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Research article

Finite groups whose coprime graphs are AT-free

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Abstract: Assume that G is a finite group. The coprime graph of G, denoted by $\Gamma(G)$, is an undirected graph whose vertex set is G and two distinct vertices x and y of $\Gamma(G)$ are adjacent if and only if $(o(x), o(y)) = 1$, where $o(x)$ and $o(y)$ are the orders of x and y, respectively. This paper gives a characterization of all finite groups with AT-free coprime graphs. This answers a question raised by Swathi and Sunitha in Forbidden subgraphs of co-prime graphs of finite groups. As applications, this paper also classifies all finite groups G such that $\Gamma(G)$ is AT-free if G is a nilpotent group, a symmetric group, an alternating group, a direct product of two non-trivial groups, or a sporadic simple group.

Keywords: coprime graph; AT-free graph; finite group

1. Introduction

In the field of algebraic graph theory, the study of graph representations according to their algebraic structures is a popular and interesting research topic. For example, a well-known graph representation from the algebraic structure group is the Cayley graph, which has a long history of research. On the other hand, graph representations of some algebraic structures have been actively studied in the literature, because of some valuable applications [1, 2].

One can define a special graph on a group, such as, power graph [3] and commuting graph [4]. Considering the order of an element in a group is one of the most basic and important concepts in group theory, we may define a graph over a group using its element order. Given a finite group G, the *coprime graph* of G, denoted by $\Gamma(G)$, is the undirected graph with vertex set G, and two distinct $x, y \in G$ are adjacent if and only if $o(x)$ and $o(y)$ are relatively prime, namely, $(o(x), o(y)) = 1$, where $o(x)$ and $o(y)$ are the orders of x and y, respectively. In 2014, the authors [5] introduced the concept of a coprime graph of a group. Afterwards, Dorbidi [6] proved that for every finite group G, the clique number of $\Gamma(G)$ is equal to the chromatic number of $\Gamma(G)$, namely, $\Gamma(G)$ is a weakly perfect graph. Also, Dorbidi [6] classified such finite groups whose coprime graph is a complete r-partite graph.

In 2017, Selvakumar and Subajini [7] obtained all finite groups G such that $\Gamma(G)$ is toroidal. In 2021, Hamm and Way [8] determined the independence number of the coprime graph on a dihedral group, and they also studied this question: which coprime graphs are perfect? Alraqad et al. [9] classified the finite groups whose coprime graph has exactly three end-vertices.

Every graph considered in our paper is undirected without loops and multiple edges. Let Γ and Δ be two graphs. If Γ contains no induced subgraph isomorphic to Δ, then Γ is called a Δ-*free* graph. This is equivalent to saying that Δ is a forbidden subgraph of Γ. Three independent vertices of a graph form an *asteroidal triple* if every two of them are connected by a path avoiding the neighborhood of the third one. A simple graph is called asteroidal triple-free (AT-free, for short) if it contains no asteroidal triple [10]. The AT-free graphs provide a common generalization of interval, permutation, trapezoid, and cocomparability graphs. In [10], Lekkerkerker and Boland demonstrated the importance of asteroidal triples in the theorem: a graph is an interval graph if and only if it is chordal and AT-free. Thus, it appears that the condition of being AT-free prohibits a chordal graph from growing in three directions at once. Later, Golumbic et al. [11] showed that cocomparability graphs (and, thus, permutation and trapezoid graphs) are also AT-free. In [12], Swathi and Sunitha studied the finite groups whose co-prime graphs are C_4-free, claw-free, cographs, split-graphs, and AT-free. Also, they proposed the following question.

Question 1.1. ([12]) *Find a characterization for the finite groups whose coprime graphs are AT-free.*

The purpose of this paper is to give a characterization of the finite groups whose coprime graphs are AT-free (see Theorem 2.3). This gives an answer to Question 1.1. As applications, we classify the finite groups G such that $\Gamma(G)$ is AT-free if G is a nilpotent group (see Corollary 2.5), a symmetric group (see Proposition 2.7), an alternating group (see Proposition 2.7), a direct product of two non-trivial groups (see Proposition 2.8), or a sporadic simple group (see Proposition 2.10).

2. Main results

This section will prove our main results. Every group considered in this paper is finite. For convenience, we always assume that G is a finite group with the identity e. Denote by $\pi_e(G)$ and $\pi(G)$ the set of orders of elements of G and the set of prime divisors of $|G|$, respectively. As usual, denote by $\mathbb{Z}_n$ the cyclic group having order n. Given a graph, say Γ, denote by $V(\Gamma)$ and $E(\Gamma)$ the vertex set and edge set of Γ, respectively. If $\{x, y\} \in E(\Gamma)$, then we also denote this by $x - y$. In a graph, we use $x_1 - x_2 - \cdots - x_n$ to denote a path of length n. The *neighborhood* of a vertex x in Γ, denoted by $N(x)$, is the set $\{v \in V(\Gamma) : v - x\}$.

We first give two results before giving the proof of our main theorem.

Observation 2.1. (1) *Suppose that $\{p_1p_2, p_1p_3, p_1p_4\} \subseteq \pi_e(G)$, where p_1, p_2, p_3, p_4 are pairwise distinct primes. Let $a, b, c \in G$ with $o(a) = p_1p_2$, $o(b) = p_1p_3$, and $o(c) = p_1p_4$. Then $a - c^{p_1} - b$ is a path such that $N(c) \cap \{a, c^{p_1}, b\} = \emptyset$, $a - b^{p_1} - c$ is a path such that $N(b) \cap \{a, b^{p_1}, c\} = \emptyset$, and $b - a^{p_1} - c$ is a path such that $N(a) \cap \{b, a^{p_1}, c\} = \emptyset$. As a result, $\{a, b, c\}$ is an* AT *in* $\Gamma(G)$.

(2) *Suppose that $\{p_1p_2, p_1p_3, p_2p_3\} \subseteq \pi_e(G)$, where p_1, p_2, p_3 are pairwise distinct primes. Let $a, b, c \in G$ with $o(a) = p_1p_2$, $o(b) = p_1p_3$, and $o(c) = p_2p_3$. Then $a - c^{p_2} - c^{p_3} - b$ is a path such that $N(c) \cap \{a, c^{p_2}, c^{p_3}, b\} = \emptyset$, $a - b^{p_1} - b^{p_3} - c$ is a path such that $N(b) \cap \{a, b^{p_1}, b^{p_3}, c\} = \emptyset$, and $b - a^{p_1} - a^{p_2} - c$ is a path such that $N(a) \cap \{b, a^{p_1}, a^{p_2}, c\} = \emptyset$. As a result, $\{a, b, c\}$ is an* AT *in* $\Gamma(G)$.

Lemma 2.2. ([12]) *Let $x, y \in G$. Then $\pi(\langle x\rangle) \subseteq \pi(\langle y\rangle)$ if and only if $N(y) \subseteq N(x)$ in $\Gamma(G)$.*

Theorem 2.3. *Let G be a finite group. Then $\Gamma(G)$ is AT-free if and only if neither $\{p_1p_2, p_1p_3, p_1p_4\}$ nor $\{p_1p_2, p_1p_3, p_2p_3\}$ is a subset of $\pi_e(G)$ where p_1, p_2, p_3, p_4 are pairwise distinct primes.*

Proof. The necessity follows trivially from Observation 2.1. We next prove the sufficiency. Suppose that neither $\{p_1p_2, p_1p_3, p_1p_4\}$ nor $\{p_1p_2, p_1p_3, p_2p_3\}$ is a subset of $\pi_e(G)$; here p_1, p_2, p_3, p_4 are pairwise distinct primes. Then it is clear that $\pi_e(G)$ has no element, which is a product of three pairwise distinct primes. Assume, to the contrary, that $\Gamma(G)$ has an AT, say $\{x_1, x_2, x_3\}$. If $\pi(\langle x_i\rangle) \subseteq \pi(\langle x_j\rangle)$ for two distinct $i, j \in \{1, 2, 3\}$ and $t \in \{1, 2, 3\} \setminus \{i, j\}$, then by Lemma 2.2, we have that every path from x_j to x_t must contain at least one vertex in $N(x_i)$; this contradicts that $\{x_1, x_2, x_3\}$ is an AT. It follows that $\pi(\langle x_i\rangle) \nsubseteq \pi(\langle x_j\rangle)$ for each two distinct $i, j \in \{1, 2, 3\}$. Since $\{x_1, x_2, x_3\}$ is an independent set of $\Gamma(G)$, it follows that for any $i \in \{1, 2, 3\}$, $o(x_i)$ is not a prime power. As a result, $|\pi(\langle x_i\rangle)| = 2$ for any $i \in \{1, 2, 3\}$.

Now let $\pi(\langle x_1\rangle) = \{p_1, p_2\}$ where p_1 and p_2 are distinct primes. Note that $(o(x_1), o(x_2)) \neq 1$. We may assume that $\pi(\langle x_2\rangle) = \{p_i, p_3\}$, where $i = 1$ or 2 and p_3 is a prime, which is different from p_1 and p_2. Similarly, we also can conclude that $\pi(\langle x_3\rangle) = \{p_j, p_4\}$, where $j = 1$ or 2 and p_4 is a prime, which is different from p_1 and p_2. If $p_3 = p_4$, then $i \neq j$, and so $\{p_1p_2, p_1p_3, p_2p_3\} \subseteq \pi_e(G)$, a contradiction. Now assume that $p_3 \neq p_4$. Since $(o(x_2), o(x_3)) \neq 1$, it must be that $i = j$. It follows that either $\{p_1p_2, p_1p_3, p_1p_4\} \subseteq \pi_e(G)$ or $\{p_1p_2, p_2p_3, p_2p_4\} \subseteq \pi_e(G)$, which is impossible. Consequently, $\Gamma(G)$ is AT-free. The proof is now complete. □

Corollary 2.4. (1) *If $|\pi(G)| \leq 2$, then $\Gamma(G)$ is AT-free;* (2) *Let $\pi(G) = \{p_1, p_2, p_3\}$. Then $\Gamma(G)$ is AT-free if and only if $\{p_1p_2, p_1p_3, p_2p_3\} \nsubseteq \pi_e(G)$;* (3) *If $\Gamma(G)$ is AT-free and $|\pi(G)| \geq 4$, then $Z(G) = \{e\}$.*

Recall that a finite group is *nilpotent* if and only if it is the direct product of its Sylow subgroups. In particular, in a finite nilpotent group, two elements a, b with $(o(a), o(b)) = 1$ must commute. Thus, if G is a nilpotent group such that $\pi(G) = \{p_1, p_2, p_3\}$ for different primes p_1, p_2, p_3, then G has elements of order $p_1p_2p_3$. The following result classifies all nilpotent groups whose coprime graphs are AT-free.

Corollary 2.5. *Let G be a nilpotent group. Then $\Gamma(G)$ is AT-free if and only if $G = P \times Q$, where P and Q are respectively a p-group and a q-group for distinct primes p and q.*

Clearly, for any subgroup H of G, $\Gamma(H)$ is an induced subgraph of $\Gamma(G)$. Thus, we have the following result.

Observation 2.6. *If $\Gamma(G)$ is AT-free, then $\Gamma(H)$ is also AT-free for any subgroup H of G.*

The *symmetric group* of order $n!$, denoted by $\mathbf{S}_n$, is the group consisting of all permutations on n objects. As we know, the symmetric group is important in many different areas of mathematics, including combinatorics and group theory, since every finite group is a subgroup of some symmetric group. In $\mathbf{S}_n$, the set of all even permutations is a group, which is called the *alternating group* on n objects and is denoted by $\mathbf{A}_n$. Note that A_n is a simple group for any $n \geq 5$.

Proposition 2.7. *For symmetric groups and alternating groups, we have the following:*

(1) *The graph $\Gamma(\mathbf{S}_n)$ is AT-free if and only if $n \leq 7$;*
(2) *The graph $\Gamma(\mathbf{A}_n)$ is AT-free if and only if $n \leq 8$.*

Proof. (1) We first prove that $\Gamma(\mathbf{S}_8)$ is not an AT-free graph. Note that the facts that $o((1,2)(3,4,5)) = 6$, $o((1,2)(3,4,5,6,7)) = 10$, and $o((1,2,3)(4,5,6,7,8)) = 15$. As a consequence, we have that $\{6,10,15\} \subseteq \pi_e(\mathbf{S}_8)$, and so by Theorem 2.3, $\Gamma(\mathbf{S}_8)$ is not AT-free, as desired. Now by Observation 2.6, it suffices to prove that $\Gamma(\mathbf{S}_7)$ is AT-free. Note that $\pi_e(\mathbf{S}_7) = \{1,2,3,4,5,6,7,10,12\}$. Since $\mathbf{S}_7$ has no elements of order 15, it follows from Theorem 2.3 that $\Gamma(\mathbf{S}_7)$ is AT-free, as desired.

(2) We first prove that $\Gamma(\mathbf{A}_9)$ is not an AT-free graph. Note that the facts that $o((1,2)(3,4,5)(6,7)) = 6$, $o((1,2)(3,4,5,6,7)(8,9)) = 10$, and $o((1,2,3)(4,5,6,7,8)) = 15$. Thus, we have that $\{6,10,15\} \subseteq \pi_e(\mathbf{A}_9)$, which implies that $\Gamma(\mathbf{A}_9)$ is not AT-free by Theorem 2.3, as desired. Now in view of Observation 2.6, it suffices to prove that $\Gamma(\mathbf{A}_8)$ is AT-free. It is easy to check that $\pi_e(\mathbf{A}_8) = \{1,2,3,4,5,6,7,15\}$, so $\pi_e(\mathbf{A}_8)$ has only two elements that are not prime powers. By Theorem 2.3, we have that $\Gamma(\mathbf{A}_8)$ is AT-free, as desired. □

Given a finite group G, if any non-trivial element of G is of prime power order, then G is a *CP-group* [13]. For example, for a prime p, any p-group is also a CP-group. Also, both the symmetric group on four letters and the alternating group of degree five are CP-groups. Given two non-trivial groups H and K, for which the direct product $H \times K$ is the coprime graph an AT-free graph? Next, we will characterize the direct products $H \times K$ whose coprime graph is AT-free.

Proposition 2.8. *Let H and K be two non-trivial groups. Then $\Gamma(H \times K)$ is AT-free if and only if one of the following holds:*

(a) $\pi(H) = \pi(K) = \{p,q\}$, *where p,q are distinct primes;*
(b) *Both H and K are CP-groups with $\pi(H) = \{p,q\}$, $\pi(K) = \{r,s\}$, where p,q,r,s are pairwise distinct primes;*
(c) *One of H and K is a p-group; without loss of generality, let $\pi(H) = \{p\}$. Also, $\pi(K) \subseteq \{p,q,r\}$ and $qr \notin \pi_e(K)$, where p,q,r are pairwise distinct primes.*

Proof. If (a) occurs, then Corollary 2.4 (1) implies that $\Gamma(H \times K)$ is AT-free. If (b) occurs, then by Theorem 2.3, it is easy to see that $\Gamma(H\times K)$ is AT-free. Now consider (c). If $\pi(K) \subseteq \{p,r\}$ or $\{p,q\}$, then Corollary 2.4 (1) also implies the desired result. Now suppose that $qr \notin \pi_e(K)$, and $\pi(K) = \{p,q,r\}$ or $\{q,r\}$. Then $\pi(H \times K) = \{p,q,r\}$. If $x \in H \times K$ and $o(x)$ is not a prime power, then $\pi(\langle x\rangle) = \{p,q\}$ or $\{p,r\}$. Hence, by Corollary 2.4 (2), we have that $\Gamma(H \times K)$ is AT-free.

Conversely, suppose that $\Gamma(H \times K)$ is AT-free. We next consider two cases.

Case 1. Neither H nor K is a p-group for some prime p.

We next prove that one of (a) and (b) holds. Firstly, by Theorem 2.3 and the fact that K is not trivial, it is easy to see that $\pi(H)$ has at most 3 pairwise prime divisors. Assume now that $\pi(H) \subseteq \{p,q,r\}$ for pairwise distinct primes p,q,r. We next consider two subcases.

Subcase 1.1. $\pi(H) = \{p,q,r\}$.

Then $\pi(K) \subseteq \{p,q,r\}$; otherwise, the case in Observation 2.1 (2) occurs, and so $\Gamma(H \times K)$ is not AT-free, a contradiction. It follows that there exist at least two distinct prime divisors in $\pi(K)$. Without loss of generality, suppose that $\{p,q\} \subseteq \pi(K)$. Then it must be that $pq, pr, qr \in \pi_e(H \times K)$. It follows from Theorem 2.3 that $\Gamma(H \times K)$ is not AT-free, a contradiction.

Subcase 1.2. $|\pi(H)| = 2$, and without loss of generality, let $\pi(H) = \{p,q\}$.

Suppose that there exists one prime divisor in $\pi(H) \cap \pi(K)$, say p. If there exists $r \in \pi(K) \setminus \{p,q\}$, then $pq, rp, rq \in \pi_e(H \times K)$, which is impossible as per Theorem 2.3. It follows that $\pi(K) \subseteq \{p,q\}$.

Since K is not a p-group, we must have $\{p, q\} = \pi(K)$, and so (a) occurs.

Suppose now that $\pi(H) \cap \pi(K) = \emptyset$. By Theorem 2.3, we clearly have $|\pi(K)| \leq 2$. As a result, we can conclude that $|\pi(K)| = 2$ since K is not a primary group. Also, Theorem 2.3 implies that any of H and K can not have elements whose order is the product of two distinct primes. Hence, both H and K are CP-groups, and so (b) holds.

Case 2. One of H and K is a p-group; without loss of generality, let $\pi(H) = \{p\}$.

In this case, we prove that (c) holds. Clearly, $\pi(K) \setminus \{p\}$ has no three pairwise distinct primes. It follows that $\pi(K) \subseteq \{p, q, r\}$, where p, q, r are pairwise distinct primes. It suffices to show that $qr \notin \pi_e(K)$. If $qr \in \pi_e(K)$, then $\{q, r\} \subseteq \pi(K)$, and so $pq, pr, qr \in \pi_e(H \times K)$, which is a contradiction by Theorem 2.3. Thus, we obtain $qr \notin \pi_e(K)$, and so (c) holds. □

By Proposition 2.8, we have the following examples.

Example 2.9. *The graph* $\Gamma(G)$ *is not* AT*-free if G is isomorphic to one of the following:*

$$\mathbf{S}_3 \times \mathbf{A}_5, D_6 \times D_{10}, \mathbf{A}_5 \times D_{10}, \mathbf{S}_4 \times L_3(2), \mathbf{S}_5 \times L_3(2), \mathbb{Z}_2 \times Sz(8).$$

Theorem 2.10. *Let G be a sporadic simple group. Then* $\Gamma(G)$ *is* AT*-free if and only if G is isomorphic to one of the following Mathieu groups:*

$$M_{11}, \quad M_{12}, \quad M_{22}, \quad M_{23}.$$

Proof. It is well known that there are precisely 26 sporadic simple groups. We first consider Mathieu groups. For M_{11}, we have that $\pi_e(M_{11}) = \{1, 2, 3, 4, 5, 6, 8, 11\}$, and so it is clear that $\Gamma(M_{11})$ is AT-free by Theorem 2.3. For M_{12}, one has $\pi_e(M_{12}) = \{1, 2, 3, 4, 5, 6, 8, 10, 11\}$, and so $\Gamma(M_{12})$ is AT-free by Theorem 2.3. For M_{22}, we have $\pi_e(M_{22}) = \{1, 2, 3, 4, 5, 6, 7, 8, 11\}$, and similarly, $\Gamma(M_{22})$ is AT-free. For M_{23}, we have $\pi_e(M_{23}) = \{1, 2, 3, 4, 5, 6, 7, 8, 11, 14, 15, 23\}$, and similarly, $\Gamma(M_{23})$ is AT-free. Note that both M_{23} and M_{12} are subgroups of M_{24} by the $\mathbb{ATLAS}$ of finite groups [14]. Hence, we have that $6, 10, 15 \in \pi_e(M_{24})$, and so $\Gamma(M_{24})$ is not AT-free by Theorem 2.3.

By [14], it follows that Janko group J_1, Janko group J_2, Janko group J_4, Held group He, Harada-Norton group HN, Thompson group Th, Baby Monster group B, Monster group M, O'Nan group $O'N$, Lyons group Ly, Rudvalis group Ru, Suzuki group Suz, Fischer group Fi_{22}, and Higman-Sims group HS contain $D_6 \times D_{10}$, $\mathbf{A}_5 \times D_{10}$, M_{24}, $\mathbf{S}_4 \times L_3(2)$, $\mathbf{A}_{12}$, $\mathbf{A}_9$, $\mathbf{S}_5 \times L_3(2)$, $\mathbf{A}_{12}$, J_1, $\mathbf{A}_{11}$, $\mathbb{Z}_2 \times \mathbb{Z}_2 \times Sz(8)$, $\mathbf{S}_3 \times \mathbf{A}_5$, $\mathbf{S}_{10}$, and $\mathbf{S}_8$ as subgroups, respectively. By Example 2.9 and Proposition 2.7, we see that the above simple groups do not have AT-free coprime graphs.

Now, note that $\{6, 10, 15\} \in \pi_e(Mcl) \cap \pi_e(J_3)$, and so both $\Gamma(Mcl)$ and $\Gamma(J_3)$ are not AT-free by Theorem 2.3. Now every of Fi_{23} and Fi'_{24} contains Fi_{22} as a subgroup, which implies that the coprime graphs of these two groups are not AT-free. Finally, note that the fact that for each $1 \leq i \leq 3$, the Conway group Co_i has a subgroup isomorphic to McL [14]. As a consequence, $\Gamma(Co_i)$ is not AT-free for each $1 \leq i \leq 3$. □

3. Conclusions

The study of graphical representations of algebraic structures, especially groups, has been an energizing and fascinating research area originating from the Cayley graphs. The coprime graph $\Gamma(G)$ of a finite G is a fairly recent development in the realm of graphs from groups. In this paper, "Forbidden subgraphs of co-prime graphs of finite groups", the authors raised the following question: what is a characterization for the finite groups whose coprime graphs are AT-free? For the above question, in this paper we give a characterization of the finite groups whose coprime graphs are AT-free. As applications, we also classify all finite groups G such that $\Gamma(G)$ is AT-free if G is a nilpotent group, a symmetric group, an alternating group, a direct product of two non-trivial groups, or a sporadic simple group.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflicts of interest.

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