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Theory article

Modeling and analysis of carbon emission-absorption model associated with urbanization process of China

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Abstract: The excessive emission of greenhouse gases leads to abnormal climate change. Under this background, China puts forward the dual carbon target. In this paper, we use the analytic hierarchy process to determine the important influencing factor of carbon emissions. Next, we establish a delayed differential equation model of carbon emission-absorption under the influence of China's urbanization. We analyze the existence and stability of the positive equilibrium. Finally, we determine the ranges of parameters and study the impact of urbanization on China's dual carbon target through numerical simulations. The numerical simulation also shows that the system may have globally asymptotically stable equilibrium. Through the simulation results, we conclude whether the dual carbon target of China can be achieved by the scheduled time and give some suggestions that could be taken to achieve this target. The projected results provide some guidance for policy adjustments and also have practical significance in protecting the ecological environment.

Keywords: dual carbon target; urbanization; delayed differential equation; stability analysis; global stability; analytic hierarchy process

1. Introduction

In recent years, abnormal climate change has increasingly become the focus of attention. This abnormal change is mainly caused by global warming, including frequent low- temperature rain and snow weather, warm winter phenomenon, and regional increases in average precipitation, etc. One of the most important causes of global warming is the excessive human emission of greenhouse gases. In 2021, a climate report issued by the U.S. National Environmental Protection Agency (EPA) recognized for the first time that the climate change caused by human factors has exceeded natural factors [1]. To actively address the issue of abnormal climate change, the world has reached a consensus on reducing carbon emissions. According to the Energy and Climate Intelligence Unit (ECIU), 132 countries and regions around the world have set carbon neutrality targets to actively respond to abnormal climate change.

Under this background, China has also put forward the dual carbon target [2]. Which is "striving to achieve carbon peak by 2030 and achieve carbon neutrality by 2060", which helps the world build a community of shared future for mankind. Among them, carbon peak and carbon neutrality are noted as "dual carbon". Carbon peak means the process that carbon dioxide emission reaches the maximum value in a certain time interval, and then decreases steadily [3]. Carbon neutrality means that man-made carbon dioxide absorption offset man-made carbon dioxide emissions globally during the specified period, which is also known as "net zero CO_2 emissions" [3]. It is a net value, which is different from the "zero emission" of carbon dioxide.

There are many factors affecting carbon emissions, including economic development, population, industrial structure, energy structure, energy efficiency, etc [4–8]. Urbanization has multi-dimensional meanings, mainly including population migration, economic development, spatial expansion, and improvement of living standards [9]. Moreover, in the process of urbanization, the labor force is transferred from the primary industry to the secondary industry and the tertiary industry. So the urbanization process is always accompanied by the above factors which affect carbon emissions. According to Emissions Gap Report 2020 issued by the United Nations Environment Programme (https://www.unep.org/emissions-gap-report-2020, *Emissions Gap Report 2020*, accessed on 11 November 2022), around two-thirds of global emissions are associated with private households. The report also shows the difference in carbon emissions between poor residents and rich residents, that is rich residents' carbon emissions are about twice that of poor residents. So cities are still the "main battlefield" of carbon emissions. It is of great significance to study the impact of urbanization on the dual carbon target of China.

There are many researches focus on the urbanization process, carbon emissions, and carbon absorption. Some research conclusions can summarize their relationship, the environmental Kuznets curve is one of the classic conclusions. Scholars' research on the delayed effect of urbanization and carbon emission usually uses econometrics methods, such as the 2SLS model, semi-parametric estimation model, Clustering model, etc [10–13]. However, there are relatively few studies that choose the differential equation model to analyze. Differential equations can describe many problems, especially after introducing time-delayed. Many problems can be described by delay differential equations, including biology, wireless electronics, ecology, and so on [14–18]. As we all know, there are relatively few studies that comprehensively discuss carbon emissions, carbon absorption, and urbanization. The majority of studies discuss carbon emissions and carbon absorption separately. So we intend to analyze them comprehensively. We consider that the emission and absorption of carbon are dynamic processes that change with time. Zhou et. al [19] found that there is a certain time delay in urban carbon emissions. The per capita carbon emissions in the previous stage have a positive influence on the per capita carbon emissions in the current stage. We consider that the carbon emissions brought by the urbanization process are related to both the current state and the past state. Thus we model a delayed differential equation, which can describe the dynamics phenomena of this content more accurately.

The rest of the paper is organized as follows. In Section 2, we use the analytic hierarchy process to analyze the influence weight of urbanization. In Section 3, we establish the delayed differential equation about carbon emissions and absorption, which considers the impact of the urbanization process of China. In Section 4, we analyze the existence and stability of the positive equilibrium of the model. We also analyze the dynamic properties near the equilibrium. In Section 5, we determine the range of parameters and carry out numerical simulations. Finally, the conclusions and suggestions are given in Section 6.

2. Analytic hierarchy process

Analytic hierarchy process (AHP) is a multi-objective decision analysis method that combines qualitative analysis with quantitative analysis [20]. The main idea is to divide the influencing factors of the problem into several criteria and compare them with each other. By calculating the weight of the criteria, we can judge the importance of each factor. We build a hierarchical structure model which is shown in Figure 1. The model shows several key factors affecting carbon emissions, which come from the classic model for predicting carbon emissions, such as the STIRPAT model, kaya identity, etc [21, 22].

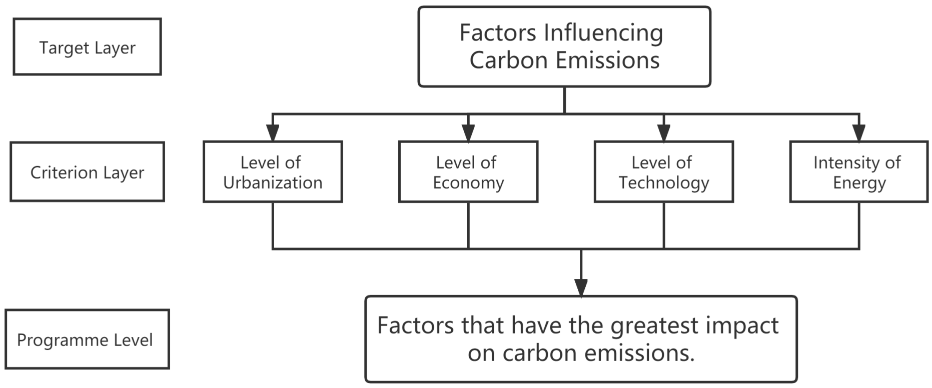


Figure 1. Hierarchical structure model of factors affecting carbon emissions.

First, we construct a judgment matrix. The value of r_{ij} is obtained according to the importance of target i compared with target j. We use the 1–9 scale method as the measurement standard. Through the current research status [23–27], we find that the four factors on the criterion layer have positive feedback on each other, then we compare the strength of the positive feedback of any two factors based on the current research. Thus, we can get the following judgment matrix R:

$$R = \begin{pmatrix} 1 & 3 & 1 & 3 \\ \frac{1}{3} & 1 & \frac{1}{3} & 2 \\ 1 & 3 & 1 & 2 \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}.$$

Second, we determine the weight of each criterion. We calculate the eigenvector of matrix R. The component of the eigenvector is the weight of the influencing factor [28]. After normalization by column, we can get the weight of each criterion as

$$\omega = (0.376 \quad 0.156 \quad 0.345 \quad 0.123)^T.$$

The level of urbanization has the greatest weighting at 0.376. Thus we can preliminarily judge that the level of urbanization has a greater impact on carbon emissions than the other three factors.

Finally, we conduct a consistency test [29, 30]. This step is to ensure that the judgment matrix is reasonable and there is no contradiction in the comparison process. We need the following data :

1) The maximum eigenvalue of the matrix R is $\lambda_{max} = \frac{1}{4}\sum_{i=1}^{4}\frac{(R\omega)_i}{\omega_i} = 4.1154$.

2) Consistency indicators $CI = \frac{\lambda_{max}-n}{n-1} = 0.0385$.

3) The random consistency index of the judgment matrix R is $RI = 0.89$.

Since $CR = \frac{CI}{RI} = 0.0433 < 0.1$, we can accept the consistency of the judgment matrix R [30]. Through the above analysis, we conclude that the impact of the level of urbanization on carbon emissions is relatively greater.

3. Mathematical modeling

In Section 2, we use the AHP method to analyze the impact of urbanization levels on carbon emissions. In this section, we establish a delayed differential equation model. Carbon emissions and carbon absorption are continuously changing quantities. Carbon peak means that the derivative of the equation describing the change of carbon emission is zero, and carbon neutrality means that the amount of carbon emissions is equal to the amount of carbon absorption. There is a time delay between urban internal construction (mainly ecological construction) and the urbanization process [19]. Therefore, in this paper, we model a delayed differential equation model, which is based on a logistic interspecific reciprocity model [31]. The absorption and emission of carbon are affected by the maximum environmental capacity, which follows the form of logistic. The model takes into account that carbon absorption is facilitated by carbon emissions, then inhibited by the maximum value of carbon absorption in a certain space; carbon emission is inhibited by environmental holding capacity, then facilitated by the urbanization process. But the promotion effect of carbon emissions on carbon absorption is not considered. The per capita carbon emissions in cities have a time delay effect [32], that is, the influence of the urbanization process on the growth rate of carbon emissions is related to both the carbon emissions $y(t)$ at the current time t and the carbon emissions $y(t-\tau)$ at a certain time $t-\tau$ in the past. We denote this impact as $v(y(t), y(t-\tau))$. To sum up, the rate of carbon emissions and carbon absorption follows the model:

$$\begin{cases} \dfrac{dx}{dt} = r_1 x(1 - \dfrac{x}{N_1} + \alpha\dfrac{y}{N_2}), \\ \dfrac{dy}{dt} = r_2 y(1 - \dfrac{y}{N_2}) + v(y(t), y(t-\tau)). \end{cases} \tag{3.1}$$

In model (3.1), x (billion tons) means total carbon absorption at time t, y (billion tons) means total carbon emissions at time t, r_1 means the annual growth rate of carbon absorption, r_2 means the annual growth rate of carbon emissions, N_1 (billion tons) means the maximum value of carbon absorption in a certain space, N_2 (billion tons) means the maximum environmental capacity of carbon emissions, α means absorption coefficient of carbon emissions corresponding to carbon absorption, and $v(y(t), y(t-\tau))$ means the impact of the urbanization development process on the rate of carbon emissions. Thus, we can describe a dynamic system in which carbon emission and carbon absorption change together in a certain time and space. We record the process that they change together as carbon emission-absorption.

Next, we discuss the specific expression of $v(y(t), y(t-\tau))$. According to the Northam curve in Figure 2, the overall trend of urbanization is "slow-fast-slow", and the shape of the curve is "S"

[33]. According to the data of the National Bureau of Statistics (http://www.stats.gov.cn/, *2021 China Statistical Yearbook*, accessed on 11 November 2022), the overall urbanization rate of China is 63%, which is in the upper middle of the curve. Therefore, the development trend curve of urbanization in the future (the second half of the "S" curve) can be regarded as the left half of the quadratic function with downward opening. In other words, the derivative form is a linear function with a negative slope.

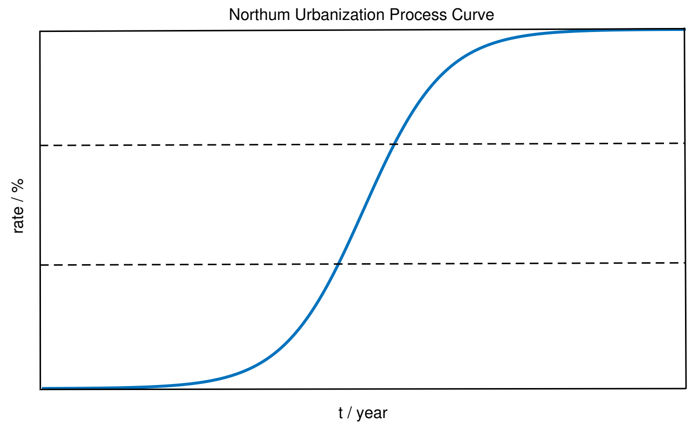


Figure 2. Northam curve showing the process of urbanization.

With the increase of the amount of carbon emissions brought by cities, the speed of urbanization will slow down. Let a be the influence coefficient of the speed of urbanization on the growth rate of carbon emissions, we can assume that the growth rate of carbon emissions caused by urbanization is a linearly decreasing function. The form is shown in the equation $v_0(y(t), y(t-\tau)) = a - sy(t)$, where $y(t)$ is the amount of carbon emissions and s is the change rate. With the increase of urbanization, carbon emissions will eventually tend to a stable constant M. At this time, the growth rate of carbon emissions is 0. So we can get the equation $a - sM = 0$. Put $a - sM = 0$ into equation $v_0(y(t), y(t-\tau))$, and we get the expression for a, M and $y(t)$. However, the speed of urban internal construction often lags behind the urbanization process. The current urban carbon emissions are promoted by the urban carbon emissions at a certain time in the past. We use time delay to represent this effect. Replace $y(t)$ with $y(t-\tau)$, we get the specific expression of $v(y(t), y(t-\tau))$ is

$$v(y(t), y(t-\tau)) = a(1 - \frac{y(t-\tau)}{M}). \tag{3.2}$$

Among them, y (billion ton) means total carbon emissions at time t, a means the influence coefficient of the speed of urbanization on the growth rate of carbon emissions, and M (billion ton) means the maximum value of urban carbon emissions.

Based on the above analysis and model (3.1), we get the following carbon emission-absorption model under the influence of urbanization,

$$\begin{cases} \dfrac{dx}{dt} = r_1 x(1 - \dfrac{x}{N_1} + \alpha \dfrac{y}{N_2}), \\ \dfrac{dy}{dt} = r_2 y(1 - \dfrac{y}{N_2}) + a(1 - \dfrac{y(t-\tau)}{M}). \end{cases} \tag{3.3}$$

The meaning of the parameters in the model (3.3) is shown in Table 1. In the next parts, we mainly discuss the model (3.3).

Table 1. Descriptions of variables and parameters in the model (3.3).

Parameter	Description	unit
x	Total carbon absorption at time t	billion tons
y	Total carbon emissions at time t	billion tons
r_1	The annual growth rate of carbon absorption	-
N_1	The maximum value of carbon absorption in a certain space	billion tons
α	The absorption coefficient of carbon emissions corresponding to carbon absorption	-
r_2	The annual growth rate of carbon emissions	-
N_2	The maximum environmental capacity of carbon emissions	billion tons
a	The influence coefficient of the speed of urbanization on the growth of carbon emissions	-
M	The maximum value of urban carbon emissions	billion tons
τ	Time of urban ecological process adapting to population migration	years

4. Model analysis

In this section, we analyze the existence and stability of the equilibrium of the model (3.3). We give the following assumption,

$$(H)\ \ \frac{a}{M} - r_2 - \sqrt{(r_2 - \frac{a}{M})^2 + 4a\frac{r_2}{N_2}} < 0.$$

When assumption (H) holds, model (3.3) has only one positive equilibrium $E^* = (x^*, y^*)$, where

$$x^* = N_1 + \alpha\frac{N_1}{N_2}y^*, \ \ y^* = \frac{N_2}{2}(-\frac{a}{Mr_2} + \frac{\sqrt{N_2(Mr_2 - a)^2 + 4ar_2M^2}}{r_2M\sqrt{N_2}} + 1).$$

4.1. Analysis of ordinary differential equation

When $\tau = 0$, model (3.3) becomes

$$\begin{cases} \dfrac{dx}{dt} = r_1x(1 - \dfrac{x}{N_1} + \alpha\dfrac{y}{N_2}), \\ \dfrac{dy}{dt} = r_2y(1 - \dfrac{y}{N_2}) + a(1 - \dfrac{y}{M}). \end{cases} \tag{4.1}$$

By linearizing system (4.1) at E^*, we get the characteristic equation at E^* as follows:

$$\lambda^2 + (h_1 + \frac{a}{M})\lambda + h_2 + h_3 = 0, \tag{4.2}$$

where

$$h_1 = -r_1 - r_2 + \frac{2r_1}{N_1}x^* + \frac{2r_2 - \alpha r_1}{N_2}y^*,$$
$$h_2 = (-r_1 + \frac{2r_1}{N_1}x^* - \frac{\alpha r_1}{N_2}y^*)(-r_2 + \frac{2r_2}{N_2}y^*),$$
$$h_3 = (-r_1 + \frac{2r_1}{N_1}x^* - \frac{\alpha r_1}{N_2}y^*)\frac{a}{M}.$$

Equation (4.2) has two roots, donated as λ_1 and λ_2. We can calculate:

$$\lambda_1 + \lambda_2 = h_1 + \frac{a}{M} < 0, \quad \lambda_1 \cdot \lambda_2 = h_2 + h_3 > 0.$$

Thus Eq (4.2) has two characteristic roots with negative real parts. According to the theory of nonlinear dynamic systems [34], we get the following lemma:

Lemma 4.1. *When assumption* (H) *holds, equilibrium* $E^* = (x^*, y^*)$ *of system* (4.1) *is locally asymptotically stable.*

By analyzing system (4.1), we get the following theorem:

Theorem 4.1. *When the initial value is nonnegative, all solutions of system* (4.1) *are nonnegative.*

Proof. Firstly, we prove that $x(t)$ is nonnegative for all $t \geq 0$ under nonnegative initial values. We use proof by contradiction. We assume that $\exists\ t_1 > 0$ such that $x(t_1) = 0$ and $x'(t_1) < 0$, then by the first equation of system (4.1) we have $\frac{dx(t_1)}{dt} = r_1 x(t_1)(1 - \frac{x(t_1)}{N_1} + \alpha\frac{y}{N_2}) = 0$, which contradicts $x'(t_1) < 0$. Similarly, note that $a > 0$, we can also prove that $y(t)$ is also nonnegative for all $t \geq 0$ under nonnegative initial values.

4.2. Analysis for delay differential equations

The characteristic equation of model (3.3) at E^* is as follows:

$$\lambda^2 + h_1\lambda + h_2 + (h_3 + \lambda\frac{a}{M})e^{-\lambda\tau} = 0, \tag{4.3}$$

where

$$h_1 = -r_1 - r_2 + \frac{2r_1}{N_1}x^* + \frac{2r_2 - \alpha r_1}{N_2}y^*,$$
$$h_2 = (-r_1 + \frac{2r_1}{N_1}x^* - \frac{\alpha r_1}{N_2}y^*)(-r_2 + \frac{2r_2}{N_2}y^*),$$
$$h_3 = (-r_1 + \frac{2r_1}{N_1}x^* - \frac{\alpha r_1}{N_2}y^*)\frac{a}{M}.$$

When $\tau > 0$,

$$h_2 + h_3 = -(r_1 + \alpha\frac{r_1}{N_2}y^*)(2r_2 + \frac{\sqrt{N_2(Mr_2 - a)^2 + 4ar_2}}{M\sqrt{N_2}}) \neq 0.$$

Thus $\lambda = 0$ is not the root of Eq (4.3). For model (3.3), there is no fixed point bifurcation at E^*.

Next, we try to discuss the existence of Hopf bifurcation. We assume that $\lambda = \pm i\omega$ is a pure imaginary root of Eq (4.3). Substituting $\lambda = \pm i\omega$ into Eq (4.3) and separating the real and imaginary parts, we can get the expression as follows:

$$\begin{cases} \omega^2 - \frac{r_1}{N_1}x^*(-r_2 + \frac{2r_2}{N_2}y^*) = \frac{a}{M}\frac{r_1}{N_1}x^* \cos(\omega\tau) + \frac{a}{M}\omega \sin(\omega\tau), \\ (-r_2 + \frac{r_1}{N_1}x^* + \frac{2r_2}{N_2}y^* + \frac{a}{M})\omega = \frac{a}{M}\frac{r_1}{N_1}x^* \sin(\omega\tau) - \frac{a}{M}\omega \cos(\omega\tau). \end{cases} \tag{4.4}$$

Add the squares of the two formulas of Eq (4.4) and let $z = \omega^2$, we get the following equation:

$$h(z) = z^2 + [(m + n + \frac{a}{M})^2 - 2mn - (\frac{a}{M})^2]z + (mn)^2 - (\frac{a}{M}n)^2 = 0, \tag{4.5}$$

where

$$m = -r_2 + \frac{2r_2}{N_2}y^* , \ n = \frac{r1}{N1}x^*.$$

Equation (4.5) has two roots, denoted as z_1 and z_2, and we can calculate:

$$z_1 + z_2 = -(m^2 + n^2 + 2m\frac{a}{M} + 2n\frac{a}{M}) < 0, \ \ z_1 \cdot z_2 = [m^2 - (\frac{a}{M})^2]n^2 > 0.$$

Equation (4.5) has two roots with negative real parts. Thus we can get the following theorem:

Theorem 4.2. *When* (H) *holds, the equilibrium* E^* *of the model* (3.3) *is locally asymptotically stable for any* $\tau > 0$*. The model* (3.3) *has no fixed point bifurcation or Hopf bifurcation at* E^*.

5. Simulations and verification

5.1. Data analysis

In this section, we collect data about China's carbon emissions, carbon absorption, and China's urbanization rate in recent years. Then we determine the range of parameters according to the trend of data change.

Based on the Chinese version of bp Statistical Review of World Energy (https://www.bp.com/, *Statistical Review of World Energy 2022−all data 1965-2021*, accessed on 11 November 2022), we can collect data on the amount of China's carbon emissions from 2010 to 2021 shown in Table 2. According to the growth of carbon emissions in recent years, we can infer that the growth rate of carbon emissions $r_2 \in (-0.01, 0.02)$.

Table 2. China's annual carbon emissions (2010–2021).

Year	2010	2011	2012	2013	2014	2015
Values/ billion ton	8.122	8.794	8.979	9.219	9.257	9.226
Annual growth rate	-	0.0764	0.0206	0.0261	0.0041	-0.0033
Year	2016	2017	2018	2019	2020	2021
Values/ billion ton	9.234	9.445	9.676	9.869	9.974	10.523
Annual growth rate	0.0009	0.0223	0.0239	0.0195	0.0106	0.0521

We fit the growth rate of carbon emissions as a cubic function, as shown in Figure 3. The zero point of abscissa corresponds to the year 2010. The fitting equation is $y_1 = -0.0003x^3 + 0.0067x^2 - 0.0486x + 0.1099$. According to the growing trend of the fitting function, N_2 can be taken as 11.760 billion tons. In addition, $y_{1max} = 0.1099$. According to the maximum growth rate y_{1max}, N_2 can be taken as 15.975 billion tons. Therefore, we consider that the range of maximum environmental capacity of carbon emissions $N_2 \in (12, 16)$.

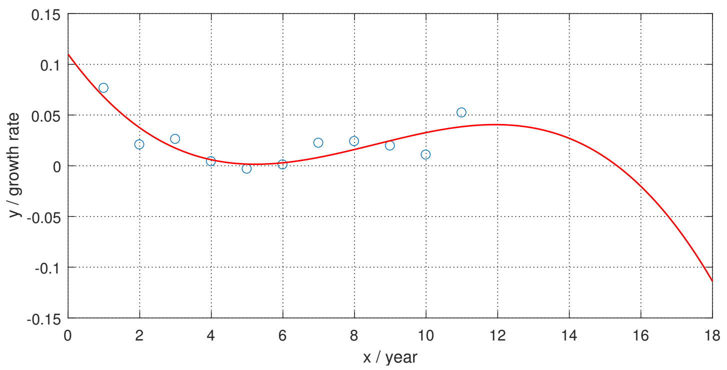


Figure 3. The fitting curve of China's annual carbon emission growth rate (2011–2021).

According to the research of the European Environmental Agency (EEA), about 69% of the carbon emitted by the world's fossil energy consumption is consumed by surface vegetation. The capacity of vegetation to absorb carbon per hectare is about $0.95t$ [35]. We can get the formula $x = 0.95S$, where x is the amount of carbon absorption and S is the vegetation coverage. Thus we select $\alpha = 0.95$. Liu et al. [36] found that the vegetation cover increased significantly from 1982 to 1997 and from 1997 to 2012, which were $1.2\%/10a$ and $0.6\%/10a$ respectively. At the same time, China is also developing advanced technologies to promote carbon absorption, such as CCUS [37]. Therefore, we can roughly estimate that the growth rate of carbon absorption $r_1 \in (0.02, 0.05)$, and the maximum value that carbon absorption $N_1 \in (7, 12)$.

Based on a report named The Homecoming of Small Town Youth: From Town to City – "Urbanization" Series No.2 issued by Haitong Securities in November 2019 (analyst: C. Jiang, X. Chen) (https://www.htsec.com/ChannelHome/2016102402/5867538.shtml, accessed on 11 November 2022), we can collect data on the urbanization rate of China from 2000 to 2020, as shown in Figure 4. The straight line of fitting result represents the development trend of the urbanization process. So we can predict that $a \in (1, 3)$.

Based on China's per capita carbon emissions from World Bank Open Data (https://data.worldbank.org/, *CO_2 emissions (metric tons per capita)*, accessed on 11 November 2022), we can collect the per capita carbon emissions data of China from 2010 to 2021 shown in Table 3.

We fit the growth rate as a cubic function to predict the range of M, as shown in Figure 5. The zero point of abscissa corresponds to the year 2010. The fitting equation is $y_2 = -0.0002x^3 - 0.0066x^2 - 0.0564x + 0.1484$. Thus we can calculate per capita carbon emissions of China will peak in 2028, with a value of 11.823t. According to the development trend of urbanization and total population, we can predict that the total urban carbon emissions $M \in (11, 12)$.

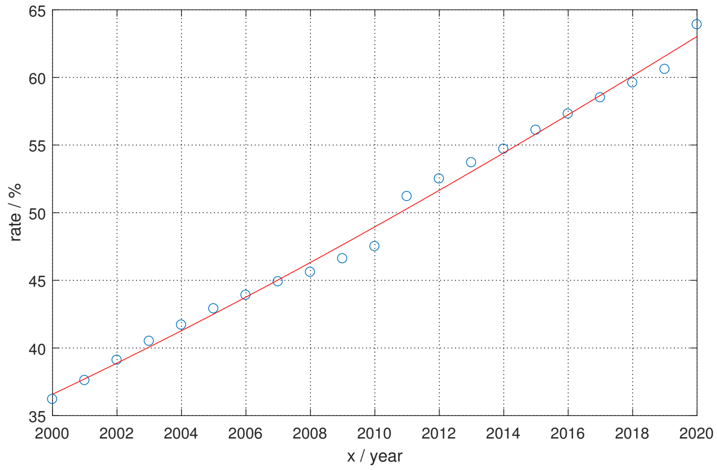


Figure 4. The fitting curve of China's urbanization rate from 2000 to 2022.

Table 3. China's per capita carbon emissions (2010–2021).

Year	2010	2011	2012	2013	2014	2015
Values/ ton	6.34	6.90	7.05	7.32	7.29	7.15
Annual growth rate	-	0.082	0.021	0.038	-0.004	-0.021
Year	2016	2017	2018	2019	2020	2021
Values/ ton	7.12	7.23	7.49	7.61	8.08	8.44
Annual growth rate	-0.004	0.016	0.034	0.016	0.059	0.043

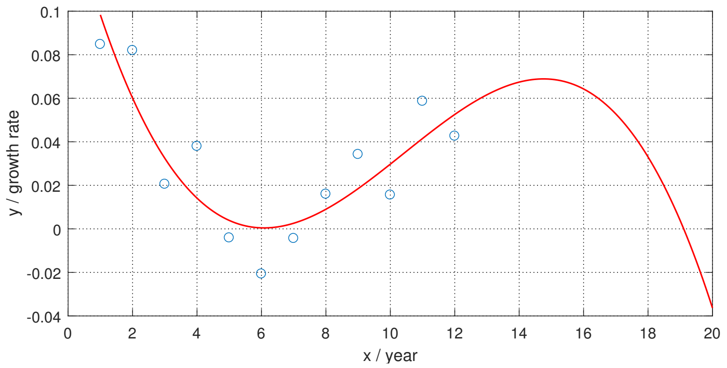


Figure 5. The fitting curve of China's per capita carbon emissions annual growth rate (2010–2021).

Based on all the above analyses, we can estimate the range of each parameter, as shown in the following Table 4.

Table 4. The range of parameters for model (3.3).

Parameters	r_1	r_2	N_1	N_2	M	α	a
Ranges	$(0.02, 0.05)$	$(-0.01, 0.02)$	$(7, 12)$	$(12, 16)$	$(11, 12)$	0.95	$(1, 3)$

5.2. Numerical simulations

1) We discuss whether the dual carbon target of China can be successfully achieved under the current conditions. We choose a group of parameters to make each parameter as close as possible to the current situation in China. In 2020, the amount of carbon emission is 9.9 billion tons and the amount of carbon absorption is 1.1 billion tons. Thus we choose $(x(t), y(t)) = (1.1, 9.9)$ for $t \in (-\tau, 0)$ as the initial function of the simulation.

$$\text{Group I}: r_1 = 0.03, r_2 = 0.01, N_1 = 10, N_2 = 16, M = 11.5, \alpha = 0.95, a = 2.5.$$

The values of parameters satisfy the assumption (H). According to Theorem 4.2, for any $\tau > 0$, the equilibrium $E^* = (16.915, 11.646)$ is locally asymptotically stable. The time delay represents the time of urban ecological process adapting to population migration. Based on the practical meaning of τ, we consider that the reasonable time delay is about 1–3 years. Thus, we choose a middle-value $\tau = 2$. We give the simulation result of the locally asymptotically stable at E^* when $\tau = 2$, as shown in Figure 6. The zero point of abscissa corresponds to the year 2020.

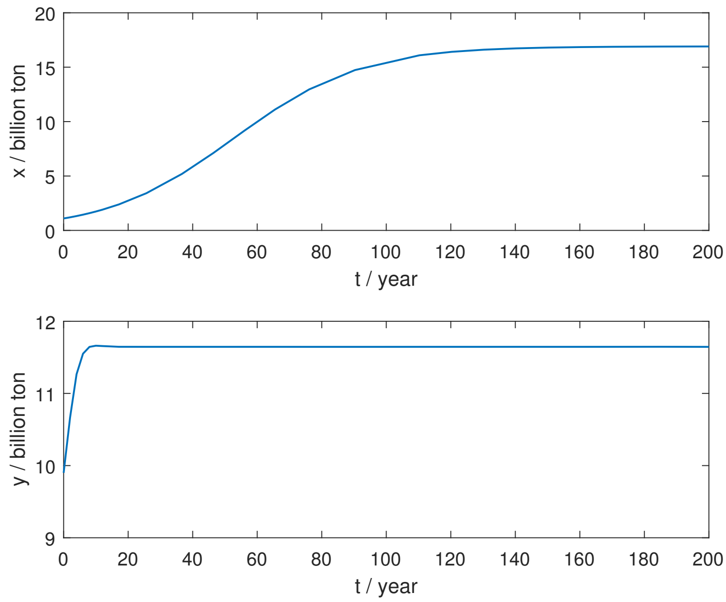


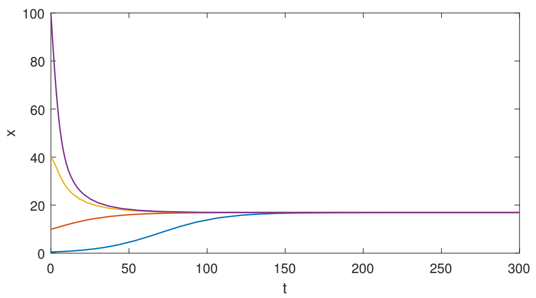
Figure 6. Under the first group of parameters, E^* of the model (3.3) with $\tau = 2$ is locally asymptotically stable.

Remark 1: From the simulation, we find that in order to reach carbon peak by 2030, the average growth rate of carbon emissions needs to be kept within 0.01. The simulation result shows that China can achieve carbon peak by 2030. The value of carbon emissions at the peak is 11.65 billion tons, which is greater than the carbon absorption of 2.32 billion tons. Therefore, we can infer that according to the current situation of carbon emission and carbon absorption, if China wants to reach carbon neutrality by 2060, some measures must be taken to increase the growth rate of carbon absorption. In other words, in 2060, the amount of carbon emission is still greater than carbon absorption.

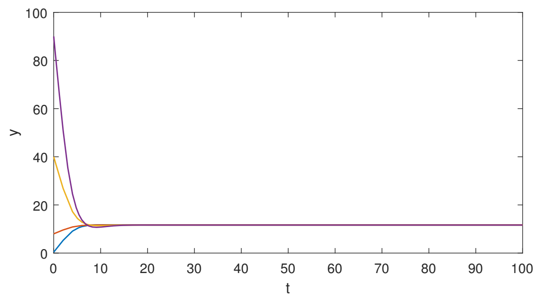
According to the simulation results in Figure 6, we perform further simulations. We choose different initial functions under the parameters Group I as follows:

$$(x(t), y(t)) = (0.5, 0.4),\ (x(t), y(t)) = (10, 8),\ (x(t), y(t)) = (40, 40),\ (x(t), y(t)) = (100, 90),$$

for $t \in (-\tau, 0)$, respectively. Then, we get the following simulation results in Figure 7.



(a)



(b)

Figure 7. The model (3.3) may have a globally asymptotically stable equilibrium under different initial functions.

Remark 2: We choose four different initial functions arbitrarily, and all the solutions of the model (3.3) eventually tend to the equilibrium $E^* = (16.915, 11.646)$. The simulation results show that model (3.3) may have a globally asymptotically stable equilibrium E^*.

2) We assume that the carbon peak of China has been achieved, and then discuss the conditions for achieving carbon neutrality in 2060. We need to choose the group of parameters that can reduce carbon emissions and increase carbon absorption. According to the current growth rate and the results of simulation 1), we estimate that the carbon absorption in 2030 is 2.32 billion tons and the carbon emissions is 11.65 billion tons. So we choose $(x(t), y(t)) = (2.32, 11.65)$ for $t \in (-\tau, 0)$ as the initial function of the simulation.

$$\text{Group II} : r_1 = 0.05, r_2 = 0.00001, N_1 = 10, N_2 = 16, M = 11.5, \alpha = 0.95, a = 1.5.$$

The values of parameters satisfy the assumption (H). At this time, we still choose $\tau = 2$. The equilibrium $E^* = (16.815, 11.478)$ is locally asymptotically stable according to Theorem 4.2. We give the simulation result of the locally asymptotically stable at E^* in Figure 8. The zero point of abscissa corresponds to the year 2030.

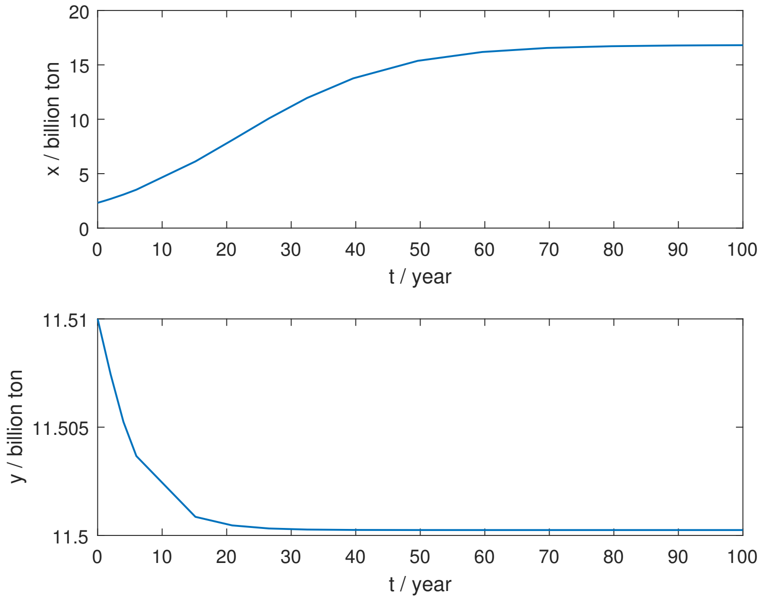


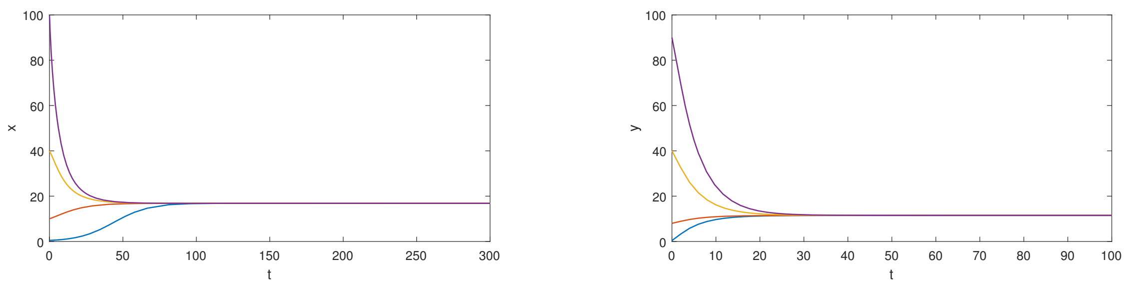
Figure 8. Under the second group of parameters, E^* of the model (3.3) with $\tau = 2$ is locally asymptotically stable.

Remark 3: We find that to achieve carbon neutrality in 2060, we need to control the average growth rate of carbon absorption to reach 0.05 or more after achieving the carbon peak. The simulation result shows that when the growth rate of carbon absorption increases and the growth rate of carbon emission decreases to almost zero, the carbon emissions and carbon absorption in 2060 are both about 11.5 billion tons. In other words, under this condition, the dynamic balance between carbon emissions and carbon absorption is achieved. And after 2060, carbon absorption will continue to increase, while carbon emissions will remain unchanged. It leads to a surplus of the amount of carbon absorption.

To further prove our guess, we select several different initial functions under the parameters Group II for numerical simulation:

$$(x(t), y(t)) = (0.5, 0.4),\ (x(t), y(t)) = (10, 8),\ (x(t), y(t)) = (40, 40),\ (x(t), y(t)) = (100, 90),$$

for $t \in (-\tau, 0)$, respectively. Then, we get the following simulation results in Figure 9.



(a) (b)

Figure 9. The model (3.3) may have a globally asymptotically stable equilibrium under different initial functions.

Remark 4: All of the solutions of the model (3.3) eventually tend to the equilibrium $E^* = (16.815, 11.478)$. The simulation results agree with our guess, that is model (3.3) may have a globally asymptotically stable equilibrium E^*.

3) We will discuss the impact of dual carbon target in a situation where the internal construction of the city lags far behind the external migration. Therefore, we assume that the time delay becomes longer and perform the simulation. We still choose the current situation of carbon emission and carbon absorption and then increase the value of time delay to $\tau = 6$ years. We choose the amount of carbon emission and the amount of carbon absorption of the year 2020 that $(x(t), y(t)) = (1.1, 9.9)$ for $t \in (-\tau, 0)$ as the initial function of the simulation.

$$\text{Group III} : r_1 = 0.03, r_2 = 0.01, N_1 = 10, N_2 = 16, M = 11.5, \alpha = 0.95, a = 2.5.$$

The parameter values are the same as in the first group of simulations. According to Theorem 4.2, the equilibrium $E^* = (16.815, 11.478)$ is also locally asymptotically stable for any $\tau > 0$. We give the simulation result of the local asymptotic stability at E^* when $\tau = 6$, as shown in Figure 10. The zero point of abscissa corresponds to the year 2020.

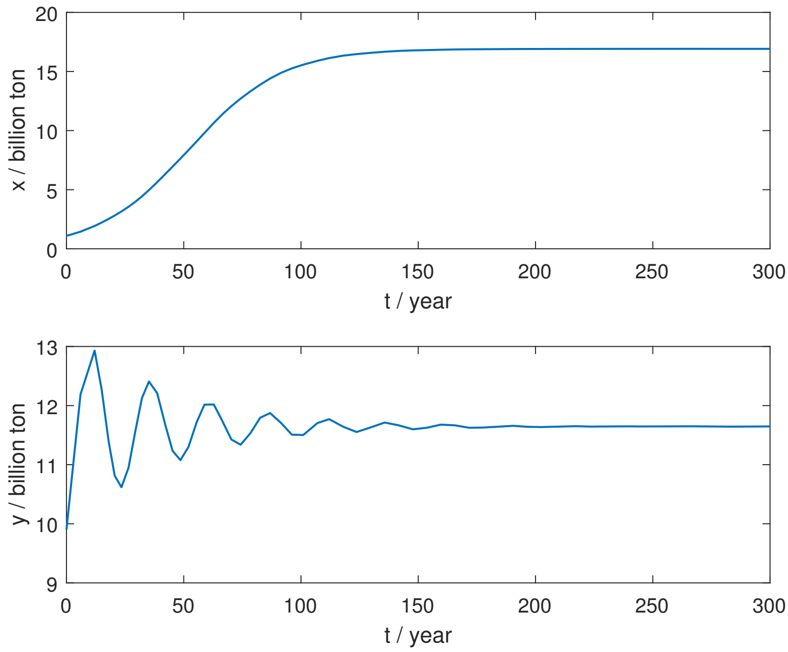


Figure 10. Under the third group of parameters, E^* of the model (3.3) with $\tau = 6$ is locally asymptotically stable.

Remark 5: We find that when the time delay is too large, carbon emissions will experience a period of large fluctuations, and then tend to stabilize slowly. In other words, when the process of urban internal construction lags behind the process of population migration too much, the achievement of China's dual carbon target will be disrupted. The cities should try to narrow the gap between internal ecological construction and the external population migration process. We give the following suggestions: (1) Improve the quality of the population. Public awareness and people's concern about carbon neutrality influence the realization of carbon neutrality [38]. So this measure can partly reduce the damage to urban ecology caused by population migration. (2) Improve the urban ecological construction level by promoting scientific and technological innovation, upgrading energy allocation, improving the greening level, and other measures. The progress of urban green development is mainly accelerated by technological development, effective energy allocation, and industrial structure upgrading [39]. Therefore, these measures could be the main factors to narrow this gap. (3) Formulate policies to regulate urban ecological development. At present, the intensity of population migration in China has decreased, but it still shows a trend of migration to cities. The development of policies that fit the Chinese context is also an important factor.

6. Conclusions

In this paper, we used mathematical modeling and numerical simulations to analyze the impact of China's urbanization process on carbon peak and carbon neutrality. According to the current development of carbon emission, carbon absorption, and urbanization, we simulated the possible situation, draw some relevant conclusions, and gave some policy recommendations.

1) Through the analytic hierarchy process, we concluded that the level of urbanization is an important influencing factor of the dual carbon target. In fact, there are many factors affecting carbon emission and absorption, such as the economy, energy, and so on. In this paper, we only selected one aspect and hope to analyze other factors in future studies.

2) We found that the equilibrium of model (3.3) with $\tau = 0$ has nonnegative solutions when the initial value is nonnegative. The equilibrium E^* of the model (3.3) is locally asymptotically stable for any $\tau \geq 0$, and the model (3.3) has no fixed point bifurcation or Hopf bifurcation at E^*. Through numerical simulation, we also guessed that the model (3.3) may have a globally asymptotically stable equilibrium.

3) We found that according to the current growth rate, the carbon peak can be achieved by 2030. However, the amount of carbon absorption at the peak is still much smaller than the amount of carbon emission. Therefore, some measures should take to control the growth rate of carbon emissions to zero or even negative after the peak, such as promoting energy reform, improving energy efficiency, vigorously advocating low-carbon life, etc. In the meanwhile, the growth rate of carbon absorption needs to be increased by expanding vegetation coverage and developing carbon capture technology, such as CCS, CCUS, etc.

4) We simulated the development of carbon emissions and carbon absorption after the carbon peak. We found that when the average growth rate of carbon absorption between 2030 and 2060 reaches 0.05 or more, carbon neutrality can be achieved by 2060. In addition, after 2060, the amount of carbon absorption will continue to increase until it reaches the equilibrium. In this case, there is a surplus of carbon absorption. According to the United Nations Framework Convention on Climate Change and the Kyoto Protocol [40,41], we suggested selling excess carbon absorption through the carbon trading market. In this way, we can create a win-win situation for the economy and ecology.

5) We increased the value of the time delay for numerical simulations. We found that urban ecological construction cannot lag too far behind the process of population migration. When the population of a city far exceeds the ecological carrying capacity of the city, the carbon balance may be destroyed. Therefore, with the steady improvement of the urbanization rate, cities should actively improve the

greening level, control the per capita carbon emissions, formulate reasonable policies, etc, in order to narrow the gap between the cities' internal ecological construction and the external population migration process.

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Conflict of interest

The authors declare that they have no competing interests.

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