_p(t)+\sum_{q=1}^{n}a_{pq}(\omega_p(t))f_q(\omega_q(t))\\ &\quad+\sum_{q=1}^{n}b_{pq}(\omega_p(t))f_q(\omega_q(t-\tau(t)))+I,\quad t\geq 0,\end{aligned} \tag{2.1}$$

where $\omega_p(t)\in Q$ stands for the state vector, $p=1,2,\ldots,n,$. C is the self-feedback matrix, $C=diag\{c_1,c_2,\ldots,c_n\}$, and $b_{pq}(\omega_p(t))$, $a_{pq}(\omega_p(t))$ are the connection weight matrices. $f(\omega(t)):Q^n\to Q^n$ denotes the activation function. $I\in Q^n$ is the external input. Moreover, $\tau(t)$ denotes a transmission delay $0\leq\tau(t)\leq\tau$. The initial conditions of system (2.1) are selected as $\omega(s)=\phi(s),-\tau\leq s\leq 0$, where $\phi(s)\in C^{(1)}([-\tau,0],Q^n)$.

Assumption 2.1. There are some positive numbers $l_q, F_q \in R$ that satisfy

$$|f_q(v) - f_q(u)| \leq m_q|v - u|, \quad |f_q(u)| < F_q,$$

for all $u, v \in Q$.

Assumption 2.2. [37] It can be defined that:

$$a_{pq}\left(u_p(t)\right) = \begin{cases} \hat{a}_{pq} = a^R_{1pq} + a^I_{1pq}i + a^J_{1pq}j + a^K_{1pq}k, & |u_p(t)| \leq T_p \\ \check{a}_{pq} = a^R_{2pq} + a^I_{2pq}i + a^J_{2pq}j + a^K_{2pq}k, & |u_p(t)| < T_p \end{cases},$$

$$b_{pq}\left(u_p(t)\right) = \begin{cases} \hat{b}_{pq} = b^R_{1pq} + b^I_{1pq}i + b^J_{1pq}j + b^K_{1pq}k, & |u_p(t)| \leq T_p \\ \check{b}_{pq} = b^R_{2pq} + b^I_{2pq}i + b^J_{2pq}j + b^K_{2pq}k, & |u_p(t)| < T_p \end{cases},$$

where the positive number T_p is the switching jumps.

Remark 1. According to Assumption 2.2, $b_{pq}(u_p(t))$ and $a_{pq}(u_p(t))$ are piecewise functions; therefore, the Quaternion-Valued Memristor-Based neural networks of system (2.1) is a discontinuous system.

Lemma 2.1. [38] Let $u, v \in Q$, $\epsilon > 0$ be a constant. Then, the following inequality is true:

$$vu + \bar{u}\bar{v} \leq \epsilon\bar{u}u + \frac{1}{\epsilon}v\bar{v}.$$

From the theory of the set valued map and differential inclusion [39], system (2.1) could be rewritten as follows:

$$\begin{aligned} \dot{\omega}_p(t) \;\; \in \;\; & -c_p\omega_p(t) + \sum_{q=1}^{n} co[a^-_{pq}, a^{TT}_{pq}]f_q\left(\omega_q(t)\right) \\ & + \sum_{q=1}^{n} co[b^T_{pq}, b^{TT}_{pq}]f_q\left(\omega_q(t - \tau(t))\right) + J_p \end{aligned} \tag{2.2}$$

where $\tilde{a}_{pq} = \max\left\{|\hat{a}_{pq}|, |\check{a}_{pq}|\right\}$, $a^{TT}_{pq} = \max\left\{\hat{a}_{pq}, \check{a}_{pq}\right\}$, $a^T_{pq} = \min\left\{\hat{a}_{pq}, \check{a}_{pq}\right\}$, $\tilde{b}_{pq} = \max\left\{|\hat{b}_{pq}|, |\check{b}_{pq}|\right\}$, $b^{TT}_{pq} = \max\left\{\hat{b}_{pq}, \tilde{b}_{pq}\right\}$, and $b^T_{pq} = \min\left\{\hat{b}_{pq}, \tilde{b}_{pq}\right\}$.

Then, there are $a'_{pq} \in co[a^T_{pq}, a^{TT}_{pq}]$, $b'_{pq} \in co[b^T_{pq}, b^{TT}_{pq}]$, which satisfies the following:

$$\dot{\omega}_p(t) \;\; = \;\; -c_p\omega_p(t) + \sum_{q=1}^{n} a'_{pq}f_q\left(\omega_q(t)\right) + \sum_{q=1}^{n} b'_{pq}f_q\left(\omega_q(t - \tau(t))\right) + J. \tag{2.3}$$

Considering (2.1) as a drive system, then the following system can be used as a response system:

$$\dot{v}_p(t) = -c_pv_p(t) + \sum_{q=1}^{n} a_{pq}(v_p(t))f_q(v_q(t)) + \sum_{q=1}^{n} b_{pq}(v_p(t))f_q(v_q(t - \tau(t))) + u_p(t) + J, \; t \geq 0, \tag{2.4}$$

where $u_p(t)$ is the controller.

From the differential inclusion and theory of the set valued map, system (2.4) can also be rewritten as:

$$\begin{aligned}\dot{v}_p(t) &\in -c_p v_p(t) + \sum_{q=1}^{n} co[a_{pq}^{T}, a_{pq}^{TT}] f_q\left(v_q(t)\right) \\ &\quad + \sum_{q=1}^{n} co[b_{pq}^{T}, b_{pq}^{TT}] f_q\left(v_q(t-\tau(t))\right) + J_p + u_p(t). \end{aligned} \tag{2.5}$$

Equivalently, there are $a_{pq}^{''} \in co[a_{pq}^{T}, a_{pq}^{TT}]$, $b_{pq}^{''} \in co[b_{pq}^{T}, b_{pq}^{TT}]$, which satisfies the following:

$$\dot{v}_p(t) = -c_p v_p(t) + \sum_{q=1}^{n} a_{pq}^{''} f_q\left(v_q(t)\right) + \sum_{q=1}^{n} b_{pq}^{''} f_q\left(v_q(t-\tau(t))\right) + J_p + u_p(t). \tag{2.6}$$

Let $e(t) = (e_1(t), \ldots, e_n(t))^T = v(t) - \omega(t)$, the following error system can be obtained:

$$\begin{aligned}\dot{e}_p(t) &= -c_p e_p(t) + \sum_{q=1}^{n}[a_{pq}^{''} f_q(v_q(t)) - a_{pq}^{'} f_q(\omega_q(t))] \\ &\quad + \sum_{q=1}^{n}[b_{pq}^{''} f_q(v_q(t-\tau(t))) - b_{pq}^{'} f_q(\omega_q(t-\tau(t)))] + u_p(t). \end{aligned} \tag{2.7}$$

Definition 2.1. If there are two constants $\pi > 0$ and $\gamma \geq 1$ that satisfy the following inequality,

$$\|e(t)\| \leq \gamma e^{-\pi t} \sup_{-\tau \leq s \leq 0} \|e(t-\tau(t))\|, \quad t \geq 0.$$

then the systems (2.1) and (2.4) are globally exponentially synchronized.

Lemma 2.2. [40] Assume α_1 and α_2 are two constants such that $\lambda_1 > \lambda_2 > 0$. $y(t)$ is a nonnegative continuous function, which is defined on $[t_0 - \tau, +\infty)$. The following inequality is true:

$$D^{+}(y(t)) \leq -\lambda_1 y(t) + \lambda_2 \bar{y}(t), \quad \text{for all } t \geq t_0,$$

where $\bar{y}(t) = \sup_{t-\tau \leq s \leq t} y(s)$, and $D^{+}(y(t)) = \overline{\lim_{h \to 0^{+}}} \frac{y(t+h)-y(t)}{h}$ is the upper-right Dini derivative. For $\forall t \geq t_0$, one can get $y(t) \leq \bar{y}(t_0) e^{-\gamma(t-t_0)}$. , which is the only positive solution to $\gamma = \lambda_1 - \lambda_2 e^{\gamma\tau}$.

3. Main results

In the following section, some sufficient criterion are acquired about the global exponential stability for the memristor-based neural network under time-varying delays. Then, the following main results are established.

Theorem 3.1. Under the Assumptions 2.1 and 2.2, the systems (2.1) and (2.4) are exponentially synchronized by the controller (3.1):

$$u_p(t) = -k_p(t) e_p(t) - \eta_p. \tag{3.1}$$

If there are two constants $\varepsilon_1 > 0, \varepsilon_2 > 0$, then the following is satisfied:

$$
\begin{aligned}
&2c_p + 2k_p - 1 - \varepsilon_1 m_p^2 - \frac{1}{\varepsilon_1}\tilde{a}_{pq}\bar{\tilde{a}}_{pq} - \frac{1}{\varepsilon_2}\tilde{b}_{pq}\bar{\tilde{b}}_{pq} > 0,\\
&\varepsilon_2 \max_p m_p^2 < 1 - \tau',
\end{aligned}
$$

where $k_p(t) \in R^n$,

$$\eta_p = \sum_{q=1}^{n}(a_{pq}^{TT} - a_{pq}^{T})F_q + \sum_{q=1}^{n}(b_{pq}^{TT} - b_{pq}^{T})F_q.$$

Proof. Construct the following auxiliary function:

$$V(t) = \sum_{p=1}^{n} \bar{e}_p(t)e_p(t) + \int_{t-\tau(t)}^{t} \bar{e}_p(s)e_p(s)ds. \tag{3.2}$$

Before continuing, the following estimation of $\dot{e}_p(t)$ and $\dot{\bar{e}}_p(t)$ can be given:

$$
\begin{aligned}
\dot{e}_p(t) &= -c_p e_p(t) + \sum_{q=1}^{n}[a_{pq}^{''}(t)f_q(v_q(t)) - a_{pq}^{'}(t)f_q(\omega_q(t))]\\
&\quad + \sum_{q=1}^{n}[b_{pq}^{''}(t)f_q(v_q(t-\tau(t))) - b_{pq}^{'}(t)f_q(\omega_q(t-\tau(t)))] + u_p(t)\\
&= -c_p e_p(t) + \sum_{q=1}^{n}(a_{pq}^{''}(t) - a_{pq}^{'}(t))f_q(\omega_q(t)) + \sum_{q=1}^{n} a_{pq}^{''}(t)[f_q(v_q(t)) - f_q(\omega_q(t))]\\
&\quad + \sum_{q=1}^{n} b_{pq}^{''}(t)[f_q(v_q(t-\tau(t))) - f_q(\omega_q(t-\tau(t)))]\\
&\quad + \sum_{q=1}^{n}(b_{pq}^{''}(t) - b_{pq}^{'}(t))f_q(\omega_q(t-\tau(t)))] - k_p(t)e_p(t) - \eta_p
\end{aligned}
$$

$$
\begin{aligned}
\dot{\bar{e}}_p(t) &= -c_p\bar{e}_p(t) + \sum_{q=1}^{n}[\bar{f}_q(v_q(t))\bar{a}_{pq}^{''}(t) - \bar{f}_q(\omega_q(t))\bar{a}_{pq}^{'}(t)]\\
&\quad + \sum_{q=1}^{n}[\bar{f}_q(v_q(t-\tau(t)))\bar{b}_{pq}^{''}(t) - \bar{f}_q(\omega_q(t-\tau(t)))\bar{b}_{pq}^{'}(t)] + \bar{u}_p(t)\\
&= -c_p\bar{e}_p(t) + \sum_{q=1}^{n} \bar{f}_q(v_q(t))[\bar{a}_{pq}^{''}(t) - \bar{a}_{pq}^{'}(t)] + \sum_{q=1}^{n}[\bar{f}_q(v_q(t)) - \bar{f}_q(\omega_q(t))]\bar{a}_{pq}^{'}(t)\\
&\quad + \sum_{q=1}^{n}[\bar{f}_q(v_q(t-\tau(t)))\bar{b}_{pq}^{''}(t) - \bar{f}_q(\omega_q(t-\tau(t)))\bar{b}_{pq}^{'}(t)] - k_p(t)\bar{e}_p(t) - \bar{\eta}_p.
\end{aligned}
$$

Next, the derivative of $V(t)$ along (1.7) is computed as follows:

$$\begin{aligned}
\dot{V}(t) &= \sum_{p=1}^{n} \dot{\bar{e}}_p(t)e_p(t) + \sum_{p=1}^{n} \bar{e}_p(t)\dot{e}_p(t) + \sum_{p=1}^{n} \bar{e}_p(t)e_p(t) \\
&\quad -(1-\dot{\tau}(t)) \sum_{p=1}^{n} \bar{e}_p(t-\tau(t))e_p(t-\tau(t)) \\
&= \sum_{p=1}^{n} \Big\{ -c_p\bar{e}_p(t) + \sum_{q=1}^{n} [\bar{f}_q(v_q(t))\bar{a}''_{pq}(t) - \bar{f}_q(\omega_q(t))\bar{a}'_{pq}(t)] \\
&\quad + \sum_{q=1}^{n} [\bar{f}_q(v_q(t-\tau(t)))\bar{b}''_{pq}(t) - \bar{f}_q(\omega_q(t-\tau(t)))\bar{b}'_{pq}(t)] \\
&\quad + \bar{u}_p(t)\Big\} e_p(t) + \sum_{p=1}^{n} \bar{e}_p(t)\Big\{ -c_pe_p(t) \\
&\quad + \sum_{q=1}^{n} [a''_{pq}(t)f_q(v_q(t)) - a'_{pq}(t)f_q(\omega_q(t))] + \sum_{q=1}^{n} [b''_{pq}(t)f_q(v_q(t-\tau(t))) \\
&\quad - b'_{pq}(t)f_q(\omega_q(t-\tau(t)))] + u(t)\Big\} + \sum_{p=1}^{n} \bar{e}_p(t)e_p(t) \\
&\quad -(1-\dot{\tau}(t)) \sum_{p=1}^{n} \bar{e}_p(t-\tau(t))e_p(t-\tau(t)) \\
&\le \sum_{p=1}^{n} \Big\{ -c_p\bar{e}_p(t) + \sum_{q=1}^{n} (\bar{a}^{TT}_{pq} - \bar{a}^{T}_{pq})F_q \\
&\quad + \sum_{q=1}^{n} [\bar{f}_q(v_q(t)) - \bar{f}_q(\omega_q(t))]\bar{a}''_{pq}(t) + \sum_{q=1}^{n} (\bar{b}^{TT}_{pq} - \bar{b}^{T}_{pq})F_q + \bar{u}_p(t) \\
&\quad + \sum_{q=1}^{n} [\bar{f}_q(v_q(t-\tau(t))) - \bar{f}_q(\omega_q(t-\tau(t)))]\bar{b}''_{pq}(t)\Big\} e_p(t) \\
&\quad + \sum_{p=1}^{n} \bar{e}_p(t)\Big\{ -c_pe_p(t) + \sum_{q=1}^{n} (a^{TT}_{pq} - a^{T}_{pq})F_q \\
&\quad + \sum_{q=1}^{n} a''_{pq}(t)[f_q(v_q(t)) - f_q(\omega_q(t))] + \sum_{q=1}^{n} (b^{TT}_{pq} - b^{T}_{pq})F_q + u_p(t) \\
&\quad + \sum_{q=1}^{n} b''_{pq}(t)[f_q(v_q(t-\tau(t))) - f_q(\omega_q(t-\tau(t)))]\Big\} \\
&\quad + \sum_{p=1}^{n} \bar{e}_p(t)e_p(t) - (1-\tau') \sum_{p=1}^{n} \bar{e}_p(t-\tau(t))e_p(t-\tau(t)).
\end{aligned} \tag{3.3}$$

Lemma 2.1 shows that there are two positive constants $\varepsilon_1, \varepsilon_2$, which satisfy the following:

$$\begin{aligned}
& [\bar{f}_q(v_q(t)) - \bar{f}_q(\omega_q(t))]\bar{a}''_{pq}(t)e(t) + \bar{e}(t)a''_{pq}(t)[f_q(v_q(t)) - f_q(\omega_q(t))] \\
\leq\ & \varepsilon_1[\bar{f}_q(v_q(t)) - \bar{f}_q(\omega_q(t))][f_q(v_q(t)) - f_q(\omega_q(t))] + \frac{1}{\varepsilon_1}\bar{e}(t)\tilde{a}_{pq}\bar{\tilde{a}}_{pq}e(t) \\
\leq\ & \varepsilon_1 m_q^2 \bar{e}_q(t)e_q(t) + \frac{1}{\varepsilon_1}\bar{e}(t)\tilde{a}_{pq}\bar{\tilde{a}}_{pq}e(t)
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
& [\bar{f}_q(v_q(t-\tau(t))) - \bar{f}_q(\omega_q(t-\tau(t)))]\bar{b}''_{pq}(t)e(t) + \bar{e}(t)b''_{pq}(t)[f_q(v_q(t-\tau(t))) - f_q(\omega_q(t-\tau(t)))] \\
\leq\ & \varepsilon_2[\bar{f}_q(v_q(t-\tau(t))) - \bar{f}_q(\omega_q(t-\tau(t)))] \\
& [f_q(v_q(t-\tau(t))) - f_q(\omega_q(t-\tau(t)))] + \frac{1}{\varepsilon_2}\bar{e}(t)\tilde{b}_{pq}\bar{\tilde{b}}_{pq}e(t) \\
\leq\ & \varepsilon_2 m_q^2 \bar{e}_q(t-\tau(t))e_q(t-\tau(t)) + \frac{1}{\varepsilon_2}\bar{e}(t)\tilde{b}_{pq}\bar{\tilde{b}}_{pq}e(t).
\end{aligned} \tag{3.5}$$

Therefore, together with systems (3.4) and (3.5), the following can be obtained:

$$\begin{aligned}
\dot{V}(t) \leq\ & -\sum_{p=1}^{n}\Big\{2c_p + 2k_p - 1 - \varepsilon_1 m_p^2 - \frac{1}{\varepsilon_1}\tilde{a}_{pq}\bar{\tilde{a}}_{pq} \\
& -\frac{1}{\varepsilon_2}\tilde{b}_{pq}\bar{\tilde{b}}_{pq}\Big\}\bar{e}_p(t)e_p(t) + \left[\varepsilon_2 \max_p m_p^2 - (1-\tau')\right]\bar{e}_q(t-\tau(t))e_q(t-\tau(t)) \\
<\ & 0.
\end{aligned} \tag{3.6}$$

Then, the error variable $e(t)$ will exponentially converge to zero; in other words, systems (2.1) and (2.4) will achieve exponentially synchronized synchronization.

Theorem 3.2. For two given Assumptions 2.1 and 2.2, the systems (2.1) and (2.4) can achieve asymptotically synchronization with the controller (3.7), if there are some constants ε_1, ε_2 that satisfy the following inequality:

$$u_p(t) = -k_p e_p(t) - \eta_p, \tag{3.7}$$

where

$$\begin{aligned}
2k_p \ &\geq\ -2c_p + \varepsilon_1 m_p^2 + \frac{1}{\varepsilon_1}\sum_{q=1}^{n}\tilde{a}_{pq}\bar{\tilde{a}}_{pq} + \frac{1}{\varepsilon_2}\sum_{q=1}^{n}\tilde{b}_{pq}\bar{\tilde{b}}_{pq}, \\
\eta_p \ &=\ \sum_{q=1}^{n}(a_{pq}^{TT} - a_{pq}^{T})F_q + \sum_{q=1}^{n}(b_{pq}^{TT} - b_{pq}^{T})F_q.
\end{aligned}$$

Moreover, the control gains satisfy the following:

$$\begin{aligned}
\rho_1 \ &=\ \min_p\Big(2c_p + 2k_p - \varepsilon_1 m_p^2 - \frac{1}{\varepsilon_1}\sum_{q=1}^{n}\tilde{a}_{pq}\bar{\tilde{a}}_{pq} - \frac{1}{\varepsilon_2}\sum_{q=1}^{n}\tilde{b}_{pq}\bar{\tilde{b}}_{pq}\Big), \\
\rho_2 \ &=\ \varepsilon_2 \max_p m_p^2, \quad \rho_1 > \rho_2 > 0.
\end{aligned}$$

Proof. Construct the suggested function $V(t)$, which is defined by

$$V(t) = \sum_{p=1}^{n} \bar{e}_p(t)e_p(t).$$

The derivative of $V(t)$ along (2.7) is computed as follows:

$$\begin{aligned}
\dot{V}(t) &= \sum_{p=1}^{n} \dot{\bar{e}}_p(t)e_p(t) + \sum_{p=1}^{n} \bar{e}_p(t)\dot{e}_p(t) \\
&= \sum_{p=1}^{n} \Big\{ - c_p\bar{e}_p(t) + \sum_{q=1}^{n} [\bar{f}_q(v_q(t))\bar{a}''_{pq}(t) - \bar{f}_q(\omega_q(t))\bar{a}'_{pq}(t)] \\
&\quad + \sum_{q=1}^{n} [\bar{f}_q(v_q(t-\tau(t)))\bar{b}''_{pq}(t) - \bar{f}_q(\omega_q(t-\tau(t)))\bar{b}'_{pq}(t)] \\
&\quad + \bar{u}(t)\Big\} e_p(t) + \sum_{p=1}^{n} \bar{e}_p(t)\Big\{ - c_p e_p(t) + \sum_{q=1}^{n} [a''_{pq}(t) f_q(v_q(t)) - a'_{pq}(t) f_q(\omega_q(t))] \\
&\quad + \sum_{q=1}^{n} [b''_{pq}(t) f_q(v_q(t-\tau(t))) - b'_{pq}(t) f_q(\omega_q(t-\tau(t)))] + u(t)\Big\} \\
&\leq -\sum_{p=1}^{n} \Big(2c_p + 2k_p - \varepsilon_1 m_p^2 - \frac{1}{\varepsilon_1}\sum_{q=1}^{n} \tilde{a}_{pq}\bar{\tilde{a}}_{pq} - \frac{1}{\varepsilon_2}\sum_{q=1}^{n} \tilde{b}_{pq}\bar{\tilde{b}}_{pq}\Big)\bar{e}_p(t)e_p(t) \\
&\quad + \varepsilon_2 \sum_{p=1}^{n} m_p^2 \bar{e}_q(t-\tau(t))e_q(t-\tau(t)) \\
&\leq -\rho_1 V(t) + \rho_2 V(t-\tau(t)).
\end{aligned} \tag{3.8}$$

where ρ_1, ρ_2 are two constants.

According to Lemma (2.2), it can be inferred that

$$V(t) = \max_{-\tau\leq\theta\leq 0} V(\theta)\exp^{-\gamma t} \tag{3.9}$$

where γ is the solution of Eq (3.10)

$$x - y\exp^{-\gamma\theta} - \gamma = 0. \tag{3.10}$$

Consequently, error system (2.7) is globally asymptotically stable. In other words, the stabilization control of the systems (2.1) and (2.4) can be achieved by controller (3.7).

Remark 2. The QVMNNs was divided into a real part and three imaginary parts in the existing work [36]. Moreover, the direct method is used to discuss the QVMNNs in this paper, which is more realistic. The results are presented in the shape of easily verifiable algebraic inequalities.

4. Numerical examples

Then, the following two numerical simulations are dedicated to verify the validity of the given theoretical results.

Example 1. Consider the two-neuron quaternion-valued memristor-based neural networks (2.1) with $c_1 = c_2 = 1$; the connection weights are as follows:

$$a_{11}(\omega_1(t)) = \begin{cases} 2.1 - 2.7i + 1.9j - 2.5k, |\omega_1(t)| < 1, \\ 2.2 - 1.6i + 2.2j - 1.5k, |\omega_1(t)| \geq 1, \end{cases}$$
$$a_{12}(\omega_1(t)) = \begin{cases} -0.2 - 0.4i - 0.2j - 0.3k, |\omega_2(t)| \geq 1, \\ -0.6 - 0.9i - 0.5j - 0.8k, |\omega_2(t)| < 1, \end{cases}$$
$$a_{21}(\omega_2(t)) = \begin{cases} 1.5 - 0.3i + 1.6j - 0.4k, |\omega_2(t)| \geq 1, \\ 1.0 + 0.7i + 1.1j + 0.6k, |\omega_2(t)| < 1, \end{cases}$$
$$a_{22}(\omega_2(t)) = \begin{cases} -1.3 - 0.1i - 1.2j - 0.3k, |\omega_2(t)| \geq 1, \\ -0.7 - 0.3i - 0.8j - 0.2k, |\omega_2(t)| < 1, \end{cases},$$
$$b_{11}(\omega_1(t)) = \begin{cases} -1.6 + 2.5i - 1.5j + 2.3k, |\omega_1(t)| \geq 1, \\ -0.1 + 3.0i - 1.4j + 2.9k, |\omega_1(t)| < 1, \end{cases}$$
$$b_{12}(\omega_1(t)) = \begin{cases} -1.4 - 0.9i - 0.1j - 0.6k, |\omega_1(t)| \geq 1, \\ -0.5 - 1.5i - 0.3j - 1.7k, |\omega_1(t)| < 1, \end{cases}$$
$$b_{21}(\omega_2(t)) = \begin{cases} -1.0 - 1.2i - 1.3j - 1.3k, |\omega_2(t)| \geq 1, \\ -0.9 - 0.3i - 0.6j - 0.1k, |\omega_2(t)| < 1, \end{cases}$$
$$b_{22}(\omega_2(t)) = \begin{cases} 1.2 - 0.6i + 1.1j - 0.4k, |\omega_2(t)| \geq 1, \\ 0.6 - 0.7i + 0.4j - 0.8k, |\omega_2(t)| < 1, \end{cases}.$$

In the response system (2.4), the activation functions are selected as $f(\omega) = 0.1tanh(\omega)$. Obviously, when $F_1 = F_2 = 0.1$, the activation functions satisfy Assumption 2.1. Figures 1 and 2 show trajectories for the error variables $e_1^\pi, e_2^\pi, \pi = R, I, J, K$ about systems (2.1) and (2.4) without a controller. The error system cannot converge to zero. Thus, the systems (2.1) and (2.4) cannot be synchronized in this situation.

According to the above parameters, the following is directly calculated from Theorem 3.1:

$$\sum_{q=1}^{2}(a_{1q}^{+} - a_{1q}^{-})F_q + \sum_{q=1}^{2}(b_{1q}^{+} - b_{1q}^{-})F_q = 0.21 + 0.2i + 0.14j + 0.33k,$$
$$\sum_{q=1}^{2}(a_{2q}^{+} - a_{2q}^{-})F_q + \sum_{q=1}^{2}(b_{2q}^{+} - b_{2q}^{-})F_q = 0.18 - 0.02j + 0.23j + 0.05k.$$

Therefore, with controller (3.1), it can pick out $\eta_1 = 0.21 + 0.2i + 0.14j + 0.33k$, $\eta_2 = 0.18 - 0.02j + 0.23j + 0.05k$, $k_1 = k_2 = 1$, $\varepsilon_1 = \varepsilon_2 = 2$ $\tau(t) = 0.65 - 0.25cos(t)$. Then, the conditions for the Theorem 3.1 are held. Figures 3 and 4 depict the error variables $e_1^\pi, e_2^\pi, \pi = R, I, J, K$ between systems (2.1) and (2.4) under the controller (3.1). Hence, the error system tends to 0. That is to say that systems (2.1) and (2.4) achieve asymptotic synchronization.

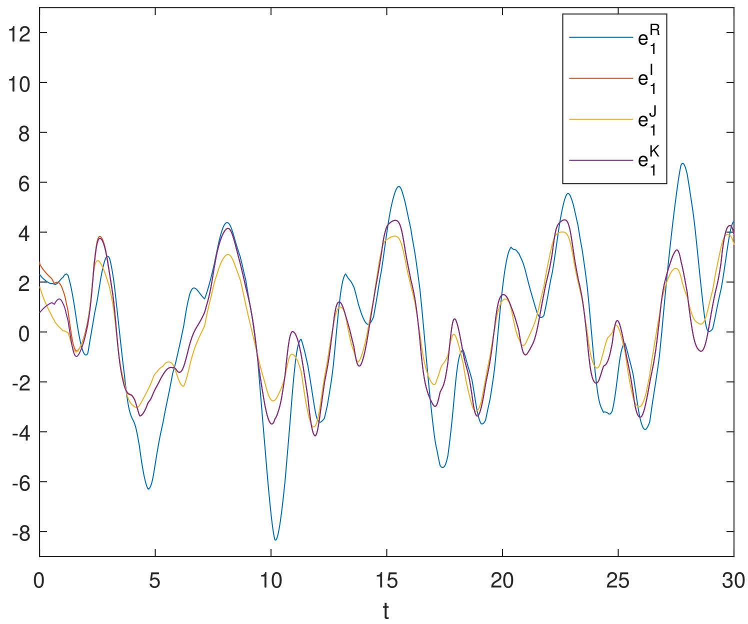

Figure 1. The error state variables $e_1^\pi, \pi = R, I, J, K$, without controller.

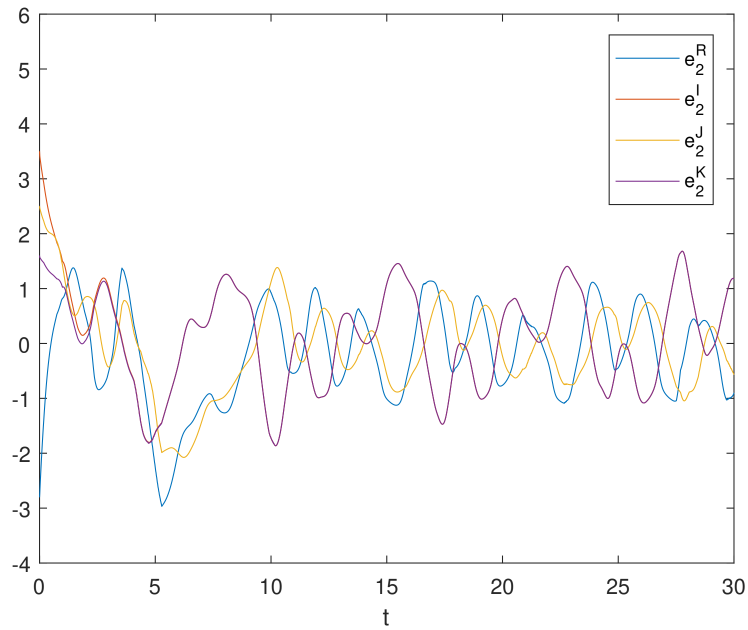

Figure 2. The error state variables $e_2^\pi, \pi = R, I, J, K$, without controller.

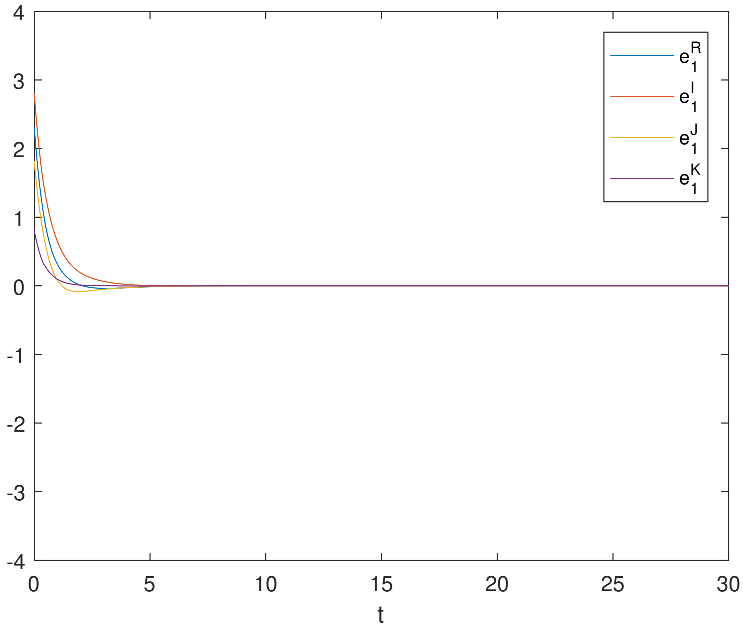

Figure 3. The error state variables$e_1^\pi, \pi = R, I, J, K$, with the controller.

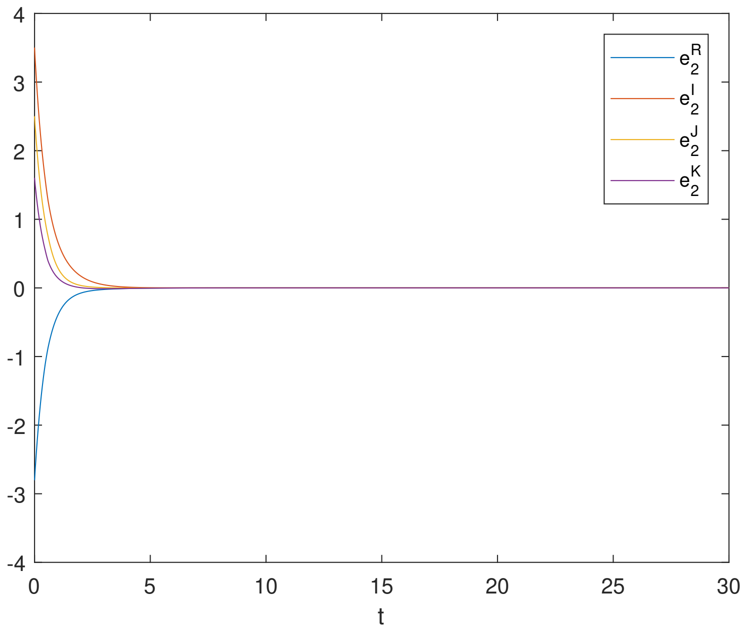

Figure 4. The error state variables $e_2^\pi, \pi = R, I, J, K$, under the controller.

Example 2. Consider the two-neuron QVMNNs (2.1) with the following:

$$a_{11}(\omega_1(t)) = \begin{cases} 2.0 - 2.7i + 2.0j - 2.5k, |\omega_1(t)| < 1, \\ 2.3 - 1.6i + 2.3j - 1.5k, |\omega_1(t)| \geq 1, \end{cases}$$
$$a_{12}(\omega_1(t)) = \begin{cases} -0.1 - 0.4i - 0.1j - 0.3k, |\omega_2(t)| \geq 1, \\ -0.5 - 0.9i - 0.5j - 0.7k, |\omega_2(t)| < 1, \end{cases}$$
$$a_{21}(\omega_2(t)) = \begin{cases} 1.6 - 0.3i + 1.5j - 0.4k, |\omega_2(t)| \geq 1, \\ 1.1 + 0.7i + 1.0j + 0.6k, |\omega_2(t)| < 1, \end{cases}$$
$$a_{22}(\omega_2(t)) = \begin{cases} -1.2 - 0.1i - 1.3j - 0.2k, |\omega_2(t)| \geq 1, \\ -0.8 - 0.3i - 0.7j - 0.3k, |\omega_2(t)| < 1, \end{cases}$$
$$b_{11}(\omega_1(t)) = \begin{cases} -1.5 + 2.6i - 1.5j + 2.3k, |\omega_1(t)| \geq 1, \\ -0.1 + 3.1i - 1.4j + 3.0k, |\omega_1(t)| < 1, \end{cases}$$
$$b_{12}(\omega_1(t)) = \begin{cases} -1.4 - 0.9i - 0.1j - 0.6k, |\omega_1(t)| \geq 1, \\ -0.5 - 1.5i - 0.5j - 1.6k, |\omega_1(t)| < 1, \end{cases}$$
$$b_{21}(\omega_2(t)) = \begin{cases} -1.2 - 1.1i - 1.3j - 1.3k, |\omega_2(t)| \geq 1, \\ -0.8 - 0.2i - 0.6j - 0.1k, |\omega_2(t)| < 1, \end{cases}$$
$$b_{22}(\omega_2(t)) = \begin{cases} 1.3 - 0.5i + 1.2j - 0.4k, |\omega_2(t)| \geq 1, \\ 0.5 - 0.8i + 0.4j - 0.7k, |\omega_2(t)| < 1, \end{cases}.$$

Moreover, $c_1 = c_2 = 2$, the time-delays are chosen as $\tau(t) = 0.6 + 0.3sin(t)$, and the active function is $f_q(\omega_q(t)) = 0.1tanh(\omega_q(t))$, which satisfy the requirements in Assumption 2.1 with $F_q = 0.1$, $m_q = 0.1$, $q = 1, 2$. Figures 5 and 6 depict the error variables $e_1^\pi, e_2^\pi, \pi = R, I, J, K$ between systems (2.1) and (2.4) without a controller. Set $\varepsilon_1 = \varepsilon_2 = 5$; then, from the conditions of Theorem 3.2 and the above given parameters, the following results can be obtained:

$$\begin{aligned} 2k_1 &\geq -2c_1 + \varepsilon_1 m_1^2 + \frac{1}{\varepsilon_1}\sum_{q=1}^{2} \tilde{a}_{1q}\bar{\tilde{a}}_{1q} + \frac{1}{\varepsilon_2}\sum_{q=1}^{2} \tilde{b}_{1q}\bar{\tilde{b}}_{1q} = 5.776, \\ 2k_2 &\geq -2c_2 + \varepsilon_1 m_2^2 + \frac{1}{\varepsilon_1}\sum_{q=1}^{2} \tilde{a}_{2q}\bar{\tilde{a}}_{2q} + \frac{1}{\varepsilon_2}\sum_{q=1}^{2} \tilde{b}_{2q}\bar{\tilde{b}}_{2q} = 1.862. \end{aligned}$$

Therefore, one could select $k_1 = 3, k_2 = 1$ and the controller can be designed as follows:

$$\begin{aligned} \rho_1 &= \min_p \Big(2c_p + 2k_p - \varepsilon_1 m_p^2 - \frac{1}{\varepsilon_1}\sum_{q=1}^{n} \tilde{a}_{pq}\bar{\tilde{a}}_{pq} - \frac{1}{\varepsilon_2}\sum_{q=1}^{n} \tilde{b}_{pq}\bar{\tilde{b}}_{pq}\Big) = 0.138, \\ \rho_2 &= \varepsilon_2 \max_p m_p^2 = 0.05, \end{aligned}$$

which means $\rho_1 > \rho_2 > 0$.

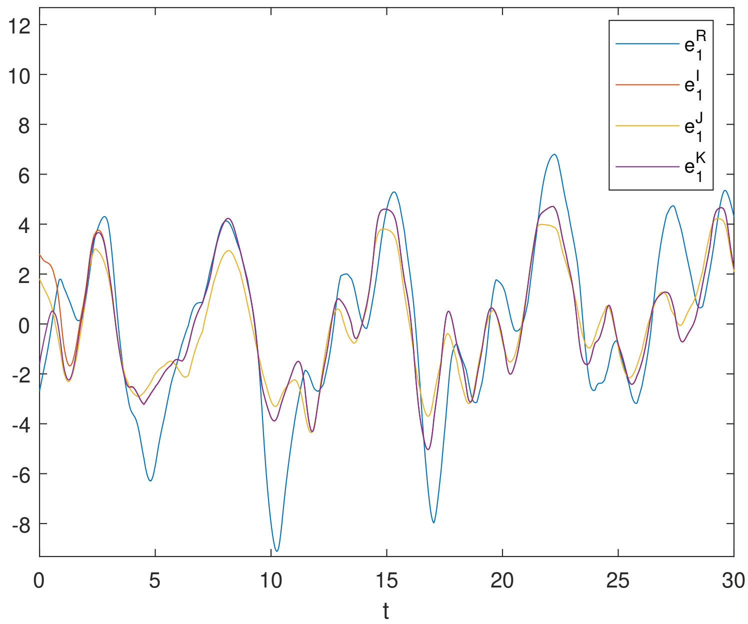

Figure 5. The error state variables $e_1^\pi, \pi = R, I, J, K$, without controller.

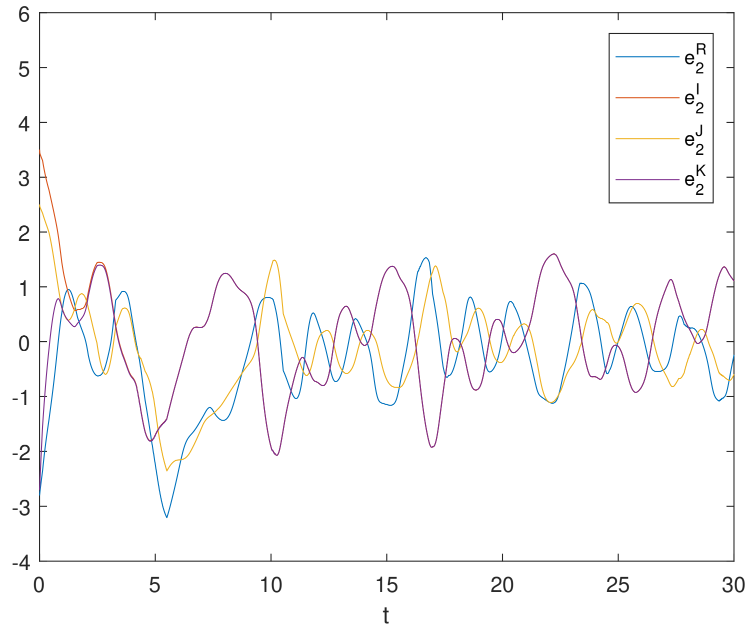

Figure 6. The error state variables $e_2^\pi, \pi = R, I, J, K$, without controller.

From the above calculation results, it can be seen that all conditions of Theorem 3.2 are met. Figures 7 and 8 depict the error variables e_1^π, $e_2^\pi, \pi = R, I, J, K$ between systems (2.1) and (2.4) under the controller (3.7). It can be seen from the above discussion that the drive system (2.1) and the response system (2.4) are synchronized, which means that the control technique achieves the desired effect.

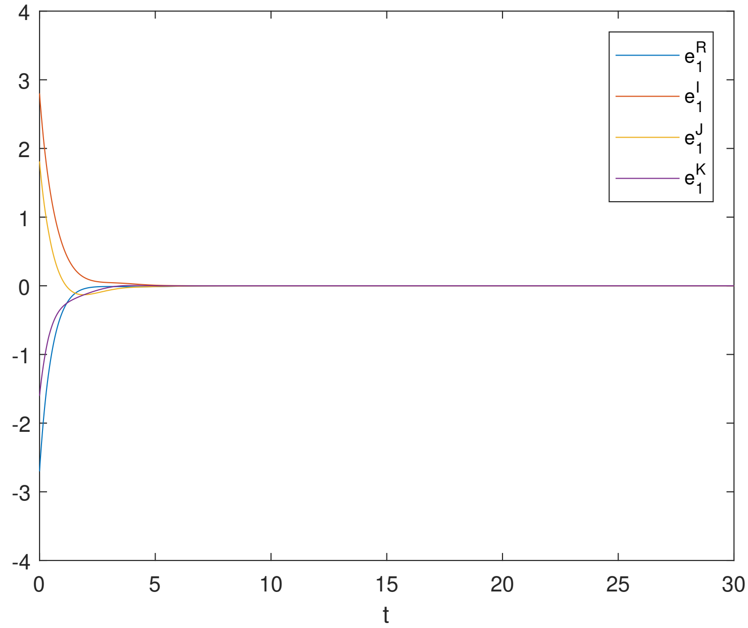

Figure 7. The error state variables $e_1^\pi, \pi = R, I, J, K$, with the controller.

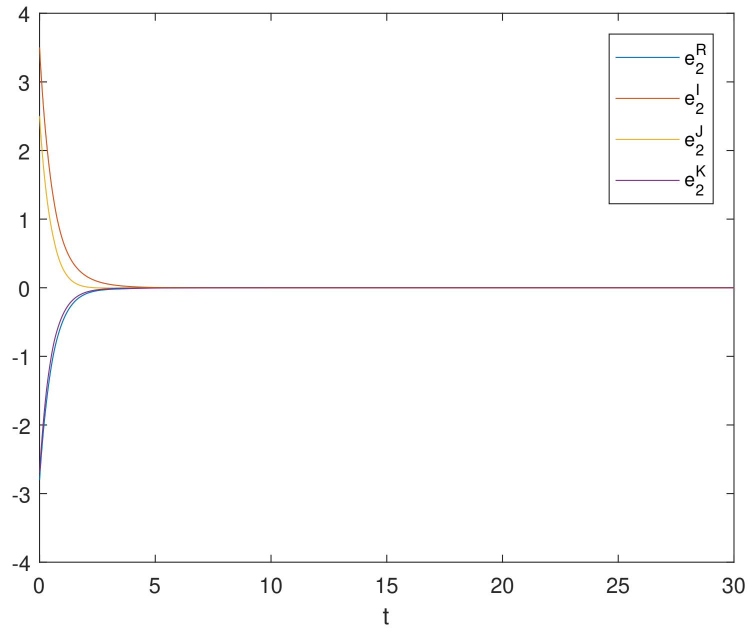

Figure 8. The error state variables $e_2^\pi, \pi = R, I, J, K$, with the controller.

5. Conclusions

In this paper, by using analytical techniques, and constructing two novel controllers, several control strategies were obtained to investigate the asymptotic synchronization and the exponential synchronization of quaternion-valued memristor-based neural networks. At the same time, the direct analysis method was given to discuss the synchronization problem, which simplified the proof process. In the end, numerical simulations were supplied to display the main theoretical results.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflicts of interest.

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