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Research article

Multi-Local-Worlds economic and management complex adaptive system with agent behavior and local configuration

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Abstract: The central focus of our investigation revolved around the convergence of agents' behavior toward a particular invariant distribution and determining the characteristics of the optimal strategies' distribution within the framework of a dynamical Multi-Local-Worlds complex adaptive system. This system was characterized by the co-evolution of agent behavior and local topological configuration. The study established a representation of an agent's behavior and local graphic topology configuration to elucidate the interaction dynamics within this dynamical context. As an illustrative example, we introduced three distinct agent types—smart agent, normal agent, and stupid agent—each associated with specific behaviors. The findings underscored that an agent's decision-making process was influenced by the evolution of random complex networks driven by preferential attachment, coupled with a volatility mechanism linked to its payment—a dynamic that propels the evolution of the complex adaptive system. Through simulation, we drew a conclusive observation that even when considering irrational behaviors characterized by limited information and memory constraints, the system's state converges to a specific attractor. This underscored the robustness and convergence properties inherent in the dynamical Multi-Local-Worlds complex adaptive system under scrutiny.

Keywords: complex adaptive system; co-evolutionary; agent behavior; game theory

1. Introduction

Generally, there are several different kinds of individuals in economic and management systems; they are homogenous individuals if they are the same, otherwise they are called in-homogenous ones.

The interaction can be divided into two categories: One happened between homogenous individuals, and the other happened between in-homogenous individuals, which makes the corresponding system more complex. In this system, the random and dynamic state, behavior, and configuration cannot only be influenced by the environment but also effect the property of the environment, which makes the system a complex adaptive system, then, more important problems should be focused on: Under evolution of this complex adaptive system, what properties would be about the individual behavior, and what could the distribution of the corresponding optimal strategy be? These problems are of much interest.

Over the past two decades, the study of complex adaptive systems has become a major field [1,2]. A very broad range of complex adaptive systems have been studied, from abstract ones, such as the evolution of economic systems [3,4] and theory of emergence [5,6], to physical ones, such as social [7,8], epidemic [9,10], characteristic of bifurcation [11], which means a minor perturbation may escalate into tectonic shifts in the system, or even abrupt changes in the property and function of the system [12], resulting in symmetry-breaking [13]. In essence, the spontaneous switches in group behavior derive from interactions between individuals [14], during which some behaviors are learned [15] or propagate [16,17], causing the structure and behaviors of the system (or of the collective) to change [18] or even to reach the critical state [19]. All have in common the property that their detailed structure cannot be explained exactly from a mathematical viewpoint. At the beginning of the 21st century, a wealth of stochastic differential game theory has emerged [20,21], which describes the interaction behavior of agents and the optimal strategy coupled with temporary deterministic structure and stochastic complex networks [22]. Kinds of literature in different fields have described the evolution law under various agent interaction rules [23]. Jiang [24] demonstrated the importance of balancing both parties' interests within an ecological compensation agreement while reducing uncertainty around unobserved environmental factors during ex-ante negotiations.

In this paper, a multi-agent model is constructed to analyze the evolution law of economic and management complex adaptive systems. Based on experimental analysis, it has been found that in the most of cases, the system behavior conforms to a specific distribution when agents work according to this model. Furthermore, some strategies for economic issues, politic events, social questions, and environmental influence would be made scientifically once this distribution law is determined. Compared with the existing works, the main contributions of the paper are summarized as follows:

1) Both the diverse behaviors of agent and the configuration of the system are all changed randomly with time, and the classic analysis method cannot deal with this dual randomness. There have been few results to discover what the system's behavior would be converged to, as time tends to infinity or a relatively large number under classic methods. We consider not only the growth of the random complex networks, but also the decline. Furthermore, in our model, the network is regarded as a multi-local-event network, and each Multi-Local-World can be regarded as a relative independent subsystem. This case is closer to reality; however, it has not been discussed before.

2) Most research results have either considered the mixed interaction of non-cooperative/cooperative games in Multi-Local-Worlds stable graph or considered the random Multi-Local-Worlds complex networks with the Boolean game between individuals, which are far away from the property of the real economic and management complex adaptive system. In this sense, new modeling methods should be introduced.

3) Furthermore, in classic research, the interaction between individuals is set as a preferential attachment with degree. In our study, this preferential attachment is designed from both income coupled with certain strategy and the local configuration, and the phase transfer equations are modeled

as a similar mechanism. So, we consider not only the behavior and adaptability of an agent, but also the interaction between environment and system, which could interpret more accurately the economic and management complex adaptive system.

The paper is organized as follows. In Section 3, the characteristics of a complex adaptive system are analyzed and a hypothesis is proposed. In Section 4, the model coevolving with both strategy and local topological structure of individual interaction is constructed, where agent behavior in the system is described as 6 subprocesses. In Section 5, a computational experiment was conducted to verify the above analysis. Section 6 provides the conclusion of this paper.

2. Literature review, need and relevance

Generally, each agent in a complex adaptive system could interact just with local agents, not global ones, meaning that an arbitrary agent can just act with limited agents in the system; their interaction relies on the system's local topological configuration [25]，Furthermore, each agent in the system can change its interacting targets (i.e., its "neighbors") to obtain more benefits; there are complex nonlinear interactions among subjects and between subjects and environments, which lead to the phenomenon of "emergence in large numbers" of the system [26–28]. Schlüter [26] has found the emergence that behavior of collective with large enough individuals is far beyond the sum of individuals behavior in social-economical systems, Steffen et al. [27] have studied the emergence in each system by invoking a similar method and draw a same conclusion, and Maia et al. [28] have studied the emergence of societal bubbles, that is, the evolution of microscopic individuals makes the macro system display a new state and a new structure [29,30]. Zhang et al. [29] have proven that the structure, property and function of supply chain is time-varied randomly, and Zou et al. [30] have proven this by introducing a complex networks model. In this sense, the local topological configuration is not stable but dynamic [31,32]. Liang et al. [31] have constructed an avalanche model to describe that the configuration of wiring-economical modular networks could be destroyed without any warning, and Fulker et al. [32] have constructed a dynamic networks model to describe the property of the economic system. Obviously, most complex adaptive systems have two characteristics: complex networks and games. Therefore, it is necessary to introduce new methods to study the properties of optimal agent strategies in complex adaptive systems. A master stability approach for a large class of adaptive networks has been developed [33]; this approach allows for reducing the synchronization problem for adaptive networks to a low-dimensional system by decoupling topological and dynamical properties. An effective new technique called the regulative norms detection technique (RNDT) to detect norms by analyzing odd events that trigger reward or penalty is demonstrated [34]. The test results showed that the RNDT performed well, although the success rate relied on the settings of the environmental variables. Some results claimed that macroeconomics should consider the economy as a complex evolving system whose far-from-equilibrium interactions continuously changed the structure of the system and that the complex interactions of agents led to the emergence of new phenomena and hierarchical structure at the macro level, having constructed a bottom-up model to describe the emergence property of multi-hierarchical structure complex systems [35].

Generally, a system can be divided into multiple subsystems, and interactions between individuals occur not only within a subsystem, but across different subsystems [36,37]. The interactions occur not merely between neighboring individuals, and the long-range interactions in spatial dimension have significant effects on the critical phase transition of the system [38–40]. Furthermore, in addition, the rules met by the interactions between individuals within an economic or management system are far more complex than the rules regulating interactions between individuals in the natural world, such as the conservation of momentum that regulates collisions of particles and the black box of biology (such as the behavior adjustment strategies defined by the Ising model and Vicsek model) [41–44]. Moreover, it has been scientifically proven that the system structure determined by interactions between individuals is a key contributing factor to the function and nature of the system [45], so the multi-layered structure of economic and management systems leads to a strong cascade effect of collective behaviors [46–48]. Furthermore, when individuals interact in varied structures, such as sparse graphs and dense graphs, random graphs and complete graphs, scale-free networks, and small-world networks [49–52], the diverse and random structures of economic and management systems, due to the dependence on the structure [53–55], make the phase transition in these systems far more complex than the critical phase transitions in physical systems comprised of mono-dimensional and simple interactions between individuals [56].

However, the economic and management system cannot be described by random complex networks models because the Boolean interaction defined in random complex networks models is much simpler than the reality, nor can it be defined by different game models because the configuration of interaction between agents is changed dynamically. So, these two properties must be considered. If this problem is resolved, a universal conclusion would be drawn and many similar economic and management events would be explained by this theory. Furthermore, there are many unknown and unseen scenarios in reality. Due to the lack of real-world data, the conclusions regarding concerted changes in collective behavior reached by classical analysis methods do not apply to unknown scenarios. So, it is necessary to draw a universal conclusion for invariable distribution about the system's behavior. In this paper, co-evolutionary complex adaptive systems with agent behavior and local topological configuration are constructed to describe the property of economic and management systems. In this paper, a Multi-Local-Worlds economic and management complex adaptive system with agent behavior and local configuration is considered. This partially complements the gap between reality and the results of previous studies.

3. Characteristic analysis of co-evolutional complex adaptive systems

A complex adaptive system can adjust its behaviors and structure according to the environment changing dynamically; on the contrary, its behaviors, functions, and structure can also react to the environment and make the environment change adaptively [57–59]. This property makes the complex adaptive system multi-hierarchical, intelligent, social, and autonomous [60]. There are several "Local-Worlds" in this system, as shown in Figure 1.

From Figure 1(1), the multi-hierarchical topological structure can be seen clearly. The details can be shown in Figure 1(2), and the topological structure of a complex adaptive system driven by the interaction between agents can be divided into several subsystems; similarly, the topological structure of each Sub-system can be divided into several sub-subsystems; and so on. In this sense, a definition of Local-World is introduced here.

Definition 1. *The connected subgraph* $G_i, i=1,2,...,m$ *of the topological structure of the complex adaptive system* G*, where* $G_i \subseteq G$*, is called Local-World.*

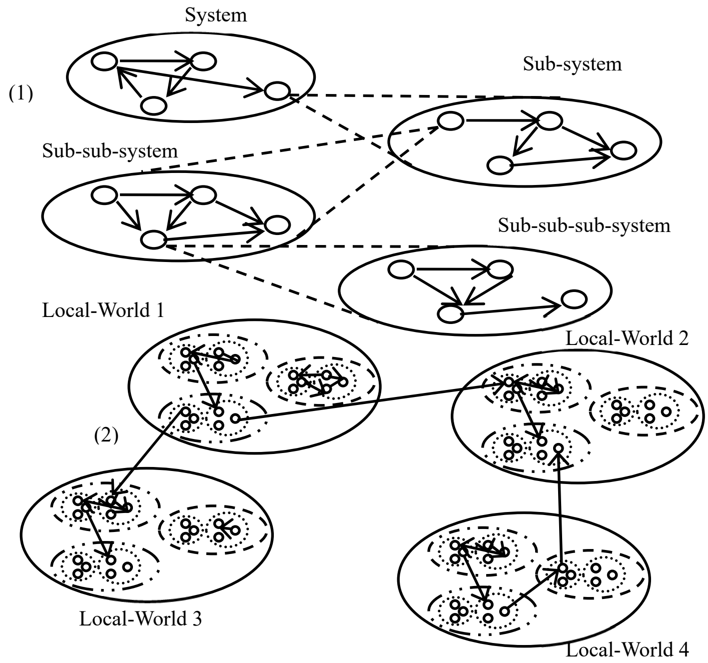

Figure 1. The universal topological structure of a complex adaptive system.

Because there are several hierarchies and there are several Local-Worlds in each corresponding system (system, subsystem and so on), the behavior of the system is more complex. Without loss of generality, we suppose that there are two hierarchies in this system. According to the dynamical property of the configuration of a complex adaptive system, the system configuration can be described in Figure 2.

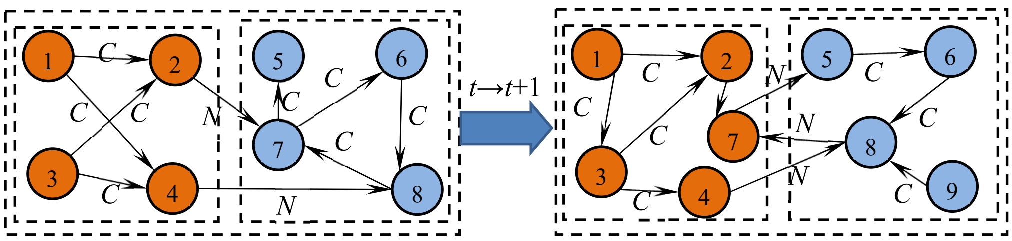

Figure 2. Interaction between agents in the dynamic topology of the complex adaptive system.

Figure 2 describes the dynamic property of the complex adaptive system with co-evolving behavior and local configuration, where C represents "Cooperation games" that happen in agents and N represents "noncooperation games" that happen in agents. There are two local worlds in this system. At time t, agents 1–4 stand in Local-World 1, and agents 5–8 are in Local-World 2. This interactive configuration is presented on the left side of Figure 2. However, at time $t+1$, agent 9 takes part in this system and interacts with agent 8, which leads to a change in the system configuration, as seen on the right side of Figure 2. There is a totally different structure, where agents 1–4 and 7 are in Local-World 1, and agents 5, 6, 8 and 9 are in Local-World 2.

Consider the complex adaptive system that satisfies the following properties:

1) There exist more than enough Local-Worlds such that the system holds in the spatial dimension. The radius is small enough from the macro spatial scale such that there are more than enough Local-Worlds existing in the system and interacting with each other. Meanwhile, from a micro spatial scale, the radius is large enough such that there are more than enough individuals existing in it and interacting with each other. Both the number of Local-Worlds and the relationships between them are constant, from a macro spatial scale, such that the system structure and function are stable. However, individuals would waver randomly from one Local-World to another, from a micro spatial scale, so that they are variable.

2) On a relatively short time scale, the interactivities of cooperative stochastic differential games between agents in a certain Local-World are defined. Conversely, the interactions of noncooperative stochastic differential games between agents standing in different Local-Worlds are also defined. Furthermore, the system's topology configuration determines the states, properties, and characters of agents.

3) On a small time scale, under preferential attachment, growth, and decline mechanisms with agents' intellective, autonomous, and social properties, agents in the system can adjust adaptively to its topological configuration such that the complex adaptive system innovates. Furthermore, on a large time scale, the interactivities between agents can be produced by pursuing more payoffs by improving the interaction structure, which consists of 6 subprocesses: self-improvement strategy; creating a new game in the same Local-World with agents with whom they have not interacted before; creating a new game relationship with agents in other Local-Worlds with whom they have not interacted before; deleting an old game relationship; creating a new game relationship with a new agent, or withdrawing from the system.

In a small time scale, an arbitrary agent should not pursue the maximum payoff but rather pursue the payoff maximized for that particular time scale. This means that an agent can give up the transitory benefit that is described by a certain integral of the objectivity function [61,62]. Furthermore, the behavior of an agent will be decided due to the corresponding resource it holds. However, there exists a dynamics equation for resources changing with time for each agent such that the objectivity of agents will change together, which should be described by some certain discount function of time.

On a long time scale, for each agent in the system, there exist 6 behaviors to be selected as a certain probability vector. Its behaviors can be controlled by the mechanisms of both preferential attachment and growth coupled with strategies, trajectory, and topological configuration coevolved together [63]. An agent can adjust its strategy according to a deterministic probability by considering its historical strategies and its neighbor's strategies—a deterministic dynamics, which makes the marginal probability of pure strategy that the agent uses proportional to the marginal income [64]. It is the preferential attachment probability that decides whether an agent creates a new link with another agent or not, and the probability relies on the ratio of the payoff for creating the new game to the payoff of the Local-World. Creating a new game relationship between different Local-Worlds is similar to creating a new game relationship within the same Local-World, except for the differences in parameters. In this sense, we consider both a cooperative and noncooperative stochastic differential game according to Property (2), which describes the system evolution mechanism in detail. The preferential attachment probabilities, in fact, satisfy the Logit properties for the last 5 of the 6 subprocesses.

This co-evolutional complex adaptive system consists of two kinds of randomness. One is the agent's behavior is random, and the other is his local topological configuration driven by the interaction is also random. Furthermore, the state would be changed dynamically. In this sense, it is a stochastic process. Suppose that homogenous agents interact in the system, i.e., the number of Local-World is equal to 1, as the system has been studied [65]. As analyzed above, the topological structure of a real economic and management complex adaptive system is a stochastic multi-hierarchical, so this model must be revised. An arbitrary agent can select each behavior from 6 subprocesses with a certain probability $p_1, \ldots, p_6$, respectively, and at the next one, he could select another behavior.

Seen as property of the system, for a certain system configuration, there must exist an optimal strategy trajectory; however, because the configurations are always changed randomly, the optimal strategies are changed with changed configuration. So, we hope these optimal strategies should satisfy certain laws, described as the following hypotheses.

Hypothesis 1. *The occurring-time of optimal strategy satisfies an invariable distribution if both behavior and local configuration of each agent in the complex adaptive system are all changed randomly.*

Hypothesis 2. *The holding-time of each optimal strategy satisfies another invariable distribution if both behavior and local configuration of each agent in the complex adaptive system are all changed randomly.*

Although similar hypotheses are given by Staudigi [65], the interaction between agents is set according to the rule of prison dilemma with four discrete strategies, but not set according to universe strategies with a continuous change state.

4. Model

In practical economic management systems, such as real estate, supply chain, regional economy, financial markets, etc., they are all complex economic management systems with the following characteristics: (i) The interaction between people changes the system structure and is influenced by the system structure, forming a trend of co evolution between behavior and topology structure; (ii) there are several subsystems in complex systems, including cooperation and competition; and (iii) an optimal strategy can be found in the system. If these economic management systems can be abstracted into a universal system model, with clear system characteristics and optimal strategies of members and their changing characteristics, then these results can be extended to more specific economic management systems.

For each agent in the system, there exist 6 behaviors to be selected as a certain probability vector. Their behaviors can be controlled by the mechanisms of both preferential attachment and growth coupled with strategies, trajectory, and topological configurations that have coevolved. Their behavior is as follows:

4.1. Adjust behavior

An arbitrary agent, j_i, will change its strategy as probability $q_1(\omega) \in [0,1]$. The probability $b^{j_i,\beta}(\cdot|\omega)$ of the agent changing its behavior in a certain system configuration should satisfy

$$b^{j_i}(a_r|\omega) \triangleq \mathbb{P}\left(a_r \in \arg\max_{a_v \in \mathscr{A}}\left(\pi^{j_i}\left(\alpha_{j_i}^{a_v}, \boldsymbol{g}\right) + \varepsilon_{a_v}^{j_i}\right)\middle|\omega\right) \tag{1}$$

where a_r is the most effective strategy within the game radius r, a_v is the strategy space, $\mathscr{A}$ is the strategy space collection, $\pi^{j_i}(\cdot)$ is the payoff of agent j_i, A is the strategy of agent j_i in the strategy space, and ε is noise. Equation (1) means that an agent transferring from one strategy to another is decided because the expected payoff coupled with the new strategy could make maximize agent j_i 's payoff, which can be regarded as random selection for strategy in a deterministic hybrid game. Furthermore, this decision relies on not only the neighbor's strategy and the topological structure $\boldsymbol{g}$, but also on the environment β.

$$-\lim_{\beta\to 0}\log b^{j_i,\beta}(a|\omega)=c_1^{j_i}(\omega,(a_{j_i}^{\alpha},\boldsymbol{g}))=v^{(t)j_i}(t,x_{j_i}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*})) \tag{2}$$

where strategy $\phi_{j_i}^{*}\left(t,x_{j_i}^{t*}\right)$ for time t and state x^t refers to the optimal strategy vector for a specific pure strategy, $\phi_{j_i}^{*}\left(t,x_{j_i}^{t*}\right)=\left(\phi_{j_1}^{*}\left(t,x_{j_1}^{t*}\right),\phi_{j_2}^{*}\left(t,x_{j_2}^{t*}\right),\cdots,\phi_{j_\vartheta}^{*}\left(t,x_{j_\vartheta}^{t*}\right)\right)^T$, and ϑ is the spatial dimension of agent j_i.

4.2. Create a new game relationship with another agent in the same Local-World

Suppose that arbitrary agent j_i creates a new game with a new agent who is not its neighbors with probability $w^{(\text{sub-process2})j_i,\beta}(\omega)\triangleq(w_{k_i}^{(\text{sub-process2})j_i,\beta}(\omega))_{k_i\in\mathscr{I}}=\lambda^{(\text{sub-process2})j_i}(\omega)/\lambda^{(\text{sub-process2})}(\omega)$, which relies on ration function $\lambda^{(\text{sub-process2})j_i}:\Omega\to\mathbb{R}_+$ that satisfies $\kappa^{j_i}(\omega)=N_i-1\Rightarrow\lambda^{j_i}(\omega)=0$ and

$$(\forall j_i\in\mathscr{I})(\forall\omega\in\Omega):\lambda^{(\text{sub-process2})j_i}=\sum_{k_i\notin\bar{\mathscr{N}}^{j_i}(\omega)}\exp\left(\tilde{W}^{(t)\{j_i,k_i\}}\left(t,x_{j_i}^{t*},x_{k_i}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{k_i}^{*}(t,x_{j_i}^{t*})\right)/\beta\right) \tag{3}$$

$$\begin{aligned}\lambda^{(\text{sub-process 2})\beta}&\triangleq\sum_{j_i\in\mathscr{I}}\lambda^{(\text{sub-process 2})j_i,\beta}(\omega)\\&=\sum_{j_i,k_i>j_i}\exp\left(\tilde{W}^{(t)\{j_i,k_i\}}\left(t,x_{j_i}^{t*},x_{k_i}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{k_i}^{*}(t,x_{j_i}^{t*})\right)/\beta\right)(1-\boldsymbol{g}_{k_i}^{j_i})\end{aligned} \tag{4}$$

$$\forall(j_i,k_i\in\mathscr{I}):w_{j_i}^{(\text{sub-process 2})k_i,\beta}(\omega)=\hat{w}_{j_i}^{(\text{sub-process 2})k_i,\beta}(\alpha)(1-\boldsymbol{g}_{j_ik_i})$$

The probability of agent j_i creating a new game with agent k_i is defined as $\lambda^{(\text{sub-process2})j_i}(\omega)/\bar{\lambda}^{(\text{sub-process2})}(\omega)$, which means that payoff of coalition of agent j_i and agent k_i is larger or equal to the other coalition's payoff affected by noise $\varsigma^{j_i}=(\varsigma_{k_i}^{j_i})_{k_i\notin\bar{\mathscr{N}}^{j_i}(\omega)}$. In this sense, we have

$$\begin{aligned}w_{k_i}^{j_i}(\omega)\triangleq\mathbb{P}\Big(&\tilde{W}^{(t)\{j_i,k_i\}}\left(t,x_{j_i}^{t*},x_{k_i}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{k_i}^{*}(t,x_{j_i}^{t*})\right)+\varsigma_{k_i}^{j_i}\\&\geq\tilde{W}^{(t)\{j_i,l_i\}}\left(t,x_{j_i}^{t*},x_{l_i}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{l_i}^{*}(t,x_{l_i}^{t*})\right)+\varsigma_{l_i}^{j_i}\forall l_i\notin\bar{\mathscr{N}}^{j_i}\,\Big|\,\omega\Big)\end{aligned} \tag{5}$$

and

$$\begin{aligned}&(\forall j_i\in\mathscr{I})(\forall\omega\in\Omega):-\lim_{\beta\to 0}\beta\log w_{k_i}^{(\text{sub-process2})j_i,\beta}(\omega)=c_2^{j_i}(\omega,(a,g\oplus(j_i,k_i)))\\&=\frac{\tilde{W}^{(t)\{j_i,k_i\}}\left(t,x_{\{j_i,k_i\}}^{(t)*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{k_i}^{*}(t,x_{k_i}^{t*}),g\oplus(j_i,k_i)\right)}{\sum_{l_i\notin\bar{\mathscr{N}}^{j_i}(\omega)}\tilde{W}^{(t)\{j_i,l_i\}}\left(t,x_{\{j_i,l_i\}}^{(t)*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{l_i}^{*}(t,x_{l_i}^{t*}),g\oplus(j_i,l_i)\right)}\end{aligned} \tag{6}$$

That is, $\exists \tilde{k}_l \neq k_l, \tilde{k}_l \notin \mathscr{N}^{-j_i}, \mathbb{P}\left\{W^{\{j_i,k_l\}}(t,x^{t*}) \geq W^{\{j_i,\tilde{k}_l\}}(t,x^{t*})\right\} = 1$ must be satisfied to create a new game relationship with agents from $N_i - \mathscr{N}_{j_i}$ with whom agent j_i did not interact. It is the link probability, also called preferential attachment mechanism, that states an agent prefers to select a game partner who can bring them more payoff than another. This causes each agent to be selected prior to their payoff coupled with the optimal strategy in the corresponding short time interval. This probability is a multi-dimension logit function, which means there exists a critical point of probability $\tilde{W}_0^{(t)\{j_i,k_i\}}$ coupled with the agent's payoff in the selected process, such that the probability a certain agent will be selected is far smaller than 0.5 if the agent's payoff is smaller than $\tilde{W}_0^{(t)\{j_i,k_i\}}$, but the selecting probability is far larger than 0.5 and close to 1 if the agent's payoff is larger than $\tilde{W}_0^{(t)\{j_i,k_i\}}$.

4.3. Create a new game relationship with another agent in a different Local-World

Similar to subprocess 2, the case where agent j_i creates a new game with agent $k_{i'}$ must satisfy the following conditions:

$$\left(\forall j_i \in \mathscr{I}\right)\left(\forall \omega \in \Omega\right): \lambda^{(\text{Sub-process 3})j_i,\beta}(\omega) = \sum_{k_{i'} \notin \mathscr{N}^{-j_i}(\omega)} \exp\left(\tilde{W}^{(t)\{j_i,k_{i'}\}}\left(t, x_{j_i}^{t*}, x_{k_{i'}}^{t*}, \phi_{j_i}^*(t,x_{j_i}^{t*}), \phi_{k_{i'}}^*(t,x_{j_{i'}}^{t*})\right)/\beta\right) \tag{7}$$

$$\lambda^{(\text{Sub-process 3})\beta}(\omega) \triangleq \sum_{j_i \in \mathscr{I}} \lambda^{(\text{Sub-process 3})j_i,\beta}(\omega)$$

$$\begin{aligned} w_{k_j}^{(\text{Sub-process 3})j_i}(\omega) &\triangleq \mathbb{P}\left(\pi\left(\alpha^{j_i},\alpha^{k_{i'}}\right) + \varsigma_{k_{i'}}^{j_i} \geq \pi\left(\alpha^{j_i},\alpha^{l_{i'}}\right) + \varsigma_{l_{i'}}^{j_i} \forall l_{i'} \notin \mathscr{N}^{-j_i}(\omega)\right) \\ &= \mathbb{P}\left(\tilde{W}^{(t)\{j_i,k_{i'}\}}\left(t, x_{j_i}^{t*}, x_{k_{i'}}^{t*}, \phi_{j_i}^*(t,x_{j_i}^{t*}), \phi_{k_{i'}}^*(t,x_{k_{i'}}^{t*})\right) + \varsigma_{k_{i'}}^{j_i} \right. \\ &\geq \left. \mathbb{P}\left(\tilde{W}^{(t)\{j_i,l_{i'}\}}\left(t, x_{j_i}^{t*}, x_{l_{i'}}^{t*}, \phi_{j_i}^*(t,x_{j_i}^{t*}), \phi_{l_{i'}}^*(t,x_{l_{i'}}^{t*})\right)\pi\left(\alpha^{j_i},\alpha^{l_{i'}}\right) + \varsigma_{l_{i'}}^{j_i}, \forall l_{i'} \notin \mathscr{N}^{-j_i}(\omega)\right)\right. \end{aligned} \tag{8}$$

So, we have

$$\begin{aligned} \left(\forall j_i \in \mathscr{I}\right)\left(\forall k_j \notin \mathscr{N}^{-j_i}(\omega)\right): w_{k_j}^{(\text{Sub-process 3})j_i,\beta}(\omega) &= \frac{\exp\left(\pi\left(\alpha^{j_i},\alpha^{k_j}\right)/\beta\right)}{\sum_{l_j \notin \mathscr{N}^{-j_i}} \exp\left(\pi\left(\alpha^{j_i},\alpha^{l_j}\right)/\beta\right)} \\ = \frac{\exp\left(\tilde{W}^{(t)\{j_i,k_{i'}\}}\left(t, x_{j_i}^{t*}, x_{k_{i'}}^{t*}, \phi_{j_i}^*(t,x_{j_i}^{t*}), \phi_{k_{i'}}^*(t,x_{k_{i'}}^{t*})\right)/\beta\right)}{\sum_{l_{i'} \notin \mathscr{N}^{-j_i}} \exp\left(\tilde{W}^{(t)\{j_i,l_{i'}\}}\left(t, x_{j_i}^{t*}, x_{l_{i'}}^{t*}, \phi_{j_i}^*(t,x_{j_i}^{t*}), \phi_{l_{i'}}^*(t,x_{l_{i'}}^{t*})\right)/\beta\right)} \end{aligned} \tag{9}$$

$$\lambda^{(\text{Sub-process 3})\beta}(\omega \to \hat{\omega}) = \lambda^{(\text{Sub-process 3})j_i}(\omega) w_{k_j}^{(\text{Sub-process 3})j_i}(\omega) + \lambda^{(\text{Sub-process 3})k_j}(\omega) w_{j_i}^{(\text{Sub-process 3})k_j}(\omega) \tag{10}$$

$$\lambda^{(\text{Sub-process 3})\beta}(\omega \to \hat{\omega}) = \exp\left(\tilde{W}^{(t)\{j_i,k_{i'}\}}\left(t, x_{j_i}^{t*}, x_{k_{i'}}^{t*}, \phi_{j_i}^*(t,x_{j_i}^{t*}), \phi_{k_{i'}}^*(t,x_{k_{i'}}^{t*})\right)/\beta\right) \tag{11}$$

4.4. Delete an existing game relationship

If an agent interacts with another one but cannot obtain the expected payoff, it will delete the game relationship without hesitation. Two kinds of agents can be deleted: one standing in the same Local-World and the other standing in another Local-World, who is denoted by k_i. Furthermore, only the weakest links are deleted as a maximum probability, which means that the probability of deleting an existing game relationship is a preferential attachment—preferential abbreviation. In this sense, suppose that an arbitrary link $(j_i, k_{i'})$ will disappear as probability $\xi > 0$. That is, if this link exists as probability $\xi h + o(h)$ during a small enough time interval $[t, t+h]$, the expected time of existence will be $1/\xi$. Therefore, starting from system state $\omega = (\alpha, g)$, the probability that system transit system state $\hat{\omega} = (\alpha, g - (j_i, k_{i'}))$ must be $\eta^{(\text{Sub-process 4})\beta}(\omega \to \hat{\omega}) = \xi$ is

$$\begin{aligned}&\left(\forall j_i \in \mathscr{I}\right)\left(\forall \omega \in \Omega\right) : \eta^{(\text{Sub-process 4}) j_i, \beta}(\omega) \\ &\quad = \sum_{k_j \in \mathscr{N}^{j_i}(\omega)} \exp\left(\tilde{W}^{(t)\{j_i, k_{i'}\}}\left(t, x_{j_i}^{t*}, x_{k_{i'}}^{t*}, \phi_{j_i}^{*}(t, x_{j_i}^{t*}), \phi_{k_{i'}}^{*}(t, x_{k_{i'}}^{t*})\right) / \beta\right)\end{aligned} \tag{12}$$

$$\eta^{(\text{Sub-process 4})\beta}(\omega) = \sum_{j_i \in \mathscr{I}} \eta^{(\text{Sub-process 4}) j_i, \beta}(\omega) \tag{13}$$

$$\begin{aligned}v_{k_{i'}}^{(\text{Sub-process 4}) j_i}(\omega) &\triangleq \mathbb{P}\left(\pi\left(\alpha^{j_i}, \alpha^{k_{i'}}\right) + \varsigma_{k_{i'}}^{j_i} \le \pi\left(\alpha^{j_i}, \alpha^{l_{i'}}\right) + \varsigma_{l_{i'}}^{j_i} \forall l_{i'} \notin \mathscr{N}^{j_i}(\omega)\right) \\ &= \mathbb{P}\left(\tilde{W}^{(t)\{j_i, k_{i'}\}}\left(t, x_{j_i}^{t*}, x_{k_{i'}}^{t*}, \phi_{j_i}^{*}(t, x_{j_i}^{t*}), \phi_{k_{i'}}^{*}(t, x_{k_{i'}}^{t*})\right) + \varsigma_{k_j}^{j_i} \right. \\ &\quad \left. \le \tilde{W}^{(t)\{j_i, l_{i'}\}}\left(t, x_{j_i}^{t*}, x_{l_{i'}}^{t*}, \phi_{j_i}^{*}(t, x_{j_i}^{t*}), \phi_{l_{i'}}^{*}(t, x_{l_{i'}}^{t*})\right) + \varsigma_{l_{i'}}^{j_i} \forall l_{i'} \notin \mathscr{N}^{j_i}(\omega)\right)\end{aligned} \tag{14}$$

$$\begin{aligned}&\left(\forall j_i \in \mathscr{I}\right)\left(\forall k_j \in \mathscr{N}^{j_i}(\omega)\right) : v_{k_j}^{(\text{Sub-process 4}) j_i, \beta}(\omega) = 1 - \frac{\exp\left(\pi\left(\alpha^{j_i}, \alpha^{k_j}\right) / \beta\right)}{\sum_{l_j \in \mathscr{N}^{j_i}} \exp\left(\pi\left(\alpha^{j_i}, \alpha^{l_j}\right) / \beta\right)} \\ &= 1 - \frac{\exp\left(\tilde{W}^{(t)\{j_i, k_{i'}\}}\left(t, x_{j_i}^{t*}, x_{k_{i'}}^{t*}, \phi_{j_i}^{*}(t, x_{j_i}^{t*}), \phi_{k_{i'}}^{*}(t, x_{k_{i'}}^{t*})\right) / \beta\right)}{\sum_{l_{i'} \in \mathscr{N}^{j_i}} \exp\left(\tilde{W}^{(t)\{j_i, l_{i'}\}}\left(t, x_{j_i}^{t*}, x_{l_{i'}}^{t*}, \phi_{j_i}^{*}(t, x_{j_i}^{t*}), \phi_{l_{i'}}^{*}(t, x_{l_{i'}}^{t*})\right) / \beta\right)}\end{aligned} \tag{15}$$

An agent always adjusts their interactive environment to maximize their payoff by adaptively adjusting the local game topological configuration, which can be expressed as adding a new link and deleting an old link with preferential attachment mechanisms. This adaptive behavior on the part of the agent, showing their intelligence by adding or deleting links, is the essence of the complex adaptive system.

It is easier to decide the probability of deleting a link than it is to decide the probability of adding a link because when considering deleting a link, only agent $k_{i'}$ is considered and the required calculation is reduced considerably. Furthermore, the probability of deleting a link is smaller than the probability of creating links either within same Local-World or between different Local-Worlds, because one agent can get the maximum payoff when they create a game relationship with a certain agent, and the payoff cannot decrease sharply with time except in the case where they can obtain more payoff if they interact with other agents (a small probability event).

4.5. Create a game relationship with a new agent in the system

Because a complex adaptive system is a dissipative system, a new agent can enter the system and, of course, it can withdraw from the system. When an agent enters the system, the first thing is to select a Local-World for it to stand in, and then it must select an agent to interact with.

In this paper, when an agent enters the system, it enters an arbitrary Local-World as an identical probability; therefore, the first step of this subprocess should be omitted. The other steps are similar to those of subprocess 2 and should be omitted, too. Therefore, when agent $N+1$ enters the complex adaptive system, it will enter Local-World i as probability $1/m$ and will be reordered to N_i+1, then it will create a game relationship with an arbitrary agent j_i with probability $w_{N_i+1}^{(\text{sub-process 5}),j_i,\beta}$, where:

$$\begin{aligned}&\left(\forall j_i\in\mathscr{I}\right)\left(\forall\omega\in\Omega\right):\lambda^{(\text{Sub-process 5})j_i,\beta}(\omega)\\&\quad=\exp\left(\tilde{W}^{(t)\{j_i,N+1\}}\left(t,x_{j_i}^{t*},x_{N+1}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{N+1}^{*}(t,x_{N+1}^{t*})\right)/\beta\right)\end{aligned}\tag{16}$$

$$\begin{aligned}w_{N_i+1}^{(\text{Sub-process 5})j_i}(\omega)&\triangleq\mathbb{P}\left(\pi\left(\alpha^{j_i},\alpha^{N_i+1}\right)+\varsigma_{N_i+1}^{j_i}\geq\pi\left(\alpha^{l_i},\alpha^{N_i+1}\right)+\varsigma_{N_i+1}^{l_i}\forall l_i\in i(\omega)\right)\\&=\mathbb{P}\left(\tilde{W}^{(t)\{j_i,N+1\}}\left(t,x_{j_i}^{t*},x_{N+1}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{N+1}^{*}(t,x_{N+1}^{t*})\right)+\varsigma_{N+1}^{j_i}\right.\\&\left.\geq\tilde{W}^{(t)\{l_i,N+1\}}\left(t,x_{l_i}^{t*},x_{N+1}^{t*},\phi_{l_i}^{*}(t,x_{l_i}^{t*}),\phi_{N+1}^{*}(t,x_{N+1}^{t*})\right)+\varsigma_{N+1}^{l_i}\forall l_i\in i(\omega)\right)\end{aligned}\tag{17}$$

$$\left(\forall j_i\in\mathscr{I}\right)\left(\forall l_i\in i(\omega),l_i\neq j_i\right):w_{N_i+1}^{(\text{Sub-process 5})j_i,\beta}(\omega)=\frac{\exp\left(\pi\left(\alpha^{j_i},\alpha^{N_i+1}\right)/\beta\right)}{\sum_{l_i\in i(\omega),l_i\neq j_i}\exp\left(\pi\left(\alpha^{l_i},\alpha^{N_i+1}\right)/\beta\right)}\tag{18}$$

$$\lambda^{(\text{Sub-process 5})\beta}(\omega\to\hat{\omega})=\lambda^{(\text{Sub-process 5})j_i}(\omega)w_{N+1}^{(\text{Sub-process 5})j_i}(\omega)\tag{19}$$

4.6. An agent is deleted from the system

As mentioned above, several agents are allowed to be deleted from the system, which reflects the property of survival of the fittest of the complex adaptive system. Obviously, when an agent is deleted, the links that expressed its game relationships must be deleted.

$$\begin{aligned}&\left(\forall j_i\in\mathscr{I}\right)\left(\forall\omega\in\Omega\right):\eta^{(\text{Sub-process 6})j_i,\beta}(\omega)=\sum_{k_j\in\mathscr{T}^{j_i}(\omega)}\exp\left(\pi\left(\alpha^{j_i},\alpha^{k_j}\right)/\beta\right)\\&\quad=\sum_{k_{i'}\in\mathscr{T}^{j_i}(\omega)}\exp\left(\tilde{W}^{(t)\{j_i,k_{i'}\}}\left(t,x_{j_i}^{t*},x_{k_{i'}}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{k_{i'}}^{*}(t,x_{k_{i'}}^{t*})\right)/\beta\right)\end{aligned}\tag{20}$$

$$\begin{aligned} v^{(\text{Sub-process 6})j_i}(\omega) &\triangleq \mathbb{P}\Big(\sum\nolimits_{k_i \in \mathscr{N}^{j_i}}\Big(\pi\Big(\alpha^{j_i},\alpha^{k_i}\Big)+\varsigma_{k_j}^{j_i}\Big) \le \sum\nolimits_{l_j \in \mathscr{N}^{k_j}}\Big(\pi\Big(\alpha^{l_j},\alpha^{k_j}\Big)+\varsigma_{k_j}^{l_j}\Big)\Big) \\ &= \mathbb{P}\Big(\sum\nolimits_{k_i \in \mathscr{N}^{j_i}}\Big(\tilde{W}^{(t)\{j_i,k_i\}}\Big(t,x_{j_i}^{t*},x_{k_i}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{k_i}^{*}(t,x_{k_i}^{t*})\Big)+\varsigma_{k_i}^{j_i}\Big) \\ &\le \sum\nolimits_{l_{i'} \in \mathscr{N}^{k_j}}\Big(\tilde{W}^{(t)\{j_i,l_{i'}\}}\Big(t,x_{j_i}^{t*},x_{l_{i'}}^{t*},\phi_{j_i}^{*}(t,x_{j_i}^{t*}),\phi_{l_{i'}}^{*}(t,x_{l_{i'}}^{t*})\Big)+\varsigma_{l_{i'}}^{j_i}\Big) \end{aligned} \tag{21}$$

$$\begin{aligned} &\Big(\forall j_i \in \mathscr{I}\Big)\Big(\forall k_i \in \mathscr{I}, k_i \in \mathscr{N}^{j_i}\Big)\Big(\forall k_j \in N \setminus j_i\Big)\Big(\forall l_j \in \mathscr{I}, l_j \in \mathscr{N}^{k_j}\Big): \\ &v^{(\text{Sub-process 6})j_i,\beta}(\omega) = 1 - \frac{\sum_{k_i \in \mathscr{N}^{j_i}} \exp\Big(\pi\Big(\alpha^{j_i},\alpha^{k_i}\Big)/\beta\Big)}{\sum_{k_j \in N \setminus j_i}\sum_{l_j \in \mathscr{N}^{k_j}} \exp\Big(\pi\Big(\alpha^{l_j},\alpha^{k_j}\Big)/\beta\Big)} \end{aligned} \tag{22}$$

Reconsidering the case of long time-scale, the binary tuple of an agent's strategy and the local topology would be considered as the system's state. The system evolves according to a state transition equation matched with the 6 subprocesses, which forms a stochastic process, a weak Markov process. By analyzing this process, the invariable distribution can be determined.

Utilizing a dynamical model on a large time-scale, the complex adaptive system exhibits distinct properties as follows: (i) The behavior of any given agent within the system necessitates synchronization with the behaviors of its neighboring agents. (ii) Decisions made by arbitrary agents, identifiable within a relatively short time-scale, should persist within this temporal interval. Notably, only a single property undergoes modification during each innovation event. (iii) This intricate process is bifurcated into two levels: a macro level delineating the system's evolution law and a micro-level. This dynamic interplay unfolds through six events triggered by alterations in behavior configuration, local topological configuration, or simultaneous changes in both aspects. Furthermore, at the macro level, the 6 basic events are observable, and they occur as an independent probability that emerges from the micro-level. Set $\tau \triangleq \dfrac{\lambda^{(\text{sub-process 2})\beta}+\lambda^{(\text{sub-process 3})\beta}+\lambda^{(\text{sub-process 5})\beta}}{\lambda^{(\text{sub-process 4})\beta}}$ to describe whether the system grows or declines. If it is far larger than 1, then the topological structure of the complex network decides the system developing, so the focus of the study is on the evolution law of complex networks. On the other hand, if it is far smaller than 1, then the changed behaviors control the system, making the stochastic differential games between agents the focus. If it is equal to or approximately equal to 1, the configuration can be regarded as stable.

On a long time scale, the system's evolution process should be defined as a continuous stochastic process analyzed above, in which each short time should be reordered to the new order of the continuous-time point in the system evolution process on a long time scale. We introduce a new preferential attachment mechanism in which each agent interacts with another as a probability that relies on the attractor of its payoff converged in the corresponding short time interval to the complex adaptive system, which is an innovation of this paper. The evolution law of this complex adaptive system must be mined if, and only if, the state transition equation of the complex adaptive system is determined and the law of the system evolution is found.

5. Example

Let's consider an example with three agents, a smart agent, a normal agent, and a stupid agent;

each agent consists of enough individuals; they interact with a homogenous individual or inhomogeneous individual in order to achieve their goal achieved. Smart individuals always have selective competitive behavior, and they are good at distributing resources, but stupid individuals have not; the normal individuals are in the middle. Furthermore, smart individuals make a decision according to Eqs (2), (6), (9), (15), (18) and (22) with larger $\bar{\mathscr{N}}$ and their behavior is denoted by behavior x; the stupid individual would not select behavior with a maximum payoff with smaller $\bar{\mathscr{N}}$, and their behavior is denoted by behavior z; the behavior of normal individual is denoted by behavior y. We construct a multi-agent computation simulation model to analyze this system.

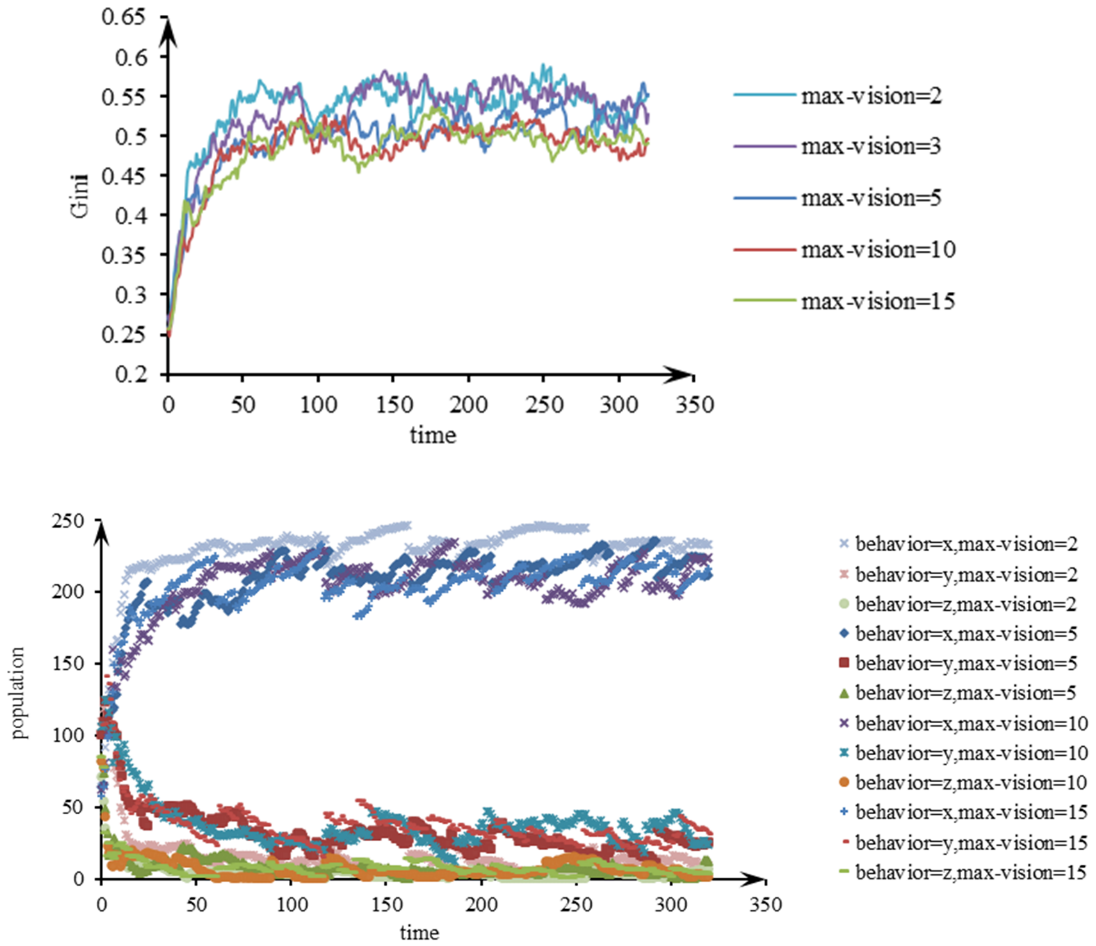

Figure 3. Average payoff distribution and strategies evolution under a different vision.

This model is adapted from Epstein & Axtell's "Sugarscape" model. Each patch has several resources and a resource capacity (the number of resources) coupled with behavior. Individuals collect resources from the program and consume some resources to survive; this amount is called their metabolism. Each individual has a fixed vision; the larger the vision is, the smarter the individual is. Furthermore, an arbitrary individual's vision comes from both $\bar{\mathscr{N}}$ and the predictive ability of the future, which relies on the noise he receives. Each individual who has the percentage of the best resources should be focused on; generally speaking, the smart one is always willing to struggle for the best resources. In our work, we consider five computational experiments in NetLogo. In the first computation experiment, we consider how different visions affect the system's behaviors. Set the minimum vision of the individual as 1, and the maximum vision as 15; we select 2, 3, 5, 10 and 15 to do this experiment, set the minimum expectation as 1, the maximum expectation 85, and the percent of best resource is 10%. So, the payoff of the average payoff of the system and the evolution process of these three strategies are shown in Figure 3.

In the second computation experiment, we consider how different metabolisms affect the system's state and behavior. Set the maximum metabolisms as 5, 10, 15, 20 and 25, respectively, and set the minimum expectation as 1, the maximum expectation as 85, the percent of best resource as 10%, and the maximum vision as 1. The payoff of the average payoff of the system and the evolution process of these three strategies are shown in Figure 4.

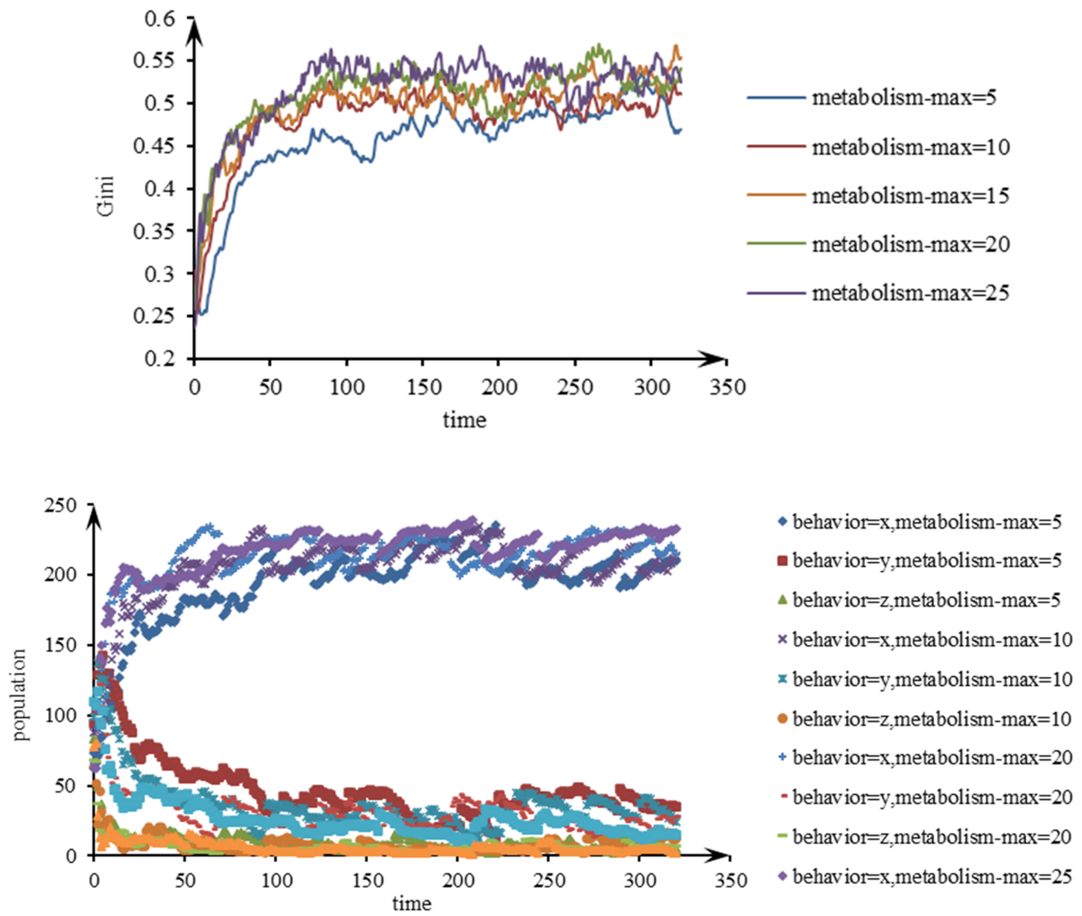

Figure 4. Average payoff distribution and strategies evolution under different metabolism.

In the third computation experiment, let's consider how different minimum expectations affect the system's state and behavior. Set the minimum expectations to be 10, 25, 50 and 70, respectively, and set the metabolisms to be 15, the maximum expected to be 85, the percent of best resource is 10%, and the maximum vision as 5. The payoff of the average payoff of the system and the evolution process of these three strategies are shown in Figure 5.

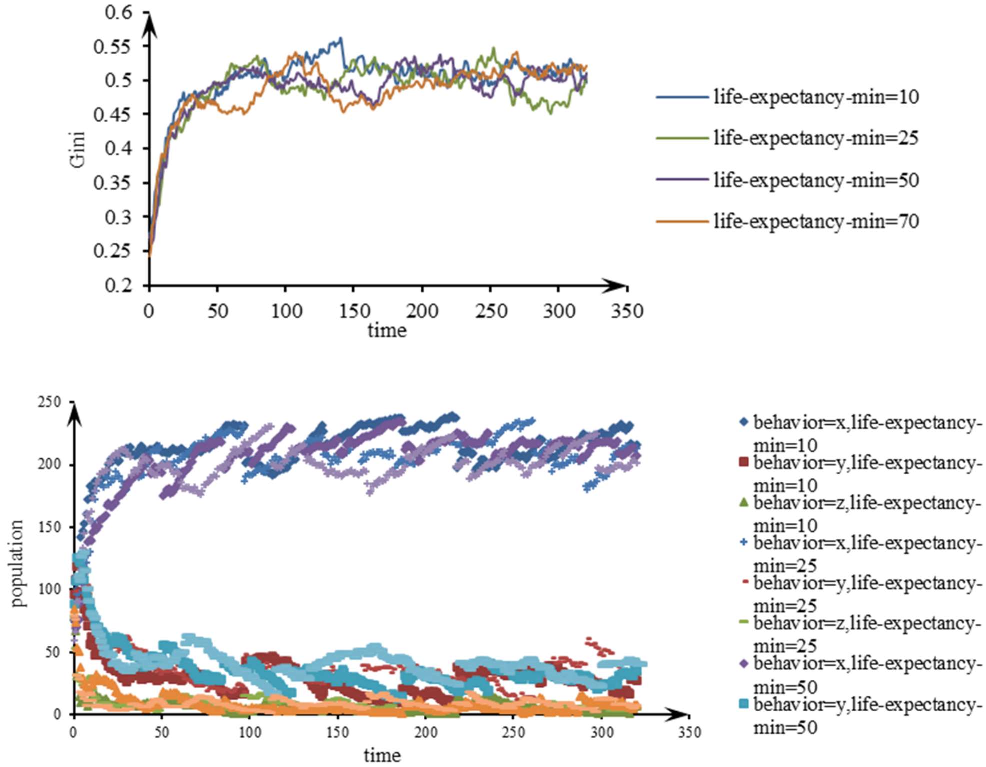

Figure 5. Average payoff distribution and strategies evolution under different minimum expectations.

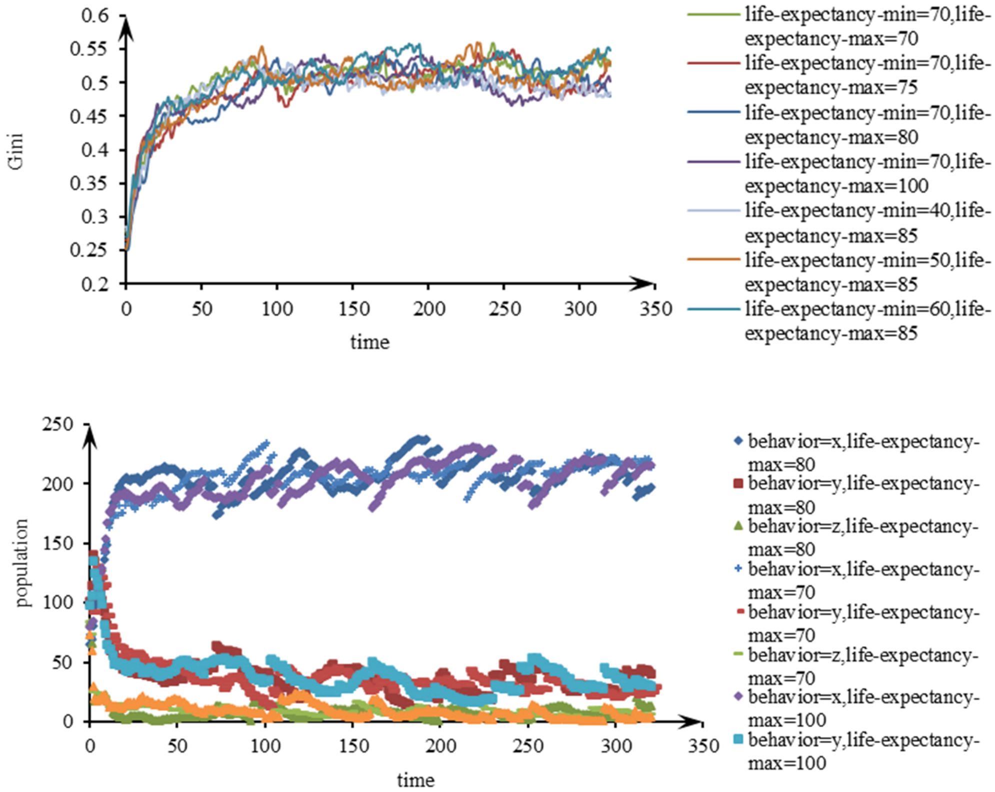

Figure 6. Average payoff distribution and strategies evolution under different minimum expectations and different maximum expectations.

In the fourth computation experiment, let's consider how both different maximum expectations and minimum expectations affect the system's state and behavior. Set the maximum expectations as 70, 75, 80, 85 and 100, respectively, the minimum expectations to be 40, 50, 60 and 70, respectively, set the metabolisms as 15, the percent of best resource as 10%, maximum vision as 5. The payoff of the average payoff of the system and the evolution process of these three strategies are shown in Figure 6.

In the fifth computation experiment, let's consider how both different best resources affect the system's state and behavior. Set the best resources are 5, 15, 20 and 25%, respectively, set the metabolisms as 15, the maximum expectation as 85, the minimum expectation as 1, and the maximum vision as 5. The payoff of the average payoff of the system and the evolution process of these three strategies are shown in Figure 7.

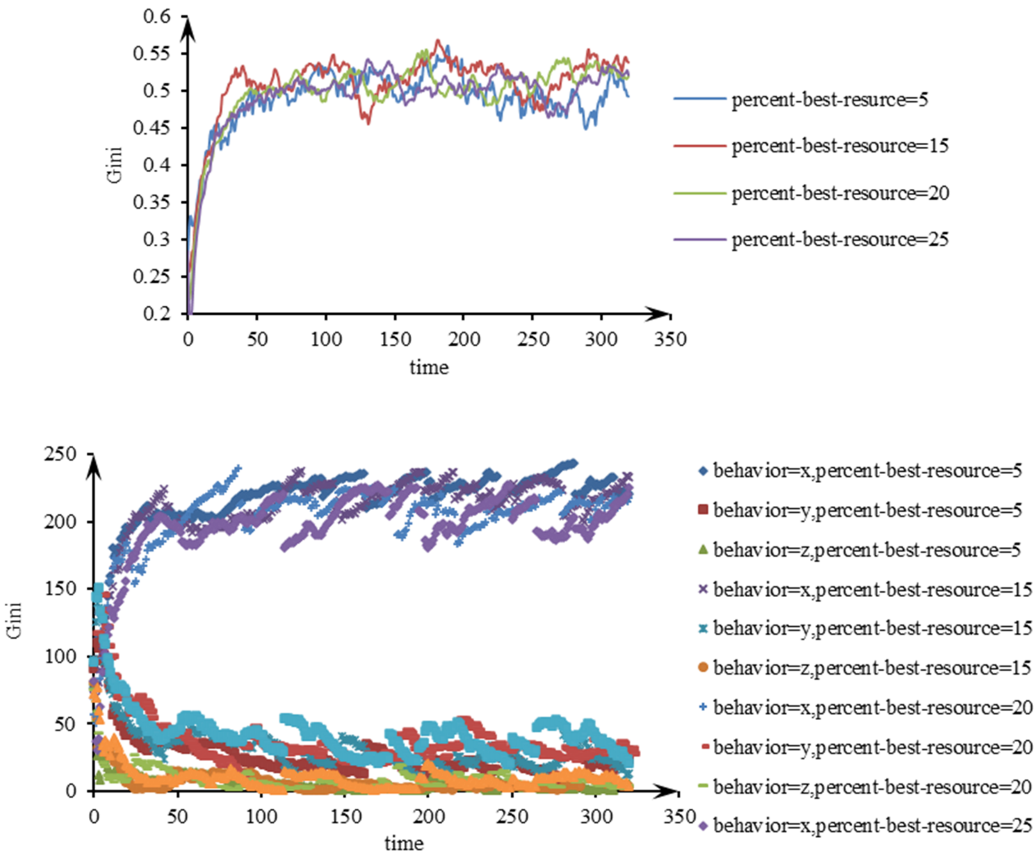

Figure 7. Average payoff distribution and strategies evolution under different best resources.

It is concluded that from Figures 3–7, the payoff of the system and the population using different strategies have been converted into a narrow interval, which means that there must exist an invariable distribution for these behaviors. Agent's decision-making process is influenced by the evolution of random complex networks driven by preferential attachment, coupled with a volatility mechanism linked to its payment—a dynamic that propels the evolution of the complex adaptive system.

It is suggested from this example that, although there are different agents coupled with certain behaviors that are reflected by different parameters, collective behavior is converged into a certain attractor. Furthermore, one or few behaviors are selected for the collective behavior; otherwise, other behaviors are abandoned, which reflects that the invariable distribution has emerged under the interaction between agents.

So, let's consider the corresponding process in this example. Observing the reachable time and holing time of this process, the following results can be described as Table 1.

Table 1. Distribution of behavior.

	Reachable time			Holing time		
Experiment	Behavior x	Behavior y	Behavior z	Behavior x	Behavior y	Behavior z
1-1	22	20	17	249	260	260
1-2	44	30	50	239	230	237
1-3	35	26	24	244	233	263
1-4	26	26	21	295	295	277
1-5	13	12	12	308	308	306
2-1	92	88	24	174	147	217
2-2	46	41	16	260	254	252
2-3	22	16	23	282	273	288
2-4	15	13	5	275	283	280
2-5	15	13	4	270	273	270
3-1	20	19	4	277	279	273
3-2	63	47	22	258	259	267
3-3	42	42	14	244	237	251
3-4	40	38	36	256	254	251
3-5	41	35	16	258	258	251
3-6	20	20	7	171	149	240
4-1	19	24	13	224	198	239
4-2	31	30	11	254	248	269
4-3	53	53	45	253	243	249
4-4	63	30	18	183	182	213
4-5	40	30	10	248	242	266
4-6	67	62	29	202	216	214
4-7	18	20	6	279	278	287
5-1	26	22	3	294	294	262
5-2	23	31	18	265	262	258
5-3	53	53	9	238	234	243
5-4	37	38	27	162	153	202

6. Results and discussion

Moreover, the model incorporates the ratios of owned best resources (representing the critical resources currently possessed by agents and anticipated in the future, constituting a pivotal constraint in decision-making) and the maximum metabolism (reflecting the agility of agents in adjusting their strategies and adaptively changing partners, thereby characterizing their volatility and stability). It is noteworthy that, due to the inherent irrationality of agents and their inability to precisely predict future system states, the minimum vision emerges as the least influential factor in behavioral collection. There are three types of agents now, and I would like to know which irrational factors influence the behavior of these three types of agents. So, we did a regression and when choosing we consider two order parameters for irrational behavior: the reachable time and duration of this behavior. We initially speculated that it was a generalized linear model. In this sense, two regressions were made. The result shows that it is basically satisfied. The square value of R is 0.87. Upon conducting a thorough statistical analysis, the principal findings are succinctly summarized in the ensuing Tables 2 and 3.

Table 2. The result of the generalized linear model of reachable time of behaviors x, y and z.

	Behavior *x*		Behavior *y*		Behavior *z*	
Parameter	B	importance	B	importance	B	importance
(Intercept)	13.583		47.491		35.867	
minimum vision	-1.523	0.0971	-1.065	0.0199	-0.670	0.1129
minimum expectation	0.212	0.1058	0.133	0.1145	0.069	0.1419
maximum expectation	0.496	0.2272	-0.026	0.1539	-0.186	0.1448
best resource	267.909	0.2395	316.388	0.2013	74.690	0.2253
maximum metabolism	-2.901	0.3304	-2.936	0.5104	-0.505	0.3751
(Scale)	186.639[a]		132.032[a]		127.743[a]	
Likelihood Ratio Chi-Square	14.397		17.041		2.069	
Sig.	0.013		0.004		0.840	

Table 3. The result of generalized linear models of holding time of behaviors x, y and z.

	Behavior *x*		Behavior *y*		Behavior *z*	
Parameter	B	importance	B	importance	B	importance
(Intercept)	211.758		150.397		204.926	
minimum vision	5.103	0.1373	5.158	0.1276	3.999	0.1482
minimum expectation	-0.400	0.2014	-0.459	0.2049	-0.266	0.2007
maximum expectation	0.625	0.2052	1.246	0.2093	0.591	0.204
best resource	-692.174	0.2062	-805.665	0.2132	-468.154	0.2104
maximum metabolism	2.784	0.2499	3.953	0.2449	2.391	0.2367
(Scale)	551.218[a]		748.422[a]		194.528[a]	
Likelihood Ratio Chi-Square	19.361		18.537		24.142	
Sig.	0.002		0.002		0.000	

Based on the analysis presented in Tables 2 and 3, the following key insights emerge: Maximum metabolism reflected by the variable $\phi(x_{\cdot}^{t^*})$ (describing the transferring resource abilities and sources quantity) stands out as the paramount factor influencing the agent's behavior. Notably, it exerts an indirect negative impact on the reachable time of arbitrary behavior, but demonstrates a positive influence on the corresponding holding time. The ratio of an agent's best resources reflected by the variable $x_{\cdot}^{t^*}$ (describing the absorbing-resource abilities and sources quantity) emerges as the second pivotal factor shaping strategy. This factor exhibits an indirect positive impact on the reachable time of arbitrary behavior while concurrently exerting a negative influence on its holding time. Additionally, the maximum expectation determined by $\max_{a_v \in \mathscr{A}} \left(\pi^{j_i} \left(\alpha_{j_i}^{a_v}, \boldsymbol{g} \right) + \varepsilon_{a_v}^{j_i} \right) | \omega$ in Eq (1) and

$\left(\tilde{W}^{(t)\{.\}}\left(t, x_{\cdot}^{t^*}, \phi_{\cdot}^{*}(t, x_{\cdot}^{t^*})\right) / \beta\right)$ in Eqs (2)–(22) ranks as the third critical factor affecting behavior, manifesting positive influences on both the reachable time and holding time of arbitrary behavior—this is because the classic irrational behavior of "every people pursue the expectation, but not the experience". In contrast, minimum vision driven by local configuration ω is identified as the least consequential factor in shaping behavior, with an indirect negative impact on the reachable time of arbitrary behavior and a positive impact on its holding time. However, the effects are significant difference among smart collective, normal collective and stupid collective.

This example explains our model well, and the behaviors of individuals and collectives are all co-evolved with local topological structure coming from individual interaction. In this model, according to the principle of "like attracts like", there occurs certain stochastic and dynamical Local-Worlds, and every individual selects the partners to pursue to maximum payment. Because the payment and location of all individuals are changed dynamically, the partners are all changed as Eqs (2)–(22). Similarily because each individual's payment relies on ω, because ω is changed dynamically and randomly, and because there are many different individuals in the system, payment is a stochastic process, and the minimum and maximum expectation are more important to set goals and strategies. Once the goal is set for each kind of people, smart individual, normal individual, and stupid individual in this example, the arbitrary individual should try their best to implement the corresponding goal by selecting the optimal strategies from his own strategies set. In this sense, a coevolutional system with behavior and local configuration is produced, as described as the model. In this model, a parameter named game radius is introduced, which is reflected by the rank of set $\overline{\mathcal{N}}$, to describe the local configuration. In this example, the parameter of vision is equal to game radius. Generally, the larger the vision, the more information he has to make an accurate decision. So, the vision size determines how smart an individual is and describes how rational a person is. As described in this example, it is concluded that the minimum vision has a negligible impact on the behavior and benefits different intelligent agents. However, the maximum vision is quite the opposite; it has a very significant impact on the behavior and benefits different intelligent agents.

In classical research, the various irrational behaviors of agents have not been discussed in depth. This paper supplements this issue, which is one of the main contributions of this study. In this example, we discussed the coexistence of individuals with different rationalities in the system, interacting with each other according to their own set rules and the model provided in this paper. We found that in any situation, there is a polarization of behavior. In other words, as the interactions continue, agents gradually form behavioral inertia and relatively fixed local structures. Consequently, behaviors with different attributes will converge to a relatively stable attractor, and the system's behavior, function, and properties will become more stable. However, this conclusion, which has been proven to be correct by evidence, has not yet been discovered in existing research. This is enough to demonstrate the correctness of our model and the value of the research presented in this paper.

7. Conclusions

Our research is notably intriguing, as the results seamlessly align with diverse social systems encompassing economic, management, political, and social domains. The system investigated in this study closely mirrors reality, offering practical solutions to numerous physical problems and yielding profound and general conclusions. Leveraging insights from the behavior evolution process and the law of invariant distribution in collective behavior enables rational decision-making. In the context of managing complex adaptive systems, the efficacy of optimal management strategies hinges upon the inherent functionality of the system. A pivotal consideration for managers is whether specific states within the complex adaptive system should be rapidly achieved or prolonged. This decision is particularly crucial for scenarios such as epidemic control, where expeditious attainment of certain states is imperative. In such cases, prioritizing the allocation of optimal resources followed by setting a high maximum expectation proves to be the most effective approach. Conversely, for situations requiring the perpetuation of specific states, such as sustainable production and political election campaigns, permitting maximum metabolism alongside establishing high maximum expectations is advised. The broad applicability and significance of this research underscores its universality, constituting a noteworthy contribution.

The emergence of a stable distribution is contingent upon the strength of decay in a complex adaptive system. A robust decay amplifies the likelihood of an invariant distribution, while a weaker decay diminishes this probability. Consequently, the stability of the complex adaptive system is inherently influenced by the noise inherent in agent behaviors. Moreover, the introduction of random changes in the properties of arbitrary agents augments the complexity of the invariant distribution. With specified parameters, the invariant distribution assumes a deterministic nature. An illustrative example is presented to elucidate the evolution of diverse behaviors, demonstrating their convergence to corresponding attractors contingent on the system's initial conditions or scenarios. Irrespective of scenario changes, our defined conditions ensure system behavior stability.

However, the introduction of irrational behaviors engenders a different behavioral pattern, a subject beyond the current scope, warranting consideration in future research. The present analysis, while comprehensive, has not delved into the intricacies of individual irrational behaviors such as preferential selection, laziness, prejudgment, and other related factors. This represents a significant and pressing area for future investigation. Understanding the nuanced dynamics of these specific irrational behaviors is paramount for a comprehensive grasp of the complex adaptive system under consideration. As these behaviors can wield substantial influence on decision-making processes and overall system dynamics, their detailed examination is warranted to enhance the depth and scope of our understanding. Consequently, delving into the impact and interplay of preferential selection, laziness, prejudgment, and similar behaviors is identified as a crucial avenue for future research within the context of this study. Addressing these individual irrational behaviors will undoubtedly contribute to a more nuanced and holistic comprehension of the overarching complex adaptive system dynamics, offering valuable insights for both theoretical and practical applications.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no competing interests.

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