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Research article

A selective evolutionary heterogeneous ensemble algorithm for classifying imbalanced data

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Abstract: Learning from imbalanced data is a challenging task, as with this type of data, most conventional supervised learning algorithms tend to favor the majority class, which has significantly more instances than the other classes. Ensemble learning is a robust solution for addressing the imbalanced classification problem. To construct a successful ensemble classifier, the diversity of base classifiers should receive specific attention. In this paper, we present a novel ensemble learning algorithm called Selective Evolutionary Heterogeneous Ensemble (SEHE), which produces diversity by two ways, as follows: 1) adopting multiple different sampling strategies to generate diverse training subsets and 2) training multiple heterogeneous base classifiers to construct an ensemble. In addition, considering that some low-quality base classifiers may pull down the performance of an ensemble and that it is difficult to estimate the potential of each base classifier directly, we profit from the idea of a selective ensemble to adaptively select base classifiers for constructing an ensemble. In particular, an evolutionary algorithm is adopted to conduct the procedure of adaptive selection in SEHE. The experimental results on 42 imbalanced data sets show that the SEHE is significantly superior to some state-of-the-art ensemble learning algorithms which are specifically designed for addressing the class imbalance problem, indicating its effectiveness and superiority.

Keywords: class imbalance learning; ensemble learning; heterogeneous ensemble; evolutionary computation; Bagging

1. Introduction

Learning from class imbalanced data is a hotspot and challenging issue in the field of machine learning [1,2]. Also, we note that the class imbalance problem exists widely in real-world applications, including biology data processing [3], business data analysis [4], industry fault detection [5], face recognition [6] and crime linkage discovery [7]. In these applications, the users generally focus on the minority class that has less training instances than the other classes. However, the conventional supervised learning algorithms always expect to minimize the overall training errors, and hence it is inevitable to favor the majority class but sacrifice the performance of the minority class.

Over the past two decades, many learning algorithms for addressing the imbalanced classification problem have been proposed. The methods mainly include sampling [8–16], cost-sensitive learning [17–19], threshold-moving [20–22], one-class learning [23] and ensemble learning [24–31] or multiple classifiers system [32–39]. Sampling is a data preprocessing strategy that is used to add the minority instances (oversampling) or remove the majority examples (undersampling) to balance the training set, further satisfying the learning rule of the conventional supervised learning algorithms. Cost-sensitive learning assigns higher penalty weights for minority instances to force the learning algorithms to pay more attention to the minority class. Threshold-moving is a post-processing solution which empirically or adaptively tunes the decision threshold to push the classification hyperplane towards the majority class. One-class learning, which describes the boundary of one class by merely training on the instances belonging to that class, is generally used to address a highly imbalanced problem. Ensemble learning aims to utilize the other class imbalance learning methods, e.g., sampling, cost-sensitive-cost learning or threshold-moving, to combine with the emerging ensemble learning paradigms, e.g., bagging or boosting, for classifying imbalanced data. In comparison to those single models, ensemble learning is expected to greatly improve the classification performance, especially the generalization ability, on class imbalanced data. In view of the merits of ensemble learning, it has been widely studied and adopted in the context of imbalanced data classification.

As we know, to make an ensemble classifier be effective, we should consider two key factors. The first one is that the performance of each base classifier should be not very poor, and the other one is that all base classifiers contained in an ensemble should be diverse as far as possible [40]. For the first factor, most class imbalance ensemble learning algorithms can guarantee it well by assigning excellently distributed training subsets and designating a robust supervised learning algorithm to train each base classifier. Meanwhile, for the second factor, i.e., diversity, many ensemble learning algorithms cannot satisfy it well, as most of them acquire diversity only by perturbing distributions of instances or costs in each training subset. Actually, only when there is a balance between the single performance and the group diversity, the corresponding ensemble model could present the strongest performance [38].

In this study, we focus on how to further improve the diversity of ensemble classifiers and how to find the best tradeoff between single performance and group diversity. For the first problem, we profit from the idea of [39] to combine multiple different sampling algorithms and multiple different types of classifiers to construct a heterogeneous ensemble learning algorithm. As indicated in [39], perturbing instances and classifiers simultaneously can help provide more diversity for an ensemble and meanwhile can improve the quality of the ensemble. As for the second problem, considering that one cannot simply estimate the potential and effect of each base classifier in ensemble by its single performance, an evolutionary genetic algorithm is adopted to adaptively select a high-quality combination of base classifiers for making the decision. According to the *many could be better than all* theory [41], the selective ensemble strategy must be helpful for searching a better tradeoff between single performance and group diversity. We call the proposed ensemble learning algorithm Selective Evolutionary Heterogeneous Ensemble, which can be called SEHE in brief. To verify the effectiveness and feasibility of the proposed SEHE algorithm, we compare it with lots of state-of-the-art class imbalance ensemble learning algorithms on 42 imbalanced data sets acquired in the Keel data repository [42] and UCI machine learning repository [43], and the results show the superiority of SEHE.

The main contribution in this study is combining the ideas of the heterogeneous ensemble and the selective ensemble to improve the performance of class imbalanced learning. To our best knowledge, it is the first time integrating these two ideas for addressing the class imbalanced learning problem. It not only increases diversity production but also helps to find the best tradeoff between single performance and group diversity

The remainder of this paper is organized as follows. Section 2 reviews the related work about class imbalance ensemble learning. Section 3 describes the proposed SEHE algorithm and its framework in detail. In Section 4, the experimental results and analysis are presented. Finally, Section 5 concludes this study and indicates the future research directions.

2. Related work

As mentioned above, lots of different techniques and solutions have been proposed to solve the class imbalance problem during the past two decades, in which the ensemble learning is most successful and popular to address such problem [1,2]. The general idea of class imbalance ensemble learning is to combine a single class imbalanced learning technique, e.g., sampling, cost-sensitive learning or threshold-moving, with one of the Bagging or Boosting ensemble learning paradigms for achieving the aim of improving the generalization capability of the classification model. Here, the adoption of a single technique aims to acquire an approximately unbiased base classifier, while the adoption of an ensemble aims to promote the robustness of the decision model.

Some primitive class imbalance ensemble learning algorithms, including UnderBagging [24], OverBagging [37], SMOTEBagging [37], RUSBoost [33], SMOTEBoost [25], EUSBoost [28], Asymmetric Bagging [36], etc., directly combine a simple undersampling or oversampling algorithm with a Bagging or Boosting ensemble framework. The merit of these algorithms lies in that they are simple and easily implemented. However, they produce diversity only by sampling different data subsets. It is clear that when adopting an undersampling strategy to generate training subsets, the diversity of training subsets could be highlighted, but the performance of each single base learning model could not be safely guaranteed. Meanwhile, oversampling safeguards the performance of a single model but always hurts the group diversity. Yu et al. [38] indicated that when and only when there exists an approximate tradeoff between single performance and group diversity, the quality of the ensemble could be maximized. They integrated several techniques, including feature selection, feature subspace, asymmetric bagging, into an ensemble algorithm called asBagging-FSS for classifying high-dimensional imbalanced biomedicine data. In contrast to Asymmetric Bagging [36], the asBagging-FSS simultaneously enhances the group diversity by inserting disturbances in feature space and the single performance by adding a feature selection procedure. However, the solution might be ineffective on low-dimensional class imbalanced data.

Another group of class imbalance ensemble learning algorithms consider coupling instance costs together with ensemble learning. In fact, the cost of an instance can be naturally combined with the Boosting ensemble learning paradigm. Some well-known algorithms integrating both cost-sensitive learning and Boosting include AdaCost [27], AdaC1 [34], AdaC2 [34] and AdaC3 [34]. These ensemble solutions gradually focus on classifying those difficult instances by the way of iterative feedback. In such fashion, the diversity between two continuous base classifiers can be seen as the variance of cost distributions.

Several algorithms also combine the threshold-moving technique with the Bagging ensemble learning paradigm, such as PT-Bagging [26] and EnSVM-OTHR [20]. Specifically, the PT-Bagging produces diversity only relying on bootstrap strategy which is a natural step in Bagging, while the EnSVM-OTHR adopts both bootstrap and a small random disturbance happening on the adjusted threshold to acquire diverse base classifiers.

To enhance the performance of a single model in an ensemble, some previous studies utilize the ensemble learning model to represent the single model, i.e., the algorithm can be seen as an ensemble of an ensemble. Such cases include EasyEnsemble [30], BalanceCascade [30] and GIR series algorithms [35]. Specifically, in GIR series algorithms, two sampling approaches convert the class imbalance problem into several balanced sub-problems for training the classifiers, and in each sub-problem, several weak classifiers are respectively trained by using Boosting method and finally combined by Bagging. These two-level ensemble algorithms ignore the effect of diversity, and hence the performance improvement is generally restricted.

In recent several years, more and more researchers began to focus on studying how to promote diversity in an ensemble. The strategies of enhancing ensemble diversity include evolutionary strategy [29,31] and heterogeneous ensemble [39]. Roshan and Asadi [31] adopted a popular evolutionary multi-objective optimization algorithm called NSGA-II to synchronously optimize the three following goals: class imbalance ratio IR in each bag, group diversity in ensemble and AUC performance metric. Lim et al. [29] presented a novel cluster-based synthetic oversampling (CSO) algorithm and integrated its five hyper-parameters in each bag together to implement chromosome encoding, finally taking advantage of a genetic algorithm (GA) to determine and produce training subsets. Due to the number of data to generate, the clustering method, the number of clusters, the number of nearest neighbors within each cluster for oversampling and the oversampling method are different in different bags, so there exists a significant diversity among those generated training subsets. Zefrehi and Altincay [39] investigated the possibility of integrating both heterogeneous sampling strategies and classification models to enhance diversity in an ensemble. A significant performance improvement has been observed by adopting heterogeneous classifiers in contrast to using homogeneous classifiers. Meanwhile, when the number of heterogeneous classifiers increases from 1 to 5, the performance of the ensemble model can continuously increase.

According to the reviews above, it is not difficult to observe that there are two major challenges for the class imbalance ensemble learning technique. The first one is how to further improve the group diversity, and the second one is how to achieve an approximately excellent tradeoff between single performance and group diversity. In this study, we wish to simultaneously focus on these two problems.

3. Methods

In this section, we first introduce the heterogeneous class imbalance ensemble learning method, and then on this basis, a selective evolutionary heterogeneous class imbalanced ensemble learning algorithm is described in detail.

3.1. Heterogeneous class imbalance ensemble learning framework

As we know, developing a successful ensemble learning method should simultaneously focus on the two following conditions: the quality of single base learning model and the group diversity among different base learning models [40]. It is not difficult to generate high-quality single classifiers; however, it is a difficult task guaranteeing the generated base classifiers to be diverse enough. The reason lies in that diversity, as an abstract notion, is difficult to accurately measure. In addition, the methods of producing and/or enhancing diversity are generally naïve. In Bagging, it always adopts a bootstrap technique to produce diverse training subsets for acquiring diversity, and hence it guarantees to produce 36.8% diverse training instances between two different subsets in theory [44]. In contrast, the Boosting acquires diverse training subsets by disturbing the distribution of training instances or the weights of training instances [45], which is totally uncontrolled in practical applications. Random forest [46] and random subspace [47] enhance the diversity of training subsets by disturbing both instance space and feature space, but the diversity promotion would be limited by the dimension of the feature space. That is to say, if the training data is low-dimension, we could not significantly improve the diversity of training subsets by these two methods.

All methods mentioned above utilize data-level disturbance to produce diversity. However, it is noteworthy that the classifier level can produce diversity for an ensemble, too. That means the diversity source can be associated with adopting multiple types of classifiers. In general, we call this strategy heterogeneous ensemble. Several previous works have indicated that the diversity among heterogeneous members is significantly higher than that of homogeneous ones [48,49], and the heterogeneous ensemble always performs better than any one single member [50,51]. As for class imbalance learning, an ensemble algorithm combining heterogeneous sampling and heterogeneous classifiers had also been proposed by Zefrehi and Altincay [39] and had acquired improving classification results in comparison to some popular class imbalance ensemble learning algorithms. Profiting from the idea of the HeteroEn algorithm proposed in [39], we propose a heterogeneous ensemble (HE) algorithm for addressing the class imbalance learning problem.

Let Φ denote the class imbalanced training data set, and $S = \{s_1, s_2, \ldots, s_L\}$ and $K = \{k_1, k_2, \ldots, k_N\}$ respectively denote the set including L different sampling algorithms and the set containing N diverse types of classifiers. Then, we can produce $L*N$ diverse combinations of training subsets and classification approaches, where each combination has one of two differences at the data level and classifier level from any other combination at least. In HE, the most popular and simplest decision rule, i.e., majority voting, is adopted to combine decisions of all base classifiers. The flow path of the HE algorithm is described below.

Algorithm 1: HE algorithm

```
Input: A class imbalanced training set $\Phi$, a sampling approaches set $S=\{s_1, s_2, \ldots, s_L\}$, a classification methods set $K = \{k_1, k_2, \ldots, k_N\}$
Output: An ensemble classifier set $E=\{C_{11}, C_{12},\ldots,C_{1N}, C_{21}, C_{22},\ldots,C_{2N}, \ldots\ldots, C_{L1}, C_{L2}\ldots C_{LN}\}$
Procedure:
1.  Initialize $E = \phi$;
2.  for $i = 1: L$
3.      for $j = 1:N$
4.          generate a training subset $\Phi_{ij}$ by adopting the sampling algorithm $s_i$ acting on the training set $\Phi$;
5.          train a base classifier $C_{ij}$ on the training subset $\Phi_{ij}$ by adopting the classification method $k_j$;
6.          put $C_{ij}$ into $E$;
7.      end
8.  end
```

A more intuitive description about the flow path of the HE algorithm can be observed in Figure 1.

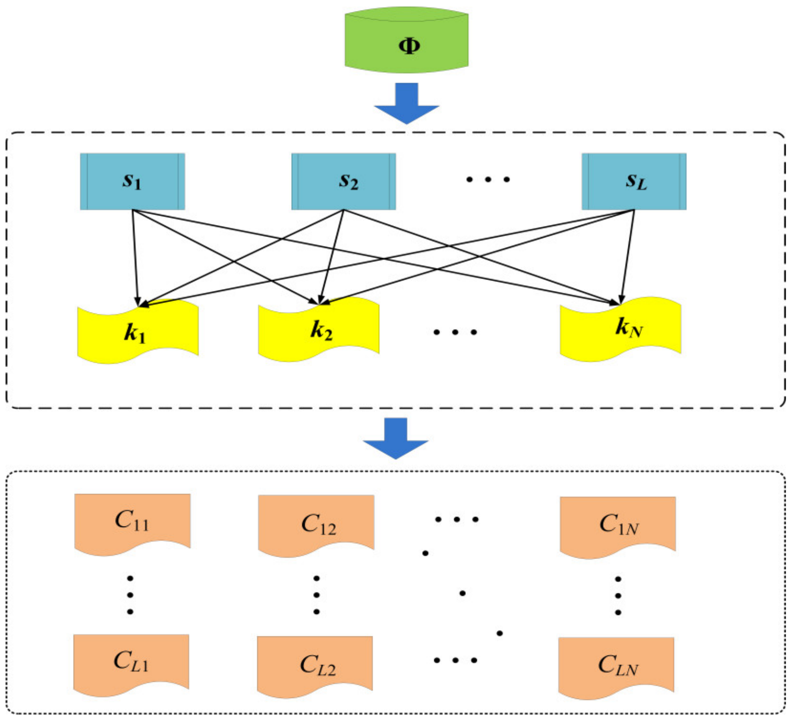

Figure 1. Description for the flow path of the HE algorithm.

It is worthy to note that although both the HeteroEn algorithm proposed in [39] and our HE algorithm adopt the same construction, that is, the combination of diverse sampling approaches and heterogeneous classifiers to construct an ensemble, there is a significant difference between them. The difference lies in that the HeteroEn algorithm integrates the heterogeneous combination into Bagging and Boosting ensemble learning frameworks, while our proposed HE algorithm only produces $L*N$ base classifiers, equal to the number of combinations constructed on the data level and classifier level. The merits of such way are two-fold: 1) It would not generate excessively redundant base classifiers which lack enough diversity with each other in the ensemble; 2) the size of the ensemble could be restricted in a small scale to help reduce time and storage consumption.

To make HE be effective, we also need to consider which sampling approaches and classification algorithms should be put into S and K, respectively. Obviously, to guarantee the quality of a single base classifier, each sampling and each classification algorithm in HE should be robust. Furthermore, to enhance the diversity, both sampling algorithms and classification models should be based on different theoretical rules and inherent assumptions. According to these two criterions, we select seven sampling approaches (including three undersampling algorithms: RUS [9], UFFDFR [12] and MPBU [13]; and four oversampling algorithms: ROS [9], SMOTE [11], MWMOTE [8] and GA-SMOTE [16]) and six classification models to constitute S and K in HE, respectively. These heterogeneous methods are shown in Table 1. In practical applications, the combination of these methods is not constant, and thus the users are encouraged to add and remove any sampling approach or classification model in this list.

Table 1. Sampling approaches and classification methods used in HE.

sampling approach		classification method	
s_1	RUS [9]	k_1	decision tree (CART)
s_2	UFFDFR [12]	k_2	K-nearest neighbors (KNN)
s_3	MPBU [13]	k_3	support vector machine (SVM) with Gaussian kernel
s_4	ROS [9]	k_4	extreme learning machine (ELM) with sigmoid activation function
s_5	SMOTE [11]	k_5	logistic regression (LR)
s_6	MWMOTE [8]	k_6	naïve Bayes classifier (NB)
s_7	GA-SMOTE [16]		

3.2. Selective evolutionary heterogeneous class imbalance ensemble learning algorithm

In HE, although we have safeguarded the performance of the ensemble by adopting robust and diverse sampling approaches and classification methods, there is still a huge potential to further improve the quality of the ensemble. This is because each classification method has its specific inherent assumption, which causes it to perform well on some data but perform poorly on some other data. This phenomenon has been observed by many previous works [8,14,16]. Therefore, it is necessary to remove some low-quality single base classifiers for further enhancing the quality of the ensemble. However, considering that diversity is an abstract notion, it is difficult to directly estimate the effect and potential of any one base classifier in HE, and thus it can be regarded as a sophisticated optimization problem. This is consistent with the idea of selective ensemble [41], i.e., *many could be better than all*.

To solve the optimization problem above, we refer the idea in [41] to conduct a procedure of selective ensemble on HE by using an evolutionary genetic algorithm. The proposed algorithm is called selective evolutionary heterogeneous ensemble, which can be also called SEHE in brief. It is clear that the SEHE is an adaptive algorithm to avoid estimating the quality of each base classifier and the diversity among them.

The reason for selecting the genetic algorithm is that the optimization problem in SEHE is discrete, which is specifically appropriate for use with this optimization technique. There are several key conditions and parameters, which are chromosome encoding strategy, fitness function, population size, crossover factor, mutation factor and the termination condition, that should be seriously considered in a genetic algorithm [52]. In our proposed SEHE algorithm, the chromosome encoding is organized as a vector of length $L*N$ to represent all base classifiers. In each position of the chromosome, it adopts a binary coding, where 1 indicates the corresponding base classifier joining into the final ensemble, while 0 expresses that the corresponding one should be excluded from the final ensemble. Each chromosome denotes an alternative of selective ensemble. As for the fitness function that is used to evaluate the quality of each selective ensemble solution, we consider that it should be directly associated with a performance evaluation metric. Here, we adopt a combination of two popular class imbalance learning performance evaluation metrics, i.e., F1-measure and G-mean, as the fitness function f. In particular, the F1-measure originating from F-measure can be calculated as below:

$$\text{F-measure}=\frac{(1+\beta^2)\times\text{recall}\times\text{precision}}{\beta^2\times\text{recall}+\text{precision}} \tag{1}$$

where β is a parameter denoting the relative importance between precision and recall. In general, β is set to be 1, and then the metric is transformed to be the widely used F1-measure, i.e.,

$$\text{F1-measure}=\frac{2\times\text{recall}\times\text{precision}}{\text{recall}+\text{precision}} \tag{2}$$

G-mean can be calculated as below:

$$\text{G-mean}=\sqrt{\text{TPR}\times\text{TNR}} \tag{3}$$

where TPR and TNR denote the true positive rate and true negative rate, respectively. Thus, the G-mean metric can be regarded as a tradeoff between TPR and TNR. Based on the F1-measure and G-mean, the fitness function f can be calculated as follows.

$$f=\frac{1}{2}\times\text{F1-measure}+\frac{1}{2}\times\text{G-mean} \tag{4}$$

It is obvious that the fitness function f represents an impartial tradeoff between F1-measure and G-mean.

In addition, to keep the tradeoff between the quality of solution and running time consumption, the population size and terminal iterative times are empirically suggested to be designated as 50 and 30, respectively. Furthermore, to accelerate the exploration process, the crossover factor and mutation factor are respectively set to be 1.0 and 0.1 in SEHE. That means all parents conduct crossover operation to generate their offspring, and random 10% of positions of all chromosomes transform their coding values.

The flow path of the proposed evolutionary algorithm is presented as below.

Algorithm 2: Evolutionary algorithm

```
Input: An ensemble classifier set $E$ which includes $L*N$ base classifiers, the population size parameter $N_{pop}$, a crossover factor $N_c$, a mutation factor $N_m$, the terminal iterative times $N_s$
Output: A selective ensemble set $SE$
Procedure:
1. Initialize a binary vector $V_{best}$ with length $L*N$ randomly;
2. Initialize a variance $f_{best}$, which is used to reserve the current best fitness value, as 0;
3. Initialize $N_{pop}$ chromosomes with the length $L*N$ using binary coding randomly;
4. for $i = 1$: $N_s$
5.     transform each chromosome to an ensemble according to its coder mapping on $E$, and estimate its quality by the fitness function, then compare it with $f_{best}$, if there exists a better fitness value, it would be used to replace $f_{best}$, and meanwhile the corresponding chromosome would be used to update $V_{best}$;
6. end
7. decode $V_{best}$ on $E$ to acquire $SE$.
8. end
```

In addition, we note that the fitness function of evolutionary algorithm associates with both F1-measure and G-mean metrics, and then if both the training procedure of HE and the calculation of fitness of evolutionary algorithm are conducted directly on the original training set Φ, the selected combination of base classifiers might be overfit. To solve the problem, we adopt an internal five-fold cross validation strategy in our SEHE algorithm, in which the original training set *E* is averagely divided into five non-overlapping subsets Φ_1, Φ_2, Φ_3, Φ_4 and Φ_5. First, we conduct a five-round iterative operation in which at each round, four subsets are integrated as the internal training set, and the remaining one is used as the internal verifying set. Then, on each internal training set, we conduct the HE algorithm to produce $L*N$ classifiers. That means $5*L*N$ classifiers are generated in total. Next, for each chromosome in SEHE, its fitness is estimated five times, in which for the *i*th time, the corresponding classifiers trained on the *i*th internal training set are used to evaluate the chromosome's fitness on the *i*th internal verifying set. Furthermore, we adopt the average of the five results as the fitness of the chromosome. Finally, to avoid wasting the training instances, it is required to retrain the best combination of classifiers on the original training set Φ after finishing the evolutionary procedure. Based on this rule, the flow path of the SEHE algorithm can be described as follows.

From the flow path of the SEHE algorithm, it is not difficult to observe that it is implemented by a two-level training procedure, where the first level runs in a five-fold cross validation environment to search for the best combination of sampling approaches and classification methods, while the second level takes charge of retraining this combination on the original training set to produce the selective ensemble result. Due to the adoption of a two-level training mode and evolutionary algorithm, it is inevitable that SEHE is a time-consuming algorithm.

Algorithm 3: SEHE algorithm

```
Input: A class imbalance training set $\Phi$, a sampling approaches set $S = \{s_1, s_2, \ldots, s_L\}$, a classification methods set $K = \{k_1, k_2, \ldots, k_N\}$, the population size parameter $N_{pop}$, a crossover factor $N_c$, a mutation factor $N_m$, the terminal iterative times $N_s$
Output: A selective ensemble set $SE$
Procedure:
1. divide randomly $\Phi$ into five average subsets $\Phi_1$, $\Phi_2$, $\Phi_3$, $\Phi_4$ and $\Phi_5$;
2. for $i$ = 1:5
3.         integrate all training subsets except $\Phi_i$ as training set $\Psi_i$;
4.         call HE algorithms to produce an ensemble classifiers set $E_i$ on $\Psi_i$;
5.         produce the output of each base classifier in $E_i$ on each instance in the verifying set $\Phi_i$,
   and record them;
6. end
7. call evolutionary algorithm to get a binary vector indicating which combinations of sampling
   approaches and classification methods should be reserved;
8. adopt the reserved combinations to retrain a selective ensemble set $SE$ on $\Phi$.
```

4. Experiments and analysis

4.1. Experimental data description

We collected 42 imbalanced data sets, including 30 ones from Keel data repository [42] and 12 ones from UCI machine learning repository [43], to verify the effectiveness and superiority of the proposed SEHE algorithm. Specifically, these datasets have different numbers of instances, numbers of features and class imbalance ratios. The information about these data sets is described in Table 2 in detail.

Table 2. Description about the used class imbalanced data sets in this study, where #Instances and #Features denote respectively the number of instances and features contained in the corresponding dataset, and IR denotes the class imbalance ratio, which is calculated by the number of majority instances / the number of minority instances.

Dataset	#Instances	#Features	IR	Dataset	#Instances	#Features	IR
glass1	214	9	1.82	ecoli-0-6-7_vs_5	220	6	10.00
wisconsin	683	9	1.86	led7digit-0-2-4-5-6-7-8-9_vs_1	443	7	10.97
pima	768	8	1.87	ecoli-0-1_vs_5	240	6	11.00
haberman	306	3	2.78	shuttle-6_vs_2-3	230	9	22.00
vehicle1	846	18	2.90	flare-F	1066	11	23.79
new-thyroid1	215	5	5.14	winequality-red-4	1599	11	29.17
yeast3	1484	8	8.10	shuttle-2_vs_5	3316	9	66.67
ecoli3	336	7	8.60	poker-8-9_vs_5	2075	10	82.00
vowel0	988	13	9.98	poker-8_vs_6	1477	10	85.88
yeast-1_vs_7	459	7	14.30	banknote	1372	4	1.25
ecoli4	336	7	15.80	ctgC5	2126	21	28.50
abalone9-18	731	8	16.40	ctgN1vsN3	2126	21	3.50
shuttle-c2-vs-c4	129	9	20.90	ctgN2	2126	21	6.20
yeast4	1484	8	28.10	ctgN3	2126	21	11.10
yeast5	1484	8	32.73	wilt	4839	5	17.50
abalone19	4174	8	129.44	mfeatmor01	2000	6	4.00

Continued on next page

ecoli-0-3-4_vs_5	200	7	9.00	mfeatmor012	2000	6	2.30
ecoli-0-6-7_vs_3-5	222	7	9.09	seeds2v13	210	7	2.00
yeast-0-3-5-9_vs_7-8	506	8	9.12	segment1	2310	19	6.00
yeast-0-3-5-9_vs_7-8	1004	8	9.14	segment12	2310	19	2.50
ecoli-0-1_vs_2-3-5	244	7	9.17	segment123	2310	19	1.30

4.2. Experimental settings

To estimate whether the proposed SEHE algorithm is effective and feasible, it is widely compared with some popular and state-of-the-art class imbalance ensemble learning algorithms, including UnderBagging [24], OverBagging [37], SMOTEBagging [37], RUSBoost [33], SMOTEBoost [25], EUSBoost [28], EnSVM-OTHR [20], EasyEnsemble [30], BalanceCascade [30], GIREnUS [35], 2Obj* [31], ECO-Ensemble [29], HeteroEn [39] and our proposed HE algorithm. Considering that the parameter settings in any algorithm could affect its performance and further influence the impartiality of experiment comparisons, we adopted the recommended parameter settings referred to in each study as default ones. For examples, in [35], we selected GIREnUS, which had been indicated to perform better than its partner, i.e., GIREnOS, while in [31], we adopted the 2Obj* version that outperforms two other algorithm versions. As for the number of base classifiers, all Bagging- and Boosting- based algorithms are uniformly designated as 100 and 20, respectively. In addition, the base classifier types in each ensemble algorithm adopted the default ones used in the corresponding study. For our proposed HE and SEHE algorithm, all sampling algorithms contained in them used the default parameters recommended in the corresponding references, and the classification algorithms used the following parameter settings: The KNN adopted a default parameter $k = 5$, the SVM used a RBF kernel function with $\sigma = 0.01$ and the penalty factor $C = 1$, the ELM adopted the sigmoid activation function with $L = 50$ hidden nodes, and the LR used L2 regularization with a default penalty factor $C = 1$. To guarantee the impartiality of experiments, if some other ensemble learning algorithms have adopted one or several sampling or classification algorithms that are the same as the ones used in our proposed algorithm, they have been designated the same parameter settings.

Both F1-measure and G-mean metrics which have been used to evaluate the fitness in our proposed SEHE algorithm are also selected as the performance evaluation metric for comparing various ensemble algorithms. In particular, both F1-metric and G-mean have been shown as the most popular metrics to evaluate the quality of a class imbalance learning algorithm.

Finally, to impartially compare the quality of all comparative algorithms, 10 random runs' external five-fold cross validation is conducted for each algorithm, and the average performance of the corresponding 50 validated results is used to represent its final result.

4.3. Results and discussion

Tables 3 and 4 present the comparative results of 15 ensemble learning algorithms in terms of F1-measure and G-mean metrics, respectively. Specifically, on each data set, the best result has been highlighted in boldface.

From the results in Tables 3 and 4, it is not difficult to observe that our proposed SEHE algorithm outperforms all other competitors, as it produces best F1-measure result on 24 data sets and best G-mean result on 22 data sets, which are much more than the number of the best results acquired by adopting other class imbalance ensemble learning algorithms. We note that no matter if it is on those data sets with low class imbalance ratio or on highly imbalanced data sets, our proposed SEHE algorithm can produce good classification results. We believe that it associates with the two following reasons: 1) The combination of heterogeneous sampling methods and heterogeneous classification models provides enough diversity for constructing ensemble learning model, and 2) the evolutionary selection procedure in SEHE algorithm adaptively meets the demands of data distribution well. Therefore, we can say that the proposed SEHE algorithm is a robust, flexible and self-adaptive class imbalance learning algorithm.

Another interesting conclusion that can be observed from the results in Tables 3 and 4 is that several state-of-the-art algorithms proposed in recent several years, including EnSVM-OTHR, GIREnUS, 2Obj*, ECO-Ensemble and HeteroEn, generally perform better than those previously popular algorithms. Of course, in contrast to those simple ensembles, these state-of-the-art algorithms often have more sophisticated constructions, and hence their training procedures are more complex, too.

We also observe that the proposed SEHE algorithm performs obviously better than the HE algorithm on almost all data sets, showing the correctness of adopting selective ensemble strategy, again. We believe that in comparison to aggregating all base classifiers, adaptively selecting some of them would be helpful for searching for a better tradeoff between single performance and group diversity.

Furthermore, we are curious which combinations between sampling approaches and classification methods perform better in HE. On each data set, we scanned the average performance of F1-measure and G-mean throughout all combinations, and the best combination was recorded based on the feedback of 10 random runs' external five-fold cross validation. Further, the top 10 combinations are presented in Table 5. One could make the following conclusion from the results in Table 5: Combining one of those sophisticated sampling approaches, e.g., MWMOTE, MPBU, UFFDFR or GA-SMOTE, and one of those robust classification methods, e.g., Support Vector Machine or Extreme Learning Machine, it is easier to produce excellent classification performance.

Table 3. F1-measure results of 15 comparative algorithms, where on each data set, the best result has been highlighted in bold.

Dataset	Under	Over	SMOTE	RUSBoost	SMOTE	EUSBoost	EnSVM	Easy	Balance	GIREnUS	2Obj*	ECO	HeteroEn	HE	SEHE
	Bagging	Bagging	Bagging		Boost		-OTHR	Ensemble	Cascade			-Ensemble			
glass1	0.6179	0.6418	0.6566	0.6531	0.6479	0.6710	0.7022	0.6937	0.6819	0.7074	0.7196	0.7338	0.7292	0.7231	**0.7479**
wisconsin	0.9496	0.9501	0.9532	0.9458	0.9526	0.9531	0.9368	0.9444	0.9496	0.9537	0.9486	0.9529	0.9501	0.9529	**0.9685**
pima	0.6074	0.6291	0.6399	0.6518	0.6436	0.6457	0.6706	0.6619	0.6538	0.6560	0.6489	**0.6917**	0.6742	0.6721	0.6898
haberman	0.4310	0.4036	0.4117	0.4425	0.4328	0.4679	0.4658	0.4631	0.4254	0.4738	0.4699	0.4852	0.4798	0.4881	**0.5122**
vehicle1	0.6829	0.6473	0.6428	0.6716	0.6632	0.6811	0.6793	0.6627	0.6519	0.6521	**0.7044**	0.6759	0.6792	0.6721	0.7022
new-thyroid1	0.9275	0.9310	0.9288	0.9219	0.9468	0.9390	0.9426	0.9410	0.9272	**0.9587**	0.9446	0.9510	0.9451	0.9439	0.9517
yeast3	0.6866	0.6922	0.7014	0.7250	0.7033	0.7315	0.7446	0.7361	0.7410	0.7398	0.7316	0.7425	0.7355	0.7398	**0.7461**
ecoli3	0.5632	0.5791	0.6318	0.6007	0.6101	0.6049	0.6251	0.5735	0.5816	0.5835	0.5961	0.6032	0.6397	0.6381	**0.6441**
vowel0	0.9489	0.9876	0.9941	0.9774	0.9986	0.9826	0.9978	0.9598	0.9689	0.9936	0.9972	**1.0000**	0.9983	0.9973	**1.0000**
yeast-1_vs_7	0.2816	0.2459	0.4109	0.3828	**0.4350**	0.3744	0.3610	0.2969	0.2871	0.3980	0.3792	0.4160	0.3967	0.4159	0.4287
ecoli4	0.6491	0.7387	0.7519	0.6393	0.7275	0.7080	0.7441	0.6146	0.7269	0.7484	0.7175	0.7290	0.7448	**0.7641**	0.7548
abalone9-18	0.3353	0.3798	0.4314	0.3722	0.3918	0.3816	0.4277	0.2934	0.2987	0.3970	0.4052	0.4329	0.4320	0.4388	**0.4530**
shuttle-c2-vs-c4	0.9964	0.9981	**1.0000**	0.9981	**1.0000**	0.9993	0.9976	0.9952	0.9971	0.9994	**1.0000**	0.9972	0.9991	**1.0000**	**1.0000**
yeast4	0.2798	0.3362	0.3517	0.3394	0.3707	0.3239	**0.3949**	0.2875	0.2987	0.3708	0.3646	0.3802	0.3569	0.3694	0.3743
yeast5	0.5967	0.6652	0.6743	0.5969	0.6837	0.6345	0.6874	0.5350	0.5989	**0.6947**	0.6528	0.6777	0.6931	0.6895	0.6933
abalone19	0.0372	0.0991	0.0869	0.0417	0.1132	0.0507	0.0962	0.0487	0.0396	0.1133	0.1521	0.1312	0.1479	0.1455	**0.1752**
ecoli-0-3-4_vs_5	0.7109	0.7983	0.8109	0.7329	0.8086	0.7751	0.8108	0.6954	0.7374	0.7930	0.8028	0.8039	0.8184	0.7999	**0.8481**
ecoli-0-6-7_vs_3-5	0.5819	0.6343	0.6420	0.5927	0.6588	0.6447	0.6736	0.5210	0.6001	0.6542	0.6571	**0.6848**	0.6352	0.6572	0.6791
yeast-0-3-5-9_vs_7-8	0.2933	0.3231	0.3490	0.2815	0.3140	0.3392	0.3431	0.3089	0.3076	0.3418	0.3396	0.3210	0.3317	0.3396	**0.3991**
yeast-0-3-5-9_vs_7-8	0.6956	0.7722	0.7630	0.6717	0.7448	0.7134	0.7520	0.6837	0.7474	0.7541	0.7498	0.7526	0.7279	0.7579	0.7890
ecoli-0-1_vs_2-3-5	0.5549	0.6627	0.6788	0.5610	0.6472	0.6081	0.6654	0.6344	0.6290	0.6708	0.6587	0.6842	0.6738	0.6688	**0.7223**
ecoli-0-6-7_vs_5	0.5345	0.6860	0.7438	0.6633	0.7297	**0.7515**	0.7337	0.6125	0.7120	0.6995	0.7498	0.7355	0.7378	0.7394	0.7511
led7digit-0-2-4-5-6-7-8-9_vs_1	0.6627	0.7413	0.7427	0.6580	0.7336	0.7189	0.7355	0.6528	0.7066	0.7209	**0.7515**	0.7320	0.7458	0.7468	**0.7515**
ecoli-0-1_vs_5	0.7440	0.7752	0.7836	0.7506	**0.7878**	0.7441	0.7642	0.6784	0.7043	0.7531	0.7752	0.7807	0.7735	0.7742	0.7835
shuttle-6_vs_2-3	0.8929	0.7304	0.7543	0.8891	0.7984	0.8894	0.8906	0.8814	0.8839	0.8539	0.8901	0.8872	0.8756	0.8795	**0.9194**
flare-F	0.1909	0.2542	0.2798	0.2590	0.2711	0.2433	0.2677	0.2516	0.2702	0.2710	0.2688	0.2854	0.2860	0.2991	**0.3796**

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winequality-red-4	0.1277	0.1792	0.2098	0.1744	0.1811	0.1795	0.1811	0.1342	0.0970	0.1979	0.1594	0.1945	0.1986	0.2333	**0.2691**
shuttle-2_vs_5	**1.0000**	0.9951	**1.0000**	0.9729	0.9982	**1.0000**	**1.0000**	0.9684	0.9797	0.9968	0.9981	**1.0000**	0.9952	0.9954	**1.0000**
poker-8-9_vs_5	0.0372	0.0661	0.0971	0.0422	0.1157	0.0679	0.1011	0.0385	0.0406	0.0920	0.1008	0.1106	0.1033	0.1176	**0.1795**
poker-8_vs_6	0.2906	0.6790	0.6929	0.2409	0.6508	0.4411	0.4076	0.1919	0.2143	0.5080	0.6945	0.6991	0.6876	0.6734	**0.7394**
banknote	0.9828	0.9816	0.9830	**0.9901**	0.9749	0.9826	0.9724	0.9849	0.9751	0.9810	0.9886	0.9815	0.9843	0.9799	0.9896
ctgC5	0.2820	0.3144	0.3462	0.2517	0.3727	0.2342	0.3109	0.3519	0.3442	0.3031	0.3572	0.3648	0.3528	0.3591	**0.4327**
ctgN1vsN3	0.6542	0.7310	0.7479	0.6968	0.7444	**0.7991**	0.7580	0.7374	0.7511	0.7332	0.7546	0.7590	0.7376	0.7521	0.7986
ctgN2	0.6378	0.7436	0.7387	0.7115	0.7220	0.7159	0.7034	0.7519	0.7226	0.7258	0.7196	0.7364	0.7419	**0.7644**	0.7428
ctgN3	0.7476	**0.8095**	0.7964	0.7555	0.7858	0.7336	0.7686	0.7587	0.7462	0.7680	0.7904	0.7725	0.7531	0.7549	0.8073
wilt	0.5625	0.6991	0.7415	0.6423	0.7404	0.6985	0.7840	0.7379	0.6581	0.8132	0.7310	0.7796	0.7529	0.7522	**0.8461**
mfeatmor01	0.9089	0.9735	0.9818	0.9527	0.9886	0.9532	0.9897	0.9436	0.9528	0.9789	**0.9901**	0.9592	0.9301	0.9459	0.9711
mfeatmor012	0.8546	0.8369	0.8226	0.8990	0.8467	0.8971	0.8858	0.8442	0.8325	0.8790	0.8687	0.8884	0.8628	0.8844	**0.9122**
seeds2v13	0.9776	0.9664	0.9685	0.9702	0.9573	0.9696	0.9668	0.9794	**0.9801**	0.9642	0.9715	0.9733	0.9598	0.9696	0.9798
segment1	0.8806	0.9428	0.9335	0.9010	**0.9552**	0.9336	0.9344	0.9522	0.9121	0.9258	0.9306	0.9357	0.9281	0.9194	0.9479
segment12	0.9562	0.9314	0.9210	0.9668	0.9506	0.9632	0.9545	0.9619	0.9454	0.9508	0.9612	0.9675	0.9449	0.9491	**0.9677**
segment123	0.9986	0.9847	0.9859	0.9987	0.9856	0.9981	**1.0000**	0.9992	0.9992	0.9983	0.9869	**1.0000**	0.9906	0.9984	**1.0000**

Table 4. G-mean results of 15 comparative algorithms, where on each data set, the best result has been highlighted in bold.

Dataset	Under Bagging	Over Bagging	SMOTE Bagging	RUSBoost	SMOTE Boost	EUSBoost	EnSVM -OTHR	Easy Ensemble	Balance Cascade	GIREnUS	2Obj*	ECO -Ensemble	HeteroEn	HE	SEHE
glass1	0.7211	0.7202	0.7198	0.7454	0.7219	0.7418	0.7529	0.7622	0.7619	0.7588	0.7641	0.7832	0.7699	0.7701	**0.8109**
wisconsin	0.9796	0.9701	0.9732	0.9769	0.9680	0.9745	0.9761	0.9687	0.9594	0.9732	**0.9858**	0.9761	0.9752	0.9747	0.9830
pima	0.6733	0.6566	0.6721	0.6831	0.6702	0.6801	0.7673	0.7328	0.7285	0.7271	0.7166	0.7330	0.7575	0.7520	**0.7793**
haberman	0.6010	0.5994	0.6004	0.6002	0.6118	0.6233	0.6219	0.6258	0.6146	0.6375	0.5998	**0.6425**	0.6333	0.6379	0.6412
vehicle1	0.7858	0.7741	0.7759	0.7901	0.7722	0.7910	0.7853	0.7766	0.7692	0.7581	0.7834	0.8022	0.7931	0.8018	**0.8356**
new-thyroid1	0.9792	0.9801	0.9847	0.9818	0.9836	0.9790	0.9829	0.9715	0.9718	0.9844	**0.9901**	0.9774	0.9855	0.9882	0.9899
yeast3	0.8993	0.8899	0.8706	0.8944	0.9052	0.8911	**0.9237**	0.8901	0.8877	0.9018	0.9134	0.9075	0.8969	0.8993	0.9105
ecoli3	0.8580	0.8681	0.8706	0.8478	0.8644	0.8590	0.8647	0.8760	0.8791	0.8632	0.8661	0.8744	0.8709	0.8776	**0.8924**
vowel0	0.9978	0.9963	0.9981	0.9972	0.9950	0.9981	**1.0000**	0.9935	0.9962	**1.0000**	0.9971	**1.0000**	0.9994	0.9991	**1.0000**
yeast-1_vs_7	0.5678	0.6882	0.7109	0.5945	0.7208	0.7044	0.6786	0.7430	0.7238	0.7196	0.7200	0.6959	0.7006	0.7159	**0.7455**
ecoli4	0.9196	0.8998	0.9106	0.9042	0.9119	0.9208	0.9198	0.9304	0.9196	0.9211	0.9307	0.9256	**0.9388**	0.9306	0.9329
abalone9-18	0.6495	0.6784	0.6513	0.6690	0.6755	0.6821	0.7492	0.7326	0.7419	0.7430	0.7374	0.7513	0.7333	0.7420	**0.7764**
shuttle-c2-vs-c4	0.9971	0.9984	**1.0000**	0.9975	**1.0000**	0.9995	0.9977	0.9941	0.9986	0.9997	**1.0000**	0.9982	0.9995	**1.0000**	**1.0000**
yeast4	0.8139	0.8057	0.8236	0.8294	0.8175	0.8277	0.8310	0.8194	0.8196	0.8079	0.8221	0.8174	**0.8395**	0.8358	0.8379
yeast5	0.9535	0.9457	0.9674	0.9521	0.9443	0.9576	0.9495	0.9478	0.9406	0.9528	0.9663	**0.9712**	0.9591	0.9544	0.9642
abalone19	0.3447	0.7240	0.7461	0.3290	0.6973	0.7276	0.5969	0.7085	0.4919	0.6738	0.7092	0.7329	0.7170	0.7181	**0.7702**
ecoli-0-3-4_vs_5	0.8781	0.8857	0.8796	0.8981	0.8848	0.8932	0.8921	0.8830	0.8829	**0.8990**	0.8998	0.8776	0.8939	0.8901	0.8957
ecoli-0-6-7_vs_3-5	0.8306	0.7998	0.8081	0.8272	0.8198	0.8255	0.8551	0.8030	0.8212	0.8187	0.8190	0.8464	0.8211	0.8186	**0.8539**
yeast-0-3-5-9_vs_7-8	0.6997	0.5687	0.5868	0.6739	0.5710	0.6845	0.6861	0.6932	0.6911	0.6954	0.6878	0.6759	0.6922	0.6878	**0.7154**
yeast-0-3-5-9_vs_7-8	0.8874	0.9028	0.9012	0.8966	0.8998	0.8775	0.8964	0.8981	0.8942	0.8911	0.8886	0.8932	0.8897	0.8933	0.8959
ecoli-0-1_vs_2-3-5	0.8377	0.8119	0.8088	0.8394	0.8299	0.8301	0.8442	0.8527	0.8504	0.8425	0.8501	0.8499	0.8455	0.8479	**0.8697**
ecoli-0-6-7_vs_5	0.8000	0.8642	0.8591	0.8320	0.8529	0.8617	**0.8832**	0.8653	0.8724	0.8695	0.8610	0.8736	0.8575	0.8531	0.8792
led7digit-0-2-4-5-6-7-8-9_vs_1	0.8534	0.8631	0.8670	0.8498	0.8692	0.8506	0.8772	0.8650	0.8762	0.8696	0.8779	0.8699	**0.8858**	0.8744	0.8821
ecoli-0-1_vs_5	0.8850	0.8952	0.8926	0.8901	0.8974	0.8833	0.8906	0.8864	0.8732	0.8830	0.8868	0.8953	0.8862	0.8934	**0.9195**
shuttle-6_vs_2-3	0.8586	0.7760	0.7793	0.8228	0.7896	0.8499	0.8934	0.8820	0.8959	0.8876	0.9030	**0.9123**	0.8896	0.8867	0.9095
flare-F	0.8218	0.7477	0.7652	0.8333	0.7501	0.8198	0.6744	0.8219	0.7946	0.8079	0.8139	**0.8448**	0.7989	0.8016	0.8337
winequality-red-4	0.5919	0.6732	0.6673	0.5792	0.6545	0.5808	0.6273	0.5898	0.5170	0.6914	0.6632	0.6500	0.6705	0.6829	**0.7173**

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shuttle-2_vs_5	**1.0000**	0.9982	**1.0000**	0.9711	0.9976	**1.0000**	**1.0000**	0.9774	0.9823	0.9954	0.9987	**1.0000**	0.9965	0.9988	**1.0000**
poker-8-9_vs_5	0.3317	0.6053	0.6149	0.3140	0.5782	0.3438	0.5562	0.3710	0.3929	0.4618	0.4491	0.5312	0.4700	0.4854	**0.6157**
poker-8_vs_6	0.5252	0.7410	0.7299	0.5740	0.7318	0.5237	0.7098	0.6536	0.6790	0.7492	0.7008	0.7129	0.7190	0.7152	**0.7733**
banknote	0.9876	0.9810	0.9809	**0.9913**	0.9804	0.9821	0.9856	0.9830	0.9777	0.9822	0.9896	0.9811	0.9834	0.9791	0.9850
ctgC5	0.5134	0.6658	0.6233	0.5730	0.6419	0.5099	0.5858	0.5736	0.6297	0.6419	0.6322	0.6785	0.6412	0.6430	**0.6859**
ctgN1vsN3	0.8297	0.8170	0.8254	0.8137	0.8350	0.8445	0.8454	0.8361	0.8299	0.8317	**0.8471**	0.8360	0.8338	0.8415	0.8447
ctgN2	0.8052	0.7969	0.8011	0.7994	0.8138	0.7876	0.8452	0.8092	0.7956	0.8004	0.8123	0.8223	0.7964	0.8558	**0.8770**
ctgN3	0.8211	0.8732	0.8536	0.8341	**0.8840**	0.8109	0.8444	0.8277	0.8309	0.8650	0.8233	0.8766	0.8427	0.8501	0.8766
wilt	0.7691	0.7038	0.6852	0.7222	0.7001	0.7559	0.7981	0.7226	0.7028	**0.8006**	0.7304	0.7578	0.7449	0.7611	0.7999
mfeatmor01	0.9521	0.9815	0.9874	0.9662	**0.9901**	0.9633	0.9884	0.9601	0.9732	0.9807	0.9874	0.9663	0.9406	0.9535	0.9715
mfeatmor012	0.8998	0.8227	0.8376	0.9015	0.8778	0.9192	0.9135	0.9004	0.8858	0.9030	0.8564	0.9002	0.8568	0.8774	**0.9323**
seeds2v13	0.9855	0.9724	0.9789	0.9802	0.9693	0.9891	0.9868	0.9780	**0.9876**	0.9905	0.9872	0.9901	0.9701	0.9743	0.9856
segment1	0.9423	0.9637	0.9716	0.9332	0.9630	0.9236	0.9579	0.9411	0.9619	0.9690	**0.9848**	0.9597	0.9481	0.9577	0.9700
segment12	0.9701	0.9539	0.9611	0.9678	0.9555	0.9690	0.9772	0.9571	0.9580	0.9747	0.9689	0.9677	0.9704	0.9708	**0.9882**
segment123	0.9991	0.9897	0.9879	0.9994	0.9834	0.9990	**1.0000**	0.9995	0.9994	0.9989	0.9881	**1.0000**	0.9935	0.9991	**1.0000**

Table 5. Top 10 combinations between single sampling approach and classification method in HE.

Rank	Combination	# data sets performing best
1	MPBU + Support Vector Machine	7
2	UFFDFR + Logistic Regression	5
3	MWMOTE + Extreme Learning Machine	4
4	MWMOTE + Support Vector Machine	3
5	UFFDER + CART	3
6	GA-SMOTE + Support Vector Machine	2
7	MPBU + KNN	2
8	MWMOTE + Support Vector Machine	2
9	GA-SMOTE + Extreme Learning Machine	2
10	MPBU + Extreme Learning Machine	2

Furthermore, we investigated an important issue, that is, how many base classifiers have been selected to organize the ensemble in our proposed SEHE algorithm? Figure 2 shows the average number of selected base classifiers in SEHE based on 50 independent runs on each data set. In Figure 2, we observed that on most data sets, SEHE integrates no more than 20 base classifiers to make final decision but produces robust results, which verifies the assumption that SEHE could adaptively find the trade-off between single quality and group diversity, again.

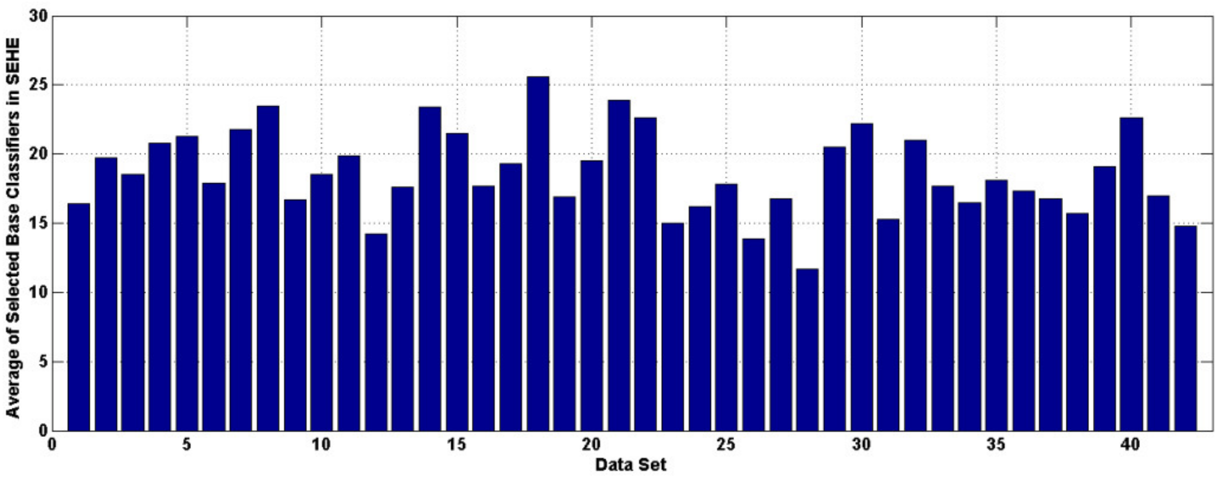
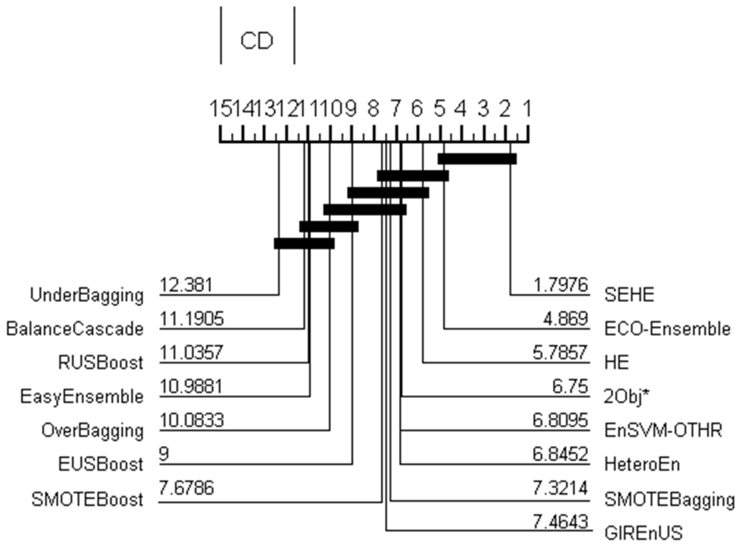


Figure 2. Average number of selected base classifiers in SEHE on each data set, where the number on the horizontal axis corresponds to the order of the data set in Table 2.

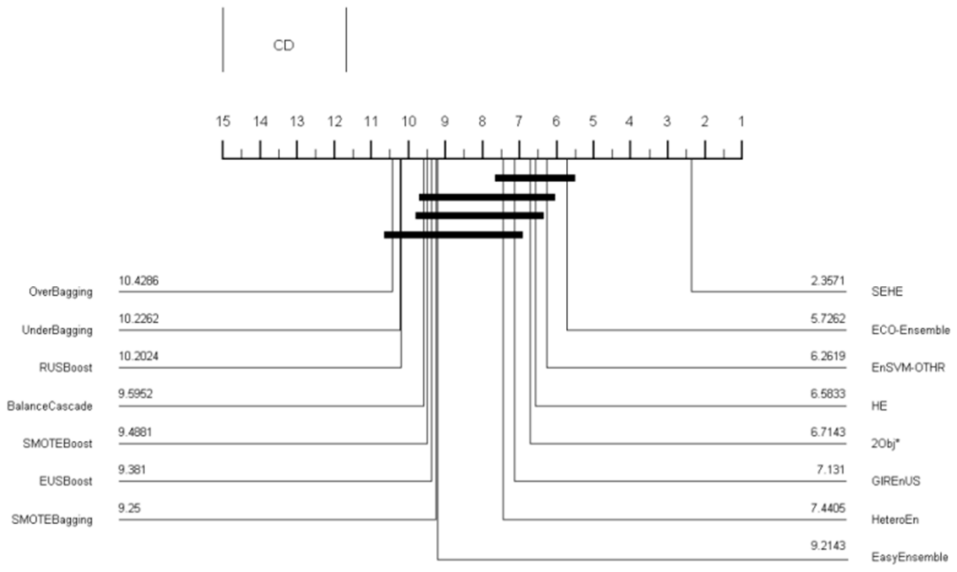
4.4. Significance analysis

Next, the Nemenyi test [53,54] is used to observe whether there exist some actual differences between the proposed SEHE algorithm and the other comparative algorithms in statistics. Specifically, the critical difference (CD) metric is used to show the differences among various algorithms. Figure 3 shows the CD diagram at a standard level of significance, α = 0.05, where the average ranking of each algorithm is marked along the axis (higher rankings to the left). In a CD diagram, if a group of algorithms are not significantly different, then these algorithms will be connected by a thick line.

In Figure 3, we observed that our proposed SEHE algorithm has acquired the lowest average rankings 1.7976 on F1-measure and 2.3571 on G-mean, which show that it is the best one among all comparative algorithms on both metrics. At a standard level of significance, α = 0.05, SEHE significantly outperforms its competitors except ECO-Ensemble on F1-measure metric. Therefore, we have to say that on the F1-metric at least, in contrast to the ECO-Ensemble algorithm, our proposed SEHE algorithm has not presented a significant superiority. In addition, another conclusion can be drawn from the results in Figure 3, i.e., several state-of-the-art algorithms proposed in several recent years generally perform better than those previously popular algorithms.



(a) CD diagram on F1-measure

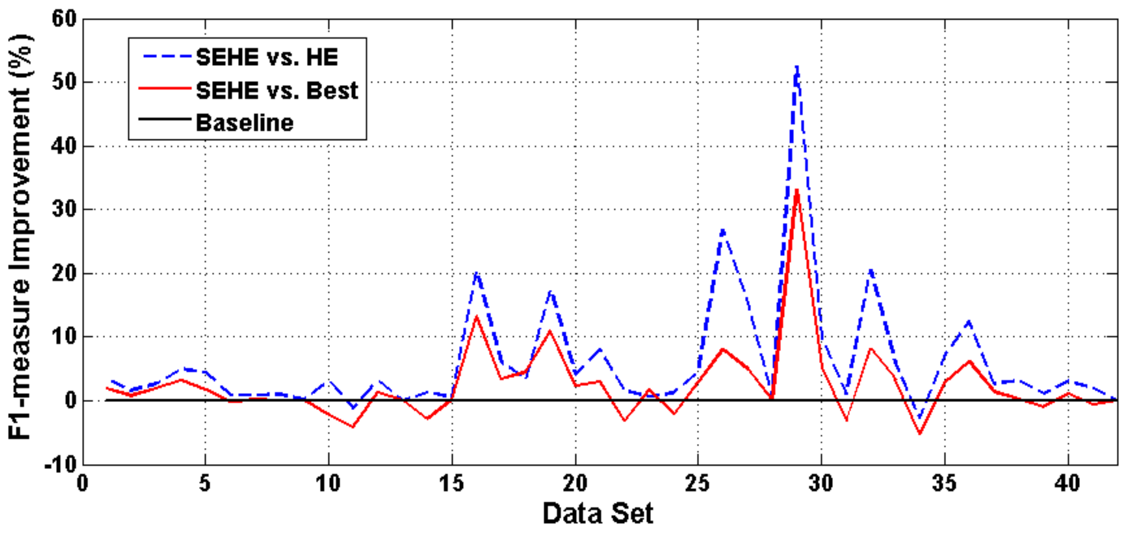


(b) CD diagram on G-mean

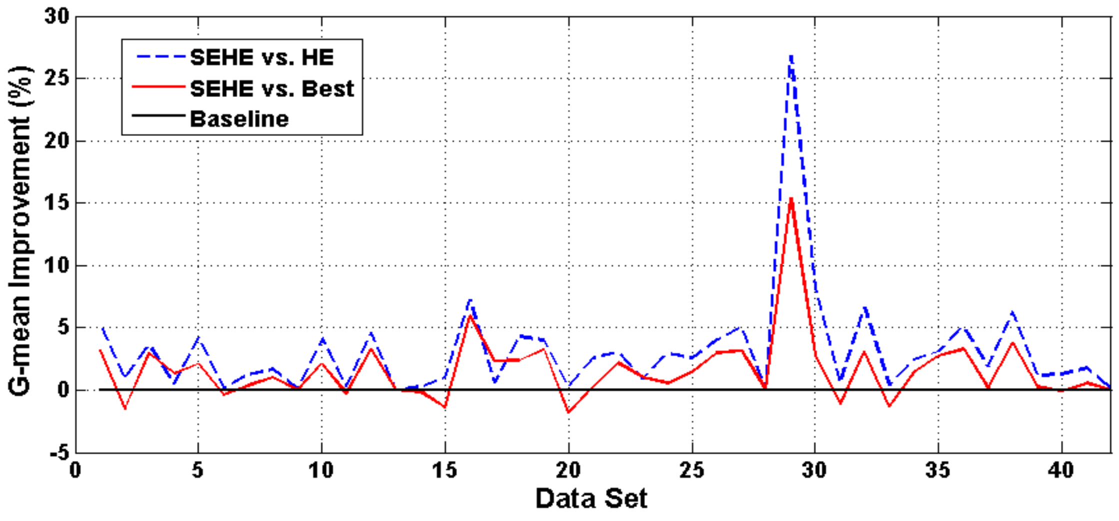
Figure 3. CD diagrams of various comparative algorithms at a standard level of significance, α = 0.05.

4.5. Performance improvement with two baselines

To make clear how much improvement the proposed SEHE algorithm has acquired, we also compared it with the two following baselines: One is the performance yielded by the best base classifier in HE, and the other one is the performance produced by HE. The percentages of performance improvement compared with the two baselines in terms of F1-measure and G-mean metrics are respectively presented in Figure 4.



(a) F1-measure improvement



(b) G-mean improvement

Figure 4. Percentage of performance improvement of SEHE in comparison with two baselines: HE and the best base classifier in HE, on each data set.

Figure 4 shows that on most data sets, the proposed SEHE algorithms can improve classification performance in comparison with two baselines to some extent, which indicates the necessity of adopting a selective ensemble, again. Meanwhile, we observed an interesting phenomenon, that is, on most data sets, the HE algorithm performs poorer than its best member (base classifier). This is acceptable, because in a practical application, it cannot foresee the best combination of sampling method and classification model. In contrast to directly integrating heterogeneous methods, the model selection is obviously a more complex task.

4.6. Ablation study

Finally, we designed two groups of ablation experiments to make clear the effect of adopting diverse sampling approaches and heterogeneous classifiers in SEHE, respectively. We tuned the ablation at the data level and classifier level, respectively. That is to say, for each test, we only reserve either one data-level or one classifier-level method in SEHE, whereas the other level remains stable. For example, if the ablation associates with SMOTE, then all other sampling algorithms would be removed, but the classifier level still maintains six heterogeneous classifiers. Considering that the ablation reduces the ensemble size and further causes an impartial comparison, we utilized the bootstrap technique before sampling to guarantee that a total number of 42 diverse base classifiers can be generated. A pairwise *t*-test at 5% significance level was adopted to compare SEHE with its ablation algorithm, and further the numbers of wins/ties/losses throughout all 42 data sets were recorded, respectively.

Table 6. Statistical comparison between SEHE and its data-level ablation algorithm based on a pairwise *t*-test at 5% significance level.

Ablation comparison	F1-measure			G-mean		
	Win	Tie	Loss	Win	Tie	Loss
SEHE vs. SEHE_RUS	15	26	1	14	25	3
SEHE vs. SEHE_UFFDFR	13	28	1	14	28	0
SEHE vs. SEHE_MPBU	11	29	2	15	26	1
SEHE vs. SEHE_ROS	10	31	1	12	28	2
SEHE vs. SEHE_SMOTE	7	33	2	9	29	4
SEHE vs. SEHE_MWMOTE	6	34	2	10	29	3
SEHE vs. SEHE_GA-SMOTE	6	32	4	8	33	1

Table 7. Statistical comparison between SEHE and its classifier-level ablation algorithm based on a pairwise *t*-test at 5% significance level.

Ablation comparison	F1-measure			G-mean		
	Win	Tie	Loss	Win	Tie	Loss
SEHE vs. SEHE_CART	17	25	0	19	22	1
SEHE vs. SEHE_KNN	20	21	1	23	19	0
SEHE vs. SEHE_SVM	15	22	5	17	22	3
SEHE vs. SEHE_ELM	13	26	3	15	24	3
SEHE vs. SEHE_LR	18	24	0	21	20	1
SEHE vs. SEHE_NB	25	17	0	23	18	1

Tables 6 and 7 provide the results of ablation experiments at data level and classifier level, respectively. Specifically, each ablation algorithm was still conducted by 10 random runs' external five-fold cross validation. The results show that reducing diversity at data level influences less the classification performance than tuning the classifier level. In other words, the adoption of heterogeneous classifiers contributes significantly more for the success of our SEHE algorithm, which illustrates that the ensemble of heterogeneous classifiers is helpful for reducing the bias of a single classifier and enhancing group diversity of ensemble, again.

5. Conclusions

In this study, a selective evolutionary heterogeneous ensemble algorithm has been proposed to address the class imbalance learning problem. First, a group of different sampling approaches run on the original training data to acquire multiple diverse balanced training subsets. Then, each subset is trained on multiple different types of classifiers, further generating some diverse base classifiers. Next, a selective ensemble procedure relying on the evolutionary algorithm is conducted to search for the best combination of base classifiers. By lots of comparative experiments, the proposed SEHE algorithm shows a superior performance in comparison to some state-of-the-art class imbalance ensemble learning algorithms.

From the experimental results, several major conclusions could be safely drawn as follows:

1) Disturbing the data level and the classifier level both help to enhance ensemble diversity, but the disturbance at the classifier level contributes significantly more.

2) Many are really better than all: Adaptively selecting a few base classifiers always performs better than aggregating all.

3) The best base classifier often outperforms the HE, but in most cases, it performs poorer than the SEHE.

4) SEHE is a flexible, robust and competitive algorithm to solve the class imbalance learning problem in real-world applications.

In future work, the issue of how to decrease the time-complexity of the SEHE algorithm will be investigated. The performance variation of SEHE by extending the library of sampling and classification methods will be observed in future study, too. Additionally, we state that although in this study, the SEHE has not been verified on a multi-class imbalanced learning problem, it is indeed compatible for both binary and multi-class classification problems when and only when all embedded sampling algorithms are simultaneously appropriate for these two problems. Therefore, extending the current SEHE version to the multi-class classification problem for estimating its effectiveness and superiority will also be studied in future work.

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Conflict of interest

The authors declare there is no conflict of interest.

References

1. P. Branco, L. Torgo, R. P. Ribeiro, A survey of predictive modeling on imbalanced domains, *ACM Comput. Surv.*, **49** (2016), 1–50. https://doi.org/10.1145/2907070
2. H. Guo, Y. Li, J. Shang, M. Gu, Y. Huang, B. Gong, Learning from class-imbalance data: Review of methods and applications, *Expert Syst. Appl.*, **73** (2017), 220–239. https://doi.org/10.1016/j.eswa.2016.12.035
3. Y. Qian, S. Ye, Y. Zhang, J. Zhang, SUMO-Forest: A Cascade Forest based method for the prediction of SUMOylation sites on imbalanced data, *Gene*, **741** (2020), 144536. https://doi.org/10.1016/j.gene.2020.144536
4. P. D. Mahajan, A. Maurya, A. Megahed, A. Elwany, R. Strong, J. Blomberg, Optimizing predictive precision in imbalanced datasets for actionable revenue change prediction, *Eur. J. Oper. Res.*, **285** (2020), 1095–1113. https://doi.org/10.1016/j.ejor.2020.02.036
5. G. Chen, Z. Ge, SVM-tree and SVM-forest algorithms for imbalanced fault classification in industrial processes, *IFAC J. Syst. Control*, **8** (2019), 100052. https://doi.org/10.1016/j.ifacsc.2019.100052
6. P. Wang, F. Su, Z. Zhao, Y. Guo, Y. Zhao, B. Zhuang, Deep class-skewed learning for face recognition, *Neurocomputing*, **363** (2019), 35–45. https://doi.org/10.1016/j.neucom.2019.04.085
7. Y. S. Li, H. Chi, X. Y. Shao, M. L. Qi, B. G. Xu, A novel random forest approach for imbalance problem in crime linkage, *Knowledge-Based Syst.*, **195** (2020), 105738. https://doi.org/10.1016/j.knosys.2020.105738
8. S. Barua, M. M. Islam, X. Yao, K. Murase, MWMOTE-majority weighted minority oversampling technique for imbalanced data set learning, *IEEE Trans. Knowl. Data Eng.*, **26** (2012), 405–425. https://doi.org/10.1109/TKDE.2012.232
9. G. E. A. P. A. Batista, R. C. Prati, M. C. Monard, A study of the behavior of several methods for balancing machine learning training data, *ACM SIGKDD Explorations Newsl.*, **6** (2004), 20–29. https://doi.org/10.1145/1007730.1007735
10. K. E. Bennin, J. Keung, P. Phannachitta, A. Monden, S. Mensah, MAHAKIL: diversity based oversampling approach to alleviate the class imbalance issue in software defect prediction, *IEEE Trans. Software Eng.*, **44** (2017), 534–550. https://doi.org/10.1109/TSE.2017.2731766
11. N. V. Chawla, K. W. Bowyer, L. O. Hall, W. P. Kegelmeyer, SMOTE: Synthetic minority over-sampling technique, *J. Artif. Intell. Res.*, **16** (2002), 321–357. https://doi.org/10.1613/jair.953
12. M. Zheng, T. Li, X. Zheng, Q. Yu, C. Chen, D. Zhou, et al., UFFDFR: Undersampling framework with denoising, fuzzy c-means clustering, and representative sample selection for imbalanced data classsification, *Inf. Sci.*, **576** (2021), 658–680. https://doi.org/10.1016/j.ins.2021.07.053
13. G. Ahn, Y. J. Park, S. Hur, A membership probability-based undersampling algorithm for imbalanced data, *J. Classif.*, **38** (2021), 2–15. https://doi.org/10.1007/s00357-019-09359-9
14. M. Li, A. Xiong, L. Wang, S. Deng, J. Ye, ACO Resampling: Enhancing the performance of oversampling methods for class imbalance classification, *Knowledge-Based Syst.*, **196** (2020), 105818. https://doi.org/10.1016/j.knosys.2020.105818
15. T. Pan, J. Zhao, W. Wu, J. Yang, Learning imbalanced datasets based on SMOTE and Gaussian distribution, *Inf. Sci.*, **512** (2020), 1214–1233. https://doi.org/10.1016/j.ins.2019.10.048

16. T. Zhang, Y. Li, X. Wang, Gaussian prior based adaptive synthetic sampling with non-linear sample space for imbalanced learning, *Knowledge-Based Syst.*, **191** (2020), 105231. https://doi.org/10.1016/j.knosys.2019.105231
17. R. Batuwita, V. Palade, FSVM-CIL: Fuzzy support vector machines for class imbalance learning, *IEEE Trans. Fuzzy Syst.*, **18** (2010), 558–571. https://doi.org/10.1109/TFUZZ.2010.2042721
18. C. L. Castro, A. P. Braga, Novel cost-sensitive approach to improve the multilayer perceptron performance on imbalanced data, *IEEE Trans. Neural Networks Learn. Syst.*, **24** (2013), 888–899. https://doi.org/10.1109/TNNLS.2013.2246188
19. S. Datta, S. Das, Near-Bayesian Support Vector Machines for imbalanced data classification with equal or unequal misclassification costs, *Neural Networks*, **70** (2015), 39–52. https://doi.org/10.1016/j.neunet.2015.06.005
20. H. Yu, C. Mu, C. Sun, W. Yang, X. Yang, X. Zuo, Support vector machine-based optimized decision threshold adjustment strategy for classifying imbalanced data, *Knowledge-Based Syst.*, **76** (2015), 67–78. https://doi.org/10.1016/j.knosys.2014.12.007
21. H. Yu, C. Sun, X. Yang, W. Yang, J. Shen, Y. Qi, ODOC-ELM: Optimal decision outputs compensation-based extreme learning machine for classifying imbalanced data, *Knowledge-Based Syst.*, **92** (2016), 55–70. https://doi.org/10.1016/j.knosys.2015.10.012
22. Z. H. Zhou, X. Y. Liu, Training cost-sensitive neural networks with methods addressing the class imbalance problem, *IEEE Trans. Knowl. Data Eng.*, **18** (2006), 63–77. https://doi.org/10.1109/TKDE.2006.17
23. D. Devi, S. K. Biswas, B. Purkayastha, Learning in presence of class imbalance and class overlapping by using one-class SVM and undersampling technique, *Connect. Sci.*, **31** (2019), 105–142. https://doi.org/10.1080/09540091.2018.1560394
24. R. Barandela, R. M. Valdovinos, J. S. Sanches, New applications of ensemble of classifiers, *Pattern Anal. Appl.*, **6** (2003), 245–256. https://doi.org/10.1007/s10044-003-0192-z
25. N. V. Chawla, A. Lazarevic, L. O. Hall, K. W. Bowyer, SMOTEBoost: Improving prediction of the minority class in Boosting, in *Knowledge Discovery in Databases: PKDD 2003*, (2003), 107–119. https://doi.org/10.1007/978-3-540-39804-2_12
26. G. Collell, D. Prelec, K. R. Patil, A simple plug-in bagging ensemble based on threshold-moving for classifying binary and multiclass imbalanced data, *Neurocomputing*, **275** (2018), 330–340. https://doi.org/10.1016/j.neucom.2017.08.035
27. W. Fan, S. J. Stolfo, J. Zhang, P. K. Chan, AdaCost: Misclassification cost-sensitive boosting, in *International Conference of Machine Learning*, (1999), 97–105. Available from: http://ids.cs.columbia.edu/sites/default/files/Adacost_Imbalanced_classes.pdf.
28. M. Galar, A. Fernandez, E. Barrenechea, F. Herrera, EUSBoost: Enhancing ensembles for highly imbalanced data-sets by eevolutionary undersampling, *Pattern Recognit.*, **46** (2013), 3460–3471. https://doi.org/10.1016/j.patcog.2013.05.006
29. P. Lim, C. K. Goh, K. C. Tan, Evolutionary Cluster-Based Synthetic Oversampling Ensemble (ECO-Ensemble) for imbalance learning, *IEEE Trans. Cybern.*, **47** (2016), 2850–2861. https://doi.org/10.1109/TCYB.2016.2579658
30. X. Y. Liu, J. Wu, Z. H. Zhou, Exploratory undersampling for class-imbalance learning, *IEEE Trans. Syst. Man Cybern. Part B Cybern.*, **39** (2008), 539–550. https://doi.org/10.1109/TSMCB.2008.2007853

31. S. E. Roshan, S. Asadi, Improvement of Bagging performance for classification of imbalanceed datasets using evolutionary multi-objective optimization, *Eng. Appl. Artif. Intell.*, **87** (2020), 103319. https://doi.org/10.1016/j.engappai.2019.103319
32. A. Roy, R. M. O. Cruz, R. Sabourin, G. D. C. Cavalcanti, A study on combining dynamic selection and data preprocessing for imbalance learning, *Neurocomputing*, **286** (2018), 179–192. https://doi.org/10.1016/j.neucom.2018.01.060
33. C. Seiffert, T. M. Khoshgoftaar, J. V. Hulse, A. Napolitano, RUSBoost: A hybrid approach to alleviating class imbalance, *IEEE Trans. Syst. Man Cybern. Part A Syst. Humans*, **40** (2009), 185–197. https://doi.org/10.1109/TSMCA.2009.2029559
34. Y. Sun, M. S. Kamel, A. K. C. Wong, Y. Wang, Cost-sensitive boosting for classification of imbalanced data, *Pattern Recognit.*, **40** (2007), 3358–3378. https://doi.org/10.1016/j.patcog.2007.04.009
35. B. Tang, H. He, GIR-based ensemble sampling approaches for imbalanced learning, *Pattern Recognit.*, **71** (2017), 306–319. https://doi.org/10.1016/j.patcog.2017.06.019
36. D. Tao, X. Tang, X. Li, X. Wu, Asymmetric bagging and random subspace for support vector machines-based relevance feedback in image retrieval, *IEEE Trans. Pattern Anal. Mach. Intell.*, **28** (2006), 1088–1099. https://doi.org/10.1109/TPAMI.2006.134
37. S. Wang, X. Yao, Diversity analysis on imbalanced data sets by using ensemble models, in *2009 IEEE Symposium on Computational Intelligence and Data Mining*, (2009), 324–331. https://doi.org/10.1109/CIDM.2009.4938667
38. H. Yu, J. Ni, An improved ensemble learning method for classifying high-dimensional and imbalanced biomedicine data, *IEEE/ACM Trans. Comput. Biol. Bioinf.*, **11** (2014), 657–666. https://doi.org/10.1109/TCBB.2014.2306838
39. H. G. Zefrehi, H. Altincay, Imbalance learning using heterogeneous ensembles, *Expert Syst. Appl.*, **142** (2020), 113005. https://doi.org/10.1016/j.eswa.2019.113005
40. J. F. Díez-Pastor, J. J. Rodríguez, C. I. García-Osorio, L. I. Kuncheva, Diversity techniques improve the performance of the best imbalance learning ensembles, *Inf. Sci.*, **325** (2015), 98–117. https://doi.org/10.1016/j.ins.2015.07.025
41. Z. H. Zhou, J. Wu, W. Tang, Ensembling neural networks: many could be better than all, *Artif. Intell.*, **137** (2002), 239–263. https://doi.org/10.1016/S0004-3702(02)00190-X
42. I. Triguero, S. González, J. M. Moyano, S. García, J. Alcalá-Fdez, J. Luengo, et al., KEEL 3.0: An open source software for multi-stage analysis in data mining, *Int. J. Comput. Intell. Syst.*, **10** (2017), 1238–1249. https://doi.org/10.2991/ijcis.10.1.82
43. C. Blake, E. Keogh, C. J. Merz, UCI repository of machine learning databases, 1998. Available from: https://cir.nii.ac.jp/crid/1572543025422228096#citations_container.
44. L. Breiman, Bagging predictors, *Mach. Learn.*, **24** (1996), 123–140. https://doi.org/10.1007/BF00058655
45. R. E. Schapire, A brief introduction to boosting, in *Proceedings of the Sixteenth International Joint Conference on Artificial Intelligence*, (1999), 1401–1406. Available from: https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=fa329f834e834108ccdc536db85ce368fee227ce.
46. L. Breiman, Random forests, *Mach. Learn.*, **45** (2001), 5–32. https://doi.org/10.1023/A:1010933404324
47. T. K. Ho, The random subspace method for constructing decision forests, *IEEE Trans. Pattern Anal. Mach. Intell.*, **20** (1998), 832–844. https://doi.org/10.1109/34.709601

48. S. A. Gilpin, D. M. Dunlavy, Relationships between accuracy and diversity in heterogeneous ensemble classifiers, 2009.
49. K. W. Hsu, J. Srivastava, Diversity in combinations of heterogeneous classifiers, in *PAKDD 2009: Advances in Knowledge Discovery and Data Mining*, (2009), 923–932. https://doi.org/10.1007/978-3-642-01307-2_97
50. R. M. O. Cruz, R. Sabourin, G. D. C. Cavalcanti, Dynamic classifier selection: Recent advances and perspectives, *Inf. Fusion*, **41** (2018), 195–216. https://doi.org/10.1016/j.inffus.2017.09.010
51. É. N. de Souza, S. Matwin, Extending adaboost to iteratively vary its base classifiers, in *Canadian AI 2011: Advances in Artificial Intelligence*, (2011), 384–389. https://doi.org/10.1007/978-3-642-21043-3_46
52. D. Whitley, A genetic algorithm tutorial, *Stat. Comput.*, **4** (1994), 65–85. https://doi.org/10.1007/BF00175354
53. J. Demsar, Statistical comparisons of classifiers over multiple data sets, *J. Mach. Learn. Res.*, **7** (2006), 1–30. Available from: https://www.jmlr.org/papers/volume7/demsar06a/demsar06a.pdf.
54. S. García, A. Fernández, J. Luengo, F. Herrera, Advanced nonparametric tests for multiple comparisons in the design of experiments in computational intelligence and data mining: Experimental analysis of power, *Inf. Sci.*, **180** (2010), 2044–2064. https://doi.org/10.1016/j.ins.2009.12.010