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Theory article

Stability analysis of discrete-time switched systems with bipartite PDT switching

Qiang Yu* **and Xiujuan Jiang**

School of Mathematics and Computer Science, Shanxi Normal University, Taiyuan 030031, China

* **Correspondence:** Email: yuqiang111111@126.com; Tel: +8613453483247;
Fax: +8603512051286.

Abstract: The stability and stabilization problems of discrete-time switched systems are studied under the so-called bipartite persistent dwell-time switching, which is proposed by relaxing some of the limitations in existing persistent dwell-time switching. This paper provides new stability criteria for discrete-time switched systems using a binary quasi-time-varying Lyapunov function. Next, the stabilizing controllers for discrete-time switched-controlled systems are designed. Finally, we give a practical example to show the effectiveness of the conclusions and less conservatism than those based on the persistent dwell-time switching.

Keywords: switched systems; bipartite persistent dwell-time; binary quasi-time-varying Lyapunov function; stabilization; controller

1. Introduction

As a special type of hybrid systems, a switched system is composed of a series of continuous or discrete subsystems and rules that coordinate the switching among these subsystems [1]. Due to the fact that many practical systems can be modeled as switched systems, such as chemical processes and mechanical systems [2], switched systems have attracted widespread attention and research in recent decades. The stability problem is the first concern of the research on switched systems. So far, there has been much literature presenting excellent results on the stability of switched systems [3–5] and the design of stabilization controllers [6–8]. Rate, anti-disturbance, and H_∞ reliable bumpless transfer control for switched systems were investigated in the literature [9–11], respectively. The designs of l_2-l_∞ and H_∞ control for semi-/hidden Markov switching systems were presented in the literature [12,13], respectively. The concept of dwell time (DT) was proposed in the literature [14]. DT refers to the running time of each subsystem being no less than a fixed constant. The concept of average dwell time (ADT) was proposed in the literature [15] by relaxing the requirement of DT, which allows subsystems to have some appropriate fast switching and compensate for it later by switching slowly enough. The persistent dwell-time (PDT) switching was first proposed in [16] and has less conservatism than DT and ADT. In the PDT switching, each stage is divided into two parts: τ-portion and $\mathcal{T}$-portion. The operating time of the subsystem activated in τ-portion is not less than a fixed constant τ to compensate for system instability caused by arbitrary switching in $\mathcal{T}$-portion. Thus, PDT allows for the existence of fast switching in its $\mathcal{T}$-portion, which makes it more general than DT and ADT. When there is no $\mathcal{T}$-portion in the PDT switching, it degenerates into DT. In addition, both ADT and DT switchings are mainly applicable to switched systems with all subsystems stable, while the PDT switching can be directly used to switched systems with some unstable subsystems. When there are unstable subsystems, the whole system can be stable by designing a suitable switching signal to make the unstable subsystem run in the $\mathcal{T}$-portion. Up to now, there have been some conclusions about the asymptotic stability of linear and nonlinear switched systems under the PDT switching strategy [16, 17]. Only when the activation time of the subsystem is no less than τ, it can be considered as τ-portion [18, 19]. This restriction is relatively strict. Therefore, the work considers relaxing the corresponding limitations by proposing a new bipartite persistent dwell-time (BPDT) switching strategy. Different from PDT, the BPDT switching defines the sum of the running times of two continuously activated subsystems is not less than τ as the τ-portion. This improvement makes it easier to design switching signals and has less conservatism than PDT switching.

As is well known, the Lyapunov function method plays an important role in the stability analysis of switched systems. In recent years, the so-called the quadratic form of quasi-time-varying Lyapunov function (QLF) was proposed in [20], which is more refined and effective than the traditional multiple Lyapunov function method in studying the stability of switched systems. During the dwell time of a certain subsystem, this function changes linearly in time, after which it is some fixed constant. It is also used to study the robust H_∞ problem of time-delay stochastic T-S fuzzy systems with sampled data [21]. In order to correspond with the BPDT switching, this article extends the corresponding QLF to the binary quasi-time-varying Lyapunov function (BQLF).

The three main contributions of this paper are as follows: First, a new switching scheme BPDT is proposed, which appropriately relaxes the τ-portion condition of the PDT switching. Second, a new BQLF method is constructed that is compatible with the PDT switching. Third, some less conservative stability and stabilization conditions have been obtained based on the BPDT switching and BQLF method.

The structure of the remainder of the paper is organized as follows: Section 2 provides some necessary preliminary knowledge. Section 3 analyzes the stability of the switched system based on the BQLF and BPDT switching, and the stabilization controller design of the system is given. Section 4 provides a practical example to demonstrate the results of this paper. Section 5 provides conclusions.

The notations used in this paper are listed in Table 1.

Table 1. The notations used in this paper.

Notation	The denotation of the notation
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^n$	the set of n-dimensional real vectors
$\mathbb{R}^{n\times n}$	the space of $n \times n$ real matrices
$\mathbb{N}$ ($\mathbb{N}_+$)	the set of nonnegative (positive) integers
$\mathfrak{G}^T$	the transpose of a matrix $\mathfrak{G}$
$\mathbb{Z}_{\geq a}$	$\{z \in \mathbb{N}_+ \mid z \geq a\}, a \in \mathbb{N}_+$
$\mathbb{Z}_{[a,b]}$	$\{z \in \mathbb{N}_+ \mid a \leq z \leq b\}, a, b \in \mathbb{N}_+$
$\|\cdot\|$	Euclidean norm of vectors
$\kappa \in \mathcal{K}_\infty$	$\kappa : [0, \infty) \to [0, \infty)$ is unbounded, continuous, strictly increasing and $\kappa(0) = 0$
$S^n_{>0}$	the set of $n \times n$ positive definite symmetric matrices
$\star$	the transpose of diagonal elements of symmetric matrices
$P > 0$ (≥ 0)	the matrix P is positive definite (semi-definite) and symmetric
$P < 0$ (≤ 0)	the matrix $-P > 0$ (≥ 0)

2. Problem statements and preliminaries

Consider the discrete-time switched system as follows:

$$x(z+1) = f_{\rho(z)}x(z), z \in \mathbb{N}, \tag{2.1}$$

where $x(z) \in \mathbb{R}^n$ is the state of the system, the switching signal $\rho(z)$ is a piecewise constant right continuous function with respect to z, taking values on $\mathfrak{M} = \{1, 2, \cdots, m\}$, where $m \in \mathbb{N}_+$ is the number of subsystems.

The switching sequence is as follows: $0 \leq z_0 < z_1 < \cdots < z_r < z_{r+1} < \cdots$, where z_0 represents the initial time of system operation and z_r represents the r^{th} switching instant. When $z \in [z_r, z_{r+1})$, the $\rho(z_r)^{th}$ subsystem is activated. The time interval between two adjacent switches is called dwell time: $\tau_r = z_{r+1} - z_r, r = 0, 1, 2, \cdots$.

This paper mainly focuses on the stability of discrete-time switched systems under BPDT switching signals. Now we will first recall several necessary definitions.

Definition 1 ([11]) For a switching signal $\rho(z)$ and two positive constants τ and $\mathcal{T}$, if there exists an infinite number of non-adjacent switching intervals with their length not smaller than τ, and the length of all other intervals shall not exceed $\mathcal{T}$. Then τ and $\mathcal{T}$ are called persistent dwell-time and period of persistence of $\rho(z)$, respectively.

Remark 1 From the above definition, it can be seen that each stage of PDT switching includes two parts: τ-portion and $\mathcal{T}$-portion (Figure 1). For the convenience of expression, we will denote the actual running time of the $\mathcal{T}$-portion of the p^{th} stage as $\mathcal{T}^{(p)}$, and the relationship between $\mathcal{T}$ and $\mathcal{T}^{(p)}$ is as follows:

$$\mathcal{T}^{(p)} = \sum_{c=1}^{\mathbb{Q}(z_{r(p)+1}, z_{r(p+1)})} \mathcal{T}_{z_{r(p)+c}} \leq \mathcal{T}, \tag{2.2}$$

where $z_{r(p)+c}$ and $\mathcal{T}_{z_{r(p)+c}}$ represent the switching instant of $\mathcal{T}$-portion and the running time of the $\rho(z_{r(p)+c})^{th}$ subsystem, respectively. $\mathbb{Q}(z_{r(p)+1}, z_{r(p+1)})$ represents the number of switching times of the system within the interval $[z_{r(p)+1}, z_{r(p+1)})$, and $z_{r(p)+1}$ and $z_{r(p+1)}$ represent the beginning and ending switching times of the $\mathcal{T}$-portion in the p^{th} stage, respectively.

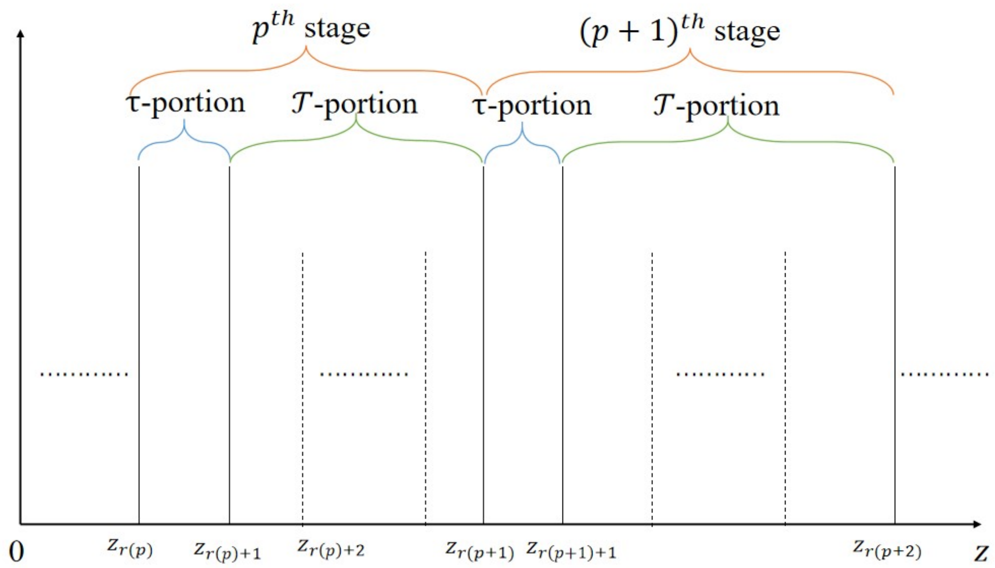

Figure 1. PDT switching.

Remark 2 Compared with the familiar DT and ADT switching, PDT switching is easier to design and less conservative because it can be switched arbitrarily in the $\mathcal{T}$-portion. The system instability caused by arbitrary switching of $\mathcal{T}$-portion can be compensated by running a stable subsystem for a certain period in τ-portion.

Definition 2 ([22]) The function $g : [0, \infty) \to [0, \infty)$ is called a class $\mathcal{K}$ function if it is continuous, satisfies $g(0) = 0$ and strictly increases. If g is also radially unbounded, it is called a class $\mathcal{K}_\infty$ function.

Definition 3 ([23]) System (2.1) is globally uniformly asymptotically stable (GUAS) under the switching signals $\rho(z)$, if for any $x(z_0) \in \mathbb{R}^n$, there exists $\kappa \in \mathcal{K}_\infty$ such that $||x(z)|| \leq \kappa(||x(z_0)||)$, $\forall z \in \mathbb{Z}_{\geq z_0}$ and $||x(z)|| \to 0$ as $z \to \infty$.

3. Main results

This section discusses the stability and stabilization of the switched system (2.1) under the BPDT switching.

Because the PDT switching requires that the running time of the subsystem in τ-portion is not less than τ, it is relatively demanding, which leads to some difficulty in designing appropriate switching signals. By relaxing the restriction of τ-portion, this paper makes it easier to design switching signals and obtain less conservative results.

Definition 4 A switching signal is called the bipartite persistent dwell-time (BPDT) switching, if it satisfies the following conditions:

(i): The sum of the lengths of two adjacent switching intervals is no less than a constant τ (called τ-portion), and there are an infinite number of such τ-portion.

(ii): All switching intervals between two adjacent τ-portions are less than τ, and the sum of them is not more than $\mathcal{T}$, which is called $\mathcal{T}$-portion.

Remark 3 Under the BPDT switching (Figure 2), the definition of $\mathcal{T}^{(p)}$ correspondingly changes to

$$\mathcal{T}^{(p)} = \sum_{c=2}^{\mathbb{Q}(z_{r(p)+2}, z_{r(p+1)})} \mathcal{T}_{z_{r(p)+c}} \leq \mathcal{T}, \tag{3.1}$$

where $z_{r(p)+c}$ and $\mathcal{T}_{z_{r(p)+c}}$ represent the switching instant of $\mathcal{T}$-portion of the p^{th} stage and the running time of the subsystem activated at switching instant $z_{r(p)+c}$, respectively. $z_{r(p)+2}$ and $z_{r(p+1)}$ represent the beginning and ending switching times of the $\mathcal{T}$-portion in the p^{th} stage, respectively.

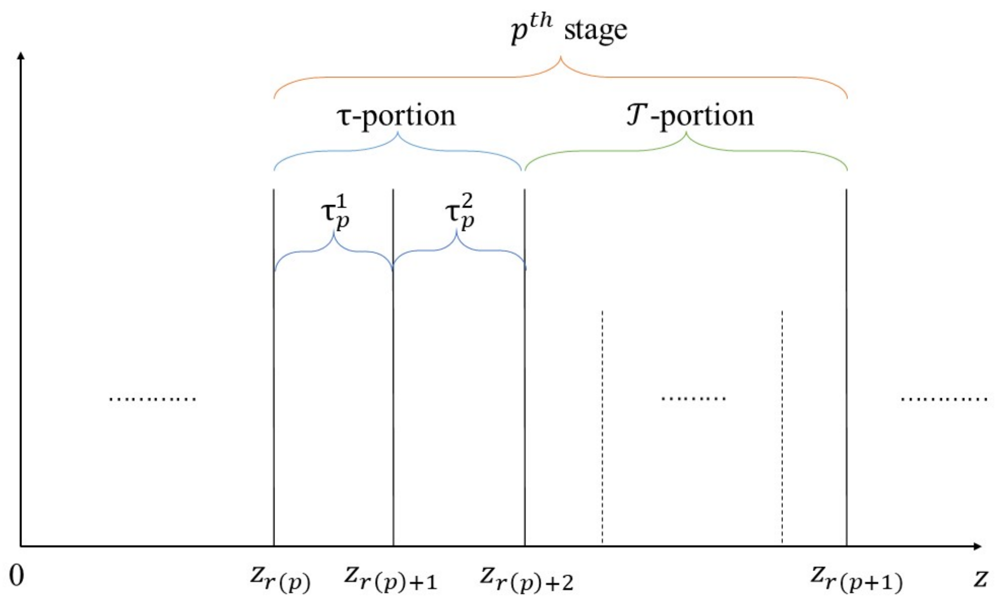

Figure 2. Improved PDT switching.

The τ-portion of the BPDT switching satisfies $\tau_p^1 + \tau_p^2 \geq \tau$, where τ_p^1 and τ_p^2 respectively represent the running time of two subsystems in the τ-portion of the p^{th} stage (as shown in Figure 2). Therefore, the BPDT switching compensates for the instability caused by the rapid switching of the $\mathcal{T}$-portion by τ-portion. This includes a wider signal range than PDT switching, thus having superiority and less conservatism. In fact, different from PDT's τ-portion only having one switching interval, every BPDT's τ-portion contains two switching intervals, which gives BPDT greater design freedom.

For the convenience of later description, some symbols are given as follows:

(i): in the τ-portion,

$$\varsigma(z)=\begin{cases} z-z_{r(p)}, & z\in[z_{r(p)},z_{r(p)}+\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau) \\ \frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau, & z\in[z_{r(p)}+\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau,z_{r(p)+1}) \\ z-z_{r(p)+1}, & z\in[z_{r(p)+1},z_{r(p)+1}+\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau) \\ \frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau, & z\in[z_{r(p)+1}+\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau,z_{r(p)+2}) \end{cases}, \tag{3.2}$$

(ii): in the $\mathcal{T}$-portion,

$$\varsigma(z)=z-H_c(z), z\in[z_{r(p)+2},z_{r(p+1)}), \tag{3.3}$$

where $H_c(z)=\arg\{\max(z_{r(p)+c}|z\geq z_{r(p)+c}, z_{r(p)+c}\in[z_{r(p)+2},z_{r(p+1)}), c\in\mathbb{Z}_{[2,\mathbb{Q}(z_{r(p)+2},z_{r(p+1)})]})\}$.

Now the following lemma is given for the nonlinear system (2.1).

Lemma 1 Consider the discrete-time switched system (2.1). Given scalars $0<\lambda<1$, $\mu>0$, if there exist functions $\mathcal{F}_{\rho(z)}(x(z),\varsigma(z))$ and $\kappa_1,\kappa_2\in\mathcal{K}_\infty$, such that

$$\forall\varsigma(z),\quad \kappa_1(\|x(z)\|)\leq\mathcal{F}_{\rho(z)}(x(z),\varsigma(z))\leq\kappa_2(\|x(z)\|), \tag{3.4}$$

$$\forall z\in[z_{r(p)},z_{r(p)}+\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau),\quad \mathcal{F}_{\rho(z)}(x(z+1),\varsigma(z)+1)\leq\lambda\mathcal{F}_{\rho(z)}(x(z),\varsigma(z)), \tag{3.5}$$

$$\forall z\in[z_{r(p)}+\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau,z_{r(p)+1}),\quad \mathcal{F}_{\rho(z)}(x(z+1),\varsigma(z))\leq\lambda\mathcal{F}_{\rho(z)}(x(z),\varsigma(z)), \tag{3.6}$$

$$\forall z\in[z_{r(p)+1},z_{r(p)+1}+\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau),\quad \mathcal{F}_{\rho(z)}(x(z+1),\varsigma(z)+1)\leq\lambda\mathcal{F}_{\rho(z)}(x(z),\varsigma(z)), \tag{3.7}$$

$$\forall z\in[z_{r(p)+1}+\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau,z_{r(p)+2}),\quad \mathcal{F}_{\rho(z)}(x(z+1),\varsigma(z))\leq\lambda\mathcal{F}_{\rho(z)}(x(z),\varsigma(z)), \tag{3.8}$$

$$\forall z\in[z_{r(p)+2},z_{r(p+1)}),\quad \mathcal{F}_{\rho(z)}(x(z+1),\varsigma(z)+1)\leq\lambda\mathcal{F}_{\rho(z)}(x(z),\varsigma(z)), \tag{3.9}$$

$$\forall\rho(z_{r(p)+j})\neq\rho(z_{r(p)+j}-1), j=1,2,\quad \mathcal{F}_{\rho(z_{r(p)+j})}(x(z_{r(p)+j}),0)\leq\mu\mathcal{F}_{\rho(z_{r(p)+j}-1)}(x(z_{r(p)+j}),\tau_p^j), \tag{3.10}$$

$$\forall\rho(z_{r(p)+c})\neq\rho(z_{r(p)+c}-1),\quad \mathcal{F}_{\rho(z_{r(p)+c})}(x(z_{r(p)+c}),0)\leq\mu\mathcal{F}_{\rho(z_{r(p)+c}-1)}(x(z_{r(p)+c}),\mathcal{T}_{z_{r(p)+c}-1}), \tag{3.11}$$

where $\mathcal{T}_{z_{r(p)+c}-1}\in[1,\min(\tau-1,\mathcal{T}^{(p)})]$, $c\in\mathbb{Z}_{[3,\mathbb{Q}(z_{r(p)+2}z_{r(p+1)})+2]}$ and $\mathcal{T}^{(p)}\in\mathbb{Z}_{[1,\mathcal{T}]}$. Then the system (2.1) is GUAS under the BPDT switching

$$(\mathcal{T}+2)\ln\mu+\mathcal{T}\ln\lambda+\tau\ln\lambda\leq 0. \tag{3.12}$$

Proof. When $\lambda\mu < 1$, it can be directly concluded that the switching system is GUAS under any switching signal. The case of $\lambda\mu > 1$ is discussed below. For the convenience of narration, let $\mathbb{Q}(z_{r(p)+2}, z_{r(p+1)}) = q$, it follows from (3.5)–(3.11) that

$$\begin{aligned}
&\mathcal{F}_{\rho(z_{r(p+1)})}(x(z_{r(p+1)}),0)\\
\leq&\mu\mathcal{F}_{\rho(z_{r(p)+q})}(x(z_{r(p+1)}),\mathcal{T}_{z_{r(p)+q}})\\
\leq&\mu\lambda^{\mathcal{T}_{z_{r(p)+q}}}\mathcal{F}_{\rho(z_{r(p)+q})}(x(z_{r(p+1)}-\mathcal{T}_{z_{r(p)+q}}),0)\\
\leq&\mu^2\lambda^{\mathcal{T}_{z_{r(p)+q}}}\mathcal{F}_{\rho(z_{r(p)+q-1})}(x(z_{r(p+1)}-\mathcal{T}_{z_{r(p)+q}}),\mathcal{T}_{\rho(z_{r(p)+q-1})})\\
\leq&\mu^2\lambda^{\mathcal{T}_{z_{r(p)+q}}}\lambda^{\mathcal{T}_{z_{r(p)+q-1}}}\mathcal{F}_{\rho(z_{r(p)+q-1})}(x(z_{r(p+1)}-\mathcal{T}_{z_{r(p)+q}}-\mathcal{T}_{z_{r(p)+q-1}}),0)\\
&\dots\\
\leq&\mu^{q+1}\lambda^{\mathcal{T}^{(p)}}\lambda^{\tau_p^1+\tau_p^2}\mathcal{F}_{\rho(z_{r(p)})}(x(z_{r(p)}),0)\\
\leq&\mu^{\mathcal{T}^{(p)}+2}\lambda^{\mathcal{T}^{(p)}}\lambda^{\tau}\mathcal{F}_{\rho(z_{r(p)})}(x(z_{r(p)}),0).
\end{aligned} \tag{3.13}$$

From $\lambda\mu > 1$, it can be concluded that $\mu^{\mathcal{T}^{(p)}+2}\lambda^{\mathcal{T}^{(p)}} = (\lambda\mu)^{\mathcal{T}^{(p)}}\mu^2 \leq (\lambda\mu)^{\mathcal{T}}\mu^2 = \mu^{\mathcal{T}+2}\lambda^{\mathcal{T}}$. Combining (3.13), it can be seen that

$$\mathcal{F}_{\rho(z_{r(p+1)})}(x(z_{r(p+1)}),0)\leq\mu^{\mathcal{T}+2}\lambda^{\mathcal{T}}\lambda^{\tau}\mathcal{F}_{\rho(z_{r(p)})}(x(z_{r(p)}),0). \tag{3.14}$$

Let $\mu^{\mathcal{T}+2}\lambda^{\mathcal{T}}\lambda^{\tau} = \varpi$. Iterating the above process yields

$$\begin{aligned}
&\mathcal{F}_{\rho(z_{r(p+1)})}(x(z_{r(p+1)}),0)\\
\leq&\varpi\mathcal{F}_{\rho(z_{r(p)})}(x(z_{r(p)}),0)\\
&\dots\\
\leq&\varpi^p\mathcal{F}_{\rho(z_{r(1)})}(x(z_{r(1)}),0)\\
\leq&\varpi^p\mu^{\mathcal{T}}\lambda^{\mathcal{T}}\mathcal{F}_{\rho(z_0)}(x(z_0),0).
\end{aligned} \tag{3.15}$$

According to the inequality (3.12), one has $\varpi \leq 1$. It can be obtained from (3.4) that $\|x(z_{r(p+1)})\| \leq a_1(\|x(z_0)\|)$, where $a_1(\cdot) = \kappa_1^{-1}(\varpi^p\mu^{\mathcal{T}}\lambda^{\mathcal{T}}\kappa_2(\cdot))$. Furthermore, from (3.4)–(3.11), we can deduce that $\|x(z)\| \leq a_2(\|x(z_0)\|)$, where $a_2(\cdot) = \kappa_1^{-1}(\varpi^p\mu^{\mathcal{T}}\lambda^{\mathcal{T}}\kappa_2(a_1(\cdot))$. By Definition 4, the GUAS of the system (2.1) can be obtained.

Remark 4 The difference between the QLF and BQLF. The $\varsigma(z)$ in the QLF is divided into two segments in the τ-portion, where $\varsigma(z)$ varies linearly with respect to z before the dwell time and remains a fixed constant after the dwell time. Unlike this, $\varsigma(z)$ in the BQLF is divided into four segments in the τ-portion, according to the proportion of the running time of the two subsystems activated in the τ-portion. $\varsigma(z)$ changes linearly with respect to z before the $\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau$ period of the first subsystem activated in the τ-portion, after which it is a fixed constant $\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau$. $\varsigma(z)$ changes linearly with respect to z before the $\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau$ period of the second subsystem activated in the τ-portion, after which it is a fixed constant $\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau$. Therefore, the BQLF is more flexible than the QLF.

Generally, one can take the BQLF as follows:

$$\mathcal{F}_{\rho(z)}(x(z), \varsigma(z)) = x^T(z) P_{\rho(z)}(\varsigma(z)) x(z), \tag{3.16}$$

where $\varsigma(z)$ is represented by Eqs (3.2) and (3.3), and $P_{\rho(z)}(\varsigma(z)) \in S^n_{>0}$.

Consider the discrete-time switched linear system

$$x(z+1) = \mathfrak{G}_{\rho(z)} x(z), \tag{3.17}$$

where $\mathfrak{G}_{\rho(z)} \in \mathbb{R}^{n\times n}$ are known constant matrices with appropriate dimensions.

Next, let's provide the stability criterion of the system (3.17) using the BQLF (3.16).

Theorem 1 Consider the system (3.17). Given scalars $0 < \lambda < 1$, $\mu > 0$, if there exist matrices $P_u(\varsigma(z)) \in S^n_{>0}$, $u \in \mathfrak{M}$, such that $\forall\, u, v \in \mathfrak{M}, u \neq v$, $\forall\, \varsigma = 0, 1, \cdots, \tau$,

$$\mathfrak{G}_u^T P_u(\varsigma + 1) \mathfrak{G}_u - \lambda P_u(\varsigma) \leq 0, \tag{3.18}$$

$$\mathfrak{G}_u^T P_u\left(\frac{\tau_p^1}{\tau_p^1 + \tau_p^2}\tau\right) \mathfrak{G}_u - \lambda P_u\left(\frac{\tau_p^1}{\tau_p^1 + \tau_p^2}\tau\right) \leq 0, \tag{3.19}$$

$$\mathfrak{G}_u^T P_u\left(\frac{\tau_p^2}{\tau_p^1 + \tau_p^2}\tau\right) \mathfrak{G}_u - \lambda P_u\left(\frac{\tau_p^2}{\tau_p^1 + \tau_p^2}\tau\right) \leq 0, \tag{3.20}$$

$$P_u(0) - \mu P_v(\tau_p^j) \leq 0, j = 1, 2, \tag{3.21}$$

$$P_u(0) - \mu P_v(\mathcal{T}_v) \leq 0 \tag{3.22}$$

hold, then the discrete-time switched linear system (3.17) is GUAS under the BPDT switching (3.12).

Proof. The BQLF is selected as the formula (3.16), so (3.4) naturally holds.

From (3.18), it follows that

$$\begin{aligned}
&\mathcal{F}_u(x(z+1), z+1-z_{r(p)}) - \lambda \mathcal{F}_u(x(z), z - z_{r(p)}) \\
=&x^T(z+1) P_u(\varsigma+1) x(z+1) - \lambda x^T(z) P_u(\varsigma) x(z) \\
=&x^T(z) \mathfrak{G}_u^T P_u(\varsigma+1) \mathfrak{G}_u x(z) - \lambda x^T(z) P_u(\varsigma) x(z) \\
=&x^T(z)[\mathfrak{G}_u^T P_u(\varsigma+1) \mathfrak{G}_u - \lambda P_u(\varsigma)] x(z) \leq 0.
\end{aligned} \tag{3.23}$$

Then it can be concluded that (3.5) and (3.7) in Lemma 1 hold.

From (3.19), it follows that

$$
\begin{aligned}
&\mathcal{F}_u\left(x(z+1), \frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau\right) - \lambda\mathcal{F}_u\left(x(z), \frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau\right)\\
=&x^T(z+1)P_u\left(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau\right)x(z+1) - \lambda x^T(z)P_u\left(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau\right)x(z)\\
=&x^T(z)\left[\mathfrak{G}_u^T P_u\left(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau\right)\mathfrak{G}_u - \lambda P_u\left(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau\right)\right]x(z) \le 0.
\end{aligned} \tag{3.24}
$$

Then one can obtain (3.6). Similarly, (3.8) follows from (3.20).

By combining (3.18)–(3.20) and (3.2), we can obtain (3.9).

It follows from (3.21) that

$$
\begin{aligned}
&\mathcal{F}_u(x(z),0) - \mu\mathcal{F}_v(x(z),\tau_p^j)\\
=&x^T(z)P_u(0)x(z) - \mu x^T(z)P_v(\tau_p^j)x(z)\\
=&x^T[P_u(0) - \mu P_v(\tau_p^j)]x(z) \le 0.
\end{aligned} \tag{3.25}
$$

Thus (3.10) holds, where $j = 1, 2$.

It follows from (3.22) that

$$
\begin{aligned}
&\mathcal{F}_u(x(z),0) - \mu\mathcal{F}_v(x(z),\mathcal{T}_v)\\
=&x^T(z)P_u(0)x(z) - \mu x^T(z)P_v(\mathcal{T}_v)x(z)\\
=&x^T[P_u(0) - \mu P_v(\mathcal{T}_v)]x(z) \le 0.
\end{aligned} \tag{3.26}
$$

One can obtain (3.11).

From the above analysis, it can be seen that all the conditions in Lemma 1 are valid. Thus, it can be concluded that the system (3.17) is GUAS under the BPDT switching (3.12).

There are two main difficulties in designing the BPDT switching to the system studied in Theorem 1. First, it is unreasonable to require the value of τ_p^1, τ_p^2 and $\mathcal{T}_v$ in advance. This issue can be resolved by the following Remarks 5 and 6. Second, parameters λ, μ, and $\mathcal{T}$ in conditions need to be determined. It is solved in the following Remark 8.

Remark 5 According to the above theorem, the conditions (3.19) and (3.20) that make the system stable are relatively strict. We need to know the values of τ_p^1 and τ_p^2 in advance in order to calculate the corresponding values of $\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau$ and $\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau$. It is difficult to achieve in practical applications. In order to make the conclusion of Theorem 1 easier to implement, we have made some appropriate simplifications to the conditions (3.19) and (3.20). Given appropriate values for λ, μ and $\mathcal{T}$, one can easily calculate the range of values for τ based on (3.12). For example, $\tau \ge 4.2$, the following situations can be taken:

(i): $\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau = 1$, $\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau = 4$;

(ii): $\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau = 2$, $\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau = 3$;

(iii): $\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau = 3$, $\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau = 2$;

(iv): $\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau = 4$, $\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau = 1$.

Remark 6 The inequalities (3.21) and (3.22) in Theorem 1 require prior knowledge of the values of τ_p^j and $\mathcal{T}_v$, which is not easy to implement in practical applications. We can simplify it as follows:

$$P_u(0) - \mu P_v(\tau) \leq 0,$$

$$P_u(0) - \mu P_v(\mathcal{T}) \leq 0.$$

This can further simplify the calculation of Theorem 1.

Remark 7 The relationship between PDT and BPDT. PDT can be seen as a special case of BPDT. When the limitation of the τ-portion of BPDT degenerates to a part where the running time of a subsystem is not less than τ, it can be considered as the τ-portion, and BPDT degenerates into PDT. At this point, inequalities (3.19) and (3.27) in Theorem 1 degenerate into

$$\mathfrak{G}_u^T P_u(\tau)\mathfrak{G}_u - \lambda P_u(\tau) \leq 0.$$

Inequality (3.21) degenerates into

$$P_u(0) - \mu P_v(\tau) \leq 0.$$

BPDT (3.12) degenerates into PDT

$$\tau \geq -((\mathcal{T} + 1)\ln\mu + \mathcal{T}\ln\lambda)/(\ln\lambda). \tag{3.27}$$

Remark 8 In Theorem 1, there are some parameters that need to be determined, for example, λ, μ, and $\mathcal{T}$. In fact, one can take a larger $\lambda \in (0, 1)$ (to ensure a larger feasible range for conditions (3.18)–(3.20) and a larger μ (to obtain a larger feasible range for conditions (3.21) and (3.22) and a smaller $\mathcal{T}$ (to make (3.12) feasible). If conditions (3.18)–(3.22) are feasible for some λ, μ but (3.12) infeasible, one can reduce λ, μ appropriately to increase (3.12) feasibility.

Remark 9 Here we present the effect of $\mathcal{T}$ on the conclusion. It follows from (3.12) that

$$\frac{\mathcal{T} + 2}{\mathcal{T} + \tau} \leq -\frac{\ln\lambda}{\ln\mu}.$$

If $\tau > 2$ and $-\frac{\ln\lambda}{\ln\mu} > 1$, then the inequality (3.12) is always feasible for any $\mathcal{T}$.

If $\tau \leq 2$ and $-\frac{\ln\lambda}{\ln\mu} < 1$, then the inequality (3.12) is infeasible for any $\mathcal{T}$.

In practical applications, the value of $\mathcal{T}$ is often less than or similar to τ.

Now we will consider the following system:

$$x(z+1) = \mathfrak{G}_{\rho(z)}x(z) + \mathfrak{B}_{\rho(z)}w(z), \tag{3.28}$$

where $w(z)$ represents controlled input. The following theorem provides the design of the stabilization controller $w(z) = K_{\rho(z)}(\varsigma)x(z)$ for the system.

Theorem 2 Consider the system (3.28). Given scalars $0 < \lambda < 1$, $\mu > 0$, if there exist matrices $C_u(\varsigma) \in S_{>0}^n$ and $D_u(\varsigma)$, $u \in \mathfrak{M}$, such that $\forall\, u, v \in \mathfrak{M}, u \neq v, \forall\, \varsigma = 0, 1, \cdots, \tau$,

$$\begin{bmatrix} -C_u(\varsigma+1) & \mathfrak{G}_u C_u(\varsigma) + \mathfrak{B}_u D_u(\varsigma) \\ \star & -\lambda C_u(\varsigma) \end{bmatrix} \leq 0, \tag{3.29}$$

$$\begin{bmatrix} -C_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau) & \mathfrak{G}_u C_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau) + \mathfrak{B}_u D_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau) \\ \star & -\lambda C_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau) \end{bmatrix} \leq 0, \tag{3.30}$$

$$\begin{bmatrix} -C_u(\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau) & \mathfrak{G}_u C_u(\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau) + \mathfrak{B}_u D_u(\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau) \\ \star & -\lambda C_u(\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau) \end{bmatrix} \leq 0, \tag{3.31}$$

$$C_v(\tau_p^j) - \mu C_u(0) \leq 0, j = 1, 2, \tag{3.32}$$

$$C_v(\mathcal{T}_v) - \mu C_u(0) \leq 0 \tag{3.33}$$

hold, then the system (3.28) is GUAS under the BPDT switching (3.12) and the controller gain $K_u(\varsigma) = D_u(\varsigma)C_u^{-1}(\varsigma)$, where $\mathcal{T}^{(p)} \in \mathbb{Z}_{[1,\mathcal{T}]}$, $\mathcal{T}_v \in \mathbb{Z}_{[1,\min(\tau-1,\mathcal{T}^{(p)})]}$.

Proof. Let $C_u(\varsigma) = P_u^{-1}(\varsigma)$, $D_u(\varsigma) = K_u(\varsigma)P_u^{-1}(\varsigma)$. According to Schur complement, the inequality (3.29) is equivalent to

$$P_u^{-1}(\varsigma)[\mathfrak{G}_u + \mathfrak{B}_u K_u(\varsigma)]^T P_u(\varsigma+1)[\mathfrak{G}_u + \mathfrak{B}_u K_u(\varsigma)]P_u^{-1}(\varsigma) - \lambda P_u^{-1}(\varsigma) \leq 0. \tag{3.34}$$

The inequality (3.34) is multiplied by left and right $P_u(\varsigma)$ respectively, to obtain

$$[\mathfrak{G}_u + \mathfrak{B}_u K_u(\varsigma)]^T P_u(\varsigma+1)[\mathfrak{G}_u + \mathfrak{B}_u K_u(\varsigma)] - \lambda P_u(\varsigma) \leq 0. \tag{3.35}$$

The inequality (3.30) is equivalent to

$$P_u^{-1}(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau)[\mathfrak{G}_u + \mathfrak{B}_u K_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau)]^T P_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau)[\mathfrak{G}_u + \mathfrak{B}_u K_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau)]P_u^{-1}(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau) - \lambda P_u^{-1}(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau) \leq 0. \tag{3.36}$$

The inequality (3.36) is multiplied by left and right $P_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau)$ respectively, to obtain

$$[\mathfrak{G}_u + \mathfrak{B}_u K_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau)]^T P_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau)[\mathfrak{G}_u + \mathfrak{B}_u K_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau)] - \lambda P_u(\frac{\tau_p^1}{\tau_p^1+\tau_p^2}\tau) \leq 0. \tag{3.37}$$

Similarly, it can be concluded from (3.31) that

$$[\mathfrak{G}_u + \mathfrak{B}_u K_u(\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau)]^T P_u(\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau)[\mathfrak{G}_u + \mathfrak{B}_u K_u(\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau)] - \lambda P_u(\frac{\tau_p^2}{\tau_p^1+\tau_p^2}\tau) \leq 0. \tag{3.38}$$

Multiplying the inequality (3.32) left and right by $P_v(\tau_p^j)$ and $P_u(0)$, respectively, yields

$$P_u(0) - \mu P_v(\tau_p^j) \leq 0, j = 1, 2. \tag{3.39}$$

Multiplying the inequality (3.33) left and right by $P_v(\mathcal{T}_v)$ and $P_u(0)$, respectively, yields

$$P_u(0) - \mu P_v(\mathcal{T}_v) \leq 0. \tag{3.40}$$

Letting $\overline{\mathfrak{G}}_u = \mathfrak{G}_u + \mathfrak{B}_u K_u(\varsigma)$, the inequalities (3.35), (3.37)–(3.40) imply (3.18)–(3.22), respectively. According to Theorem 1, the system (3.28) is GUAS under the BPDT switching (3.12) with designed controller gain $K_u(\varsigma) = D_u(\varsigma)C_u^{-1}(\varsigma)$.

Remark 10 Compared to the traditional multiple Lyapunov function for switched systems, the BQLF here allows for greater freedom in selecting the parameter matrices, thereby reducing the conservatism of the stability and controller design of the system studied. The fact can be seen from the Lyapunov function (3.16) that both the parameter matrices $P_{\rho(z)}(\varsigma(z))$ and the controller gain $K_u(\varsigma) = D_u(\varsigma)C_u^{-1}(\varsigma)$ are binary quasi-time-varying.

4. A practical example

This section demonstrates the practicality and effectiveness of the proposed method through a practical example of controlling the growth of harmful flying insects.

Spodoptera frugiperda is an invasive alien species that causes serious harm to various crops, such as corn and sorghum, due to its strong reproductive ability and great destructive effects on crops. In studying the explosive growth pattern of harmful species populations and their impact on crops and ecosystems, the transition period of flying insects from a loose solitary stage to a group living stage is a crucial period for controlling their population size [24]. In order to control the number of insects in a timely manner before their eclosion, using a discrete single population model [25, 26] and considering the impact of insect predation on their species outbreak [27], the following model is proposed:

$$x(z+1) = e^{\left[c(1-\frac{x(z)}{L})-\frac{bx(z)}{a^2+x(z)^2}\right]}x(z), \tag{4.1}$$

where $x(z)$ represents the density of the z^{th} generation insect population, c represents the intrinsic growth rate, L represents the environmental carrying capacity, and parameters a and b are constants. After linearizing the above model, it can be simplified as:

$$x(z+1) = \mathfrak{G}_{\rho(z)}x(z), \tag{4.2}$$

where the system's matrices with a certain constructiveness are selected as follows:

$$\mathfrak{G}_1 = \begin{bmatrix} -0.5 & 0.2 \\ 0.1 & -0.4 \end{bmatrix}, \mathfrak{G}_2 = \begin{bmatrix} -0.1 & 0.3 \\ 0.4 & -0.2 \end{bmatrix}.$$

The eigenvalues of $\mathfrak{G}_1$ are $a_1 = -0.4732$ and $a_2 = 0.1268$; the eigenvalues of $\mathfrak{G}_2$ are $a_1 = 0.2$ and $a_2 = -0.5$. As we know, the stability of a discrete-time linear system (4.2) is equivalent to the Schur stability of the system matrix $\mathfrak{G}_{\rho(z)}$, that is, whether all eigenvalues of matrix $\mathfrak{G}_{\rho(z)}$ are located within the unit circle of the complex plane. Obviously, both subsystems are stable.

For the convenience of calculation, we select some appropriate parameters under the constraint of Theorem 1: $\lambda = 0.6$, $\mu = 2.1$, $\mathcal{T} = 3$. By substituting the above parameters into inequalities (3.12) and (3.27), it can be calculated that: BPDT is $\tau \geq 4.2621$; we take $\tau = 5$; PDT not less than 2.8098; select 3. Suppose the initial state of the system is $x(0) = [3, 1]^T$.

Select the appropriate switching instants: 0, 2, 5, 6, 7, 9, 12, 13, 14, 16 $\cdots$. Under the BPDT switching strategy, $\underbrace{0,\ 2,\ 5,}_{\tau-\text{portion}}\underbrace{\ 6,\ 7,}_{\mathcal{T}-\text{portion}}\underbrace{\ 9,\ 12,}_{\tau-\text{portion}}\underbrace{\ 13,\ 14,}_{\mathcal{T}-\text{portion}}\cdots$, that is, within each complete stage, the running time of τ-portion and $\mathcal{T}$-portion is 5 and 2, respectively. Under the PDT switching strategy, $\underbrace{0, \quad 2,}_{\mathcal{T}-\text{portion}}\underbrace{\qquad 5,}_{\tau-\text{portion}}\underbrace{\quad 6, \quad 7, \quad 9,}_{\mathcal{T}-\text{portion}}\underbrace{\qquad 12,}_{\tau-\text{portion}}\underbrace{\quad 13, \quad 14, \quad 16,}_{\mathcal{T}-\text{portion}}\cdots$, that is, within each complete stage, the running time of τ-portion and $\mathcal{T}$-portion is 3 and 4, respectively.

From the above analysis, it can be seen that the τ-portion of the BPDT switching strategy has a longer running time than the $\mathcal{T}$-portion, while the PDT switching strategy has the opposite situation. This indicates that it is easier to compensate for system instability caused by arbitrary switching in the $\mathcal{T}$-portion. This also proves the superiority of the BPDT switching strategy proposed in this paper.

Under the switching sequence given above, in order to simplify the calculation, take the special case (i) in Remark 5, and the conditions of Theorem 1 become:

$$\mathfrak{G}_u^T P_u(1)\mathfrak{G}_u - \lambda P_u(0) \leq 0, \tag{4.3}$$

$$\mathfrak{G}_u^T P_u(1)\mathfrak{G}_u - \lambda P_u(1) \leq 0, \tag{4.4}$$

$$\mathfrak{G}_u^T P_u(3)\mathfrak{G}_u - \lambda P_u(2) \leq 0, \tag{4.5}$$

$$\mathfrak{G}_u^T P_u(4)\mathfrak{G}_u - \lambda P_u(3) \leq 0, \tag{4.6}$$

$$\mathfrak{G}_u^T P_u(5)\mathfrak{G}_u - \lambda P_u(4) \leq 0, \tag{4.7}$$

$$P_u(0) - \mu P_v(5) \leq 0, \tag{4.8}$$

$$P_u(0) - \mu P_v(3) \leq 0, \tag{4.9}$$

where $u,\ v \in \mathfrak{M} = \{1, 2\}$. Substitute the values of λ and μ into inequalities (4.3)–(4.9). By using MATLAB to solve the above inequalities, it can be obtained that $P_u(1)$–$P_u(5)$, their values are given in the second column of Table 2. For ease of expression, denote $P_u(i)$ as $P_{u,i}$, where $u \in \mathfrak{M}$, $i \in \{1, 2, 3, 4, 5\}$.

Under the PDT switching strategy, inequality (4.4) becomes

$$\mathfrak{G}_u^T P_u(2)\mathfrak{G}_u - \lambda P_u(1) \leq 0, \tag{4.10}$$

inequality (4.8) becomes (4.9). The values of $P_u(1)$–$P_u(5)$ can be calculated using MATLAB, and their values are given in the third column of Table 2.

Select $\lambda = 0.6$, $\mu = 2.1$, $\mathcal{T} = 3$. It can be concluded from inequalities (3.12) and (3.27) that BPDT $\tau \geq 4.2621$ and PDT $\tau \geq 2.9098$. We can take BPDT switching $\tau = 5 > 4.2621$ and PDT switching $\tau = 3 > 2.9098$. The state responses under the PDT switching in [18] and the designed BPDT switching in the paper are shown in Figures 3 and 4, respectively. Here, the running times of the τ-portion and $\mathcal{T}$-portion under the two switching strategies are equal, respectively. It can be clearly observed from Figures 3 and 4 that the BPDT switching strategy can stabilize the system faster.

When the selected parameters are $\lambda = 0.6$, $\mu = 2.3$, $\mathcal{T} = 3$, it follows from inequalities (3.12) and (3.27) that BPDT $\tau \geq 5.1610$ and PDT $\tau \geq 3.5279$. If we take $\tau_p^1 = \tau_p^2 = 3$, then $\tau = \tau_p^1 + \tau_p^2 = 3 + 3 = 6 > 5.1610$, which implies the system is stable under such BPDT switching. However, $\tau_p^1 = \tau_p^2 = 3 < 3.5279$, thus we can not obtain the stability of the system under the same signal by the PDT switching. Therefore, the BPDT switching is more general than the PDT [18].

Table 2. Comparison of BPDT and PDT switching strategies.

Switching strategy	BPDT in Theorem 1	PDT in [18]
λ	0.6	0.6
μ	2.1	2.1
$\mathcal{T}$	3	3
$P_{1,0}$	$\begin{bmatrix} 212.4547 & -43.3794 \\ -43.3794 & 178.1463 \end{bmatrix}$	$\begin{bmatrix} 8.4689 & -2.6514 \\ -2.6514 & 7.7258 \end{bmatrix} *$
$P_{1,1}$	$\begin{bmatrix} 206.4627 & 265.3486 \\ -354.0830 & 165.7543 \end{bmatrix}$	$\begin{bmatrix} 8.0089 & -2.2567 \\ -2.2567 & 7.3375 \end{bmatrix} *$
$P_{1,2}$	$\begin{bmatrix} 175.9481 & -26.7140 \\ -26.7140 & 153.7497 \end{bmatrix}$	$\begin{bmatrix} 7.1591 & -1.6892 \\ -1.6892 & 6.6613 \end{bmatrix} *$
$P_{1,3}$	$\begin{bmatrix} 142.3370 & -91.5902 \\ 48.9129 & 125.3799 \end{bmatrix}$	$\begin{bmatrix} 5.3566 & -1.2626 \\ -1.2626 & 5.0028 \end{bmatrix} *$
$P_{1,4}$	$\begin{bmatrix} 160.2338 & -33.5240 \\ -6.4107 & 149.9520 \end{bmatrix}$	$\begin{bmatrix} 0.3408 & -3.7304 \\ -3.7304 & -2.4305 \end{bmatrix} *$
$P_{1,5}$	$\begin{bmatrix} 138.7184 & -3.7261 \\ -37.2291 & 122.7773 \end{bmatrix}$	$\begin{bmatrix} -19.5540 & -28.0980 \\ -28.0980 & -49.6990 \end{bmatrix} *$
$P_{2,0}$	$\begin{bmatrix} 212.4547 & -299.9207 \\ 213.1619 & 178.1463 \end{bmatrix}$	$\begin{bmatrix} 8.4689 & -2.6514 \\ -2.6514 & 7.7258 \end{bmatrix} *$
$P_{2,1}$	$\begin{bmatrix} 206.4627 & -86.0807 \\ -2.6537 & 165.7543 \end{bmatrix}$	$\begin{bmatrix} 8.0089 & -2.2567 \\ -2.2567 & 7.3375 \end{bmatrix} *$
$P_{2,2}$	$\begin{bmatrix} 175.9481 & -26.7140 \\ -26.7140 & 153.7497 \end{bmatrix}$	$\begin{bmatrix} 7.1591 & -1.6892 \\ -1.6892 & 6.6613 \end{bmatrix} *$
$P_{2,3}$	$\begin{bmatrix} 142.3370 & -17.4150 \\ -25.2623 & 125.3799 \end{bmatrix}$	$\begin{bmatrix} 5.3566 & -1.2626 \\ -1.2626 & 5.0028 \end{bmatrix} *$
$P_{2,4}$	$\begin{bmatrix} 160.2338 & -297.6663 \\ 257.7316 & 149.9520 \end{bmatrix}$	$\begin{bmatrix} 0.3408 & -3.7304 \\ -3.7304 & -2.4305 \end{bmatrix} *$
$P_{2,5}$	$\begin{bmatrix} 138.7184 & -60.8570 \\ 19.9019 & 122.7773 \end{bmatrix}$	$\begin{bmatrix} -19.5540 & -28.0980 \\ -28.0980 & -49.6990 \end{bmatrix} *$
τ	4.2621	2.9098
State response	Figure 3	Figure 4

Note: $*$ represents 10^7.

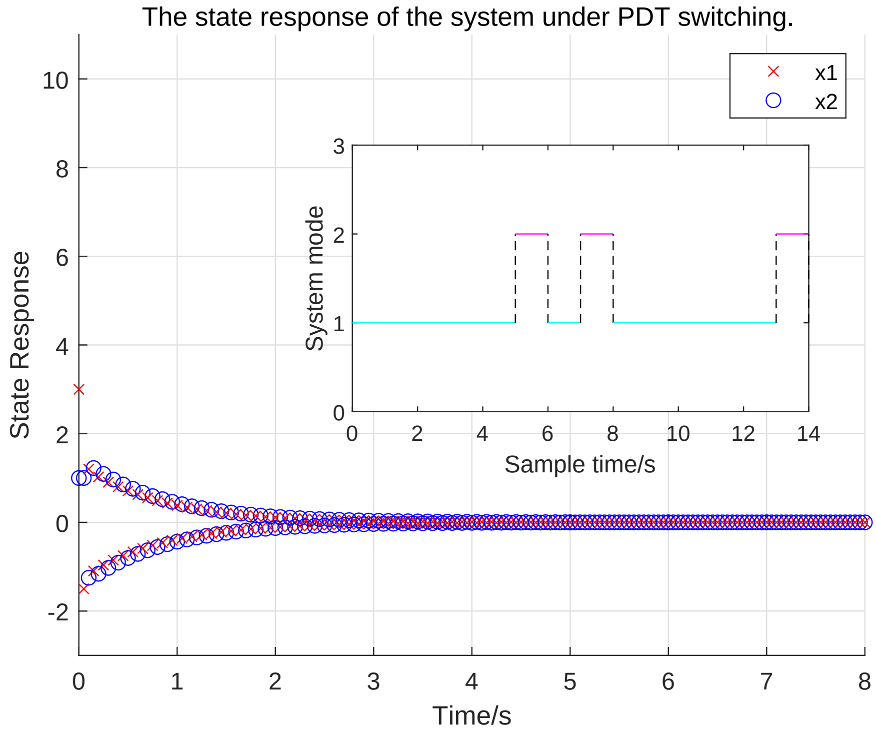

Figure 3. State response of the system under PDT switching signal.

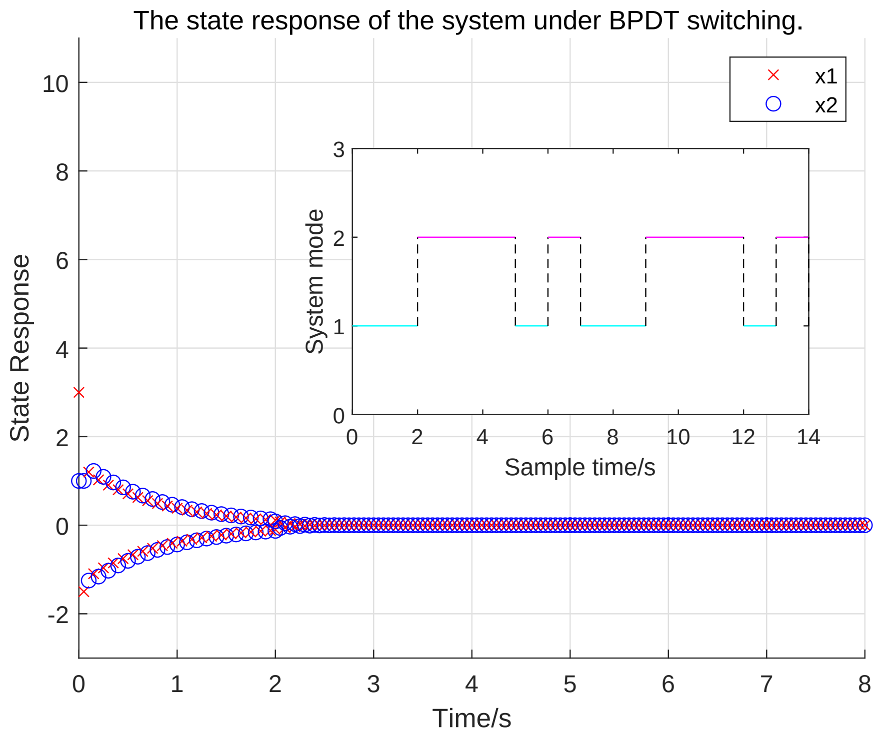

Figure 4. State response of the system under BPDT switching signals.

5. Conclusions

This paper investigates the stability and stabilization controller design of the discrete-time switched systems under BPDT switching. By relaxing the constraints in PDT switching, this article proposes the concept of BPDT switching. It has a wider switching signal range than traditional PDT switching. In addition, we analyze the stability of switched systems by extending the QLF to the BQLF. The stability criterion for the discrete-time switched system is given in the form of linear matrix inequalities. Subsequently, a stabilizing controller is designed to stabilize the system. Finally, a practical example is used to illustrate the validity of the conclusions given in this paper.

Since BPDT can give greater design freedom than PDT, it and its adapted BQLF are expected to be extended to those systems that are suitable for PDT switching, such as discrete-time switched GRNs with time delays [18], fuzzy switched systems [28], and positive switched systems [29]. It is worth noting that the BPDT strategy proposed in this article for discrete-time switched systems may be extended to the continuous case. The difficulty of this extension lies in how to partition τ_p^1 and τ_p^2 in the construction for $\varsigma(z)$ in (3.2).

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there are no conflicts of interest.

References

1. D. Liberzon, A. S. Morse, Basic problems in stability and design of switched systems, *IEEE Control Syst. Mag.*, **19** (1999), 59–70. https://doi.org/10.1109/37.793443
2. S. Zheng, S. Wang, X. Chen, Y. Xie, P. Shi, Z. Jiang, Logic-based switching finite-time stabilization with applications in mechatronic systems, *Int. J. Robust Nonlinear Control*, **33** (2023), 3064–3085. https://doi.org/10.1002/rnc.6565
3. U. Boscain, G. Charlot, M. Sigalotti, Stability of planar nonlinear switched systems, *Discrete Contin. Dyn. Syst.*, **15** (2006), 415–432. https://doi.org/10.3934/dcds.2006.15.415
4. P. Shi, X. Su, F. Li, Dissipativity-based filtering for fuzzy switched systems with stochastic perturbation, *IEEE Trans. Autom. Control*, **61** (2016), 1694–1699. https://doi.org/10.1109/TAC.2015.2477976

5. W. Xiang, J. Xiao, Stabilization of switched continuous-time systems with all modes unstable via dwell time switching, *Automatica*, **50** (2014), 940–945. https://doi.org/10.1016/j.automatica.2013.12.028

6. G. Zong, H. Ren, L. Hou, Finite-time stability of interconnected impulsive switched systems, *IET Control Theory Appl.*, **10** (2015), 648–654. https://doi.org/10.1049/iet-cta.2015.0617

7. D. Li, Y. Lin, Z. Pu, Non-autonomous stochastic lattice systems with Markovian switching, preprint, arXiv:2204.00776.

8. X. M. Sun, J. Zhao, D. J. Hill, Stability and L_2-gain analysis for switched delay systems: A delay-dependent method, *Automatica*, **42** (2006), 1769–1774. https://doi.org/10.1016/j.automatica.2006.05.007

9. Y. Zhao, J. Zhao, J. Fu, Y. Shi, C. Chen, Rate bumpless transfer control for switched linear systems with stability and its application to aero-engine control design, *IEEE Trans. Ind. Electron.*, **67** (2019), 4900–4910. https://doi.org/10.1109/TIE.2019.2931222

10. Y. Zhao, S. H. Yu, J. Lian, Anti-disturbance bumpless transfer control for switched systems with its application to switched circuit model, *IEEE Trans. Circuits Syst. II Express Briefs*, **67** (2020), 3177–3181. https://doi.org/10.1109/TCSII.2020.2970068

11. Y. Zhao, J. Zhao, H_∞ reliable bumpless transfer control for switched systems with state and rate constraints, *IEEE Trans. Syst. Man Cybern.: Syst.*, **50** (2020), 3925–3935. https://doi.org/10.1109/TSMC.2018.2871335

12. H. Shen, M. Xing, H. Yan, J. Cao, Observer-based $l_2 - l_\infty$ control for singularly perturbed semi-Markov jump systems with an improved weighted TOD protocol, *Sci. China Inf. Sci.*, **65** (2022), 199204. https://doi.org/10.1007/s11432-021-3345-1

13. J. Wang, D. Wang, H. Yan, H. Shen, Composite Anti-disturbance H_∞ control for hidden Markov jump systems with multi-sensor against replay attacks, *IEEE Trans. Autom. Control*, **69** (2024), 1760–1766. https://doi.org/10.1109/TAC.2023.3326861

14. A. S. Morse, Supervisory control of families of linear set-point controllers - Part 1: Exact matching, *IEEE Trans. Autom. Control*, **41** (1996), 1413–1431. https://doi.org/10.1109/9.539424

15. J. P. Hespanha, A. S. Morse, Stability of switched systems with average dwell time, in *Proceedings of the 38th IEEE Conference on Decision and Control*, IEEE, (1999), 2655–2660. https://doi.org/10.1109/CDC.1999.831330

16. J. P. Hespanha, Uniform stability of switched linear systems extensions of Lasalle's invariance principle, *IEEE Trans. Autom. Control*, **49** (2004), 470–482. https://doi.org/10.1109/TAC.2004.825641

17. J. Liu, L. Kang, New results on stability and L_1-gain characterization for switched positive systems: A persistent dwell time approach, *Trans. Inst. Meas. Control*, **44** (2022), 1288–1296. https://doi.org/10.1177/01423312211053325

18. T. Yu, Y. Zhao, Q. Zeng, Stability analysis for discrete-time switched GRNs with persistent dwell-time and time delays, *J. Franklin Inst.*, **357** (2020), 11730–11749. https://doi.org/10.1016/j.jfranklin.2019.09.039

19. T. Han, S, Ge, T. Lee, Persistent dwell-time switched linear systems: Variation paradigm and gauge design, *IEEE Trans. Autom. Control*, **55** (2010), 321–337. https://doi.org/10.1109/TAC.2009.2034927

20. L. I. Allerhand, U. Shaked, Robust stability and stabilization of linear switched systems with dwell time, *IEEE Trans. Autom. Control*, **56** (2011), 381–386. https://doi.org/10.1109/TAC.2010.2097351

21. S. Q. Li, F. Q. Deng, M. L. Xing, Aperiodic sampled-data robust H_∞ control for delayed stochastic fuzzy systems with quasi-periodical multi-rate approach, *J. Franklin Inst.*, **356** (2019), 4530–4553. https://doi.org/10.1016/j.jfranklin.2019.01.051

22. Y. E. Wang, H. R. Karimi, D. Wu, Conditions for the stability of switched systems containing unstable subsystems, *IEEE Trans. Circuits Syst. II Express Briefs*, **66** (2019), 617–621. https://doi.org/10.1109/TCSII.2018.2852766

23. D. Liberzon, *Switching in Systems and Control*, Birkhäuser Boston, 2003. https://doi.org/10.1007/978-1-4612-0017-8

24. C. Xiang, S. Tang, R. A. Cheke, W. Qin, A locust phase change model with multiple switching states and random perturbation, *Int. J. Bifurcation Chaos*, **26** (2016), 1630037. https://doi.org/10.1142/S0218127416300378

25. W. E. Ricker, Stock and recruitment, *J. Fish. Res. Board Can.*, **11** (1954), 559–623. https://doi.org/10.1139/f54-039

26. P. Moran, Some remarks on animal population dynamics, *Biometrics*, **6** (1950), 250–258. https://doi.org/10.2307/3001822

27. D. Ludwig, D. D. Jones, C. S. Holling, Qualitative analysis of insect outbreak systems: The spruce budworm and forest, *J. Anim. Ecol.*, **47** (1978), 315–332.

28. C. Wei, X. Xie, J. Sun, J. H. Park, Attack-resilient dynamic-memory event-triggered control for fuzzy switched systems with persistent dwell-time, *IEEE Trans. Fuzzy Syst.*, **32** (2024), 3154–3164. https://doi.org/10.1109/TFUZZ.2024.3364754

29. Q. Yu, X. Yuan, Stability analysis for positive switched systems having stable and unstable subsystems based on a weighted average dwell time scheme, *ISA Trans.*, **136** (2023), 275–283. https://doi.org/10.1016/j.isatra.2022.10.019