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Research article

The application of an optimized fractional order accumulated grey model with variable parameters in the total energy consumption of Jiangsu Province and the consumption level of Chinese residents

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Abstract: Fractional order imply the idea of "in between", the grey model generated by fractional accumulation has better prediction and adaptability than that generated by first-order accumulation. General grey model of the differential equation of the left is a cumulative function derivative of time, in order to improve the adaptability of the model and prediction ability, general fractional order differential equation model is presented. In this paper, on the basis of the derivation of time t extensions to the derivation of t^u, added a variable coefficient, and through the integral differential equation and tectonic background value. We establish an optimized fractional order cumulative grey model with variable parameters, i.e., optimized fractional order accumulated grey model (FOGM (1,1)). By using the Particle swarm optimization (PSO) algorithm, we search for the order and variable parameters of the optimal fractional order. Then we apply the proposed model to predict the total energy consumption of Jiangsu province and the consumption level of Chinese residents. The results indicate that the proposed model has high fitting and prediction accuracy compared to other classical grey prediction models, such as grey model (GM (1,1)), non-homogeneous grey model (NGM (1,1)) and fractional order accumulated grey model (FGM (1,1)). It also validates that the proposed model is a practical and promising model for forecasting the energy consumption as well as the consumption level of Chinese residents.

Keywords: FOGM (1,1) model; variable parameter; PSO algorithm; prediction accuracy

1. Introduction

Per capita energy consumption is an important index to measure the development of the economic for a country and people's living standards. The more energy is consumed by per person, the bigger of the GNP, and the people in this country get richer and richer. Usually, the changes of the energy intensity is closely related to the process of industrialization, especially in developed countries. As is known to all, during the early and middle stages of industrialization, the energy consumption generally grows slowly with the economies grow. And the pattern of economic growth changes significantly as the economic development enters into the post-industrial stage, at the same time, the energy consumption intensity begins to decline. So far, many researchers have done a lot of meaningful work on the energy consumption. For example, in order to predict the demand of the building energy more accuracy, Kim et al. adopted a modified dataset approach to deal with the limitations of the prediction methods which have used for solving the existing building energy [1]. Based on historical data (such as climate, capacity) of air conditioning operating enterprises, Liu et al. established a scientific model of air conditioning energy consumption and the prediction model for solving the air condition starting time, then they constructed a demand prediction model according to the long short-term, that is, the sequence-to-sequence (seq2seq) model. Finally, they applied their theory and technology proposed in the research to design and implement the air-condition energy control system in smart factory, and predict the starting time of air-condition in the production workshop of enterprises [2]. It is essential to develop the efficient forecasting system for more accurate modeling the electrical energy. To achieve this goal, Bendaoud et al. proposed an innovative load forecasting approach using the Load Profiles (LPs) in reference [3]. According to the low accuracy of charging station load forecasting caused by the number of EVs, temperature and electricity price as well as other factors, Feng et al. presented a load forecasting method of EVCSs based on a combination of multivariable residual correction grey model (EMGM) and long short-term memory (LSTM) network [4].

It is vital for forecasting nuclear energy accurately to ensure both the reliable electricity supply and the problems of alleviate environmental degradation problems. However, it is difficult to model the nuclear time series because of its complexity, nonlinearity, and uncertainty. Therefore, by providing the generalized time response function theoretically and modifying the background value based on Simpson's rule accurately, Ding et al. put forward an optimized structure-adaptive grey model in [5]. As is known to all, it is vital to predict the carbon emissions accurately for controlling its growth from the source. In order to solve the growth of carbon emissions, Gao et al. established a Gompertz differential equation and transformed it into a fractional accumulation grey Gompertz model by using the differential information principle and the fractional accumulation operator [6]. In the forecasting of time series, mobile holiday effect usually bring about some difficulties in obtaining accurate forecasts for monthly and quarterly series, because it can lead to enormous disturbances for modeling, especially for the sequences with limited information and greater uncertainty. Zhou et al. constructed a discrete grey seasonal model by cycle accumulation generation as an alternative method for forecasting seasonal time series and effectively dealing with this problem [7]. In order to predict natural gas consumption more accurately and understand the future supply situation clearly as well as to optimize the allocation of resources, Xiong presented a new fractional-order accumulation-based incomplete gamma grey forecasting model. To further optimize the traditional grey action quantity, he chosen the dynamic nonlinear action-based incomplete gamma functions as the grey action quantity and combined with fractional-order accumulation, and fully considered the role of new information, then a detailed modeling process is given out, including both the computational steps and the intelligent optimization algorithms [8].

Chinese household consumption expenditure refers to the total expenditure of individual and family living consumption and collective individual consumption of urban and rural residents, it includes the purchase of commodity expenditure, enjoy cultural services and life services and other non-commodity expenditure. The consumption level of resident's refers to the degree to satisfy their needs for survival, development and enjoyment in the process of consumption for material products and service. This mainly reflects in the quantity and quality on the material goods and services consumed, and for rural residents, it also includes the expenditure of self-supporting products for their daily consumption. Collective consumption of individuals refers to the expenditure of goods and services which is provided by collective to individuals, none of non-consumption expenses are included. Collective consumption of individuals refers to the expenditure of goods and services which is provided by collective to individuals, none of non-consumption expenses are included. It is reflected comprehensively by the living consumption level of urban and rural residents through the average annual consumption expenditure index of inhabitant. There have been a lot of abundant research results on the income and consumption level of residents. For instance, based on residents' income level and various consumption expenditures, Xiang established a grey model to predict and analyze the income level and various consumption expenditures of residents in China [9]. As we all know, tourism is a high consumption field, Yan pointed out that there exists a specific relationship between the tourism and the income level of residents as well as the change of Tourism Engel coefficient [10].

However, in the previous research work, there are few literatures to predict energy consumption and household consumption expenditure by applying grey model, much less fractional grey model. In 1982, Deng proposed the grey model and pointed out that the model is a method suitable for forecasting "poor information and small sample" [11]. After more than 30 years of the development, the grey system theory has become the theoretical structure of an emerging discipline, especially for the grey prediction model, which has been widely applied in economy, energy and so on [12–14]. The intrinsic concept of grey system is "partial information known, partial information unknown", which provides the approach to solve the problem under the condition of poor information. The grey model can fully reveal the laws which is contained in separated and scattered original data by accumulating the original data, then we can see the trend of the development for the quantity of grey [15]. Many researchers have made a systemic analysis for different accumulation methods. Yang et al. improved the background value of the grey opposite-direction model (GOM (1,1)) according to the characteristics of the reverse sequence accumulation [16]. Qian et al. pointed out that the accumulation sequence was the weighted sum of the original data. However, traditional accumulation regarded the original data as equally important and did not reflect the importance of new information in the original data. Therefore, they proposed the GM (1,1) model generated by weighted accumulation [17]. Based on new information priority accumulation, Zhou et al. constructed a grey discrete model, and used examples to verify this model has better generalization ability [18]. In recent years, the fractional accumulation generation method has attracted more and more attention because of its widely application in many science and engineering [19]. Fractional order involve the idea of "in between", it replaces the integer order with a fractional order in grey model, which weakens the randomness of the original data sequence and makes the disturbance smaller of the solution of grey prediction model. Many scholars have applied the fractional cumulative generation method into different grey models. For example, Xie et al. analyzed the "non-homology" between the form of the discrete and the basic for the traditional

GM (1,1) model, and proposed the gray discrete model [20]. Cui et al. reconstructed the NGM(1,1,k,c) model in their study [21]. Chen et al. extended the idea of the grey modeling to the more general Nonlinear Grey Bernoulli Model (NGBM (1,1)) [22]. Liu and Xie first combined the Weibull distribution in probability statistics with the nonlinear grey Bernoulli model, then introduced variable parameters into the NGBM (1,1) model, and used this model to forecast the number of integrated circuit (IC) invention patent applications in China from 2007 to 2017 [23]. In the energy consumption a grey multivariate convolution model with adjacent accumulation (AGMC (1,N)) with better prediction performance is used to study 13 provinces (cities) in 7 different regions of China in detail. The results show that only the energy consumption in Central China will decrease in the short term [24]. A dynamic adaptive four-parameter discrete grey forecasting model suitable for forecasting China's Coalbed methane (CBM) production from 2019 to 2023 was constructed and derived by Zeng and Li [25]. Their research provides a reference basis for the Chinese government to formulate CBM import policies and optimize energy structure, as well as a feasible modeling method for the prediction of CBM production in the future.

According to previous research works, this paper will construct a fractional order cumulative grey model with variable parameters, that is the FOGM (1,1) model. Then, we search both the optimal fractional order and variable parameters through the PSO algorithm. The main contributions of this paper are as follows: 1) it is the first time for us to introduce variable parameters into the FGM (1,1) model, and present the detailed proof process of the solution of the proposed model; 2) we apply the PSO algorithm to search for the optimal variable parameters and the number of fractional order; 3) A classical literature data and some sample data are chosen to validity the effectiveness of the proposed model.

The rest part of the structure is as follows: in the next part, the solution of the FOGM (1,1) model is constructed; The optimization search process of both the variable parameter and the fractional order by using PSO method are presented in part three. In the fourth part, a classical literature data and an example data is applied to verify the effectiveness of the proposed model.

2. The FOGM (1,1) model

Definition 1: Let $X^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n))$ be the non-negative sequence, then the fractional accumulation operator is

$$X^{(r)}=(x^{(r)}(1),x^{(r)}(2),\cdots,x^{(r)}(n)), \tag{1}$$

where $$x^{(r)}(k)=\sum_{i=1}^{k}C_{k-i+r-1}^{k-i}x^{(0)}(i).$$

(2)

Definition 2: Assume that the non-negative sequence is defined as in Definition 1, the cumulative decreasing sequence with order r is defined as following,

$$X^{(-r)}=(x^{(-r)}(1),x^{(-r)}(2),\cdots,x^{(-r)}(n)) \tag{3}$$

where

$$x^{(-r)}(k)=\sum_{i=1}^{k}(-1)^{i}\frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)}x^{(0)}(k-i),k=1,2,\cdots,n. \tag{4}$$

Definition 3: let

$$\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t^{\mathrm{u}}}+ax^{(r)}(t)=b \tag{5}$$

be the whitening equation of the FOGM (1,1) model. We define

$$x^{(r)}(k)-x^{(r)}(k\text{-}1)+az^{(r)}(k)=b(k^{u}-(k-1)^{u}) \tag{6}$$

as the basic form of the FOGM (1,1) model.
where

$$z^{(r)}(k)=0.5(uk^{u-1}x^{(r)}(\mathrm{k})+u(k-1)^{u-1}x^{(r)}(k\text{-}1)). \tag{7}$$

Next we will give out the detail analysis of the prove process for the Eq (6).

Prove: Take the definite integral on both sides of Eq (5), we get $\int_{k-1}^{k}\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t}+a\int_{k-1}^{k}ut^{u-1}x^{(r)}(t)dt=b\int_{k-1}^{k}ut^{u-1}dt$.

Assuming that $z^{(r)}(k)=0.5(uk^{u-1}x^{(r)}(k)+u(k-1)^{u-1}x^{(r)}(k\text{-}1))$,

We can obtain $x^{(r)}(k)-x^{(r)}(k\text{-}1)+az^{(r)}(k)=b(k^{u}-(k-1)^{u})$.

Theorem 1: Assume that $X^{(0)},X^{(r)},Z^{(r)}$ are defined as in Definition 2. Let $\theta=[a,b,c]^{T}$ be the parameters column, set

$$Y=\begin{bmatrix} x^{(r)}(2)-x^{(r)}(1)\\ x^{(r)}(3)-x^{(r)}(2)\\ \vdots\\ x^{(r)}(n)-x^{(r)}(n-1)\end{bmatrix},\quad B=\begin{bmatrix} -z^{(r)}(2) & 2^{u}-1^{u}\\ -z^{(r)}(3) & 3^{u}-2^{u}\\ \vdots & \vdots\\ -z^{(r)}(n) & n^{u}-(n-1)^{u}\end{bmatrix}. \tag{8}$$

Then by applying the least square method, the parameters of the FOGM (1,1) model: $x^{(r)}(k)-x^{(r)}(k\text{-}1)+az^{(r)}(k)=b(k^{u}-(k-1)^{u})$

can be estimated as $\theta=(B^{T}B)^{-1}B^{T}Y$.

Theorem 2: B,Y are defined as in Theorem 1, $\theta=[a,b]^{T}=(B^{T}B)^{-1}B^{T}Y$, taking $\hat{x}^{(r)}(1)=x^{(0)}(1)$, then the solution of the whitening equation $\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t^{\mathrm{u}}}+ax^{(r)}(t)=b$ (which also known as the time response function) is

$$\hat{x}^{(r)}(\mathrm{t})=e^{a(1-t^{u})}x^{(0)}(1)-\frac{b}{a}e^{a(1-t^{u})}+\frac{b}{a}. \tag{9}$$

And the time response sequence of the equation $x^{(r)}(k)-x^{(r)}(k\text{-}1)+az^{(r)}(k)=b(k^{u}-(k-1)^{u})$ which contained in FOGM (1,1) model is

$$\hat{x}^{(r)}(k)=e^{a(1-k^{u})}x^{(0)}(1)-\frac{b}{a}e^{a(1-k^{u})}+\frac{b}{a}, \tag{10}$$

The reduced value of the Eq (10) is

$$\hat{x}^{(0)}(k)=\begin{cases}\hat{x}^{(\mathrm{r})}(k),k=1\\ \sum_{i=1}^{k}(-1)^{i}\frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)}\hat{x}^{(\mathrm{r})}(k-i),k=2,\cdots,n\end{cases}. \tag{11}$$

Proof: According to the derivative formula

$$\frac{\mathrm{d}t^{\mathrm{u}}}{\mathrm{d}t}=ut^{u-1},$$

We have,

$$\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t}=\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t^{\mathrm{u}}}\frac{\mathrm{d}t^{\mathrm{u}}}{\mathrm{d}t}=\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t^{\mathrm{u}}}\mathrm{u}t^{u-1}. \tag{12}$$

Then the Eq (5) can be written as

$$\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t}+a\mathrm{u}t^{u-1}x^{(r)}(t)=b\mathrm{u}t^{u-1}. \tag{13}$$

Multiply the $e^{at^{u}}$ on the two sides of Eq (13),

$$e^{at^{u}}[\frac{\mathrm{d}x^{(r)}(t)}{\mathrm{d}t}+a\mathrm{u}t^{u-1}x^{(r)}(t)]=e^{at^{u}}b\mathrm{u}t^{u-1}. \tag{14}$$

$$\frac{\mathrm{d}e^{at^{u}}x^{(r)}(t)}{\mathrm{d}t}=e^{at^{u}}b\mathrm{u}t^{u-1}. \tag{15}$$

Take the definite integral on both sides of the above equation on the interval [1, t], we obtain

$$\int_1^t \frac{\mathrm{d}e^{at^u} x^{(r)}(t)}{\mathrm{d}t} = \int_1^t e^{at^u} b u t^{u-1}, \tag{16}$$

$$e^{at^u} x^{(r)}(t)\Big|_1^t = \frac{b}{a} e^{at^u}\Big|_1^t, \tag{17}$$

$$e^{at^u} x^{(r)}(t) - e^a x^{(r)}(1) = \frac{b}{a}(e^{at^u} - e^a). \tag{18}$$

We get the time response function as following

$$x^{(r)}(t) = e^{a(1-t^u)} x^{(r)}(1) - \frac{b}{a} e^{a(1-t^u)} + \frac{b}{a}. \tag{19}$$

And the time response sequence is

$$\hat{x}^{(r)}(k) = e^{a(1-k^u)} x^{(0)}(1) - \frac{b}{a} e^{a(1-k^u)} + \frac{b}{a}, \tag{20}$$

Make the cumulative decrease of order r to the time response sequence, we obtain the final prediction value as follow

$$\hat{x}^{(0)}(k) = \begin{cases} \hat{x}^{(\mathrm{r})}(k), k = 1 \\ \sum_{i=1}^{k} (-1)^{i} \dfrac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} \hat{x}^{(\mathrm{r})}(k-i), k = 2, \cdots, n \end{cases}. \tag{21}$$

This completes the proof the Theorem 2.

3. The optimal settings for the FOGM (1,1) model based on particle swarm optimization algorithm

3.1. PSO algorithm

In 1995, Kennedy and Eberhart first proposed the Particle swarm optimization (PSO) algorithm [26], which is a swarm intelligence optimization algorithm in the field of computational intelligence excepted to the ant colony algorithm and the fish swarm algorithm. The PSO algorithm has many advantages, it is not only easier to understand and to adjust less parameters, but also easier to implement the programming and more stable etc., which has a wide range of applications on neural network training and function optimization.

In this section, we mainly apply the PSO algorithm to minimize the mean relative error, then we search for the optimal Weibull parameter u and the optimal nonlinear parameter r of the FOGM (1,1) model as follow,

$$\min f(u, r) = \frac{1}{n-1} \sum_{k=2}^{n} \left| \frac{\hat{x}^{(0)}(k) - x^{(0)}(k)}{x^{(0)}(k)} \right|. \tag{22}$$

The following is the detail steps of the search procedure.

a) We assume that the parameters are $c_1=1, c_2=2$, the minimum and maximum of inertia factors are $w_{\min}=0.2$, $w_{\max}=0.9$, respectively, the initial iteration value is $k=1$ as well as the maximum iteration value is $\max ite=300$, the initial velocity is 10% of the position of each particle and α_1, α_2 are the random numbers between 0–1.

b) We distribute 50 particles in the search space randomly, then each particle has the initial position $v_i=(v_{i1}, v_{i2})$, $i=1,2,\cdots;\zeta$ as well as the initial velocity $\phi_i=(\varphi_{i1}, v_{i2}), i=1,2,\cdots;\zeta$.

c) We calculate the applicability of each particle $MAPE_i^k = MAPE(v_i^k)$, find the index b_k of the best particle and assume that,

$$pbest_i^k = v^k, gbest_i^k = v_{b_k}^k. \tag{23}$$

d) Now, we update both the position and velocity of each particle according to the formulation as below

$$\varphi_{i,j}^{k+1} = w\varphi_{i,j}^k + c_1\alpha_1(pbest_{i,j}^k - v_{i,j}^k) + c_2\alpha_2(gbest_{i,j}^k - v_{i,j}^k), j = 1,2,\cdots. \tag{24}$$

$$\mathrm{v}_{i,j}^{k+1} = v_{i,j}^k + \varphi_{i,j}^{k+1} j = 1,2. \tag{25}$$

e) Then, we update the $pbest$ as well as the $gbest$

If $MAPE_i^{k+1} < MAPE_i^k$, then $pbest_i^k = v_i^{k+1}$; and if $pbest_i^{k+1} = pbest_i^k$, then the optimal index of the best particle is b_{k+1};

If $MAPE_{b_{k+1}}^{k+1} < MAPE_{b_k}^k$, then $gbest^{k+1} < pbest_{b_{k+1}}^{k+1}$, otherwise, we have $gbest^{k+1} = gbest$;

f) Break out the iteration loop

If $k < \max ite$, then $k = k+1$; otherwise, we output the optimal parameters and the fitness degree of each particle.

3.2. The evaluation of the model

In order to verify both the simulation and prediction effect of the model, we apply the root mean square error (RMSE) and the mean absolute percentage error (MAPE) to evaluate them.

Assume that the fitting sequence is as following,

$$\hat{X}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(n)). \tag{26}$$

$$e(i) = \hat{x}^{(0)}(i) - x^{(0)}(i). \tag{27}$$

Then the root mean square error (RMSE) is:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e^2(i)}. \tag{28}$$

And the mean absolute percentage error (MAPE) is:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{e(i)}{x^{(0)}(i)}\right| . \tag{29}$$

4. **The validation and application of the FOGM (1,1) model**

In this section, in order to verify the validation of the FOGM (1,1) model, we used this model to model a classical literature data. Then, we adopted the FOGM (1,1) model to analyze the consumer price index data of per capita in China.

4.1. The validation of the FOGM (1,1) model

Economic development in the context of the progress of human civilization, showed a growing dependence on energy. With the rapid economic development, energy storage continues to decline, and the environmental pollution caused by the massive development and utilization of fossil energy has aroused people's concern. Under the background of resource shortage and environmental pollution, it is of great practical significance to reduce environmental pollution and realize sustainable economic development while ensuring economic development. Scientific prediction of regional energy is of practical significance for rational control of energy supply.

In this part, the proposed prediction model is used to analyze the total energy consumption data (unit: tons of standard coal) from 2001 to 2012 of Jiangsu province in China, and the adopted data mainly comes from literature [8]. First, we divided the data into two groups such as training sets (from 2001 to 2010) and test sets (from 2011 to 2012), respectively. Then we built the FOGM (1,1) model according to the training data, and we obtained the Weibull parameters u (is equal to 1.3) and the order of the fractional order r (is equal to 0.64) of the model by using the PSO algorithm. Finally, we used the test data to predict the results and compared them with both the actual values and the results forecasted by various grey models such as GM (1,1), NGM (1,1) and FGM (1,1). Table1 listed out the fitting accuracy as well as the prediction accuracy of each model. The results showed that the FOGM (1,1) model has higher fitting accuracy and more prediction accuracy than other various grey models. Figure 1 presented the comparison results of each model intuitively.

As can be seen from the results in Table 1, the results predicted by FOGM (1,1) model is closer to the real value, and the relative error is smaller than that of other models. By using the FOGM (1,1) model to forecast the test data and estimate the training data, we also can obtain the smallest value of the RMSE as well as the MAPE.

Table 1. The comparison of prediction results for total energy consumption of Jiangsu province in China with different models.

Year	Raw	FOGM (1,1)		GM (1,1)		NGM (1,1)		FGM (1,1)		ARIMA	
		Predicted value	Relative error	Predicted value	Relative error	Predicted value	Relative error	Predicted value	Relative error	Predicted value	Relative error
2001	8881	--	--	--	--	--	--	--	--	--	--
2002	9593	9584.67	0.09%	11,349.64	18.31%	8195.72	14.57%	10,208.32	6.41%	9593.00	0%
2003	11,950	11,937.84	0.10%	12,644.28	5.81%	10,623.84	11.10%	12,202.66	2.11%	10,975.56	8.15%
2004	14,207	14,223.62	0.12%	14,086.6	0.85%	12,939.77	8.92%	14,212.80	0.04%	14,584.88	2.66%
2005	16,360	16,380.31	0.12%	15,693.44	4.07%	15,148.68	7.40%	16,192.05	1.03%	16,775.35	2.545
2006	18,412	18,420.20	0.04%	17,483.58	5.04%	17,255.52	6.28%	18,143.19	1.46%	18,786.83	2.04%
2007	20,369	20,360.79	0.04%	19,477.91	4.37%	19,265.01	5.42%	20,075.37	1.44%	20,635.37	1.31%
2008	22,235	22,217.04	0.08%	21,699.73	2.41%	21,181.64	4.74%	21,997.55	1.07%	22,333.36	0.44%
2009	24,010	24,000.87	0.04%	24,175	0.69%	23,009.71	4.17%	23,917.43	0.39%	23,886.14	0.52%
2010	25,711	25,721.78	0.04%	26,932.62	4.75%	24,753.31	3.72%	25,841.49	0.51%	25,292.76	1.63%
		RMSE	13.18	RMSE	912.18	RMSE	1172.26	RMSE	281.10	RMSE	409.42
		MAPE	0.08%	MAPE	5.15%	MAPE	7.37%	MAPE	1.61%	MAPE	1.93%
2011	27,329	27,387.37	0.21%	30,004.79	9.79%	26,416.34	3.34%	27,775.22	1.63%	26,582.04	2.735
2012	28,872	29,003.85	0.46%	33,427.41	15.78%	28,002.53	3.01%	29,723.33	2.95%	26,515.67	8.16%
		RMSE	101.96	RMSE	3735.75	RMSE	891.33	RMSE	679.66	RMSE	1747.9
		MAPE	0.34%	MAPE	12.78%	MAPE	3.18%	MAPE	2.29%	MAPE	5.45%

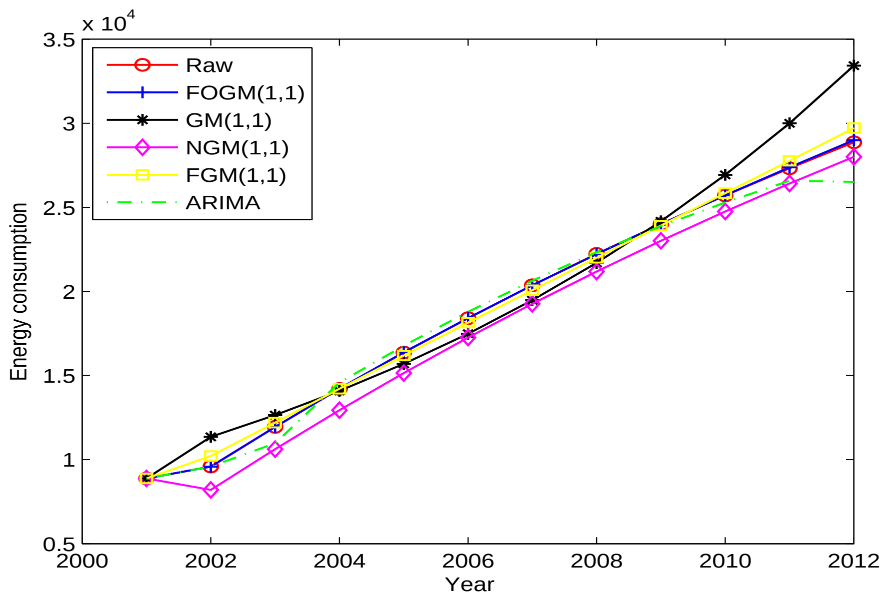

Figure 1. The forecast results of grey model for total energy consumption of Jiangsu Province in China.

4.2. *The application of the FOGM (1,1) model*

The importance of predicting resident consumption lies in its supporting effect on the national economy. A country cannot depend on investing to drive growth forever. The purpose of investment is to produce more goods for people to consume. If household consumption falters, investment will lose its meaning. In addition, resident consumption plays a decisive role in the national living standard, which depends on the level of consumption. If people do not spend, or have no money to spend, they cannot benefit from economic growth. Then economic growth is meaningless.

Table 2. Comparison of Chinese residents consumption level forecast by various models.

		FOGM (1,1)		GM (1,1)		NGM (1,1)		FGM (1,1)		ARIMA	
Year	Raw	Predicted value	Relative error	Predicted value	Relative error	Predicted value	Relative error	Predicted value	Relative error	Predicted value	Relative error
2001	3968	--	--	--	--	--	--	--	--	---	--
2002	4270	4122.26	3.46%	4236.34	0.79%	3685.55	13.69%	3962.73	7.20%	4270	0%
2003	4555	4507.79	1.04%	4758.24	4.46%	4289.73	5.82%	4487.87	1.47%	3973.05	12.78%
2004	5071	5056.10	0.29%	5344.45	5.39%	4954.47	2.30%	5158.28	1.72%	4326.36	14.68%
2005	5688	5745.10	1.00%	6002.87	5.54%	5685.82	0.04%	5920.71	4.09%	5392.10	5.20%
2006	6319	6563.12	3.86%	6742.40	6.70%	6490.47	2.71%	6764.24	7.05%	5978.62	5.39%
2007	7454	7502.51	0.65%	7573.04	1.60%	7375.77	1.05%	7689.11	3.15%	6867.92	7.86%
2008	8505	8557.69	0.62%	8506.02	0.01%	8349.79	1.82%	8699.85	2.29%	8223.42	3.31%
2009	9249	9724.28	5.14%	9553.94	3.30%	9421.43	1.86%	9803.14	5.99%	9048.60	2.17%
2010	10,575	10,998.68	4.01%	10,730.96	1.47%	10,600.47	0.24%	11,007.13	4.09%	10,236.47	3.20%
2011	12,668	12,377.84	2.29%	12,052.98	4.85%	11,897.68	6.08%	12,321.10	2.74%	12,256.49	3.25%
2012	14,074	13,859.13	1.53%	13,537.87	3.81%	13,324.89	5.32%	13,755.44	2.26%	13,631.05	3.15%
2013	15,586	15,440.23	0.94%	15,205.70	2.44%	14,895.14	4.43%	15,321.59	1.70%	15,281.40	1.95%
2014	17,220	17,119.04	0.59%	17,078.99	0.82%	16,622.77	3.47%	17,032.18	1.09%	17,009.18	2.22%
2015	18,857	18,893.69	0.19%	19,183.07	1.73%	18,523.54	1.77%	18,901.01	0.23%	18,640.21	1.15%
2016	20,801	20,762.45	0.19%	21,546.37	3.58%	20,614.80	0.90%	20,943.22	0.68%	20,417.85	1.84%
		RMSE	210.13	RMSE	297.71	RMSE	344.98	RMSE	125.75	RMSE	384.93
		MAPE	1.72%	MAPE	3.10%	MAPE	3.43%	MAPE	3.05%	MAPE	4.20%
2017	22,969	22,723.74	1.07%	24,200.82	5.36%	22,915.66	0.23%	23,175.37	0.90%	22,544.94	1.85%
2018	25,245	24,776.09	1.86%	27,182.29	7.67%	25,447.12	0.80%	25,615.62	1.47%	24,264.03	3.89%
2019	27,504	26,918.11	2.13%	30,531.07	11.01%	28,232.27	2.65%	28,283.79	2.84%	26,385.25	4.07%
2020	27,438	29,148.53	6.23%	34,292.41	24.98%	31,296.57	14.06%	31,201.61	13.72%	28,612.47	4.28%
		RMSE	941.97	RMSE	3445.45	RMSE	1306.54	RMSE	1321.83	RMSE	971.22
		MAPE	2.82%	MAPE	12.26%	MAPE	4.44%	MAPE	4.73%	MAPE	3.52%

In this part, we applied the FOGM (1,1) model to analyze the Chinese household consumption level (yuan) data which comes from the website of China's National Bureau of Statistics. Similarly, we first splitted the data into two part such as the training (the resident's consumption price of 2003–2014) and the test (the resident's consumption price of 2015 to 2018), then by using the PSO algorithm, we got the value of the Weibull parameter u which is equal to 1.65, and the order of the fractional order r which is equal to 0.01. Finally, in order to verify the efficient of the FOGM (1,1) model, we compared the results for the real value and the predicted values which are obtained from various grey model, for example, the GM (1,1) model, the NGM (1,1) model and the FGM (1,1) model, and we listed all the forecast results in Table 2 and drew out the corresponding graph in Figure 2 respectively. As is can be seen from the Table 2, both the RMSE and MAPE values are smaller than that of other three grey models. This indicated that the FOGM (1,1) not only has higher fitting precision but also more accuracy of the prediction both in the fitting and prediction stage. According to the Figure 2, we known that the consumer level of per person in China has been increasing year by year.

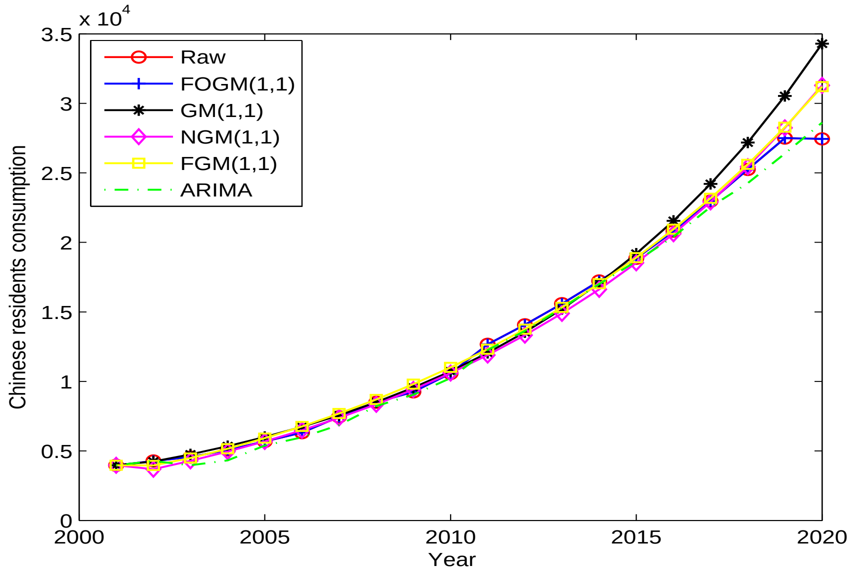

Figure 2. The prediction results of the consumption level of Chinese residents with different Grey models.

5. Conclusions

In this paper, we constructed an optimized fractional gray model with variable parameters (FOGM (1,1)). We used the PSO algorithm to minimize the mean correlation error and search for the order of variable parameters as well as the fractional order. Two cases are simulated by the proposed prediction model and compared it to other grey prediction models, we obtained that the FOGM (1,1) model has both the better fitting and prediction accuracy. Finally, the experiment results show that both the energy consumption of Jiangsu province and the per capita consumption level of Chinese people has increased year by year, this indicated that the economy of China is developed well. The scalability of our model is very good. For example, the grey model based on the proposed method can be directly extended to non-homogeneous grey model. In addition, the background value coefficient can be adjusted dynamically. Accordingly, the method proposed in this paper can be directly extended to other new types of models. The grey model has a simple structure and relatively few parameters with a smaller data set. Thus, the time consumption is usually tiny. For example, our model is implemented in MATLAB environment on a computer with windows 10 system, Intel i7 CPU and 16GB of RAM. The time costs of the first case and the second case are only 0.0253 and 0.0354s, respectively. Accordingly, the model does not have the problem of occupying large computing resources. In our future work, we will consider to introduce the variable parameters into some other grey models, such as the grey model with time term and nonlinear time fractional wave model [27].

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Conflicts of interest

The authors declare that they have no conflicts of interest.

References

1. K. C. Ho, K. Marie, Y. J. Song, Sequence-to-sequence deep learning model for building energy consumption prediction with dynamic simulation modeling, *J. Build. Eng.*, **43** (2021), 102577. https://doi.org/10.1016/J.JOBE.2021.102577
2. R. Liu, Z. H. Wang, H. H. Chen, J. Yang, Research on energy consumption prediction based on machine learning, *IOP Conf. Ser.: Earth Environ. Sci.*, **791** (2021), 012100. https://doi.org/10.1088/1755-1315/791/1/012100
3. N. M. M. Bendaoud, N. Farah, S. Ben Ahmed, Applying load profiles propagation to machine learning based electrical energy forecasting, *Electr. Power Syst. Res.*, **203** (2022), 107635. https://doi.org/10.1016/J.EPSR.2021.107635
4. J. Feng, J. Yang, Y. Li, H. Wang, H. Ji, W. Yang, et al., Load forecasting of electric vehicle charging station based on grey theory and neural network, *Energy Rep.*, **7** (2021), 487–492. https://doi.org/10.1016/J.EGYR.2021.08.015
5. S. Ding, Z. Tao, H. H. Zhang, Y. Li, Forecasting nuclear energy consumption in China and America: An optimized structure-adaptative grey model, *Energy*, **239** (2022), 121928. https://doi.org/10.1016/J.ENERGY.2021.121928
6. M. Y. Gao, H. L. Yang, Q. Z. Xiao, G. Mark, A novel method for carbon emission forecasting based on Gompertz's law and fractional grey model: Evidence from American industrial sector, *Renewable Energy*, **181** (2022), 803–819. https://doi.org/10.1016/J.RENENE.2021.09.072
7. W. J. Zhou, J. Pan, H. H. Tao, S. Ding, L. Chen, X. Zhao, A novel grey seasonal model based on cycle accumulation generation for forecasting energy consumption in China, *Comput. Ind. Eng.*, **163** (2022), 107725. https://doi.org/10.1016/J.CIE.2021.107725
8. P. Xiong, K. Li, H. Shu, J. Wang, Forecast of natural gas consumption in the Asia-Pacific region using a fractional-order incomplete gamma grey model, *Energy*, **237** (2021), 121533. https://doi.org/10.1016/J.ENERGY.2021.121533

9. H. F. Xiang, Grey correlation analysis and forecast of residents' income and consumption structure, *Acad. J. Bus. Manage.* **2** (2020), 6–14. https://doi.org/10.25236/AJBM.2020.020302
10. X. Yan, M. Y. Wu, Study of Shanghai residents tourism consumption structure based on the tourism engel coefficient, *Bus. Manage. Res.*, **2** (2013), 136–148. https://doi.org/10.5430/bmr.v2n4p136
11. J. L. Deng, Control problem of grey systems, *Syst. Control Lett.*, **1** (1982), 288–294. https://doi.org/10.1016/S0167-6911(82)80025-X
12. F. P. Zhang, X. K. Liu, K. Wang, Prediction of postgraduate education scale in China based on grey model, *Math. Pract. Theory*, **49** (2019), 318–323. https://doi.org/CNKI:SUN:SSJS.0.2019-15-037
13. R. X. Suo, X. Y. Wang, J. Shen, Prediction of Chinese coal demand based on dynamic unbiased grey markov model, *Math. Pract. Theory*, **49** (2019), 179–186. https://doi.org/CNKI:SUN:SSJS.0.2019-13-019
14. B. Wu, M. Xin, Z. D. Bo, Y. Wang, W. Cai, Forecasting short-term renewable energy consumption of China using a novel fractional nonlinear grey Bernoulli model, *Renewable Energy*, **140** (2019), 70–87. https://doi.org/10.1016/j.renene.2019.03.006
15. J. L. Deng, Grey exponential law for accumulative generation, *J. huazhong Univ. Sci. Technol.*, **15** (1987), 7–12. https://doi.org/10.13245/j.hust.1987.05.002
16. Z. Yang, P. Ren, Y. G. Dang, Grey opposite-direction accumulated generating and optimization of GOM (1,1) model, *Syst. Eng.-Theory Pact.*, **29** (2009), 162–166. https://doi.org/CNKI:SUN:XTLL.0.2009-08-022
17. W. Y. Qian, Y. G. Dang, Y. M. Wang, GM (1,1) Model Based on Weighting Accumulated Generating Operation and Its Application, *Math. Pract. Theory*, **39** (2009), 49–-53. https://doi.org/CNKI:SUN:SSJS.0.2009-15-008
18. W. J. Zhou, H. R. Zhang, Y. G. Dang, Z. X. Wang, New information priority accumulation grey discrete model and its application, *Chin. Manage. Sci.*, **25** (2017), 140–148. https://doi.org/10.16381/j.cnki.issn1003-207x.2017.08.015
19. L Wu, S Liu, L Yao, S Yan, D Liu, Grey system model with the fractional order accumulation, *Commun. Nonlinear Sci. Numer. Simul.* **18** (2013), 1775–1785. https://doi.org/10.1016/ j.cnsns.2012.11.017
20. N. M. Xie, S. F. Liu, Discrete grey forecasting model and its optimization, *Appl. Math. Modell.*, **33** (2009), 1173–1186. https://doi.org/10.1016/j.apm.2008.01.011
21. J. Cui, S. F. Liu, B. Zeng, N. M. Xie, A novel grey forecasting model and its optimization, *Appl. Math. Modell.*, **37** (2013), 4399–4406. https://doi.org/10.1016/j.apm.2012.09.052
22. Y. S. Chen, H. L. Chen, S. P. Chen, Forecasting of foreign exchange rates of Taiwan's major trading partners by novel nonlinear Grey Bernoulli model NGBM (1,1), *Commun. Nonlinear Sci. Numer. Simul.* **13** (2008), 1194–1204. https://doi.org/10.1016/j.cnsns.2006.08.008
23. X. Liu, N. Xie, A nonlinear grey forecasting model with double shape parameters and its application, *Appl. Math. Comput.* **360** (2009), 203–212. https://doi.org/10.1016/j.amc.2019.05.012
24. W. A. Meng, W. A. Wei, L. F. Wu, Application of a new grey multivariate forecasting model in the forecasting of energy consumption in 7 regions of China, *Energy*, **243** (2022), 123024. https://doi.org/10.1016/j.energy.2021.123024

25. B. Zeng, H. Li, Prediction of coalbed methane production in China based on an optimized grey system model, *Energy Fuels*, **35** (2021), 4333–4344. https://doi.org/10.1021/acs.energyfuels.0c04195
26. J. Kennedy, R. Eberhart, Particle swarm optimization, in *Icnn95-international Conference on Neural Networks IEEE*, 1995.
27. Y. N. Yang, Y. Liu, C. Wen, H. Li, J. F. Wang, Efficient time second-order SCQ formula combined with a mixed element method for a nonlinear time fractional wave model, *Electron. Res. Arch.*, **30** (2022), 440–458. https://doi.org/10.3934/era.2022023