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Research article

Hopf bifurcation analysis in a delayed diffusive predator-prey system with nonlocal competition and schooling behavior

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Abstract: We consider a delayed diffusive predator-prey system with nonlocal competition in prey and schooling behavior in predator. We mainly study the local stability and Hopf bifurcation at the positive equilibrium by using time delay as the parameter. We also analyze the property of Hopf bifurcation by center manifold theorem and normal form method. Through the numerical simulation, we obtain that time delay can affect the stability of the positive equilibrium and induce spatial inhomogeneous periodic oscillations of prey and predator's population densities. In addition, we observe that the increase of space area will not be conducive to the stability of the positive equilibrium (u_*, v_*), and may induce the inhomogeneous periodic oscillations of prey and predator's population densities under some values of the parameters.

Keywords: predator-prey; delay; Hopf bifurcation; nonlocal competition

1. Introduction

Predator-prey relationship exists widely in nature, and many scholars explore this relationship between populations by studying predator-prey model [1–5]. In the real world, the schooling behavior occurs for various reasons among both predator and prey population [6]. By schooling behavior, prey can effectively avoid the capture of predators, and predators can increase the success rate of predation. For example, the wolves [7], African wild dogs and lions [8] are famous examples who have the schooling behavior among predator individuals. To reflect this effect in predator, Cosner et al. [9] proposed the following functional response

$$\eta(u, v) = \frac{Ce_0 uv}{1 + t_h Ce_0 uv},$$

where u, v, C, e_0, and t_h represent density of prey, density of predator, capture rate, encounter rate, and handling time, respectively. The functional response $\eta(u, v)$ monotonically increases with respect to the predator. This reflects that the increase in the number of predators will be conducive to the success rate of predation.

The reaction diffusion equation is widely used in many fields, such as vegetation-water models [10, 11], bimolecular models [12,13], population models [14–16]. By introducing time and space variables, the reaction-diffusion model can better describe the development law of things. Incorporating the group cooperation in predator and the group defense behavior in prey, J. Yang [17] proposed the following reaction diffusion predator-prey model

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = D_1\Delta u + ru\left(1-\dfrac{u}{K}\right) - \dfrac{Ce_0\sqrt{u}v^2}{1+t_h\sqrt{u}v}, \\ \dfrac{\partial v(x,t)}{\partial t} = D_2\Delta v + v\left(\dfrac{\varepsilon Ce_0\sqrt{u(x,t-\tau)}v(x,t-\tau)}{1+t_h\sqrt{u(x,t-\tau)}v(x,t-\tau)} - d\right), \quad x\in\Omega,\ t>0 \\ \dfrac{\partial u(x,t)}{\partial \bar{v}} = \dfrac{\partial v(x,t)}{\partial \bar{v}} = 0, \quad x\in\partial\Omega,\ t>0 \\ u(x,\theta) = u_0(x,\theta)\geq 0, v(x,\theta) = v_0(x,\theta)\geq 0, \quad x\in\bar{\Omega}, \theta\in[-\tau,0], \end{cases} \tag{1.1}$$

where $u(x,t)$ and $v(x,t)$ represent prey and predator's densities, respectively. r, K, ε, τ and d represent growth rate, environmental capacity, conversion rate, gestation delay and death rate, respectively. The terms $\sqrt{u}$ and $\sqrt{u(t-\tau)}$ represent the herd behavior (or group defense behavior) in prey. They studied saddle-node, Hopf and Bogdanov-Takens types of bifurcations, and discussed the effect of diffusion and time delay on this model through numerical simulations [17].

In the model (1.1), the competition in prey is reflected by the term $-\frac{u}{K}$, which supposes this type competition is spatially local. In fact, the resources is limited in nature, and competition within the population always exist. This competition is usually nonlocal. In [18,19], the authors suggested that the consumption of resources in spatial location is related not only to the local population density, but also to the number of nearby population density. Some scholars have studied the predator-prey models with nonlocal competition [20–22]. S. Chen et al studied the existence and uniqueness of positive steady states and Hopf bifurcation in a diffusive predator-prey model with nonlocal effect [20]. J. Gao and S. Guo discussed the steady-state bifurcation and Hopf bifurcation in a diffusive predator-prey model with nonlocal effect and Beddington-DeAngelis Functional Response [21]. S. Djilali studied the pattern formation in a diffusive predator-prey model with herd behavior and nonlocal prey competition, and showed rich dynamic phenomena through numerical simulations [23]. These works suggest that the predator-prey models with nonlocal competition will exhibit different dynamic phenomena compared with the model without nonlocal competition, for example the stably spatially inhomogeneous periodic solutions are more likely to appear.

Based on the model (1.1), we assume there is spatially nonlocal competition in prey. Then, we proposed the following model.

$$
\begin{cases}
\dfrac{\partial u(x,t)}{\partial t} = d_1\Delta u + u\left(1 - \displaystyle\int_\Omega G(x,y)u(y,t)dy\right) - \dfrac{\alpha\sqrt{u}v^2}{1+\sqrt{u}v}, \\
\dfrac{\partial v(x,t)}{\partial t} = d_2\Delta v + v\left(\dfrac{\beta\sqrt{u(t-\tau)}v(t-\tau)}{1+\sqrt{u(t-\tau)}v(t-\tau)} - \gamma\right), \quad x\in\Omega,\ t>0, \\
\dfrac{\partial u(x,t)}{\partial \bar{\nu}} = \dfrac{\partial v(x,t)}{\partial \bar{\nu}} = 0, \quad x\in\partial\Omega,\ t>0, \\
u(x,\theta) = u_0(x,\theta) \geq 0, v(x,\theta) = v_0(x,\theta) \geq 0, \quad x\in\bar{\Omega}, \theta\in[-\tau,0].
\end{cases}
\tag{1.2}
$$

The model (1.2) has been changed by $\tilde{t} = r\tilde{t}$, $\tilde{u} = \frac{\tilde{u}}{K}$, $\tilde{v} = t_hCe_0\sqrt{K}v$, $\alpha = \frac{1}{rt_h^2Ce_0K^{3/2}}$, $\beta = \frac{\varepsilon}{rt_h}$ and $\gamma = \frac{d}{r}$, then drop the tilde. $\int_\Omega G(x,y)u(y,t)dy$ represents the nonlocal competition effect in prey. We also choose the Newman boundary condition, which is based on the hypothesis that the region is closed and no prey and predator can leave or enter the boundary.

With the scope of our knowledge, there is no work to study the dynamics of the predator-prey model (1.2) with the nonlocal competition in prey, schooling behavior in predator, reaction diffusion and gestation delay, although it seems more realistic. The aim of this paper is to study the effect of time delay and nonlocal competition on the model (1.2). Whether there exist stable spatially inhomogeneous periodic solutions?

The paper is organized as follows. In Section 2, the stability of coexisting equilibrium and existence of Hopf bifurcation are considered. In Section 3, the property of Hopf bifurcation is studied. In Section 4, some numerical simulations are given. In Section 5, a short conclusion is obtained.

2. Stability analysis

For convenience, we choose $\Omega = (0, l\pi)$. The kernel function $G(x,y) = \frac{1}{l\pi}$, which is based on the assumption that the competition strength among prey individuals in the habitat is the same, that is the competition between any two prey is the same. $(0,0)$ and $(1,0)$ are boundary equilibria of model (1.2). The existence of positive equilibria of model (1.2) has been studied in [17], that is

Lemma 2.1. *[17] Assume $\beta > \gamma$, then the model* (1.2) *has*

- *two distinct coexisting equilibria $E_1 = (u_1, v_1)$ and $E_2 = (u_2, v_2)$ with* $0 < u_1 < \frac{3}{5} < u_2 < 1$ *when* $\alpha < \alpha_c(\beta,\gamma) := \frac{6\sqrt{15}\beta(\beta-\gamma)}{125\gamma^2}$;
- *a unique coexisting equilibrium denoted by $E_3 = (u_3, v_3)$ when $\alpha = \alpha_c(\beta,\gamma)$;*
- *no coexisting equilibrium when $\alpha > \alpha_c(\beta,\gamma)$.*

Make the following hypothesis

$$
(\mathbf{H_0}) \qquad \beta > \gamma, \ \ \alpha \leq \alpha_c(\beta,\gamma). \tag{2.1}
$$

If $(\mathbf{H_0})$ holds, then model (1.2) has one or two coexisting equilibria. Hereinafter, for brevity, we just denote $E_*(u_*, v_*)$ as coexisting equilibrium. Linearize model (1.2) at $E_*(u_*, v_*)$

$$
\frac{\partial u}{\partial t}\begin{pmatrix} u(x,t) \\ u(x,t) \end{pmatrix} = D\begin{pmatrix} \Delta u(t) \\ \Delta v(t) \end{pmatrix} + L_1\begin{pmatrix} u(x,t) \\ v(x,t) \end{pmatrix} + L_2\begin{pmatrix} u(x,t-\tau) \\ v(x,t-\tau) \end{pmatrix} + L_3\begin{pmatrix} \hat{u}(x,t) \\ \hat{v}(x,t) \end{pmatrix}, \tag{2.2}
$$

where

$$D = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}, \quad L_1 = \begin{pmatrix} a_1 & a_2 \\ 0 & 0 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 0 & 0 \\ b_1 & b_2 \end{pmatrix}, \quad L_3 = \begin{pmatrix} -u_* & 0 \\ 0 & 0 \end{pmatrix},$$

and $a_1 = 1 - u_* - \frac{v_*^2\alpha}{2\sqrt{u_*}(1+\sqrt{u_*}v_*)^2}$, $a_2 = -\frac{(2\sqrt{u_*}v_*+u_*v_*^2)\alpha}{(1+\sqrt{u_*}v_*)^2} < 0$, $b_1 = \frac{v_*^2\beta}{2\sqrt{u_*}(1+\sqrt{u_*}v_*)^2} > 0$, $b_2 = \frac{\sqrt{u_*}v_*\beta}{(1+\sqrt{u_*}v_*)^2} > 0$, $\hat{u} = \frac{1}{l\pi}\int_0^{l\pi} u(y,t)dy$. The characteristic equation is

$$\lambda^2 + A_n\lambda + B_n + (C_n - b_2\lambda)e^{-\lambda\tau} = 0, \;\; n \in \mathbb{N}_0, \tag{2.3}$$

where

$$\begin{aligned} &A_0 = u_* - a_1, \;\; B_0 = 0, \;\; C_0 = -b_2(u_* - a_1) - a_2 b_1, \\ &A_n = (d_1 + d_2)\frac{n^2}{l^2} - a_1, \;\; B_n = d_1 d_2 \frac{n^4}{l^4} - a_1 d_2 \frac{n^2}{l^2}, \\ &C_n = -b_2 d_1 \frac{n^2}{l^2} + a_1 b_2 - a_2 b_1, \;\; n \in \mathbb{N}. \end{aligned} \tag{2.4}$$

When $\tau = 0$, the characteristic Eq (2.3) is

$$\lambda^2 + (A_n - b_2)\lambda + B_n + C_n = 0, \quad n \in \mathbb{N}_0, \tag{2.5}$$

where

$$\begin{cases} A_0 - b_2 = -a_1 + u_* - b_2, \;\; B_0 + C_0 = -b_2(u_* - a_1) - a_2 b_1, \\ A_n - b_2 = (d_1 + d_2)\frac{n^2}{l^2} - a_1 - b_2, \\ B_n + C_n = d_1 d_2 \frac{n^4}{l^4} - (a_1 d_2 + b_2 d_1)\frac{n^2}{l^2} + a_1 b_2 - a_2 b_1, \;\; n \in \mathbb{N}. \end{cases} \tag{2.6}$$

Make the following hypothesis

$$(\mathbf{H_1}) \qquad A_n - b_2 > 0, \; B_n + C_n > 0, \; \text{for } n \in \mathbb{N}_0. \tag{2.7}$$

Theorem 2.2. *For model* (1.2)*, assume $\tau = 0$ and* ($\mathbf{H_0}$) *holds. Then $E_*(u_*, v_*)$ is locally asymptotically stable under* ($\mathbf{H_1}$).

Proof. If ($\mathbf{H_1}$) holds, we can obtain that the characteristic root of (2.5) all have negative real parts. Then $E_*(u_*, v_*)$ is locally asymptotically stable.

Let $\mathrm{i}\omega$ ($\omega > 0$) be a solution of Eq (2.3), then

$$-\omega^2 + \mathrm{i}\omega A_n + B_n + (C_n - b_2\mathrm{i}\omega)(\cos\omega\tau - \mathrm{i}\sin\omega\tau) = 0.$$

We can obtain $\cos\omega\tau = \frac{\omega^2(b_2 A_n + C_n) - B_n C_n}{C_n^2 + b_2^2\omega^2}$, $\sin\omega\tau = \frac{\omega(A_n C_n + B_n b_2 - b_2\omega^2)}{C_n^2 + b_2^2\omega^2}$. It leads to

$$\omega^4 + \omega^2\left(A_n^2 - 2B_n - b_2^2\right) + B_n^2 - C_n^2 = 0. \tag{2.8}$$

Let $z = \omega^2$, then (2.8) becomes

$$z^2 + z\left(A_n^2 - 2B_n - b_2^2\right) + B_n^2 - C_n^2 = 0, \tag{2.9}$$

and the roots of (2.9) are $z^{\pm} = \frac{1}{2}[-P_n \pm \sqrt{P_n^2 - 4Q_nR_n}]$, where $P_n = A_n^2 - 2B_n - b_2^2$, $Q_n = B_n + C_n$, and $R_n = B_n - C_n$. If $(\mathbf{H_0})$ and $(\mathbf{H_1})$ hold, $Q_n > 0$ $(n \in \mathbb{N}_0)$. By direct calculation, we have

$$\begin{aligned} P_0 &= (a_1 - u_*)^2 - b_2^2 > 0, \\ P_k &= \left(a_1 - d_1\frac{k^2}{l^2}\right)^2 + d_2^2\frac{n^4}{l^4} - b_2^2, \\ R_0 &= a_2b_1 + b_2(u_* - a_1) \\ R_k &= d_1d_2\frac{k^4}{l^4} + (b_2d_1 - a_1d_2)\frac{k^2}{l^2} + a_2b_1 - a_1b_2, \quad \text{for } k \in \mathbb{N}. \end{aligned} \tag{2.10}$$

Define

$$\begin{aligned} \mathbb{W}_1 &= \{n|R_n < 0,\ n \in \mathbb{N}_0\}, \\ \mathbb{W}_2 &= \{n|R_n > 0,\ P_n < 0,\ P_n^2 - 4Q_nR_n > 0,\ n \in \mathbb{N}\}, \\ \mathbb{W}_3 &= \{n|R_n > 0,\ P_n^2 - 4Q_nR_n < 0,\ n \in \mathbb{N}_0\}, \end{aligned} \tag{2.11}$$

and

$$\begin{aligned} &\omega_n^{\pm} = \sqrt{z_n^{\pm}}, \qquad \tau_n^{j,\pm} = \begin{cases} \frac{1}{\omega_n^{\pm}}\arccos(V_{cos}^{(n,\pm)}) + 2j\pi, & V_{sin}^{(n,\pm)} \geq 0, \\ \frac{1}{\omega_n^{\pm}}\left[2\pi - \arccos(V_{cos}^{(n,\pm)})\right] + 2j\pi, & V_{sin}^{(n,\pm)} < 0. \end{cases} \\ &V_{cos}^{(n,\pm)} = \frac{(\omega_n^{\pm})^2(b_2A_n + C_n) - B_nC_n}{C_n^2 + b_2^2(\omega_n^{\pm})^2}, \quad V_{sin}^{(n,\pm)} = \frac{\omega_n^{\pm}\left(A_nC_n + B_nb_2 - b_2(\omega_n^{\pm})^2\right)}{C_n^2 + b_2^2(\omega_n^{\pm})^2}. \end{aligned} \tag{2.12}$$

We have the following lemma.

Lemma 2.3. *Assume* $(\mathbf{H_0})$ *and* $(\mathbf{H_1})$ *hold, the following results hold.*
- *Eq* (2.3) *has a pair of purely imaginary roots* $\pm i\omega_n^+$ *at* $\tau_n^{j,+}$ *for* $j \in \mathbb{N}_0$ *and* $n \in \mathbb{W}_1$.
- *Eq* (2.3) *has two pairs of purely imaginary roots* $\pm i\omega_n^{\pm}$ *at* $\tau_n^{j,\pm}$ *for* $j \in \mathbb{N}_0$ *and* $n \in \mathbb{W}_2$.
- *Eq* (2.3) *has no purely imaginary root for* $n \in \mathbb{W}_3$.

Lemma 2.4. *Assume* $(\mathbf{H_0})$ *and* $(\mathbf{H_1})$ *hold. Then* $Re(\frac{d\lambda}{d\tau})|_{\tau=\tau_n^{j,+}} > 0$, $Re(\frac{d\lambda}{d\tau})|_{\tau=\tau_n^{j,-}} < 0$ *for* $n \in \mathbb{W}_1 \cup \mathbb{W}_2$ *and* $j \in \mathbb{N}_0$.

Proof. By Eq (2.3), we have

$$(\frac{d\lambda}{d\tau})^{-1} = \frac{2\lambda + A_n - b_2e^{-\lambda\tau}}{(C_n - b_2\lambda)\lambda e^{-\lambda\tau}} - \frac{\tau}{\lambda}.$$

Then

$$\begin{aligned} [\mathrm{Re}(\frac{d\lambda}{d\tau})^{-1}]_{\tau=\tau_n^{j,\pm}} &= \mathrm{Re}[\frac{2\lambda + A_n - b_2e^{-\lambda\tau}}{(C_n - b_2\lambda)\lambda e^{-\lambda\tau}} - \frac{\tau}{\lambda}]_{\tau=\tau_n^{j,\pm}} \\ &= [\frac{1}{C_n^2 + b_2^2\omega^2}(2\omega^2 + A_n^2 - 2B_n - b_2^2)]_{\tau=\tau_n^{j,\pm}} \\ &= \pm[\frac{1}{C_n^2 + b_2^2\omega^2}\sqrt{(A_n^2 - 2B_n - b_2^2)^2 - 4(B_n^2 - C_n^2)}]_{\tau=\tau_n^{j,\pm}}. \end{aligned}$$

Therefore $\mathrm{Re}(\frac{d\lambda}{d\tau})|_{\tau=\tau_n^{j,+}} > 0$, $\mathrm{Re}(\frac{d\lambda}{d\tau})|_{\tau=\tau_n^{j,-}} < 0$.

Denote $\tau_* = min\{\tau_n^0 | \, n \in \mathbb{W}_1 \cup \mathbb{W}_2\}$. We have the following theorem.

Theorem 2.5. *Assume* $(\mathbf{H_0})$ *and* $(\mathbf{H_1})$ *hold, then the following statements are true for model* (1.2).

- $E_*(u_*, v_*)$ *is locally asymptotically stable for* $\tau > 0$ *when* $\mathbb{W}_1 \cup \mathbb{W}_2 = \varnothing$.
- $E_*(u_*, v_*)$ *is locally asymptotically stable for* $\tau \in [0, \tau_*)$ *when* $\mathbb{W}_1 \cup \mathbb{W}_2 \neq \varnothing$.
- $E_*(u_*, v_*)$ *is unstable for* $\tau \in (\tau_*, \tau_* + \varepsilon)$ *for some* $\varepsilon > 0$ *when* $\mathbb{W}_1 \cup \mathbb{W}_2 \neq \varnothing$.
- *Hopf bifurcation occurs at* (u_*, v_*) *when* $\tau = \tau_n^{j,+}$ $(\tau = \tau_n^{j,-})$, $j \in \mathbb{N}_0$, $n \in \mathbb{W}_1 \cup \mathbb{W}_2$. *The bifurcating periodic solutions are spatially homogeneous when* $\tau = \tau_0^{j,+}$ $(\tau = \tau_0^{j,-})$, *and spatially inhomogeneous when* $\tau = \tau_n^{j,+}$ $(\tau = \tau_n^{j,-})$ *for* $n \in \mathbb{N}$.

3. Property of Hopf bifurcation

By the works [24,25], we study the property of Hopf bifurcation. For fixed $j \in \mathbb{N}_0$ and $n \in \mathbb{W}_1 \cup \mathbb{W}_2$, we denote $\tilde{\tau} = \tau_n^{j,\pm}$. Let $\bar{u}(x,t) = u(x,\tau t) - u_*$ and $\bar{v}(x,t) = v(x,\tau t) - v_*$. Drop the bar, (1.2) can be written as

$$\begin{cases} \dfrac{\partial u}{\partial t} = \tau[d_1 \Delta u + (u + u_*)\left(1 - \dfrac{1}{l\pi}\displaystyle\int_0^{l\pi} (u(y,t) + u_*)dy\right) - \dfrac{\alpha\sqrt{u+u_*}(v+v_*)^2}{1 + \sqrt{u+u_*}(v+v_*)}], \\ \dfrac{\partial v}{\partial t} = \tau[d_2 \Delta v + \left(\dfrac{\beta\sqrt{u(t-1)+u_*}(v(t-1)+v_*)}{1 + \sqrt{u(t-1)+u_*}(v(t-1)+v_*)} - \gamma\right)(v+v_*)]. \end{cases} \tag{3.1}$$

Rewrite the model (3.1) as

$$\begin{cases} \dfrac{\partial u}{\partial t} = \tau[d_1\Delta u + a_1 u + a_2 v - u_*\hat{u} + \alpha_1 u^2 - u\hat{u} + \alpha_2 uv + \alpha_3 v^2 + \alpha_4 u^3 + \alpha_5 u^2 v + \alpha_6 uv^2 \\ \qquad + \alpha_7 v^3] + h.o.t., \\ \dfrac{\partial v}{\partial t} = \tau[d_2\Delta v + b_1 u(t-1) + b_2 v(t-1) + \beta_1 u^2(t-1) + \beta_2 u(t-1)v(t-1) + \beta_3 u^2(t-1) \\ \qquad + \beta_4 u^3(t-1) + \beta_5 u^2(t-1)v(t-1)] + \beta_6 u(t-1)v^2(t-1) + \beta_7 v^3(t-1)] + h.o.t., \end{cases} \tag{3.2}$$

where $\alpha_1 = \frac{v_*^2(1+3\sqrt{u_*}v_*)\alpha}{8u_*^{3/2}(1+\sqrt{u_*}v_*)^3}$, $\alpha_2 = -\frac{v_*\alpha}{\sqrt{u_*}(1+\sqrt{u_*}v_*)^3}$, $\alpha_3 = -\frac{\sqrt{u_*}\alpha}{(1+\sqrt{u_*}v_*)^3}$, $\alpha_4 = -\frac{v_*^2(1+4\sqrt{u_*}v_*+5u_*v_*^2)\alpha}{16u_*^{5/2}(1+\sqrt{u_*}v_*)^4}$, $\alpha_5 = \frac{v_*(1+4\sqrt{u_*}v_*)\alpha}{u_*^{3/2}(1+\sqrt{u_*}v_*)^4}$, $\alpha_6 = \frac{(-1+2\sqrt{u_*}v_*)\alpha}{2\sqrt{u_*}(1+\sqrt{u_*}v_*)^4}$, $\alpha_7 = \frac{u_*\alpha}{(1+\sqrt{u_*}v_*)^4}$, $\beta_1 = -\frac{v_*^2(1+3\sqrt{u_*}v_*)\beta}{2u_*^{3/2}(1+\sqrt{u_*}v_*)^3}$, $\beta_2 = -\frac{v_*(-1+\sqrt{u_*}v_*)\beta}{2\sqrt{u_*}(1+\sqrt{u_*}v_*)^3}$, $\beta_3 = -\frac{u_*v_*\beta}{(1+\sqrt{u_*}v_*)^3}$, $\beta_4 = \frac{v_*^2(1+4\sqrt{u_*}v_*+5u_*v_*^2)\beta}{16u_*^{5/2}(1+\sqrt{u_*}v_*)^4}$, $\beta_5 = \frac{v_*(-1-4\sqrt{u_*}v_*+3u_*v_*^2)\beta}{2u_*^{3/2}(1+\sqrt{u_*}v_*)^4}$, $\beta_6 = \frac{v_*(-2+\sqrt{u_*}v_*)\beta}{2(1+\sqrt{u_*}v_*)^4}$, $\beta_7 = \frac{u_*^{3/2}v_*\beta}{(1+\sqrt{u_*}v_*)^4}$.

Define the real-valued Sobolev space $X := \left\{(u,v)^T : u, v \in H^2(0, l\pi),\ (u_x, v_x)|_{x=0,l\pi} = 0\right\}$, the complexification of X $X_{\mathbb{C}} := X \oplus \mathrm{i}X = \{x_1 + \mathrm{i}x_2 | \, x_1, x_2 \in X\}$. and the inner product $< \tilde{u}, \tilde{v} > := \int_0^{l\pi} \overline{u_1} v_1 dx + \int_0^{l\pi} \overline{u_2} v_2 dx$ for $\tilde{u} = (u_1, u_2)^T$, $\tilde{v} = (v_1, v_2)^T$, $\tilde{u}, \tilde{v} \in X_{\mathbb{C}}$. The phase space $\mathscr{C} := C([-1,0], X)$ is with the sup norm, then we can write $\phi_t \in \mathscr{C}$, $\phi_t(\theta) = \phi(t+\theta)$ or $-1 \le \theta \le 0$. Denote $\beta_n^{(1)}(x) = (\gamma_n(x), 0)^T$, $\beta_n^{(2)}(x) = (0, \gamma_n(x))^T$, and $\beta_n = \{\beta_n^{(1)}(x), \beta_n^{(2)}(x)\}$, where $\{\beta_n^{(i)}(x)\}$ is an an orthonormal basis of X. We define the subspace of $\mathscr{C}$ as $\mathbb{B}_n := \mathrm{span}\{< \phi(\cdot), \beta_n^{(j)} > \beta_n^{(j)} | \phi \in \mathscr{C}, j = 1, 2\}$, $n \in \mathbb{N}_0$. There exists a 2×2 matrix function $\eta^n(\sigma, \tilde{\tau})$ $-1 \le \sigma \le 0$, such that $-\tilde{\tau} D \frac{n^2}{l^2} \phi(0) + \tilde{\tau} L(\phi) = \int_{-1}^0 d\eta^n(\sigma, \tau)\phi(\sigma)$ for $\phi \in \mathscr{C}$. The bilinear form on $\mathscr{C}^* \times \mathscr{C}$ is defined by

$$(\psi, \phi) = \psi(0)\phi(0) - \int_{-1}^0 \int_{\xi=0}^{\sigma} \psi(\xi - \sigma) d\eta^n(\sigma, \tilde{\tau})\phi(\xi) d\xi, \tag{3.3}$$

for $\phi \in \mathscr{C}, \psi \in \mathscr{C}^*$. Define $\tau = \tilde{\tau}+\mu$, then the system undergoes a Hopf bifurcation at $(0,0)$ when $\mu = 0$, with a pair of purely imaginary roots $\pm \mathrm{i}\omega_{n_0}$. Let A denote the infinitesimal generators of semigroup, and A^* be the formal adjoint of A under the bilinear form (3.3). Define the following function

$$\delta(n_0) = \begin{cases} 1 & n_0 = 0, \\ 0 & n_0 \in \mathbb{N}. \end{cases} \tag{3.4}$$

Choose $\eta_{n_0}(0,\tilde{\tau}) = \tilde{\tau}[(-n_0^2/l^2)D + L_1 + L_3\delta(n_{n_0})]$, $\eta_{n_0}(-1,\tilde{\tau}) = -\tilde{\tau}L_2$, $\eta_{n_0}(\sigma,\tilde{\tau}) = 0$ for $-1 < \sigma < 0$. Let $p(\theta) = p(0)e^{\mathrm{i}\omega_{n_0}\tilde{\tau}\theta}$ $(\theta \in [-1,0])$, $q(\vartheta) = q(0)e^{-\mathrm{i}\omega_{n_0}\tilde{\tau}\vartheta}$ $(\vartheta \in [0,1])$ be the eigenfunctions of $A(\tilde{\tau})$ and A^* corresponds to $i\omega_{n_0}\tilde{\tau}$ respectively. We can choose $p(0) = (1,p_1)^T$, $q(0) = M(1,q_2)$, where $p_1 = \frac{1}{a_2}(\mathrm{i}\omega_{n_0} + d_1n_0^2/l^2 - a_1 + u_*\delta(n_0))$, $q_2 = a_2/(\mathrm{i}\omega_{n_0} - b_2e^{\mathrm{i}\tau\omega_{n_0}} + \frac{d_2n^2}{l^2})$, and $M = (1 + p_1q_2 + \tilde{\tau}q_2(b_1 + b_2p_1)e^{-\mathrm{i}\omega_{n_0}\tilde{\tau}})^{-1}$. Then (3.1) can be rewritten in an abstract form

$$\frac{dU(t)}{dt} = (\tilde{\tau}+\mu)D\Delta U(t) + (\tilde{\tau}+\mu)[L_1(U_t) + L_2U(t-1) + L_3\hat{U}(t)] + F(U_t,\hat{U}_t,\mu), \tag{3.5}$$

where

$$F(\phi,\mu) = (\tilde{\tau}+\mu)\begin{pmatrix} \alpha_1\phi_1(0)^2 - \phi_1(0)\hat{\phi}_1(0) + \alpha_2\phi_1(0)\phi_2(0) + \alpha_3\phi_2(0)^2 + \alpha_4\phi_1^3(0) + \alpha_5\phi_1^2(0)\phi_2(0) \\ +\alpha_6\phi_1(0)\phi_2^2(0) + \alpha_7\phi_2^3(0) \\ \beta_1\phi_1^2(-1) + \beta_2\phi_1(-1)\phi_2(-1) + \beta_3\phi_2^2(-1) + \beta_4\phi_1^3(-1) + \beta_4\phi_1^2(-1)\phi_2(-1) \\ +\beta_6\phi_1(-1)\phi_2^2(-1) + \beta_7\phi_2^3(-1) \end{pmatrix} \tag{3.6}$$

respectively, for $\phi = (\phi_1,\phi_2)^T \in \mathscr{C}$ and $\hat{\phi}_1 = \frac{1}{l\pi}\int_0^{l\pi}\phi dx$. Then the space $\mathscr{C}$ can be decomposed as $\mathscr{C} = P \oplus Q$, where $P = \{zp\gamma_{n_0}(x) + \bar{z}\bar{p}\gamma_{n_0}(x) | z \in \mathscr{C}\}$, $Q = \{\phi \in \mathscr{C} | (q\gamma_{n_0}(x),\phi) = 0 \text{ and } (\bar{q}\gamma_{n_0}(x),\phi) = 0\}$. Then, model (3.6) can be rewritten as $U_t = z(t)p(\cdot)\gamma_{n_0}(x) + \bar{z}(t)\bar{p}(\cdot)\gamma_{n_0}(x) + \omega(t,\cdot)$ and $\hat{U}_t = \frac{1}{l\pi}\int_0^{l\pi}U_tdx$, where

$$z(t) = (q\gamma_{n_0}(x),U_t), \quad \omega(t,\theta) = U_t(\theta) - 2\mathrm{Re}\{z(t)p(\theta)\gamma_{n_0}(x)\}. \tag{3.7}$$

then, we have $\dot{z}(t) = \mathrm{i}\omega)n_0\tilde{\tau}z(t) + \bar{q}(0) < F(0,U_t),\beta_{n_0} >$. There exists a center manifold C_0 and ω can be written as follow near $(0,0)$.

$$\omega(t,\theta) = \omega(z(t),\bar{z}(t),\theta) = \omega_{20}(\theta)\frac{z^2}{2} + \omega_{11}(\theta)z\bar{z} + \omega_{02}(\theta)\frac{\bar{z}^2}{2} + \cdots. \tag{3.8}$$

Then, restrict the system to the center manifold is $\dot{z}(t) = \mathrm{i}\omega_{n_0}\tilde{\tau}z(t) + g(z,\bar{z})$. Denote $g(z,\bar{z}) = g_{20}\frac{z^2}{2} + g_{11}z\bar{z} + g_{02}\frac{\bar{z}^2}{2} + g_{21}\frac{z^2\bar{z}}{2} + \cdots$. By direct computation, we have

$$g_{20} = 2\tilde{\tau}M(\varsigma_1 + q_2\varsigma_2)I_3, \quad g_{11} = \tilde{\tau}M(\varrho_1 + q_2\varrho_2)I_3, \quad g_{02} = \bar{g}_{20},$$

$$g_{21} = 2\tilde{\tau}M[(\kappa_{11} + q_2\kappa_{21})I_2 + (\kappa_{12} + q_2\kappa_{22})I_4],$$

where $I_2 = \int_0^{l\pi}\gamma_{n_0}^2(x)dx$, $I_3 = \int_0^{l\pi}\gamma_{n_0}^3(x)dx$, $I_4 = \int_0^{l\pi}\gamma_{n_0}^4(x)dx$, $\varsigma_1 = -\delta\mathrm{n} + \alpha_1 + \alpha_2\xi + \alpha_3\xi^2$, $\varsigma_2 = e^{-2i\tau\omega_n}(\beta_1 + \xi(\beta_2 + \beta_3\xi))$, $\varrho_1 = \frac{1}{4}(2\alpha_1 - 2\delta\mathrm{n} + \alpha_2\bar{\xi} + \alpha_2\xi + 2\alpha_3\bar{\xi}\xi)$, $\varrho_2 = \frac{1}{4}(2\beta_1 + 2\beta_3\bar{\xi}\xi + \beta_2(\bar{\xi}+\xi))$, $\kappa_{11} = 2W_{11}^{(1)}(0)(-1+2\alpha_1-\delta\mathrm{n}+\alpha_2\xi) + 2W_{11}^{(2)}(0)(\alpha_2+2\alpha_3\xi) + W_{20}^{(1)}(0)(-1+2\alpha_1-\delta\mathrm{n}+\alpha_2\bar{\xi}) + W_{20}^{(2)}(0)(\alpha_2+2\alpha_3\bar{\xi})$, $\kappa_{12} = \frac{1}{2}(3\alpha_4+\alpha_5(\bar{\xi}+2\xi)+\xi(2\alpha_6\bar{\xi}+\alpha_6\xi+3\alpha_7\bar{\xi}\xi))$, $\kappa_{21} = 2e^{-i\tau\omega_n}W_{11}^{(1)}(-1)(2\beta_1+\beta_2\xi) + 2e^{-i\tau\omega_n}W_{11}^{(2)}(-1)(\beta_2+$

$2\beta_3\xi) + e^{i\tau\omega_n}W_{20}^{(1)}(-1)(2\beta_1 + \beta_2\bar{\xi}) + e^{i\tau\omega_n}W_{20}^{(2)}(-1)(\beta_2 + 2\beta_3\bar{\xi})$, $\kappa_{22} = \frac{1}{2}e^{-i\tau\omega_n}(3\beta_4 + \beta_5(\bar{\xi} + 2\xi) + \xi(2\beta_6\bar{\xi} + \beta_6\xi + 3\beta_7\bar{\xi}\xi))$.

Now, we compute $W_{20}(\theta)$ and $W_{11}(\theta)$ for $\theta \in [-1, 0]$ to give g_{21}. By (3.7), we have

$$\dot{\omega} = \dot{U}_t - \dot{z}p\gamma_{n_0}(x) - \dot{\bar{z}}\bar{p}\gamma_{n_0}(x) = A\omega + H(z, \bar{z}, \theta), \tag{3.9}$$

where

$$H(z, \bar{z}, \theta) = H_{20}(\theta)\frac{z^2}{2} + H_{11}(\theta)z\bar{z} + H_{02}(\theta)\frac{\bar{z}^2}{2} + \cdots. \tag{3.10}$$

Compare the coefficients of (3.8) with (3.9), we have

$$(A - 2\mathrm{i}\omega_{n_0}\tilde{\tau}I)\omega_{20} = -H_{20}(\theta), \;\; A\omega_{11}(\theta) = -H_{11}(\theta). \tag{3.11}$$

Then, we have

$$\begin{aligned} \omega_{20}(\theta) &= \frac{-g_{20}}{\mathrm{i}\omega_{n_0}\tilde{\tau}}p(0)e^{\mathrm{i}\omega_{n_0}\tilde{\tau}\theta} - \frac{\bar{g}_{02}}{3\mathrm{i}\omega_{n_0}\tilde{\tau}}\bar{p}(0)e^{-\mathrm{i}\omega_{n_0}\tilde{\tau}\theta} + E_1e^{2\mathrm{i}\omega_{n_0}\tilde{\tau}\theta}, \\ \omega_{11}(\theta) &= \frac{g_{11}}{\mathrm{i}\omega_{n_0}\tilde{\tau}}p(0)e^{\mathrm{i}\omega_{n_0}\tilde{\tau}\theta} - \frac{\bar{g}_{11}}{\mathrm{i}\omega_{n_0}\tilde{\tau}}\bar{p}(0)e^{-\mathrm{i}\omega_{n_0}\tilde{\tau}\theta} + E_2, \end{aligned} \tag{3.12}$$

where $E_1 = \sum_{n=0}^{\infty} E_1^{(n)}$, $E_2 = \sum_{n=0}^{\infty} E_2^{(n)}$,

$$\begin{aligned} E_1^{(n)} &= (2\mathrm{i}\omega_{n_0}\tilde{\tau}I - \int_{-1}^{0} e^{2\mathrm{i}\omega_{n_0}\tilde{\tau}\theta}d\eta_{n_0}(\theta, \tilde{\tau}))^{-1} < \tilde{F}_{20}, \beta_n >, \\ E_2^{(n)} &= -(\int_{-1}^{0} d\eta_{n_0}(\theta, \tilde{\tau}))^{-1} < \tilde{F}_{11}, \beta_n >, \;\; n \in \mathbb{N}_0, \end{aligned} \tag{3.13}$$

$$< \tilde{F}_{20}, \beta_n >= \begin{cases} \frac{1}{l\pi}\hat{F}_{20}, & n_0 \neq 0, n = 0, \\ \frac{1}{2l\pi}\hat{F}_{20}, & n_0 \neq 0, n = 2n_0, \\ \frac{1}{l\pi}\hat{F}_{20}, & n_0 = 0, n = 0, \\ 0, & other, \end{cases} \qquad < \tilde{F}_{11}, \beta_n >= \begin{cases} \frac{1}{l\pi}\hat{F}_{11}, & n_0 \neq 0, n = 0, \\ \frac{1}{2l\pi}\hat{F}_{11}, & n_0 \neq 0, n = 2n_0, \\ \frac{1}{l\pi}\hat{F}_{11}, & n_0 = 0, n = 0, \\ 0, & other, \end{cases}$$

and $\hat{F}_{20} = 2(\varsigma_1, \varsigma_2)^T$, $\hat{F}_{11} = 2(\varrho_1, \varrho_2)^T$.

Thus, we can obtain

$$\begin{aligned} c_1(0) &= \frac{\mathrm{i}}{2\omega_n\tilde{\tau}}(g_{20}g_{11} - 2|g_{11}|^2 - \frac{|g_{02}|^2}{3}) + \frac{1}{2}g_{21}, \quad \mu_2 = -\frac{\mathrm{Re}(c_1(0))}{\mathrm{Re}(\lambda'(\tilde{\tau}))}, \\ T_2 &= -\frac{1}{\omega_{n_0}\tilde{\tau}}[\mathrm{Im}(c_1(0)) + \mu_2\mathrm{Im}(\lambda'(\tau_n^j))], \quad \beta_2 = 2\mathrm{Re}(c_1(0)). \end{aligned} \tag{3.14}$$

Theorem 3.1. *For any critical value τ_n^j ($n \in \mathbb{S}$, $j \in \mathbb{N}_0$), we have the following results.*

• *When $\mu_2 > 0$ (resp.¡0), the Hopf bifurcation is forward (resp. backward).*

• *When $\beta_2 < 0$ (resp. ¿0), the bifurcating periodic solutions on the center manifold are orbitally asymptotically stable (resp. unstable).*

• *When $T_2 > 0$ (resp. $T_2 < 0$), the period increases (resp. decreases).*

4. Numerical simulations

To verify our theoretical results, we give the following numerical simulations. Fix parameters

$$\alpha = 1.07, \ \gamma = 0.2, \ l = 2, \ d_1 = 1, \ d_2 = 1. \tag{4.1}$$

The bifurcation diagram of model (1.2) with parameter β is given in Figure 1. We can see that with the increase of parameter β, the stable region of positive equilibrium (u_*, v_*) will decrease.

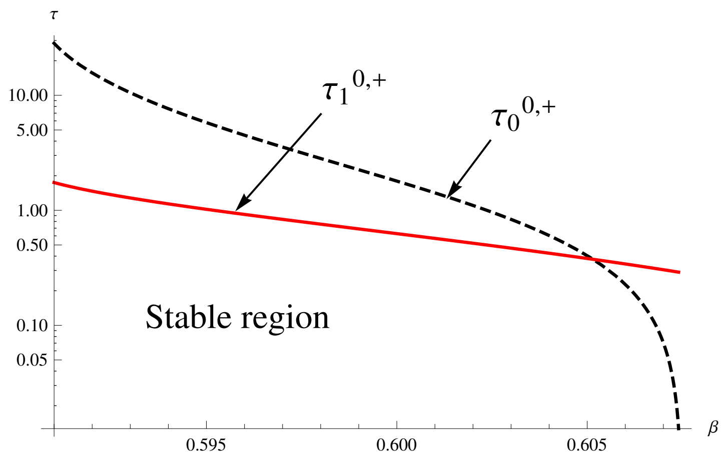

Figure 1. Bifurcation diagram of model (1.2) with parameter β.

Especially, fix $\beta = 0.595$, we can obtain $E_1 \approx (0.5362, 0.6914)$ and $E_2 \approx (0.6616, 0.6225)$ are two positive equilibria. It is easy to obtain that E_2 is always unstable. Then we mainly consider the stability of E_1. It can be verified that $(\mathbf{H_1})$ holds. By direct calculation, we have $\tau_* = \tau_1^{0,+} \approx 0.6271 < \tau_0^{0,+} \approx 5.7949$. When $\tau = \tau_*$, we have $\mu_2 \approx 147.6936$, $\beta_2 \approx -6.770$ and $T_2 \approx 48.7187$, then E_1 is locally asymptotically stable for $\tau < \tau_*$ (shown in Figure 2). And the stable inhomogeneous periodic solutions exists for $\tau > \tau_*$ (shown in Figure 3). To compare our result with the work in [17], we give the numerical simulations of model (1.2) without nonlocal competition same with the model in [17] under the same parameter $\tau = 4$ in Figure 4. We can see that nonlocal competition is the key to the existence of stable inhomogeneous periodic solutions.

To consider the effect of space length on the stability of the positive equilibrium (u_*, v_*), we give the bifurcation diagram of model (1.2) with parameter l (Figure 5) as other parameters fixed in (4.1) and $\beta = 0.595$. We can see that when the parameter l smaller than the critical value, stable region of the positive equilibrium (u_*, v_*) remains unchanged. This means that the spatial diffusion will not affect the stability of the positive equilibrium (u_*, v_*). When the parameter l is larger than the critical value, increasing of parameter l will cause the stable region of positive equilibrium (u_*, v_*) decrease. This means that the increase of space area will not be conducive to the stability of the positive equilibrium (u_*, v_*), and the inhomogeneous periodic oscillations of prey and predator's population densities may occur.

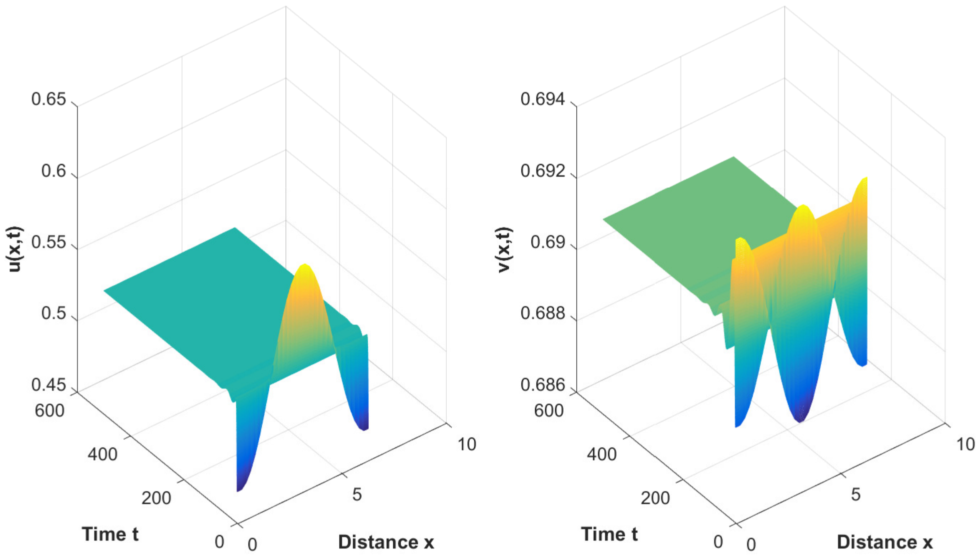

Figure 2. The numerical simulations of model (1.2) with $\tau = 0.5$. The positive equilibrium E_1 is asymptotically stable.

5. Conclusions

In this paper, we study a delayed diffusive predator-prey system with nonlocal competition and schooling behavior in prey. By using time delay as parameter, we study the local stability of the positive equilibrium and Hopf bifurcation at the positive equilibrium. We also analyze the property of Hopf bifurcation by center manifold theorem and normal form method. Through numerical simulation, we consider the effect of nonlocal competition on the model (1.2). Our results suggest that time delay can affect the stability of the positive equilibrium. When time delay is smaller than the critical value, the positive equilibrium is locally stable, and becomes unstable when time delay larger than the critical value. Then the prey and predator's population densities will oscillate periodically. But under the same parameters, spatial inhomogeneous periodic oscillations of prey and predator's population densities will appear in the model with nonlocal competition, and prey and predator's population densities will tend to the positive equilibrium in the model without nonlocal competition. This means that time delay can induce spatial inhomogeneous periodic oscillations in the predator-prey model with the nonlocal competition term, which is different from the model without the nonlocal competition term. In addition, we obtain that the increase of space area will not be conducive to the stability of the positive equilibrium (u_*, v_*), and may induce the inhomogeneous periodic oscillations of prey and predator's population densities under some parameters.

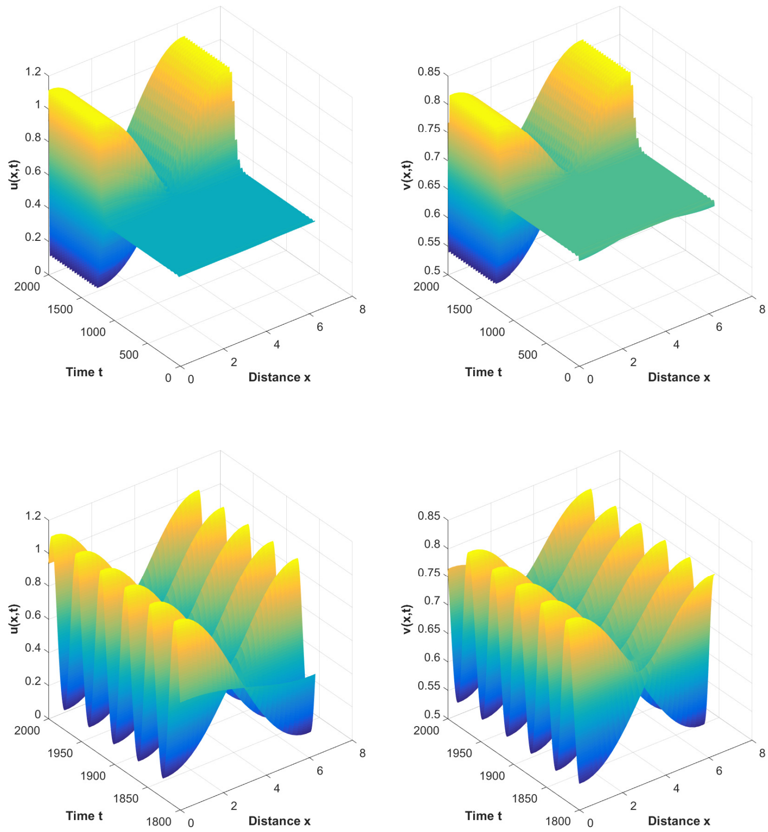

Figure 3. The numerical simulations of model (1.2) with $\tau = 4$. The positive equilibrium E_1 is unstable and there exists a spatially inhomogeneous periodic solution with mode-1 spatial pattern.

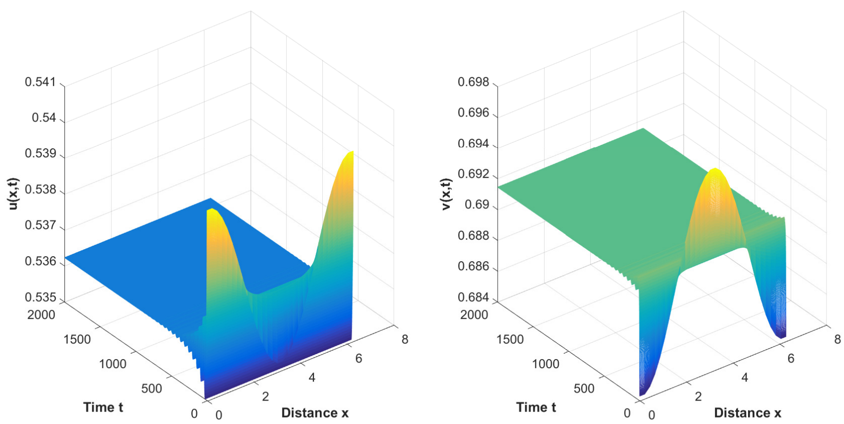

Figure 4. The numerical simulations of model (1.2) without nonlocal competition, and with $\tau = 4$. The positive equilibrium E_1 is asymptotically stable.

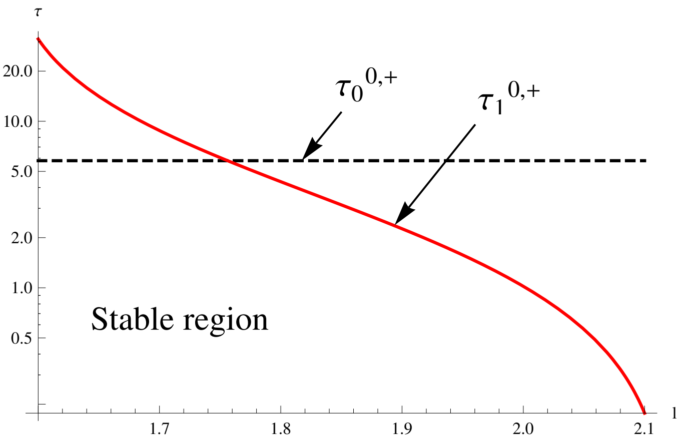

Figure 5. Bifurcation diagram of model (1.2) with parameter l.

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Conflict of interest

The authors declare there is no conflicts of interest.

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