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ERA, 31 (5): 2925–2939.
DOI: 10.3934/era.2023147
Received: 21 June 2022
Revised: 17 February 2023
Accepted: 19 February 2023
Published: 17 March 2023

Research article

Dynamic behavior of stochastic predator-prey system

Wan Pinglan*

The Hong Kong Polytechnic University (PolyU), China

* **Correspondence:** Email: 21051033g@connect.polyu.hk; Tel: +8618811505960.

Abstract: In this paper, a stochastic predator-prey system with mutual interference is studied, which provides guidance on creating appropriate biodegradable environments. By utilizing Mawhin's coincidence degree theorem and constructing a suitable Lyapunov function, a Volterra model with mutual interference is studied. A few sufficient conditions are obtained for existence, extinction and global asymptomatic stability of the positive solution of the model. Then we perform various numerical simulations to verify the stochastic and deterministic systems are global asymptotically stable. It is significant that such a model is firstly proposed with stochastic mutual interference.

Keywords: predator-prey model; Lyapunov function; asymptotic stability; mutual interference

1. Introduction

The biodegradable materials have been a dominant theme due to the in-depth study of the ESG [1]. Under properly controlled conditions, the materials will be decompose into little to no harmful residue. In order to provide guidance on how to create suitable biodegradable [2–5] environments, this article will investigate a two kinds of populations stochastic prey-predator model.

There are three relationships between two kinds of populations: cooperation, competition, and predator-prey. And the predator-prey relationship is the most popular. In this paper, we consider the dynamic behavior of predator-prey model. The classical deterministic predator-prey model has been widely studied [6–11]. In the early 20th century, Lotka and Volterra jointly proposed the classic Lotka-Volterra model in the field of chemistry and the observation on the competitive behavior of fish which have an important role in the modeling of special classes of nonlinear systems such as interaction of microorganisms [12] and population ecology [13]. Through the continuous efforts of scientists, plenty of improved predator-prey models have been obtained which can analyze the complex processes of life phenomena more accurately. The immediate advantage is that individuals can better predict and control the dynamics of the population and then numerous practical problems that human beings encounter in life can be solved [14,15]). Ahmad [17,18] investigates the critical normal form coefficients for two discrete-time Bazykin-Berezovskaya prey-predator models with a strong Allee effect and Mixed Functional Response respectively. There are not a sea of papers researching the mutual interference between the predators and prey while the research on the interaction between predatory predators is of great application value and research meanings. The idea of mutual interference was first proposed by Hassell [19,20] in 1971.

$$\begin{cases} \dot{x} = xg(t,x) - \varphi(t,x)y^m) \\ \dot{y} = y(-d(t) + k\varphi(t,x)y^{m-1} - q(t,y)) \end{cases} \tag{1}$$

This can also be generalized to include group or herd defense, toxin production and mimicry [16]. Kai Wang [21] studied the existence, global attractivity and uniqueness of a Volterra model with mutual interference, which can be described by the following form:

$$\begin{cases} \dot{x} = x\left(r_1 - b_1 x - \frac{c_1 y^m}{k+x}\right) \\ \dot{y} = y\left(-r_2 - b_2 y + \frac{c_2 x y^{m-1}}{k+x}\right) \end{cases} \tag{2}$$

where $x(t)$ and $y(t)$ denote the densities of the prey and the predator at time t , respectively. The parameters r_1 , r_2 ; b_1 , b_2 ; c_1 , c_2 ; m and k are positive constants from the viewpoint of ecology which denote the intrinsic growth rate for the prey, intrinsic death rate of the predator; effect of the density of species on the rate of growth rate of themselves; efficiency of biomass conversion from prey to predator; mutual interference parameter and value of population density at which per capita removal rate is half of x, respectively.

Meanwhile, the noise in the real world always exists. In order to describe clearly, one should consider the effect of noise. On the other hand, by the theory of central limit theorem, we know that the error term obeys the normal distribution, and the stochastic nonautonomous system can be obtained from the deterministic system [22].

$$r_1 \to r_1 + \sigma_1(t)dB_1(t) \qquad -r_2 \to -r_2 + \sigma_2(t)dB_2(t)$$

$B_i(t), i = 1,2$ are Brownian motion defined on a complete probability space(Ω, $\mathcal{F}$, $\mathbb{P}$), then corresponding to the deterministic system (2), the similar stochastic nonautonomous is as follow.

$$\begin{cases} dx = x\left(r_1 - b_1 x - \frac{c_1 y^m}{k+x}\right)dt + \sigma_1(t)x dB_1(t) \\ dy = y\left(-r_2 - b_2 y + \frac{c_2 x y^{m-1}}{k+x}\right)dt + \sigma_2(t)y dB_2(t) \\ x(0) = x_0 > 0, y(0) = y_0 > 0 \end{cases} \tag{3}$$

For stochastic predator-prey model with mutual interference, Wang and Zhu [23,24] consider the following system

$$\begin{cases} dx = x(r_1 - b_1 x - c_1 y^m)dt + \sigma_1(t)xdB_1(t) \\ dy = y(-r_2 - b_2 y + c_2 x y^{m-1})dt + \sigma_2(t)ydB_2(t) \\ x(0) = x_0 > 0, y(0) = y_0 > 0 \end{cases} \tag{4}$$

In this paper, the dynamic behavior of the system (3) is studied. Recently, the predator-prey system with different function response has been studied by [25–30]. But the predator-prey of the system (3) has not been studied until now. Generally speaking, coincidence degree theory, Jacobi matrix and Fokker-Planck equation are the approaches for studying stochastic system. However, these classical approaches are not useful for the system (3). Therefore, the novelty of this paper is unique system and the extinction and global asymptotic stability characteristic.

The rest of this paper is organized as follows. In Section 2, the existence and uniqueness of the system (3) are given. we will prove that the positive solution of system (3) is globally existent which is the solution is not explosive at finite time. In Section 3, we will carry out the survival analysis for the model and obtain a sufficient condition for the extinction of each species in many cases. In Section 4, sufficient conditions for global asymptotical stability of system (3) are established. In Section 5, the numeric simulations are shown to support the result. Finally, the conclusions and discussions are given in Section 6.

2. Globally positive solution

In this section, we present a few results on the existence and uniqueness of global positive solution of system (SM).

$$\begin{cases} dx = x\left(r_1(t) - b_1(t)x - \dfrac{c_1(t)y^m}{k+x}\right)dt + \sigma_1(t)xdB_1(t) \\ dy = y\left(-r_2(t) - b_2(t)y + \dfrac{c_2(t)xy^{m-1}}{k+x}\right)dt + \sigma_2(t)ydB_2(t) \\ x(0) = x_0 > 0, y(0) = y_0 > 0. \end{cases} \tag{SM}$$

In the SM, $x(t)$ and $y(t)$ are the densities of prey population and predator population respectively, only the non-negative solution has ecological significance. In order to make the stochastic differential equation have a global solution, generally speaking, the equation needs to satisfy the linear growth condition and the local Lipschitz condition. However, SM neither satisfies the linear growth condition nor the local Lipschitz condition. Therefore, we use the following method which is a combination of a few stochastic calculus and marting. If $f(t)$ is a continuous bounded function on $\mathbb{R}_+$, define

$$f^u = \sup_{t\in\mathbb{R}_+} f(t), \qquad f^l = \inf_{t\in\mathbb{R}_+} f(t)$$

Throughout this paper, suppose $r_i(t)$, $b_i(t)$ and $c_i(t)$, $i = 1,2$ are continuous bounded functions on $\mathbb{R}_+$ and $r_i(t) > 0, b_i(t) > 0, c_i(t) > 0, i = 1,1$.

The following theorems give a few sufficient conditions for the existence and uniqueness of the global positive solution.

Theorem 1. [22] *There is a unique positive local solution* $(x(t), y(t))$ *for* $t \in [0, \tau_e)$ *to SM almost surely (a. s.) for the initial* $x_0 > 0, y_0 > 0$, *where* τ_e *is the explosion time.*

Proof. Let us suppose $x(t) = e^{u(t)}, y(t) = e^{v(t)}$. then by using Itô's formula, we obtain

$$\begin{cases} du = \left(r_1(t) - b_1(t)e^u - \dfrac{c_1(t)e^{mv}}{k + e^u}\right)dt + \sigma_1(t)dB_1(t) \\ dv = \left(-r_2(t) - b_2(t)e^v + \dfrac{c_2(t)e^u e^{(m-1)v}}{k + e^u}\right)dt + \sigma_2(t)dB_2(t) \end{cases} \qquad \text{(SM')}$$

Along with $u(0) = ln\, x_0\, , v(0) = ln\, y_0$,

We denote the above system along with the initial conditions as (SM). Note that the coefficients of (SM') satisfy the local *Lipschitz* condition, then for given initial values $u_0 > 0, v_0 > 0$ there is a unique maximal local solution $(u(t), v(t), \tau_e)$, where τ_e is the explosion time of the solution. By Ito's formula, $x(t) = e^{u(t)}, y(t) = e^{v(t)}$ is the positive local solution to (SM') with initial value $x_0 > 0, y_0 > 0$.

Theorem 2. *For any given initial value* $(x_0, y_0) \in \mathbb{R}_+^2$ *, there is a unique solution* $(x(t), y(t))$ *on* $t \geq 0$ *to (SM), and this solution will remain in* $\mathbb{R}_+^2$ *with probability 1, where* $\mathbb{R}_+^2 = \{(x, y) \in \mathbb{R}^2 | x, y > 0, i = 1,2\}$.

Proof. For convenience, let

$$F(x, y) = r_1(t) - b_1(t)x - \frac{c_1(t)y^m}{k + x}$$

$$G(x, y) = -r_2(t) - b_2(t)e^v + \frac{c_2(t)e^u e^{(m-1)v}}{k + e^u}$$

Let $n_0 > 0$, be large enough, such that for x_0 and y_0 given, they lie in the interval $[\frac{1}{n_0}, n_0]$. For each integer $n > n_0$, define the stopping times:

$$\tau_n = inf\left\{t \in [0, \tau_n]: \ x(t) \notin \left(\frac{1}{n}, n\right) \ or \ y(t) \notin \left(\frac{1}{n}, n\right)\right\}$$

Obviously, τ_n is increasing as $n \to \infty$.Let $\tau_\infty = lim_{n\to\infty}\tau_n$, whence $\tau_\infty < \tau_e$ a.s. We only need to show $\tau_\infty = \infty$. If this statement is false, there exist constants $T > 0$ and $\varepsilon \in (0,1)$ such that $P\{\tau_\infty \leq T\} > \varepsilon$. Consequently, there exists an integer $n_1 \geq n_0$ such that

$$P\{\tau_n \leq T\} > \varepsilon; \ n \geq n_1 \qquad (5)$$

Define a C^2 function V: $\mathbb{R}_+^2 \to \mathbb{R}_+$ by

$$V(x, y) = \left(\sqrt{x} - 1 - 0.5\, ln\, x\right) + \left(\sqrt{y} - 1 - 0.5\, ln\, y\right)$$

If $(x, y) \in \mathbb{R}^2$, then Ito's formula [8] yields:

$$dV(x, y) = (x^{0.5} - 1)F(x, y)dt + \frac{1}{8}(-x^{0.5} + 2)\sigma_1{}^2(t)dt + (x^{0.5} - 1)\sigma_1(t)dB_1(t)$$
$$+ (y^{0.5} - 1)G(x, y)dt + \frac{1}{8}(-y^{0.5} + 2)\sigma_2{}^2(t)dt + (y^{0.5} - 1)\sigma_2(t)dB_2(t)$$

Compute

$$
\begin{aligned}
(x^{0.5}-1)F(x,y) &= (x^{0.5}-1)(r_1(t)-b_1(t)x-\frac{c_1(t)y^m}{k+x}) \\
&= r_1(t)x^{0.5}-b_1(t)x^{1.5}-r_1(t)+b_1(t)x+\frac{c_1(t)x^{0.5}y^m}{k+x}-\frac{c_1(t)y^m}{k+x} \\
&\le r_1(t)x^{0.5}-b_1(t)x^{1.5}-r_1(t)+b_1(t)x \\
&\le K_1
\end{aligned}
$$

And

$$
\begin{aligned}
(y^{0.5}-1)G(x,y) &= (y^{0.5}-1)(r_2(t)-b_2(t)y-\frac{c_2(t)xy^{m-1}}{k+x}) \\
&= r_2(t)y^{0.5}-b_2(t)y^{1.5}-r_2(t)+b_2(t)y+\frac{c_2(t)xy^{m-0.5}}{k+x}-\frac{c_2(t)y^{m-1}}{k+x} \\
&\le r_2(t)y^{0.5}-b_2(t)y^{1.5}-r_2(t)+b_2(t)y \\
&\le K_2
\end{aligned}
$$

where K_1 and K_2 are given positive constants. Integrating both sides of the above inequality from 0 to $\tau_n \wedge T$ and then taking the expectation leads to

$$EV(x(\tau_n \wedge T), y(\tau_n \wedge T)) \le V(x_0, y_0) + (K_1+K_2)T \tag{6}$$

Set $\Omega_n = \{\tau_n \le T\}$, by (5) we have $P(\Omega_n) \ge \varepsilon$. Note that for each $\omega \in \Omega_n$, there are a few i such that $x_i(\tau_n, \omega)$ equals n or $1/n$ for i = 1, 2. Hence $V(x(\tau_n \wedge T), y(\tau_n \wedge T))$ is no less than

$$min\left\{\sqrt{n}-1-0.5\,ln\,n\,,\sqrt{\frac{1}{n}}-1-0.5\,ln\frac{1}{n}\right\}$$

Thus by (6) we have

$$
\begin{aligned}
V(x_0,y_0)+(K_1+K_2)T &\ge E[1_{\Omega_n(\omega)}V(x(\tau_n \wedge T), y(\tau_n \wedge T))] \\
&\ge \varepsilon\, min\left\{\sqrt{n}-1-0.5\,ln\,n\,,\sqrt{\frac{1}{n}}-1-0.5\,ln\frac{1}{n}\right\}
\end{aligned}
$$

where 1_{Ω_n} is the indicator function of Ω_n. Letting $n \to \infty$, leads to the contraction

$$\infty > V(x_0,y_0)+(K_1+K_2)T = \infty$$

This completes the proof.

3. Extinction

In this section, we investigate extinction criteria in the model, finding that the stochastic forcing can be responsible for extinction of both species, under certain conditions.

Theorem 3. [30,31] *Let conditions* $r_1 - 0.5\sigma_1^2 < 0$ *and* $c_2 - r_2 - 0.5\sigma_1^2 < 0$ *hold. The* *populations x and y by (SM) will become extinct exponentially with probability one.*

Proof. Define $V(x) = c_1 x + c_2 y$. By the Itô formula, we have

$$d[\ln x(t)] = \left(r_1 - b_1 x - \frac{c_1(t)y^m}{k+x} - \frac{1}{2}\sigma_1{}^2\right)dt + \sigma_1(t)dB_1(t) \tag{7}$$

Integrating both sides of (7) leads to

$$\frac{\ln\left(\frac{x(t)}{x_0}\right)}{t} = \frac{\int_0^t [r_1 - 0.5\sigma_1^2]ds}{t} - \frac{\int_0^t b_1 x ds}{t} - \frac{\int_0^t \frac{c_1 y^m}{k+x} ds}{t} + \frac{\int_0^t \sigma_1 dB_1(s)}{t} \tag{8}$$

In the same way

$$\frac{\ln\left(\frac{y(t)}{y_0}\right)}{t} = \frac{\int_0^t [r_2 - 0.5\sigma_2^2]ds}{t} - \frac{\int_0^t b_2 y ds}{t} - \frac{\int_0^t \frac{c_2 x y^{m-1}}{k+x} ds}{t} + \frac{\int_0^t \sigma_2 dB_2(s)}{t} \tag{9}$$

Set $M_i(t) = \int_0^t \sigma_i dB_i(t), i = 1,2$. Then $M_i(t)$ is a local martingale whose quadratic variation is

$$\langle M_i,\ M_i\rangle_t = \int_0^t \sigma_i^2\, ds \le (\sigma_i^2)^u t \tag{10}$$

By the strong law of large numbers for Martingales [8] we have

$$\lim_{t\to\infty} \frac{M_i(t)}{t} = 0 \quad \text{a.s.} \tag{11}$$

From (8), (10) and $r_1(t) - 0.5\sigma_1^2 < 0$, we have

$$\lim_{t\to+\infty} \sup \frac{\ln(x(t))}{t} \le \frac{\int_0^t [r_1 - 0.5\sigma_1^2]ds}{t} < 0 \quad \text{a.s.}$$

Consider m = 1, by the Itô formula, we have

$$d[\ln y(t)] = \left(-r_2 - b_2 y + \frac{c_2 x}{k+x} - \frac{1}{2}\sigma_2{}^2\right)dt + \sigma_2 dB_2(t) \tag{12}$$

Integrating both sides of (12) leads to

$$\ln y(t) - \ln y(0) = (-r_2 - \frac{1}{2}\sigma_2{}^2)t - \int_0^t b_2 y ds + \int_0^t \frac{c_2(k + x - k)}{k+x} ds + \int_0^t \sigma_2 dB_2(s)$$

$$= (c_2 - r_2 - \frac{1}{2}\sigma_2{}^2)t - \int_0^t b_2 y(s)ds - \int_0^t \frac{c_2 k}{k+x(s)}ds + \sigma_2 dB_2(s) \quad (13)$$

By the strong law of large numbers for Martingales, similarly we have

$$ln\, y(t) - ln\, y(0) \leq (c_2 - r_2 - \frac{1}{2}\sigma_2{}^2)t - \int_0^t b_2 y ds - \int_0^t \frac{c_2 k}{k+x}ds \quad (14)$$

From (14) and $c_2 - r_2 - \frac{1}{2}\sigma_2{}^2 < 0$, we have

$$ln\, y(t) - ln\, y(0) \leq (c_2 - r_2 - \frac{1}{2}\sigma_2{}^2)t < 0$$

As $t \to +\infty$.
Thus $lim_{\to+\infty} sup\, ln\, y(t) < 0$ a.s.

Consider 0<m<1, and the term $\frac{\int_0^t \frac{c_2(s)x(s)y^{m-1}}{k+x}ds}{t}$ in (9).
From $y(t) \geq 1$, (9) and $c_2 - r_2 - 0.5\sigma_1^2 < 0$, we have

$$-\frac{\int_0^t b_2(s)y(s)ds}{t} - \frac{\int_0^t \frac{c_2(s)x(s)y^{m-1}}{k+x}ds}{t} \leq \frac{\int_0^t (c_2(s) - b_2(s))y(s)ds}{t} < 0$$

As $t \to +\infty$.
We obtain the same result $lim_{t\to+\infty} sup\, ln\, y(t) < 0$ a.s.

4. Global asymptotic stability

Definition 4. *SM is said to be globally asymptotical stable if*

$$\lim_{t\to+\infty} |x_1(t) - x_2(t)| = \lim_{t\to+\infty} |y_1(t) - y_2(t)| = 0\ a.s.$$

for any two positive solutions $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ of SM, we first give a few lemmas.

Lemma 5. [18] *Let* $x(t)$, $y(t)$ *be a solution to (SM) with initial value*(x_0, y_0). *If* $b_2^l - c_2^u > 0$, *then for all* $p > 1$, *there exist constants* $L(p)$ and $G(p)$ *such that*

$$E[x^p(t)] \leq L(p);\ \ E[y^p(t)] \leq G(p)$$

Proof. Define $V(x) = x^p$ for all $x \in \mathbb{R}_+, p > 1$. By the Itô formula, we have

$$\begin{aligned} dV(x) = &\quad px^{p-1}dx + 0.5p(p-1)x^{p-2}(dx)^2 \\ = &\quad px^p\left[\left(r_1(t) - b_1(t)x - \frac{c_1(t)y^m}{k+x}\right) + 0.5(p-1)\sigma_1^2(t)\right]dt \\ &\quad + px^p\sigma_1(t)dB_1(t) \end{aligned}$$

Using Itô formula again to $e^tV(x)$ results in

$$\begin{aligned} d[e^tV(x)] = &\quad e^tV(x)dt + e^tdV(x) \\ = &\quad \left\{e^tx^p + pe^tx^p\left[\begin{matrix}\left(r_1(t) - b_1(t)x - \dfrac{c_1(t)y^m}{k+x}\right) + \\ 0.5(p-1)\sigma_1^2(t)\end{matrix}\right]\right\}dt \\ &\quad + pe^tx^p\sigma_1(t)dB_1(t) \end{aligned}$$

Taking expectations on both sides, we have

$$E[e^tx^p(t)] \le x_0^p + pE\int_0^t e^s\left\{x^p(s)\left[\frac{1}{p} + r_1^u + 0.5(p-1)(\sigma_1^2)^u - b_1^l x(s)\right]\right\}ds$$

$$\le x_0^p + \int_0^t e^sL_1(p)ds = x_0^p + L_1(p)(e^t - 1)$$

$$L_1(p) = \frac{\left[1 + r_1^\mu p + 0.\ 5p(p-1)\left(\sigma_1^2\right)^u\right]^{p+1}}{(p+1)^{p+1}\left(b_1'\right)^p}.$$

There exists a $T > 0$ such that $E[x^p(t)] \le 1.5L_1(p)$ for all $t \ge T$. At the same time, an application of the continuity of $E[x^p(t)]$results in that there exists $\tilde{L}_1(p) > 0$ such that $E[x^p(t)] \le \tilde{L}_1(p)$ for all $t \le T$. Let $L(p) = max\{1.5L_1(p), \tilde{L}_1(p)\}$, then for all $t \ge 0$, we have $E[x^p(t)] \le L(p)$.

On the other hand, we can show that

$$\begin{aligned} d[e^tV(y)] = &\quad e^tV(y)dt + e^tdV(y) \\ = &\quad \left\{e^ty^p + pe^ty^p\left[\left(\begin{matrix}-r_2(t) - b_2(t)y + \dfrac{c_2(t)xy^{m-1}}{k+x} \\ +0.5(p-1)\sigma_2^2(t)\end{matrix}\right]\right\}dt \\ &\quad + pe^ty^p\sigma_2(t)dB_2(t) \end{aligned}$$

Consider $k = 1$, Taking expectations on both sides results in

$$
\begin{aligned}
E[e^t y^p(t)] \leq\ & y_0^p + pE\int_0^t e^s\left\{y^p(s)\left[\frac{1}{p} + c_2^u x(s) + 0.5(p-1)(\sigma_2^2)^u - b_2^l y(s)\right]\right\} ds \\
\leq\ & y_0^p + p\int_0^t e^s\left\{E\left[y^p(s)\left[\frac{1}{p} + 0.5(p-1)(\sigma_2^2)^u - b_2^l y(s)\right]\right]\right. \\
& \left. + c_2^u E[y^p(s)x(s)]\right\} ds, \\
\leq\ & y_0^p + p\int_0^t e^s E\left[y^p(s)\left[\frac{1}{p} + 0.5(p-1)(\sigma_2^2)^u - b_2^l y(s)\right]\right] ds \\
& + pc_2^u\int_0^t e^s E[y^{p+1}(s)] + \frac{pc_2^u}{p+1}\int_0^t e^s E[x^{p+1}(s)] ds \\
=\ & y_0^p + pE\int_0^t e^s y^p(s)\left[\frac{1}{p} + 0.5(p-1)(\sigma_2^2)^u - \left(b_2^l - c_2^u\right)y(s)\right] ds \\
& + \frac{pc_2^u}{p+1}\int_0^t e^s E[x^{p+1}(s)] ds \\
\leq\ & y_0^p + \int_0^t e^s L_2(p) ds + \frac{pc_2^u}{p+1} L(p+1)\int_0^t e^s ds \\
=\ & y_0^p + \left[L_2(p) + \frac{pc_2^u}{p+1} L(p+1)\right](e^t - 1)
\end{aligned}
$$

where $L_2(p) = \frac{\left[1+0.5p(p-1)\left(\sigma_2^2\right)u\right]^{p+1}}{(p+1)^{p+1}\left(b_2^l - c_2^\mu\right)^p}$. Thus we get

$$
\lim_{t\to+\infty} sE[y^p(t)] \leq L_2(p) + \frac{pc_2^u}{p+1} L(p+1) =: L_3(p)
$$

Then there exists a $T > 0$ such that $E(y^p(t)) \leq 1.5L_3(p)$ for all $t \geq T$. There also exists $E(y^p(t)) \leq \tilde{L}_3(p)$ for $t < T$. Let $G(p) = max\{1.5L_3(p), \tilde{L}_3(p)\}$, then for all $t \geq 0$, $E(y^p(t)) \leq G(p)$.

Lemma 6. [32] *Suppose that an n-dimensional stochastic process $X(t)$ on $t \geq 0$ satisfies the condition*

$$
E|X(t) - X(s)|^{\alpha_1} \leq c|t-s|^{1+\alpha_2},\ 0 \leq s, \qquad t < \infty
$$

for a few positive constants α_1, α_2 and c. Then there exists a continuous modification $\tilde{X}(t)$ of $X(t)$ which has the property that for every $\vartheta \in (0, \alpha_2/\alpha_1)$, there is a positive random variable $h(\omega)$ such that

$$
\mathcal{P}\left\{\omega: \sup_{0<|t-s|<h(\omega),\ 0\leq s,\ t<\infty} \frac{\tilde{X}(t,\omega) - X(t,\omega)}{|t-s|^{\vartheta}} \leq \frac{2}{1-2^{-\vartheta}}\right\} = 1
$$

Lemma 7. [33] *Let $(x(t), y(t))$ be a solution of (SM) on $t \geq 0$. If $b_2^l - c_2^u > 0$, then almost every sample path of $(x(t), y(t))$ is uniformly continuous.*

Proof. The first equation of SM is equivalent to the following stochastic integral equation

$$x(t)=x_0+\int_0^t x(s)\left(r_1(s)-b_1(s)x(s)-\frac{c_1(s)y^m}{k+x}\right)ds+\int_0^t \sigma_1(s)x(s)dB_1(s)$$

Note that

$$\begin{aligned}
&E\left|x\left(r_1(s)-b_1(s)x-\frac{c_1(s)y^m}{k+x}\right)\right|^p\\
&\quad=E|x|^p\left|\left(r_1(s)-b_1(s)x-\frac{c_1(s)y^m}{k+x}\right)\right|^p\\
&\quad\le 0.5E|x|^{2p}+0.5E\left|\left(r_1(s)-b_1(s)x-\frac{c_1(s)y^m}{k+x}\right)\right|^{2p}\\
&\le 0.5\{L(2p)+3^{2p-1}[(|r_1|^u)^{2p}+b_1^uE|x(t)|^{2p}+c_1^uE|y(t)|^{2p}]\}\\
&\le 0.5\{L(2p)+3^{2p-1}[(|r_1|^u)^{2p}+b_1^uL(2p)+c_1^uG(2p)]\}:=K_2(p)
\end{aligned}$$

Moreover, in the view of the moment inequality for stochastic integrals we can obtain that for $0\le t_1\le t_2$ and $p>2$,

$$\begin{aligned}
E\left|\int_{t_1}^{t_2}\sigma_1(s)x(s)dB_1(s)\right|^p &\le [(\sigma_1^2)^u]^p\left[\frac{p(p-1)}{2}\right]^{p/2}(t_2-t_1)^{(p-2)}\int_{t_1}^{t_2}E|x(s)|^p ds\\
&\le [(\sigma_1^2)^u]^p\left[\frac{p(p-1)}{2}\right]^{p/2}(t_2-t_1)^{p/2}L(p)
\end{aligned}$$

Then for $0<t_1<t_2<\infty$, $t_2-t_1\le 1$, $1/p+1/q=1$, we have

$$\begin{aligned}
&E(|x(t_2)-x(t_1)|^p)\\
&= E\left|\int_{t_1}^{t_2}x(s)\left[r_1(s)-b_1(s)x-\frac{c_1(s)y^m}{k+x}\right]ds+\int_{t_1}^{t_2}\sigma_1(s)x(s)dB_1(s)\right|^p\\
&\le 2^{p-1}E\left|\int_{t_1}^{t_2}x(s)\left[r_1(s)-b_1(s)x-\frac{c_1(s)y^m}{k+x}\right]ds\right|^p\\
&\quad +2^{p-1}E\left|\int_{t_1}^{t_2}\sigma_1(s)x(s)dB_1(s)\right|^p\\
&\le 2^{p-1}(t_2-t_1)^{p/q}\left|\int_{t_1}^{t_2}E\left|x(s)\left[r_1(s)-b_1(s)x-\frac{c_1(s)y^m}{k+x}\right]^p ds\right.\right.\\
&\quad +2^{p-1}\left[\frac{p(p-1)}{2}\right]^{p/2}(t_2-t_1)^{p/2}[(\sigma_1^2)^u]^pL(p)\\
&\le 2^{p-1}(t_2-t_1)^{p/q+1}K_2(p)+2^{p-1}\left[\frac{p(p-1)}{2}\right]^{p/2}(t_2-t_1)^{p/2}[(\sigma_1^2)^u]^pL(p)
\end{aligned}$$

$$\leq 2^{p-1}(t_2-t_1)^{p/2}\left[(t_2-t_1)^{p/2}+\left(\frac{p(p-1)}{2}\right)^{p/2}\right]K_3(p)$$

$$\leq 2^{p-1}(t_2-t_1)^{p/2}\left[1+\left(\frac{p(p-1)}{2}\right)^{p/2}\right]K_3(p)$$

where $K_3(p)=max\{K_2(p),[(\sigma_1^2)^u]^p L(p)\}$. By Lemma 6, variable ϑ for every $\vartheta\in(0,(p-2)/2p)$ and each sample path of $x(t)$ is uniformly continuous on $t\geq 0$. Similarly, we can almost verify $y(t)$ is uniformly continuous.

Lemma 8. [33] *Let f be a non-negative function defined on $\mathbb{R}_+$ such that f is integrable and is uniformly continuous. Then* $\lim_{t\to\infty} f(t)=0$.

Theorem 9. *If $b_2^l-c_2^u>0$, then SM is globally asymptotically stable.*

Proof. Define $V(t)=|ln\ x_1(t)-ln\ x_2(t)|+|ln\ y_1(t)-ln\ y_2(t)|$

For $d^+V(t)$ of $V(t)$, applying Itö's formula we obtain

$$d^+V(t)=sgn(x_1-x_2)\left\{\left[\frac{dx_1}{x_1}-\frac{(dx_1)^2}{2x_1^2}\right]-\left[\frac{dx_2}{x_2}-\frac{(dx_2)^2}{2x_2^2}\right]\right\}$$

$$+sgn(y_1-y_2)\left\{\left[\frac{dy_1}{y_1}-\frac{(dy_1)^2}{2y_1^2}\right]-\left[\frac{dy_2}{y_2}-\frac{(dy_2)^2}{2y_2^2}\right]\right\}$$

$$= sgn(x_1-x_2)\left\{-b_1(t)(x_1-x_2)-\frac{c_1(t)(y_1^m-y_2^m)}{k+x}\right\}dt$$

$$+sgn(y_1-y_2)\left\{-b_2(t)(y_1-y_2)+\frac{c_2(t)(x_1y_1^{m-1}-x_2y_2^{m-1})}{k+x}\right\}dt$$

$$\leq \{-b_1(t)|x_1-x_2|+c_1(t)|y_1^m-y_2^m|-b_2(t)|y_1-y_2|$$
$$+c_2(t)|x_1y_1^{m-1}-x_2y_2^{m-1}|\}dt$$

Integrating both sides lead to

$$V(t)\leq V(0)+\int_0^t\left\{-b_1(s)|x_1-x_2|+\frac{c_1(s)|y_1^m-y_2^m|}{k+x}-b_2(s)|y_1-y_2|\right.$$
$$\left.+\frac{c_2(s)|x_1y_1^{m-1}-x_2y_2^{m-1}|}{k+x}\right\}ds$$

thus

$$V(t)+\int_0^t\left\{b_1(s)|x_1-x_2|-\frac{c_1(s)|y_1^m-y_2^m|}{k+x}+b_2(s)|y_1-y_2|-\frac{c_2(s)|x_1y_1^{m-1}-x_2y_2^{m-1}|}{k+x}\right\}ds$$
$$\leq V(0)<\infty$$

It then follows from $V(t)\geq 0$ that

$$|x_1(t)-x_2(t)|\in L^1[0,\quad \infty),\ |y_1(t)-y_2(t)|\in L^1[0,\quad \infty)$$

This completes the proof.

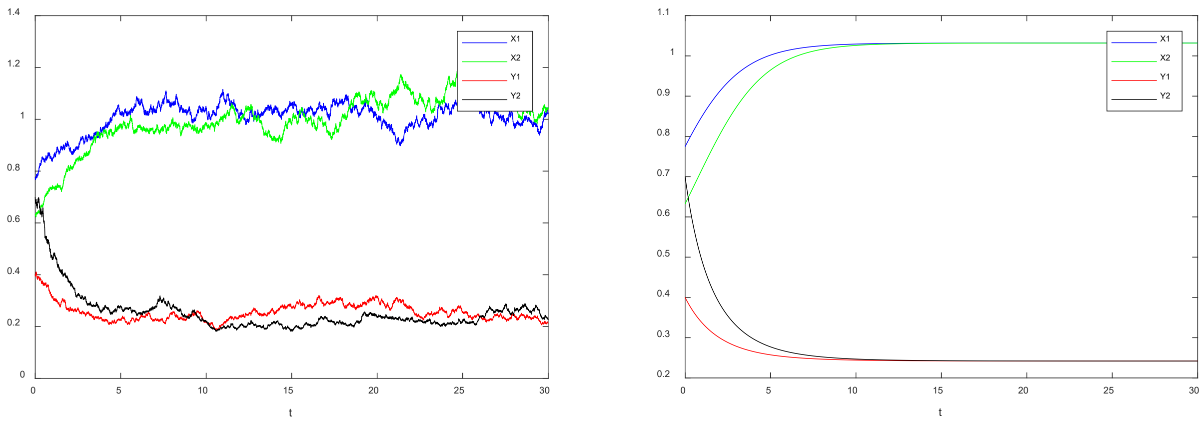
5. Examples and numerical simulations

In this section, we will use Milstein method mentioned in Higham [34] to illustrate our main results, we get the discretization equation of (2) as

$$\begin{cases} x^{(k+1)} = & x^{(k)} + x^{(k)}\left(r_1 - b_1 x^{(k)} - \frac{c_1\left(y^{(k)}\right)^m}{k+x^{(k)}}\right)\Delta t \\ & +\sigma_1 x^{(k)}\sqrt{\Delta t}\xi^{(k)} - \frac{\sigma_1^2}{2}x^{(k)}\left[\left(\xi^{(k)}\right)^2 - 1\right]\Delta t \\ y^{(k+1)} = & y^{(k)} + y^{(k)}\left(-r_2 - b_2 y^{(k)} + \frac{c_2 x^{(k)}\left(y^{(k)}\right)^{m-1}}{k+x^{(k)}}\right)\Delta t \\ & +\sigma_2 y^{(k)}\sqrt{\Delta t}\eta^{(k)} - \frac{\sigma_2^2}{2}y^{(k)}\left[\left(\eta^{(k)}\right)^2 - 1\right]\Delta t \end{cases} \tag{15}$$

where $\xi^{(k)}$ and $\eta^{(k)}$ are the Gaussian random variable which follow $N(0,1)$.

Choosing the suitable parameters, the numerical simulation has been given as follow. Without loss of generality, we choose the parameters $r_1 = 0.65$, $b_1 = 0.55$, $c_1 = 0.27$, $r_2 = 0.15$, $b_2 = 0.55$, $c_2 = 0.27$, $m = 0.5$, $k = 1, \Delta t = 0.0001$. and initial value $(x_1(0), y_1(0)) = (0.6, 0.4)$ and $(x_2(0), y_2(0)) = (0.4, 0.7)$. The numerical simulation results are shown in Figure 1, the vertical axis represents the population sizes and the horizontal axis represents the time *t*.



$\sigma_1 = \sigma_2 = 0.1$ (a) $\sigma_1 = \sigma_2 = 0$ (b)

Figure 1. The solution of system (SM), (a) $\sigma_1 = \sigma_2 = 0.1$; (b) $\sigma_1 = \sigma_2 = 0$.

In Figure 1(a), we choose $\sigma_1 = \sigma_2 = 0.1$. By Theorem 5, SM is global asymptotically stable. In Figure 1(b), we choose $\sigma_1 = \sigma_2 = 0$. Similarly, system (2) is global asymptotically stable. By Figure 1(a), (b), if the positive equilibrium state of the deterministic system is asymptotic stable, then the corresponding positive equilibrium state of the stochastic system will be globally asymptotic stable when the parameter *b2* > *c2* and the effect of noise is sufficiently small.

6. Conclusions and future directions

In this paper, we investigate the existence of globally positive solution, extinction and global asymptomatic stability of the populations to Hassell-Varley model with mutual interference in the form (SM), some interest results are obtained under a few natural and simple conditions. The biosphere environment in which the population located is often highly stochastic, and stochastic noise is one of the reasons leading to the extinction of individuals. The impact of mutual interference and environmental perturbation is important so it is of practical significance to study the non-autonomous model with white noise disturbance. We have added white Gaussian noise in the growth rate parameter of the Predator and prey population and observe that the stochastic system preserves the property of the deterministic system when the noise is sufficiently small while the intensity of environmental disturbance is large enough, it may lead to the extinction of the population. Therefore, the model can be used to study the effects of small disturbances as sunlight and rain on population dynamics, as well as to study large disturbances as hail and greenhouse effect on population dynamics. Furthermore, the results of this paper can be applied to biological communities on biodegradable materials, which provides reference value for the sustainable development of ESG and offers advice on creating suitable biodegradable environment.

In the studies of models with white noise and mutual interference, current research concerning optimal predation strategy is not sufficient. Therefore, In future studies we hope to consider this kind of questions to the crowley-Martin or Beddington-DeAngelis models. The main challenge of how to apply this model to biodegradable communities now is to effectively combine and conjugate the requisite cross disciplinary approaches.

Acknowledgments

The author wishes to acknowledge The Hong Kong Polytechnic University (PolyU).

Conflict of interest

The author declares no conflict of interest.

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