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Research article

Huizhou GDP forecast based on fractional opposite-direction accumulating nonlinear grey bernoulli markov model

Meilan Qiu[1,2], Dewang Li[2], Zhongliang Luo[3,*] and Xijun Yu[4,*]

[1] Division of Applied and Computational Mathematics, Beijing Computational Science Research Center, Beijing 100193, China
[2] School of Mathematics and Statistics, Huizhou University, Guangdong, Huizhou 516007, China
[3] School of Electronic and Information Engineering, Huizhou University, Guangdong, Huizhou 516007, China
[4] Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing 100088, China

***Correspondence:** Email: hk3333@163.com, yuxj@iapcm.ac.cn; Tel: +8613910178921.

Abstract: In this paper, a fractional opposite-direction accumulating nonlinear grey Bernoulli Markov model (FOANGBMKM) is established to forecast the annual GDP of Huizhou city from 2017 to 2021. The optimal fractional order number and nonlinear parameters of the model are determined by particle swarm optimization (PSO) algorithm. An experiment is provided to validate the high fitting accuracy of this model, and the effect of prediction is better than that of the other four competitive models such as autoregressive integrated moving average model (ARIMA), grey model (GM (1,1)), fractional accumulating nonlinear grey Bernoulli model (FANGBM (1,1)) and fractional opposite-direction accumulating nonlinear grey Bernoulli model (FOANGBM (1,1)), which proves the robustness of the opposite-direction accumulating nonlinear Bernoulli Markov model. This research will provide a scientific basis and technical references for the economic planning industries.

Keywords: FOANGBMKM (1,1); GDP; PSO algorithm; differential equation; forecast

1. Introduction

GDP is a key index in the system of the national economic accounts, which reflects the economic power and market scale of a country (or region). It is more and more important to forecast and analyze annual GDP for the economic planning and development, which is of great significance for the economic development of a country. In order to forecast the GDP more effectively, a great deal of efforts have been devoted to the precision of prediction. For example, in [1], combining SPSS with EVIEWS 6.0, some mathematical statistics methods such as correlation analysis, regression analysis and combination prediction model were used to analyze the significant influencing factors of GDP, i.e., the total agricultural output value and residents' consumption level, quantitatively. In order to simulate the time series data of China's real GDP from 1952 to 2005, the auto-regressive and moving average model (ARMA) as well as Holter-Winter non-seasonal short-term forecast models were established, respectively, in [2] and then were used to forecast the national GDP from 2006 to 2010. Given the range of lag order and given different polynomial weight functions to high-frequency explanatory variables, the optimal mixed data sampling (MIDAS) model was selected based on Akaike information criterion (AIC) to forecast China's quarterly GDP [3]. According to the modeling theory and analysis technology of the mixed data econometric model, a MIDAS regression prediction model and non-restricted MIDAS model of five different weight functions were constructed in [4]. The least square identification method of MIDAS model was deduced with the traditional distributed lag model to forecast China's quarterly GDP in the short-term and analyze the change effects on hysteresis order of high frequency explanatory variables as well as the influence effects on low frequency variable GDP. In [5], the data of Nanjing GDP from 2000 to 2015 were transformed by wavelet transform, and the auto-regressive model (AR) and GM (1,1) model were established to predict high frequency information as well as the low frequency information after transformation, respectively.

The fractional GM in the grey system has attracted considerable research attention in recent years. The classical GM was based on 1-AGO. Wu et al. [6] proposed the fractional GM based on fractional accumulated grey operation (FAGO), followed by the FAGO discrete (Ma et al. [7]) and FAGO grey Bernoulli models being constructed and applied to the energy forecast by Wu et al. [8]. To solve the prediction problem with memory characteristics, Mao et al. [9] started from the memory principle, proposed to improve the integral differential equation into a fractional differential equation and established a single-variable fractional derivative grey prediction model. Mao et al. [10] introduced the fractional derivative based on FAGO. Kang et al. [11] proposed a variable-order fractional GM. Xie et al. [12] developed a generalized fractional GM by introducing a generalized fractional derivative that conforms to the memory effect.

A self-adaptive intelligence grey prediction model with fractional accumulation was discussed in [13]. In [14], a smooth generation method was used to weaken the influence of the extreme value on the performance of GM (1,N), and a novel multi-variable grey forecasting model NMGM (1,N) based on the smooth generation of independent variable sequences with variable weights was constructed. A novel fractional time delayed GM with grey wolf optimization algorithm was established and applied to forecast the natural gas consumption of Chongqing city [15]. A novel definition of conformable fractional accumulation was proposed, which was more feasible and simpler compared to the traditional fractional GM [16]. In [17], the accumulating generation operator and the inverse accumulating generation operator were extended to the field of positive real numbers by Gamma function. Then, the analytic expressions were given out, and the inverse property was proved between the two operators. In [18], the primary data was processed by using reverse accumulating and extended to the field of fractional order on the basis of integer order. Then, the opposite-direction accumulating fractional Verhulst model was established based on the fractional opposite-direction accumulating generation operator and the fractional opposite-direction inverse accumulating generation operator. Although many excellent works have been done in the above areas, the GM (1,1) model based on forward direction sequence accumulation cannot satisfy the priority principle of the new information, and there is no theory to prove that the opposite-direction sequence accumulation satisfies the new information principle for the GM (1,1) model with inverse accumulation sequence [6]. A novel Grey system model with fractional accumulation was proposed and the priority of new information can be better reflected as the fractional accumulation order number becomes smaller in the in-sample model [19].

The new information priority principle was embodied in the FAGO grey Bernoulli model [20]. Referring to the practice of Gao et al. [21], we list these representative complementary approaches to GDP in Table 1.

Table 1. Contemporary methods for forecasting GDP.

Source	Model	Study focus	City/region
Statistical econometric method			
Bian [22]	Multiple logarithmic model Factor-	GDP growth	China
Tang et al. [23]	MIDAS	GDP nowcasting Per	China
Sun and Liu [24]	Dynamic combination model	Capital GDP	Hebei/China
Machine learning/Neural Network			
Yu [25]	RBF Neural Network Neural Network	GDP Forecasting	China
Hua [26]	and ARIMA	GDP Trend	China
Jiang [27]	Machine learning	Local GDP	China
GM			
Wu et al. [28]	NGBM(1,1)	GDP	China
Wang [29]	GM/Few-Shot Learning	Regional economics GDP	China
Lu and Wang [30]	GM	GDP	Guangdong
Liu and Cheng [31]	Extended GM	GDP	Suzhou/China

Although a great deal of efforts are devoted to the grey prediction model, it is easy to generate random error among these models in fact. To capture the nonlinear trend in annual GDP data from Huizhou of China and obtain an appreciate prediction accuracy, this paper proposes a FOANGBMKM (1,1)), and the main contributions can be summarized as follows:

1) The FOANGBM (1,1) model is established based on the PSO algorithm. Under the condition of minimizing mean relative errors, we search for the optimal order and nonlinear parameters of the FOANGBM (1,1) model.

2) According to Markov transition probability matrix and state division, we construct concrete expressions for the estimated and predicted values of the FOANGBMKM (1,1) model.

3) The validity of this proposed model is verified by numerical examples and applied to forecast Huizhou's annual GDP.

2. FOANGBMKM (1,1)

2.1. FOANGBM (1,1)

Let the non-negative sequence be

$$U^{(0)}=\{u^{(0)}(1),u^{(0)}(2),\cdots,u^{(0)}(n)\}. \tag{1}$$

Accumulating the original sequence by the fractional opposite-direction accumulation, the accumulation sequence is obtained as follows:

$$U^{(r)}=\{u^{(r)}(1),u^{(r)}(2),\cdots,u^{(r)}(n)\}, \tag{2}$$

where

$$u^{(r)}(k)=\sum_{i=k}^{n}C_{k-i+r-1}^{k-i}u^{(0)}(i)\,. \tag{3}$$

Then, the whitening differential equation of the model FOANGBMKM (1,1) is

$$\frac{du^{(r)}}{dt}+au^{(r)}(t)=b(u^{(r)}(t))^{\gamma}, \tag{4}$$

and the grey differential equation is

$$u^{(r)}(k)-u^{(r)}(k-1)+av^{(r)}(k)=b(v^{(r)}(k))^{\gamma}, \tag{5}$$

where $v^{(r)}(k)=[u^{(r)}(k)+u^{(r)}(k-1)](k=1,2,\cdots,n)$.

We obtain the following matrixes

$$\mathrm{A}=\begin{bmatrix}-v^{(r)}(2) & (v^{(r)}(2))^{\gamma}\\ -v^{(r)}(3) & (v^{(r)}(3))^{\gamma}\\ \vdots & \vdots\\ -v^{(r)}(n) & (v^{(r)}(n))^{\gamma}\end{bmatrix},\ D=\begin{bmatrix}u^{(r)}(2)-u^{(r)}(1)\\ u^{(r)}(3)-u^{(r)}(2)\\ \vdots\\ u^{(r)}(n)-u^{(r)}(n-1)\end{bmatrix}, \tag{6}$$

Let $\theta=(a,b)^{T}$,

By using the least square algorithm, we obtain

$$\theta=(A^{T}A)^{-1}A^{T}D, \tag{7}$$

where Eq (7) is obtained by the least squares formula.

Assume that $u^{(r)}(n)=u^{(0)}(n)$, and then the solution of Eq (4) is

$$\hat{u}^{(r)}(k)=[((u^{(0)}(n))^{1-\gamma}-\frac{\hat{b}}{\hat{a}})e^{-\hat{a}(1-\gamma)(k-n)}+\frac{\hat{b}}{\hat{a}}]^{\frac{1}{1-\gamma}}, \tag{8}$$

where

$$\hat{U}^{(r)}=\{\hat{u}^{(r)}(1),\hat{u}^{(r)}(2),\cdots,\hat{u}^{(r)}(n)\}. \tag{9}$$

Applying the inverse accumulating on Eq (9), the result of simulation is as follows

$$\alpha^{(r)}U^{(0)}=\{\alpha^{(1)}\hat{u}^{(1-r)}(1),\alpha^{(1)}\hat{u}^{(1-r)}(2),\cdots,\alpha^{(1)}\hat{u}^{(1-r)}(n),\cdots\}. \tag{10}$$

Next, we will calculate the fitting values $\hat{u}^{(0)}(1),\hat{u}^{(0)}(2),\cdots,\hat{u}^{(0)}(n)$ and the prediction value $\hat{u}^{(0)}(n+1),\ \hat{u}^{(0)}(n+2),\cdots$

Let

$$\hat{U}^{(r)}=\{\hat{u}^{(r)}(n+1),\hat{u}^{(r)}(n+2),\cdots,\}, \tag{11}$$

and we define the inverse accumulating operator with r order of the prediction sequence as follows:

$$u^{(r)}(t)=\sum_{i=n}^{t}C_{n-i+r-1}^{n-i}\hat{u}^{(0)}(t). \tag{12}$$

By applying the fractional opposite-direction inverse accumulating on Eq (12), we obtain the prediction values

$$\hat{u}^{(r)}(t)=\sum_{i=n}^{t}(-1)^{i-t}\frac{\Gamma(r+1)}{\Gamma(i-n+1)\Gamma(n+r-i+1)}\hat{u}^{(0)}(i). \tag{13}$$

2.2. Markov model

The Markov model is a general tool for data, statistics and analysis, which predicts the latest state according to the state transition probability of the previous time. Markov process has the characteristics of non-aftereffect property and good short-term prediction, which is suitable to be applied to analyze the fluctuation data. It has been widely applied in military, biology, meteorology and so on [32–34].

2.2.1. State partition

According to the Markov chain, the data sequence can be partitioned into multiply different states, which is denoted by $\mathrm{E}_1,\ \mathrm{E}_2,\cdots,\ \mathrm{E}_m$. The state transition occurs only at countable moments such as $\mathrm{t}_1,\ \mathrm{t}_2,\ \cdots,\ \mathrm{t}_m$, and state partitions are

$$\mathrm{E}_i=[Q_{i1},Q_{i2}]\ (i=1,2,\cdots,j). \tag{14}$$

where Q_{i1}, Q_{i2} represents the lower and upper limits of relative error of state partition respectively, j denotes the number of state partition.

2.2.2. State transition probability matrix

The transition probability of Markov chains from state $\mathrm{E_i}$ to state $\mathrm{E_j}$ after k steps is expressed by $\mathrm{p_{ij}}(k)$:

$$\mathrm{p_{ij}}(k) = \frac{m_{ij}(k)}{M_i}, \tag{15}$$

where M_{i} denotes the total number for the occurrence of status $\mathrm{E_i}$, $m_{\mathrm{ij}}(k)$ represents the number of state $\mathrm{E_i}$ to state $\mathrm{E_j}$ after k steps, m is the number of state partition.

The state transition probability matrix of one step is as following:

$$\mathrm{P}(1) = \begin{bmatrix} p_{11}(1) & p_{12}(1) & \cdots & p_{1m}(1) \\ p_{21}(1) & p_{22}(1) & \cdots & p_{2m}(1) \\ \vdots & \vdots & \cdots & \vdots \\ p_{m1}(1) & p_{m2}(1) & \cdots & p_{mm}(1) \end{bmatrix}. \tag{16}$$

Using the Chapman-Kolmogorov equation repeatedly, let $\mathrm{V}(0)$ be the initial vector for the original state $\mathrm{E_i}$ of one variable, and we get the transition probability matrix after k steps and the state vector as the following, respectively.

$$\mathrm{P(k)} = (\mathrm{P}(1))^{\mathrm{k}}, \tag{17}$$

$$\mathrm{V(k)} = \mathrm{V}(0) \cdot (\mathrm{P}(1))^{\mathrm{k}}. \tag{18}$$

2.2.3. Determination of the value of prediction

Select j groups of data which are closest to the predicted data. According to the order of data groups from near to far, the number of the step t is determined as $1, 2, \cdots, j$. Then, a new matrix is constructed by choosing the row vectors of the t-step state transition matrix corresponding to each data, and the most probable state of prediction value is determined by the sum of the column vectors of the new matrix. The state partition can be determined after confirming the status. Choosing the midpoint of the state interval $\frac{1}{2}(Q_{i1}+Q_{i2})$ as the Markov corrected value, the forecast value with Markov model is

$$\hat{u}_M(k) = \frac{\hat{u}_{FOANGBM}(k)}{1+\frac{1}{2}(Q_{i1}+Q_{i2})}. \tag{19}$$

2.3. The validation of model errors

In this part, the mean absolute percentage error (MAPE) and root mean square error (RMSE) are used to evaluate the model errors. With [35], we calculate statistics STD and R^2, and their calculation formulas are listed as follows.

$$MAPE = \frac{1}{\mathrm{n}}\sum_{t=1}^{n}\left|\frac{(\hat{u}_t - u_t)}{u_t}\right|, \tag{20}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{u}_i - u_t)^2}, \tag{21}$$

$$\mathrm{S}TD = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\frac{|\hat{u}_i - u_t|}{u_t} - \mathrm{M}APE)^2}, \tag{22}$$

$$R^2 = 1 - \frac{\sum_{t=1}^{n}(\hat{u}_t - u_t)^2}{\sum_{\mathrm{t}=1}^{\mathrm{n}}(\hat{u}_t - \overline{u})^2}, \tag{23}$$

where $\overline{u}$ is the average of training data, and $\overline{u} = \frac{1}{n}\sum_{t=1}^{n} u_t$.

2.4. Determination the optimal order r and nonlinear parameters γ of the model

PSO algorithm is a swarm intelligence optimization algorithm in the field of computational intelligence excepted to the ant colony algorithm and the fish swarm algorithm, which is originated from the research on predation problems of birds and first proposed by Kennedy and Eberhart in 1995. The PSO algorithm has many advantages such as definite physical concept, good convergence, more stability, etc. In this section, we will use PSO algorithm to search for the optimal order r and nonlinear parameter γ of the FOANGBM (1,1) model under the condition of minimizing mean relative errors, the mathematical expression of the PSO algorithm is

$$\min f(r,\gamma) = \frac{1}{n-1}\sum_{k=2}^{n}\left|\frac{\hat{u}^{(0)}(k) - u^{(0)}(k)}{u^{(0)}(k)}\right|. \tag{24}$$

3. The prediction research on Huizhou GDP

3.1. Estimation and prediction of the model FOANGBM (1,1)

Importing the statistical yearbook data of Huizhou into R, and combing the PSO algorithm with (2.2), we get the optimal order $r = 0.01$ and the nonlinear parameter $\gamma = 0.99$ of the accumulating generation operator, respectively. The accumulating generation operator is

$$\hat{u}^{(0.01)}(k)=[((u^{0}(n))^{0.01}-1.1974)e^{-10.15562(k-n)}+1.1974]^{100}, \tag{25}$$

where $k=2,3,\cdots$, $u^{0}(n)=33{,}595{,}203$.

Applying the opposite-direction inverse accumulation with order $r=0.01$ on Eq (25), we obtain the estimate value as $k=2,\ 3,\ \cdots,\ 12$ and the prediction value as $k=13,\ 14,\ 15,\ 16,\ 17$. Taking GM (1,1) model and fractional nonlinear grey Bernoulli model (FANGBM (1,1)) as the comparison model, the prediction results are listed in Table 2.

Table 2. The comparison results among GM (1,1), FANGBM (1,1) and FOANGBM (1,1).

Year	Raw	FOANGBM (1,1) Predicted value	Relative error	FANGBM (1,1) Predicted value	Relative error	GM (1,1) Predicted value	Relative error
2005	8,051,130	7,614,028	−5.43%	8,051,130	0%	8,051,130	0%
2006	9,309,277	9,425,517	1.25%	8,899,122	−4.41%	10,811,274	16.13%
2007	11,217,080	11,417,136	1.79%	11,203,745	−0.12%	12,190,465	8.68%
2008	13,095,122	13,565,876	3.60%	13,428,613	2.55%	13,745,599	4.97%
2009	14,130,759	15,845,538	12.14%	15,666,871	10.87%	15,499,121	9.68%
2010	17,235,617	18,228,457	5.76%	17,968,577	4.25%	17,476,339	1.40%
2011	20,801,369	20,687,249	−0.55%	20,366,854	−2.09%	19,705,790	−5.27%
2012	23,524,573	23,196,650	−1.39%	22,886,982	−2.71%	22,219,652	−5.55%
2013	26,745,036	25,735,859	−3.77%	25,550,327	−4.47%	25,054,207	−6.32%
2014	29,591,103	28,292,849	−4.39%	28,376,306	−4.11%	28,250,364	−4.53%
2015	30,902,218	30,877,291	−0.08%	31,383,479	1.56%	31,854,255	3.08%
2016	33,595,203	33,595,203	0%	34,590,214	2.96%	35,917,892	6.91%
		RMSE	755,540	RMSE	811200	RMSE	1,272,100
		MAPE	3.34%	MAPE	3.34%	MAPE	6.04%
		STD	3.28%	STD	2.71%	STD	4.02%
		R^2	99.12%	R^2	99.08%	R^2	97.71%
2017	37,457,511	35,182,264	−6.07%	38,015,126	1.48%	40,499,927	8.12%
2018	40,033,312	37,507,682	−6.30%	41,677,391	4.10%	45,666,490	14.07%
2019	41,929,295	39,816,692	−5.03%	45,596,987	8.74%	51,492,151	22.80%
2020	42,217,852	42,254,647	0.08%	49,794,889	17.94%	58,060,989	37.52%
2021	49,773,600	44,161,175	−11.27%	54,293,245	9.08%	65,467,810	31.53%
		RMSE	3,082,800	RMSE	4342900	RMSE	11,223,000
		MAPE	5.76%	MAPE	8.27%	MAPE	22.81%
		STD	3.56%	STD	5.62%	STD	10.81%
		R^2	42.62%	R^2	59.04%	R^2	28.92%

As can be seen from Table 2, the prediction results of the FOANGBM (1,1) model are closer to the real values, and the relative error is smaller than that of other models. We also can obtain the smallest value of RMSE, STD and the MAPE when forecasting the test data and estimate the training data by using the FOANGBM (1,1) model. However, FOANGBM (1,1) model has a higher value of R^2 than that of the other models, such as FANGBM (1,1), GM (1,1) and ARIMA (in Table 3).

Table 3. The comparison results among ARIMA, FOANGBM (1,1) and FOANGBMKM (1,1).

Year	Raw	ARIMA		FOANGBM (1,1)		FOANGBMKM (1,1)	
		Predicted value	Relative error	Predicted value	Relative error	Predicted value	Relative error
2005	8,051,130	8,043,079	−0.10%	7,614,028	−5.43%	1	7,912,322
2006	9,309,277	8,756,966	−5.93%	9,425,517	1.25%	2	9,438,731
2007	11,217,080	10,465,731	−6.70%	11,417,136	1.79%	3	11,041,717
2008	13,095,122	12,769,877	−2.48%	13,565,876	3.60%	3	13,119,802
2009	14,130,759	14,837,179	5.00%	15,845,538	12.14%	5	14,341,151
2010	17,235,617	15,354,878	−10.91%	18,228,457	5.76%	4	17,045,499
2011	20,801,369	19,439,938	−6.54%	20,687,249	−0.55%	2	20,716,251
2012	23,524,573	23,943,896	1.78%	23,196,650	−1.39%	2	23,229,170
2013	26,745,036	26,305,582	−1.64%	25,735,859	−3.77%	1	26,719,122
2014	29,591,103	29,557,422	−0.11%	28,292,849	−4.39%	1	29,373,805
2015	30,902,218	32,336,129	4.64%	30,877,291	−0.08%	2	30,920,579
2016	33,595,203	32,601,380	−2.96%	33,595,203	0%	2	33,642,302
		RMSE	925520	RMSE	755,540	RMSE	156,190
		MAPE	4.07%	MAPE	3.34%	MAPE	0.85%
		STD	3.04%	STD	3.28%	STD	0.62%
		R^2	98.88%	R^2	99.12%	R^2	99.97%
2017	37,457,511	35,638,803	−4.86%	35,182,264	−6.07%	1	35,231,588
2018	40,033,312	37,731,237	−5.75%	37,507,682	−6.30%	1	37,560,266
2019	41,929,295	39,646,009	−5.45%	39,816,692	−5.03%	1	39,872,513
2020	42,217,852	41,469,128	−1.77%	42,254,647	0.08%	2	42,313,886
2021	49,773,600	43,180,258	−13.25%	44,161,175	−11.27%	1	44,223,087
		RMSE	3,401,600	RMSE	3,082,800	RMSE	3,037,100
		MAPE	6.21%	MAPE	5.76%	MAPE	5.68%
		STD	3.79%	STD	3.56%	STD	3.48%
		R^2	21.04%	R^2	42.62%	R^2	43.47%

3.2. Markov model forecasting Huizhou GDP

3.2.1. Determination the value of prediction

Generally, the state interval is partitioned by the relative error of FAONGBM model, it can be seen from Table 2 that the minimum and maximum relative error of the first 12 fitting data of this model are −5.43 and 12.14% respectively. Hence, according to the equal spacing rule, five state partitions are divided as follow, $E_1(-5.45\%,-1.91\%]$, $E_2(-1.91\%,1.63\%]$, $E_3(1.63\%,5.17\%]$, $E_4(5.17\%,8.71\%]$, $E_5(8.17\%,12.25\%]$.

Combining these state partitions with the probability of the current state transferring to the next state, we obtain the state transition matrix of steps one

$$\mathrm{P}(1)=\begin{bmatrix} 0.34 & 0.66 & 0 & 0 & 0 \\ 0.33 & 0.33 & 0.34 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

3.2.2. Huizhou GDP prediction

Constructing the new state transition matrix by using the several most recent groups of data, we get the state of 2017, which is listed in Table 4.

Table 4. The prediction status of 2017.

Year	Initial status	Transferring steps	P_{ij}	E_1	E_2	E_3	E_4	E_5
2016	2	1	P_{12}	0.33	0.33	0.34	0	0
2015	2	2	P_{22}	0.2211	0.3267	0.2822	0	0.17
2014	1	3	P_{31}	0.2593	0.3660	0.2625	0	0.1122
2013	1	4	P_{41}	0.2089	0.2919	0.2557	0.1122	0.1313
2012	2	5	P_{52}	0.1732	0.3160	0.2335	0.1261	0.1062
Total				1.1925	1.6765	1.3739	0.2383	0.5197

According to the results of Table 4, since E_2 is the maximum in summation, we deduce that the most likely state of Huizhou GDP in 2017 is E_2. The predicting values are 35182264 by FOANGBM model and 35231588 by Markov model, respectively. Applying the same method, we obtain the results of GDP from 2018–2021 forecasted by Markov model, which are listed in Table 3.

As can be seen from Table 3, the MAPE, RMSE, STD and R^2 fitted by FOANGBM (1,1) model are 3.34%, 755540, 3.28% and 99.12%, respectively, while these values are 0.85%, 156190, 0.62% and 99.97%, respectively, by applying FOANGBMKM (1,1) model. This indicates that the fitting effect of FOANGBMKM (1,1) model is better than FOANGBM (1,1) model. At the same time, the MAPE, RMSE, STD and R^2 predicted by FOANGBM (1,1) model are 5.76%, 3082800, 3.56% and 42.62%, respectively, while these values forecasted by FOANGBMKM (1,1) model are 5.68%, 3037100, 3.48% and 43.47%, respectively. This shows that the prediction accuracy of the FOANGBMKM (1,1) model is better than the FOANGBM (1,1) model.

The results in Figure 1 show that the curve of FOANGBMKM (1,1) model is closer to the true values than that of FOANGBM (1,1) model.

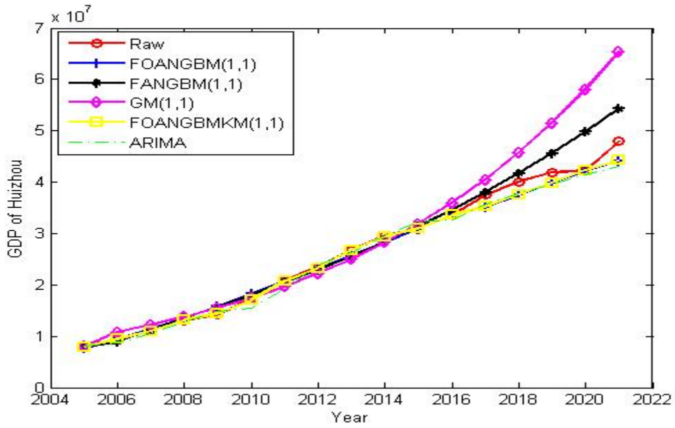

Figure 1. The forecast results of different models for GDP of Huizhou in China.

4. Conclusions and future directions

In this paper, a novel FOANGBMKM is proposed to predict the annual GDP of Huizhou city. The suitable states are determined by using the transition matrix of Markov. PSO algorithm is used to search for the optimal order as well as the optimal nonlinear parameters of the accumulating generation operator. According to the results of prediction of the statistical yearbook data from 2005 to 2016, we calculate four statistics, MAPE, RMSE, STD and R^2, for different models. According to the size of the values of these statistics, we find that the estimation accuracy of FOANGBM model is higher than that of GM, ARIMA and FANGBM models. At the same time, the fitting effect FOANGBMKM model is superior to FOANGBM model. Finally, the proposed model is applied to forecast the GDP of Huizhou city from 2017 to 2021. Compared to the opposite-direction accumulating linear Bernoulli model, the new model can more accurately and effectively to evaluate the development level of Huizhou GDP. The results show that the prediction effect of the proposed new model is better than that of the other four competitive models such as GM (1,1), ARIMA, FANGBM (1,1) and FOANGBM (1,1), which proves the greater accuracy and efficiency of the FOANGBMKM (1,1) model.

We will focus on the multi-variable GMs of electricity consumption that fully utilize potential factors. In addition, the other cutting-edge optimization algorithms are used to seek for the optimal parameters, such as ant lion optimization algorithm [36], grey wolf algorithm [37] and whale optimization algorithm [38]. Further, it is well known that the Optimal fractional accumulation GM with variable parameters is an efficient method to improve the prediction accuracy [39] and fractional time-varying grey traffic flow model based on viscoelastic fluid [40], which can be used for forecasting shorten prediction period, thus respectively establishing the Optimal fraction accumulation grey Markov model with variable parameters and fractional time-varying grey traffic flow Markov model will receive extensive attention in our next work.

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Conflicts of interest

The authors declare that they have no conflicts of interest.

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