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Research article

A comprehensive high pure momentum equity timing framework using the Kalman filter and ARIMA forecasting

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Abstract: The pursuit of higher returns has led to a growing interest in factor timing as a strategy to enhance portfolio returns. Momentum is a popular factor, which involves buying securities that have shown consistent price appreciation over the past 3 to 12 months or past few years, with the expectation that the trend will continue and reducing exposure to those that consistently declined. An important part of a factor timing strategy is in the portfolio optimization process. This article aimed to first construct a large capitalization pure momentum portfolio, which included a dynamic stringent portfolio construction process criteria for selecting stocks estimated from historical data. Second, as a part of the portfolio's risk management strategy, the Kalman filter was applied to the historical performance of this portfolio. Lastly, the ARIMA forecast was used to estimate expected performance and the confidence intervals. The empirical results showed that this pure equity momentum factor timing framework with the Kalman filter together with the ARIMA (autoregressive integrated moving average) forecasting methodology was iterative and incorporated new information as it became available and further enhanced the monitoring and rebalancing process. This adaptive approach enabled the portfolio to capitalize on time-varying return anomalies as they occured.

Keywords: momentum; factor timing; Kalman filter; return predictability; factor investing; investment style factors

JEL Codes: G11, G12, G17, C22

1. Introduction

Momentum is a popular factor, which involves buying securities that have shown consistent price appreciation over the past 3 to 12 months or past few years, with the expectation that the trend will continue and reducing exposure to those that consistently declined. This factor takes into consideration the historical trends of stocks, as documented by Jegadeesh and Titman (1993), where they examined a strategy of buying stocks that performed well in the past using a sample that spanned over a period of 24 years (from 1965 to 1989) and found that past winners continued on a winning streak and past losers continued on a losing streak; buying past winners was rewarded with outperformance which was not attributable to systematic risk and investors' reactions to certain stock factors. Their results also imply that there is a compensation for investors' slow reaction to factor prices, where short-term underreaction and long-term overreactions might play a role in the prices of stocks, especially in the short-term (Hong and Stein, 1999).

Factors have a long history and have evolved since the introduction of the capital asset pricing model (CAPM) by William Sharpe (1964) and John Lintner (1965), where it was believed that only the market factor was the driver of investment returns. The CAPM set the groundwork for further developments of factor models, which later led to the arbitrage pricing theory (APT) by Stephen Ross in 1976. The APT is an alternative model to the CAPM, and it is one of the foundational models which showed that investment returns could be modeled by more than one factor, due to the existing linear relationship between expected investment returns and other response variables (factors) (Huberman and Wang, 2005). This further led to the Fama and French (1992) three-factor model, an expansion of the earlier CAPM, which included size, value, and market excess returns. Carhart (1997) further expanded on the Fama and French three-factor model by adding momentum as a fourth factor.

Timing strategies aim to limit risk, minimize losses and maximize returns by limiting exposures to certain stocks with the consideration of future market trends. Treynor and Mazuy (1966) conceptualized the idea of market timing, where they attribute a manager's skill to be a contributing factor to the success, or lack thereof, of obtaining favorable returns. The expectations of the market factor increasing or decreasing would lead to the manager adjusting the portfolio positions by either increasing exposure or decreasing exposure based on market expectations. They included 57 mutual funds in the study, and ultimately found that at the time, none of the funds were successful at outperforming the market. The main difference between market and factor timing is that a market timing strategy includes the overall market trend analysis, and factor timing is rule based and focuses on a portfolio's factor exposure.

Equity factor timing aims to potentially increase returns by buying or holding equity factors that are expected to outperform and selling those that are expected to underperform, essentially taking advantage of market anomalies and inefficiencies. This is an important part of factor investing, presenting unique opportunities to generate alpha. Though timing can be a challenging endeavor, in this article the Kalman filter is applied as an iterative aspect of the process.

This article starts with the introduction of momentum and factor timing, followed by factor timing overview in Section 2. Methodology is in Section 3, the framework is in Section 4, the empirical results are in Section 5, the transaction cost is in Section 6, and finally, the conclusion is in Section 7.

2. Factor timing overview

There is a growing interest in factor timing as a strategy to enhance portfolio returns. This is supported by a growing level of research consisting of varying opinions and findings, where some find factor timing to be an effective strategy and others find it does not produce abnormal returns, especially when faced with high transaction costs and market movements amongst other factors that impact returns. Traditional momentum strategies may not yield the desired returns, and after Gupta and Kelly (2019) studied momentum extensively to prove that factors can be timed based on recent historical performance, they found that adding factor timing as a strategy was beneficial with positive returns. Further comparing the returns from a traditional momentum strategy showed that adding time series momentum factor timing into a portfolio as a strategy also yielded better results. Momentum equity factor timing is based on the assumption that during market upswings this strategy will outperform, enhance portfolio returns, and reduce risk.

A portfolio manager's skill can be attributed to returns, leading to the assumption that skill is also an important aspect of a successful factor timing strategy. For example, Osinga et al. (2020), found that hedge fund managers are better suited for incorporating factor timing as a strategy, which is the main contributor to positive factor timing returns. Daniel et al. (1997) analyzed returns from 125 passive portfolios in comparison to their respective benchmarks and found that factor timing was unsuccessful and it might not be a strategy that could be applied successfully to reach optimal results for passive funds, noting that the manager skill would be the main contributing factor if funds produce desired results. Aiken and Kang (2023) used a holdings-based approach to assess the impact a manager's skill might have on the performance of a fund. This was used for portfolio selection and factor timing skills, and found that the manager's skill plays an important role in the generation of alpha. Clare et al. (2022) found that experience may be a contributing factor to timing returns, as managers that combine experience with skill are better equipped to bring positive portfolio returns than managers who rely solely on skill. Another contributing factor is that a manager with a stake in the fund tends to be more careful when managing the fund, as this will directly affect their income. Hence, skill with experience is much better than skill without much experience.

However, an overreliance on a manager's skill might be detrimental and will have the opposite results, as shown by Drew et al. (2005), where the portfolio selection process, accuracy of forecasting future returns and their factor timing abilities were assessed. The reliance on managers' skills produced disappointing results, and might not be beneficial to the general investor. Forecasting based on past performance is not the best way to estimate future returns, especially if the time period is short. The relationship between portfolio construction and stock selection process does not result in a successful factor timing strategy, implying that the manager's stock selection ability is not directly linked to a successful factor timing strategy. Hedge funds are different from passive investments because they are actively managed, whereas, passive investment strategies have restrictive rules compared to active management, especially how frequently trading can occur within a portfolio. Davies et al. (2019) took a passive investment equity momentum and value factor strategies approach, which are widely popular especially for the ease of accessibility and affordability, and found that the Sharpe ratio decreased significantly for both momentum and value strategies within the passive investment space. Due to the rules in place, a passive factor investing strategy may perform differently and may even underperform as opposed to an active factor investing strategy for value and momentum, which may outperform and provide a higher Sharpe ratio. Fergis et al. (2019) suggested a framework which might be more beneficial for passive funds which intends to maintain and reserve capital in periods such as recessions and market downturns and use methods of factor diversification to minimize the risk of being exposed to certain factors. In a multifactor portfolio market, signals can be used for a successful defensive factor timing strategy, which is more passive as opposed to most proposed factor timing strategies. In order to successfully apply a defensive factor timing strategy, there has to be a number of measurements such as the level of risk that can be tolerated and a factor diversification strategy. Zheng et al. (2024) showed another approach to factor timing, which relies on sentiment, however, executing a momentum sentiment timing is challenging as momentum is rule based and requires more than sentiment for it to work as a strategy. A timing strategy works because of the predictability of factors, implying that factor timing can be added to factor exposed funds as a strategy, adding that expected factor premiums may be due to the reward of systematic risk (Souza, 2020).

Momentum outperformance can, for example, be attributed to the industry of stocks. Moskowitz and Grinblatt (1999) found that industry-specific momentum showed significant differences in returns and improved profitability, and this might be used to explain persistence in momentum strategies returns anomalies. George and Hwang (2004), on the other hand, found that momentum investing strategies' outperformance can be attributed to the 52-week high price that takes into account the performance of underlying stocks, which also plays a large role in the forecasting results of momentum returns. A momentum strategy can be impacted by behavioral biases and market sentiment, which persists during market downturns (Karki and Khadka, 2024). Short-term momentum can be subject to reversals and affect the short-term performance of a momentum investment strategy, overall, this strategy tends to persist (Huang et al., 2023).

There are many factor timing strategies and frameworks that can be used, such as a factor rotation strategy, which aims to maximize the benefits of factor exposure. Kwon (2022) found that there were exploitable return differences in equity factors when combined with economic factor analysis, which may be used as a factor timing strategy by rotating the factors in accordance with economic outlooks. Aked (2021) used three methods, namely, historical returns, economic cycle, and factor discount with momentum, and found that information within the economic cycle was already reflected in the factor discount and momentum. Factor's discount and momentum strategies should provide better limited returns, and there is a greater need for continuous improvement of forecasting methodologies of future returns which remains a great challenge in investment management. Chin and Gupta (2020) set out a framework that seeks to assess a factor timing strategy that can attribute and contribute to returns by taking the difference between long- and short-term factor investing strategies, where the significant contributor to returns was the stock selection process. However, ultimately a factor timing strategy failed to outperform.

The main question might be this: can factor timing be used in practice? Asness et al. (2018) attempted to answer this question using momentum, value, and style premia, which showed that though factors are expensive, they are not as expensive as they used to be, and that factor timing cannot not be justified as a strategy as there is still lack of substantial evidence to prove that it can outperform. Value showed some promise if it is within a single factor portfolio as opposed to a multifactor portfolio.

Time series predictors and cross-sectional tilting presents some benefits, however, the benefits of fundamental and technical time series predictability are not enough to offset the transaction costs required for a factor timing strategy, which makes cost effectiveness difficult (Dichtl et al., 2019).

Asness et al. (2017) examined a value factor timing strategy, noted that a basic performance forecasting method might be enough to predict future returns and an attempt to outperform a traditional passive buy and hold strategy is not easy and furthermore attempted to answer the question of whether a factor timing strategy will be beneficial or detrimental to returns. There are challenges to value investing as it is not enough for forecasting and timing of the market. Further cautions against the simplicity of ex-post contrarian which might be applied in a factor timing strategy, and concludes that a value timing strategy will be improved by adding a momentum component into it, which produces minimal performance levels.

3. Methodology

3.1. Portfolio construction, diversification and monitoring

One of the most commonly used portfolio construction methods is the Markowitz (1952) mean-variance optimization model, in which a risk averse investor is inclined to expect compensation for the risk taken. Expected returns can be optimized by diversification, in a momentum equity portfolio construction process of selecting stocks and allocating a percentage to achieve the stipulated investment objectives with a specified risk tolerance. Diversification of a momentum equity portfolio involves spreading exposure across different industries for risk reduction.

Suppose a momentum portfolio only has two stocks with weights w_1 and w_2. The portfolio will have a variance as,

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2 \cdot w_1 w_2 \sigma_{12}, \tag{1}$$

where σ_p^2 is the variance of the portfolio and σ_{12} is the covariance between the returns of assets 1 and 2.

This portfolio will be considered well-diversified if the two stocks have a low correlation coefficient estimated by

$$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \cdot \sigma_2}, \tag{2}$$

where ρ_{12} is the correlation coefficient between the returns of assets 1 and 2, $-1 < \rho_{12} < 1$.

This two stock portfolio will have a diversification benefit (DB), where there is a reduction in portfolio risk, given by

$$DB = \sigma_1 + \sigma_2 - \sigma_p. \tag{3}$$

To monitor a momentum equity only portfolio, returns will have to be estimated on a regular basis, together with the standard deviation and Sharpe ratio (i.e., a measure for the risk-adjusted performance), as well as the tracking error which is the difference between the portfolio return and the benchmark return. Included as well is the information ratio to estimate the active return of the portfolio compared to the benchmark.

The portfolio return is given by

$$R_p = \sum_{i=1}^{n} w_i R_i, \tag{4}$$

where R_p is the portfolio return, w_i is the weight of the i-th asset in the portfolio, and R_i is the return of the i-th asset.

The Sharpe ratio is defined as,

$$SR = \frac{R_p - R_f}{\sigma_p}, \tag{5}$$

where R_p is the portfolio return, R_f is the risk-free rate, and σ_p is the standard deviation of the portfolio return.

The tracking error is defined by

$$TE = \sqrt{\mathrm{Var}(R_p - R_b)}, \tag{6}$$

where R_b is the benchmark return.

The information ratio is defined by

$$IR = \frac{R_p - R_b}{TE}. \tag{7}$$

The portfolio variance on its own is limited and there are a number of different ways in which risk can be estimated. Some of the ways to estimate risk includes the downside risk, shortfall probabilities, value at risk (VaR), conditional value at risk (CVaR), and the maximum drawdown.

Although investors are interested in the balance between risk and return in most cases, they are more interested in the maximum value they could lose, which is the downside risk. The downside risk is also known as the semi-variance defined by

$$SV = \int_{-\infty}^{\mu} (r - \mu)^2 f(r) dr, \tag{8}$$

where SV is the semi-variance of return, μ is the expected return, r is the return on the investment, and $f(r)$ is the probability density function of the return.

In some cases, the level of tolerable losses is specified to estimate the shortfall risk, which is the probability that the loss will fall below the specified benchmark level (L). In this case an investor will not accept a risk that the loss will fall below what has been specified. The shortfall probability is given by

$$SP = P(r < L) = \int_{-\infty}^{L} f(r) dr, \tag{9}$$

where $f(r)$ is the probability density function of portfolio returns and r is the return on the investment.

The VaR is the maximum loss of a portfolio at a particular confidence level and it is defined by

$$\mathrm{VaR}_\alpha = \inf\{r \in \mathbb{R} : F(r) \geq \alpha\}, \tag{10}$$

where α is the significance level.

The CVaR is the expected value of losses exceeding the VaR defined by

$$\mathrm{CVaR}_\alpha = \frac{1}{1-\alpha} \int_{-\infty}^{-\mathrm{VaR}_\alpha} f(r)\, dr, \tag{11}$$

The maximum drawdown (MD) is the difference between the peak and trough before the next peak occurs defined by

$$MD = \frac{\text{Peak} - \text{Trough}}{\text{Peak}} \times 100, \tag{12}$$

where the peak is the highest point and trough is the lowest point.

3.2. Kalman filter and ARIMA forecasting

The Kalman filter was developed by Rudolf E. Kalman in 1960, where the Wiener problem was formulated and solved to obtain the attributes of a linear state of a system where vector spaces were taken into consideration. The results were a set of equations that are stochastic optimal estimators, iterative and recursive in nature, which are used to estimate the state of a system.

New information can be incorporated into the model as it becomes available, making it better suited for a factor timing strategy. An investment in a state space dynamic may be affected by new information which affects returns causing them to significantly vary with time, where the state space evolves in line with the discrete time stochastic model.

The ARIMA (autoregressive integrated moving average model) is a time series model, which uses historical data to forecast future values. The Kalman filter and ARIMA model are both used for prediction, and combining them improves prediction accuracy.

Using the mean square error estimation method found in Shumway and Stoffer (2017), the state transition from time t to $t+1$ is given by

$$A_{t+1} = \varphi A_t + \theta_t, \tag{13}$$

where A_t is the state vector A at time t i.e., the state vector A_t represents the current state of the momentum factor and can also be considered to be a simple autoregressive time series model (AR(1)). φ is the state transition matrix from time t to $t+1$. θ_t is the Gaussian white noise process with a known covariance matrix. The observation equation is given by

$$B_t = HA_t + \delta_t, \tag{14}$$

where B_t is the actual measurement of A at time t. H is stationary and does not contain noise, as well as shows the relationship between the state vector and the measurement vector. δ_t is the Gaussian white noise process with a known covariance matrix. θ_t and δ_t are independent of each other. B_t is the observed momentum factor values or returns, and H represents the relationship between the observed momentum factor and the underlying momentum state.

$$\zeta = \mathbb{E}[\theta\theta^T]_t. \tag{15}$$

$$\xi = \mathbb{E}[\delta\delta^T]_t, \tag{16}$$

where ζ and ξ are the uncertainty, i.e., volatility in the state transition, which captures the uncertainty in how momentum evolves over a period of time.

$$P_t = \mathbb{E}[\eta_t\eta_t^T] = \mathbb{E}[(A_t - \hat{A}_t)(A_t - \hat{A}_t)^T], \tag{17}$$

where the covariance matrix of the estimation error quantifies the uncertainty in the estimated state. This is related to the mean squared error, the expected value of the squared estimation error is equal to the error covariance matrix. The covariance matrix quantifies the uncertainty in the state estimates. The mean square error is essential in quantifying the uncertainty in the estimated state and is crucial in the update and prediction steps of the filter.

Let Equation (18) represent the update step of the Kalman filter, where the prior estimate is combined with the measurement data.

$$\hat{A}_t = \hat{A}_t' + K_t(HA_t + \delta_t - H\hat{A}_t'), \tag{18}$$

where $\hat{A}_t'$ is the prior estimate of $\hat{A}_t$ gained by knowledge of the system and K_t is the Kalman gain.

Hence,

$$P_t = \mathbb{E}\left[\left((I - K_tH)(A_t - \hat{A}_t') - K_t\delta_t\right)\left((I - K_tH)(A_t - \hat{A}_t') - K_t\delta_t\right)^T\right]. \tag{19}$$

$$P_t = (I - K_tH)P_t'(I - K_tH)^T + K_t\xi(K_t)^T, \tag{20}$$

where P_t' is the prior estimate of P_t and $A_t - \hat{A}_t'$ is the prior error estimate at time t.

$$P_t = P_t' - K_tHP_t' - P_t'H^TK_t^T + K_t(HP_t'H^T + \xi)K_t^T. \tag{21}$$

The derivative of the trace matrix is

$$\frac{d\mathrm{Tr}(P_t)}{dK_t} = -2(HP_t')^T + 2K_t(H^TP_t'H + \xi) = 0. \tag{22}$$

The Kalman gain is, therefore,

$$K_t = P_t'H^T(HP_t'H^T + \xi)^{-1}. \tag{23}$$

The update equation is, therefore,

$$P_t = P_t' - P_t'H^T(HP_t'H^T + \xi)^{-1}HP_t'. \tag{24}$$

$$P_t = (I - K_tH)P_t'. \tag{25}$$

Let η_{t+1}' be the prior error at time $t + 1$ and defined by

$$\eta_{t+1}' = \varphi(A_t - \hat{A}_t') + \theta_t. \tag{26}$$

$$\eta_{t+1}' = \varphi\eta_t' + \theta_t. \tag{27}$$

Prior error at the next time step $(t + 1)$ is related to the prior error at the current time step (t) and the process noise θ_t. Let P_{t+1}' be the error covariance matrix at time $t + 1$:

$$P_{t+1}' = E[(\varphi\eta_t' + \theta_t)(\varphi\eta_t' + \theta_t)^T]. \tag{28}$$

$$P_{t+1}' = \varphi^2E[\eta_t'\eta_t'^T] + \varphi E[\eta'_t\theta_t^T] + \varphi E[\theta_t\eta_t'^T] + E[\theta_t\theta_t^T], \tag{29}$$

where η_t' and θ_t have no cross-correlation, $E[\eta_t'\theta_t^T] = E[\theta_te_t'^T] = 0$. Therefore,

$$P_{t+1}' = \varphi P_t\varphi^T + \zeta. \tag{30}$$

The error covariance matrix at the next time step $(t + 1)$ is related to the error covariance matrix at the current time step (t), the state transition matrix φ, and the process noise covariance matrix ζ.

The final Kalman filter equations are illustrated by the following five steps,

1. State Prediction:

$$\hat{A}_{t+1|t} = \varphi \hat{A}_{t|t}, \tag{31}$$

Predicts the state at time $t+1$ ($\hat{A}_{t+1|t}$) based on the state estimate at time t ($\hat{A}_{t|t}$) and the state transition matrix (φ).

2. State covariance prediction:

$$P_{t+1|t} = \varphi P_{t|t} \varphi^T + \zeta, \tag{32}$$

Predicts the error covariance matrix at time $t+1$ ($P_{t+1|t}$) based on the error covariance matrix at time t ($P_{t|t}$), the state transition matrix (φ), and the process noise covariance matrix (ζ).

3. Kalman gain:

$$K_{t+1} = P_{t+1|t} H^T (H P_{t+1|t} H^T + \xi)^{-1}, \tag{33}$$

Estimates the Kalman gain (K_{t+1}) based on the predicted error covariance matrix ($P_{t+1|t}$), the measurement matrix (H), and the measurement noise covariance matrix (ξ).

4. State update:

$$\hat{A}_{t+1|t+1} = \hat{A}_{t+1|t} + K_{t+1}(B_{t+1} - H\hat{A}_{t+1|t}), \tag{34}$$

Updates the state estimate at time $t+1$ ($\hat{A}_{t+1|t+1}$) based on the predicted state at time $t+1$ ($\hat{A}_{t+1|t}$), the Kalman gain (K_{t+1}), and the difference between the actual measurement (B_{t+1}) and the predicted measurement ($H\hat{A}_{t+1|t}$).

5. Error covariance update:

$$P_{t+1|t+1} = (I - K_{t+1}H)P_{t+1|t}, \tag{35}$$

Updates the error covariance matrix at time $t+1$ ($P_{t+1|t+1}$) based on the predicted error covariance at time $t+1$ ($P_{t+1|t}$) and the Kalman gain (K_{t+1}).

The Kalman filter steps here do not include a multistep ahead predictor, thus, as a part of the continuous risk monitoring process, the following addition to the Kalman filter is added for forecasting and signal purposes. The state transition and observation equations are in a simple autoregressive model form (i.e., $AR(1)$), therefore, the proposal here is to add an $ARIMA(p, d, q)$ (where p - autoregressive term, d - integrated order required for stationarity, q - lagged forecast errors moving average term) process for a multistep prediction (forecast) after the Kalman filter has been applied, as well as estimate the confidence intervals.

The general form of an ARIMA(p,d,q) process is

$$\varphi(E)(1 - E)^d A_t = \theta(E) dW_t, \tag{36}$$

where E is the back shift operator in the form $EA_t = A_{t-1}$ and W_t white noise process with a known covariance matrix, $(1 - E)^d$ is integrated of order d.

$AR(p)$

$$\varphi(E) = 1 - \varphi_1 E - \varphi_2 E^2 - \ldots - \varphi_p E^p. \tag{37}$$

and $MA(q)$:

$$\theta(E) = 1 + \theta_1 E + \theta_2 E^2 + \ldots + \theta_q E^q. \tag{38}$$

The Kalman filter starts off with the state transition and the observation equation as $AR(1)$ processes. Thus, the MA part of the model is set to be of order $q = 0$, and integrated of order $d = 1$ for stationarity of the returns. The historical prices only need to be differenced once to achieve stationarity, resulting in an $ARIMA(1, 1, 0)$. The ARIMA model will be applied to the Kalman filtered data.

Estimating the multistep prediction interval is preceded by determining the forecast error and the prediction error.

The multistep forecast error is defined by

$$\eta^n_{n+m} = A_{n+m} - A^n_{n+m}. \tag{39}$$

$$A^n_{n+m} = \varphi^m A_n. \tag{40}$$

The multistep prediction error is defined by

$$P^n_{n+m} = Var(A_{n+m} - A^n_{n+m}) = Var(\eta^n_{n+m}). \tag{41}$$

The multistep prediction interval is, therefore,

$$A^n_{n+m} \pm z_{\frac{\alpha}{2}} \sqrt{P^n_{n+m}}, \tag{42}$$

where n is the number of observations and m is the number of prediction steps.

4. Framework

The proposed high pure momentum equity timing framework is illustrated by the flow chart in Figure 1. The process starts with the overall stock data, from which the population of large capitalization stocks will be selected. This process ends with the portfolio rebalancing, followed by the portfolio performance evaluation.

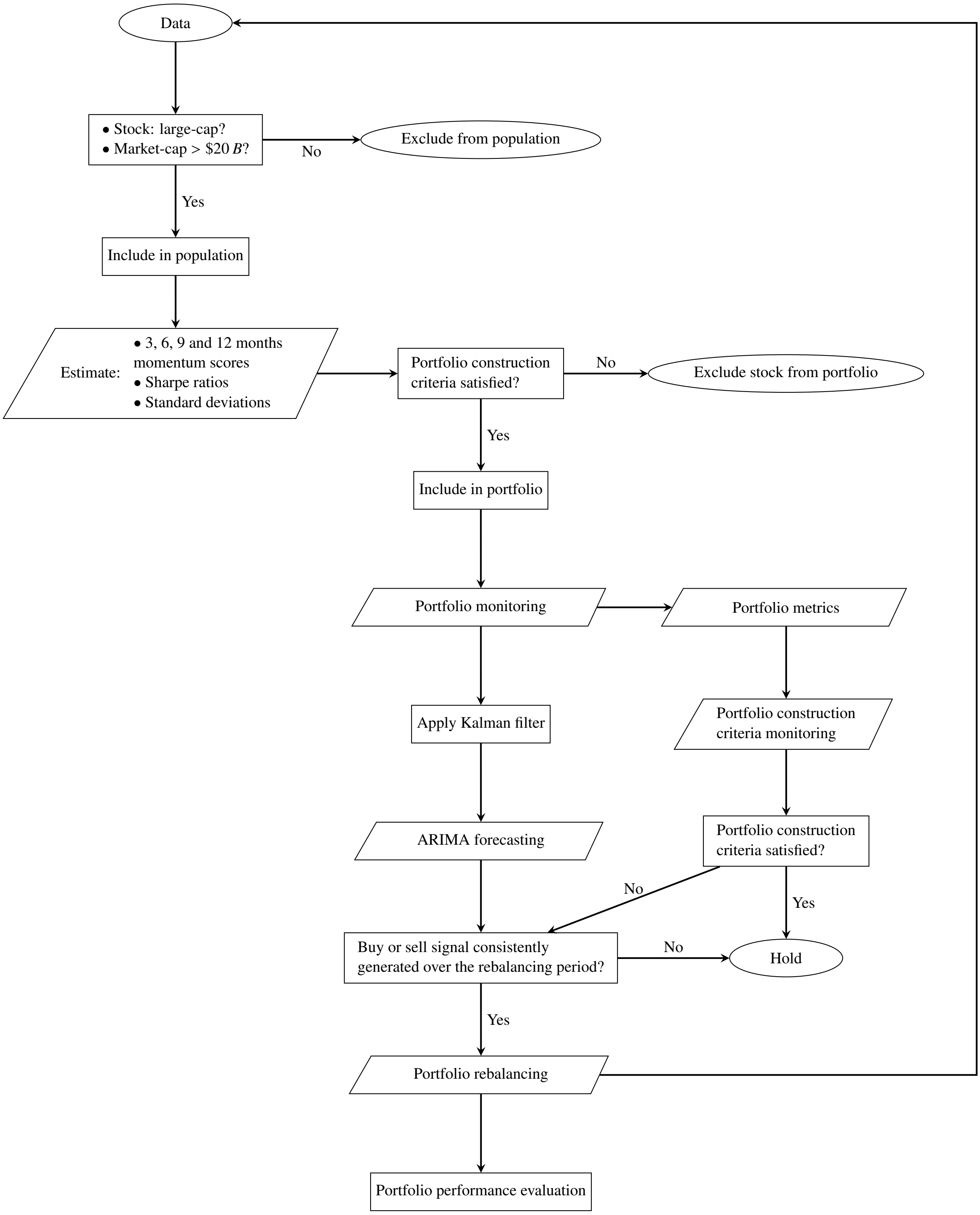

Figure 1. Momentum equity timing framework.

4.1. Data

The population consists of 540 large capitalization US listed stocks, with more than $20 billion dollars in market capitalization. The historical data period was set to be from 01 January 2013 to

31 December 2023, taken from the Yahoo finance database. The portfolio criteria was used to select momentum stocks. This resulted in an initial portfolio of 54 stocks.

4.2. Portfolio construction criteria

The portfolio construction process is an important part of a factor timing framework, and starts off with defining the investable universe population of stocks, followed by estimating the momentum scores, Sharpe ratio and standard deviation.

The momentum score is estimated by:

$$MS = \frac{P_t - P_{t-s}}{P_{t-s}} \times 100, \tag{43}$$

where P_t is the current price and P_{t-s} is the price at the beginning of the period.

The market capitalization is estimated by

$$MC = PS \times OS, \tag{44}$$

where PS is the current market price per share and OS is the total number of outstanding shares.

After all the metrics above have been estimated, the next part is defining the portfolio selection criteria. The stocks that fit the below criteria are included in the portfolio.

1. 3-months momentum score > 10%
2. 6-months momentum score > 15%
3. 9-months momentum score > 20%
4. 12-months momentum score > 30%
5. Sharpe ratio > 0.7
6. Standard deviations < 0.25

Momentum gains are subject to short-term reversals, therefore the thresholds are conservative. A Sharpe ratio above 0.7 is considered good. The standard deviation of below 0.25 is low risk and low volatility. The above criteria is set to limit the risk of investing in momentum stocks.

4.3. Portfolio rebalancing and performance evaluation

Portfolio rebalancing is a process where some stocks will be bought (added into the portfolio) and sold, to ensure that the stocks are still within the initial criteria. The market upturns and downturns may change the metrics used in the construction process to change significantly; rebalancing maintains the asset allocation criteria (Kimball et al, 2020; Fischer et al., 2021). The rebalancing period is followed by the portfolio performance evaluation, which involves assessing returns earned within the rebalance period (Plastira, 2014).

5. Empirical results

Table 1 presents the initial portfolio constructed using the selection criteria.

Table 1. Initial portfolio constructed using the selection criteria.

Symbol	Description	3-Months	6-Months	9-Months	12-Months	Sharpe Ratio	Standard Deviation
NVDA	NVIDIA Corporation	10.59433136	16.78126421	80.43768882	246.0983373	1.26710737	0.028147058
AMZN	Amazon.com, Inc.	17.36443269	16.6794663	46.16643281	77.04498134	0.851726283	0.020567677
META	Meta Platforms, Inc.	15.36404764	23.75359013	64.84724951	183.758223	0.795465724	0.02412872
AVGO	Broadcom Inc.	34.30537794	28.64431864	78.66948701	106.2686067	1.187161527	0.021668064
NVO	Novo Nordisk A/S	12.69062621	30.39886221	31.09779796	53.00867092	0.839034754	0.016593725
COST	Costco Wholesale Corporation	18.38221012	25.24164456	36.42349095	50.06235101	1.028850178	0.013021139
AMD	Advanced Micro Devices, Inc.	42.74233413	27.27508552	53.76029982	130.2561877	0.926067692	0.036203874
ADBE	Adobe Inc.	14.4819853	22.95706727	54.90068416	77.07466218	0.950663157	0.019874772
INTU	Intuit Inc.	20.91908102	38.89376618	42.8613522	60.94612887	0.886367935	0.018753059
PDD	PDD Holdings Inc.	46.79442184	105.2321443	99.87705418	73.08647778	0.77998403	0.048813747
NOW	ServiceNow, Inc.	27.24276698	25.51566016	48.40668172	83.26588592	0.911687887	0.025500119
BX	Blackstone Inc.	23.24061869	40.44177126	65.45178543	78.09046953	0.887277843	0.0214946
MU	Micron Technology, Inc.	25.95177023	34.20639753	50.04574591	70.59995941	0.748027812	0.028166555
ETN	Eaton Corporation plc	15.00282665	20.99063704	50.37858267	55.43148966	0.718579387	0.017132361
LRCX	Lam Research Corporation	24.49205203	21.01097475	56.64416157	91.36082903	0.931538062	0.024530173
KLAC	KLA Corporation	26.54408107	20.55738631	49.28312541	56.24836143	0.920050579	0.022711055
SHOP	Shopify Inc.	44.25926208	20.12336678	63.68985349	118.3295988	1.018964952	0.039144939
PANW	Palo Alto Networks, Inc.	24.54280507	15.80270405	50.02035437	112.986646	0.867442583	0.024271038
KKR	KKR & Co. Inc.	36.18028349	47.40475551	63.81022106	79.47745348	0.730641813	0.021258249
DELL	Dell Technologies Inc.	13.63644415	41.85501394	91.18587756	92.59426258	0.901406608	0.022556047
SNPS	Synopsys, Inc.	11.0150414	18.64011464	33.76717402	61.07044074	1.065284995	0.017099892
ANET	Arista Networks, Inc.	25.39800733	47.28580581	41.24384476	94.7812318	0.897128435	0.027619942
CDNS	Cadence Design Systems, Inc.	15.17188413	15.70518359	28.77405427	70.61513165	1.05750787	0.01873592
SHW	The Sherwin-Williams Company	23.73412144	18.61901006	40.72000842	31.59449596	0.803611683	0.016106649
STLA	Stellantis N.V.	23.32099507	30.9376759	41.48762857	74.36241033	0.753285988	0.025164742
RELX	RELX PLC	18.84926884	20.73084076	24.18272439	45.77388631	0.809707056	0.012892264
CRWD	CrowdStrike Holdings, Inc.	50.59573991	74.92463386	86.55560879	147.211474	0.842925719	0.037694839
MAR	Marriott International, Inc.	16.68543983	23.02565505	38.30927896	54.23834213	0.701551537	0.019730663
MELI	MercadoLibre, Inc.	24.78382706	31.64733312	20.71126888	90.23375421	0.804798822	0.02995831
PH	Parker-Hannifin Corporation	20.01806558	18.68797821	45.39440952	60.39240001	0.71811907	0.018713831
APH	Amphenol Corporation	20.20587212	18.45162967	25.3927607	31.05243903	0.85351241	0.014524717
CTAS	Cintas Corporation	24.60678347	24.29478328	33.50394459	35.61233196	1.141890667	0.015809445
TDG	TransDigm Group Incorporated	27.2220435	16.99404634	42.56879154	67.60040166	0.902680183	0.020196651
TT	Trane Technologies plc	23.53579994	28.46180007	41.9914447	44.43744943	0.927943241	0.016294082
CEG	Constellation Energy Corporation	11.86941724	28.24692605	55.0810585	44.44362031	1.589334207	0.024262035
TEAM	Atlassian Corporation	20.45983618	41.92971288	42.86743514	88.03162104	0.840574422	0.032610153
DHI	D.R. Horton, Inc.	43.47039855	27.02567595	57.06760037	69.14983165	0.723872265	0.022227888
URI	United Rentals, Inc.	31.55034448	28.63989792	61.39941906	63.02588499	0.753044739	0.026398223
DDOG	Datadog, Inc.	32.16463643	23.49170427	75.10097777	68.37287199	0.744583765	0.040413051
IR	Ingersoll Rand Inc.	21.63842538	18.61625561	40.08534617	46.10885678	0.732573065	0.022311443
IT	Gartner, Inc.	30.37860848	29.97291771	43.62901319	33.66618223	0.86736747	0.017808524
MPWR	Monolithic Power Systems, Inc.	37.53918296	16.70525232	31.34530319	85.36402381	0.977600083	0.02517973
VRT	Vertiv Holdings Co	25.11485268	93.23124282	266.8443661	261.3279128	0.838094781	0.031530406
NET	Cloudflare, Inc.	33.19469584	26.49650945	35.97909972	93.53789226	0.850583626	0.046429274
FICO	Fair Isaac Corporation	34.78733815	46.41635343	68.87087392	96.89270877	1.06509056	0.020985282
ZS	Zscaler, Inc.	37.70899273	51.14264486	100.4886375	101.0708709	0.840145677	0.038594287
ICLR	ICON Public Limited Company	16.42264047	16.47533586	34.25821108	46.22140228	0.835171098	0.019419194
ARES	Ares Management Corporation	16.8772408	24.97193959	50.37064775	79.7513977	0.863940276	0.021670163
BR	Broadridge Financial Solutions, Inc.	16.73261031	26.60157385	45.2276038	55.73197355	1.071776412	0.014227325
BLDR	Builders FirstSource, Inc.	35.72357922	22.45287185	91.0943238	155.4552507	0.835582112	0.03276591
PTC	PTC Inc.	24.22608014	24.66866986	37.11599436	46.22649769	0.74578355	0.019649932
DECK	Deckers Outdoor Corporation	29.1527405	25.77239398	46.82378824	71.73136033	0.850109584	0.0252146
DKNG	DraftKings Inc.	21.38429369	34.28571429	84.74842841	219.0045194	0.765326057	0.043951352
CBOE	Cboe Global Markets, Inc.	14.22484439	30.83153957	33.41978594	43.38262867	0.889541723	0.015152917

The initial portfolio in Table 1 contains 54 stocks. The selection criteria determines the portfolio size. Hence, the maximum number of stocks that should be included in the portfolio will vary, after each rebalancing period.

- 3 months momentum scores are mostly the lowest scores compared to other periods, and this might be subject to reversals.
- 6 months momentum scores will either be greater than the 3 months momentum scores or they will be on a reversal trajectory (i.e., decrease).
- 9 months momentum scores are higher compared to the 6 months time period.
- 12 months momentum scores indicate the persistence of the price momentum with high scores.
- Sharpe ratios are obtained by subtracting the risk-free-rate of 0.05, indicating that each stock had significant excess returns.
- Standard deviation is lower than the threshold of 0.25, indicating that the stocks within the portfolio have low volatility.

5.1. Population and portfolio comparison

Table 2 compares the metrics of both the population of stocks and the back tested portfolio.

Table 2. Population of Stocks and Portfolio Metrics Comparison.

Metric	**Population**	**Portfolio**
Total Return	4.5377	14.6402
Annualized Return	0.1686	0.2845
Volatility	0.0111	0.0146
Sharpe Ratio	10.6844	16.0229
Max Drawdown	−0.3608	-0.3699
Downside Risk	0.0092	0.0117
Shortfall Probability	0.4434	0.4257
VaR (95%)	−0.0176	−0.0230
CVaR (95%)	−0.0280	−0.0343

- Total and annualized returns: indicate that even though the population contained more stocks, this did not automatically result in higher returns over a period of 10 years (2013–2023).
- Volatility: was relatively stable when spread out through the 10 year period for both the constructed portfolio and the population of stocks.
- Sharpe ratio is relatively high, indicating better risk adjusted performance.
- Max drawdown: the largest peak-to-trough decline was negative indicating that the population could lose −36.08% and portfolio −36.99% in value.
- Downside risk: the volatility of returns below zero was also relatively low.
- Shortfall probability: the likelihood of not achieving projected positive returns was 0.4424 and 0.4311.
- VaR (95%): the value at risk, which is the maximum value that could be lost for the population, is −0.0176 and for the portfolio it is −0.0230.
- CVaR (95%): the conditional value at risk is −0.0280 and −0.0343 for the population and portfolio, respectively.

5.2. Portfolio sector exposure

Sector exposure is a crucial aspect of portfolio risk management (Keisler and Linkov, 2011). A portfolio will have a wide range of sector exposures. Market sentiment is another driving force within the stock market and herd mentality can be an influential factor in the liquidity of stocks; these are the overall expectations from investors (Aggarwal, 2022).

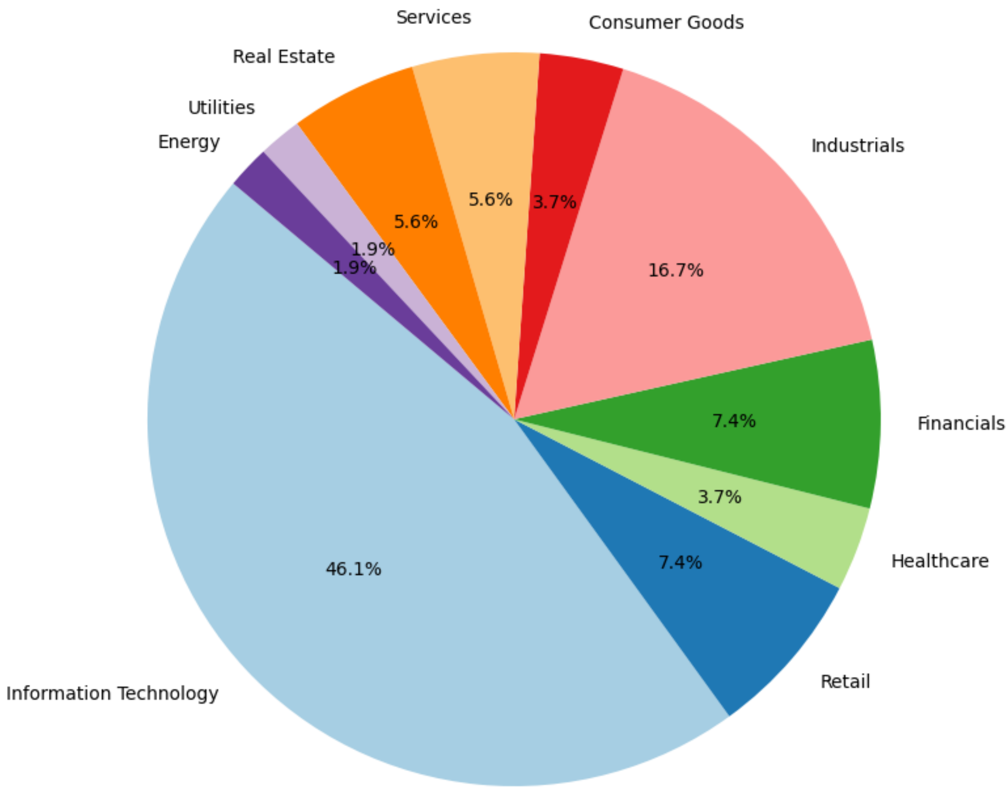

Figure 2. Portfolio industry exposure.

Figure 2 shows that the initial portfolio constructed by the selection criteria has high exposure of information technology stocks that tend to have high momentum due to their market sentiment, perceived future potential growth, and their liquidity. This is reflected by the overall portfolio exposure of 46.1%. This indicates that information technology stocks have been high performers and are one of the most popular stocks in the market. This drives up their value, further increasing their momentum scores at a higher rate as compared to other sectors.

The next largest exposure is industrial which includes manufacturing, construction and others. Financials and retail are tied with an exposure of 7.4% each, and finance stocks are popular due to their stability in the overall market. The rest of the sectors make up the remaining exposure.

5.3. Kalman filter, ARIMA forecasting and trading signal

The portfolio construction is followed by the risk management process. The Kalman filter is applied to the portfolio as a part of the risk management process. The Kalman filter incorporates information as it becomes available and is better equipped to handle missing data. This is then followed by the ARIMA forecasting and the confidence interval for each underlying stock.

Figure 3 illustrates the trading signals over a period of 6 months for the first four stocks within the initial portfolio. The trading signals are generated for each underlying stock. The Kalman filter, and ARIMA forecast with the confidence intervals of the forecast are also depicted in Figure 3.

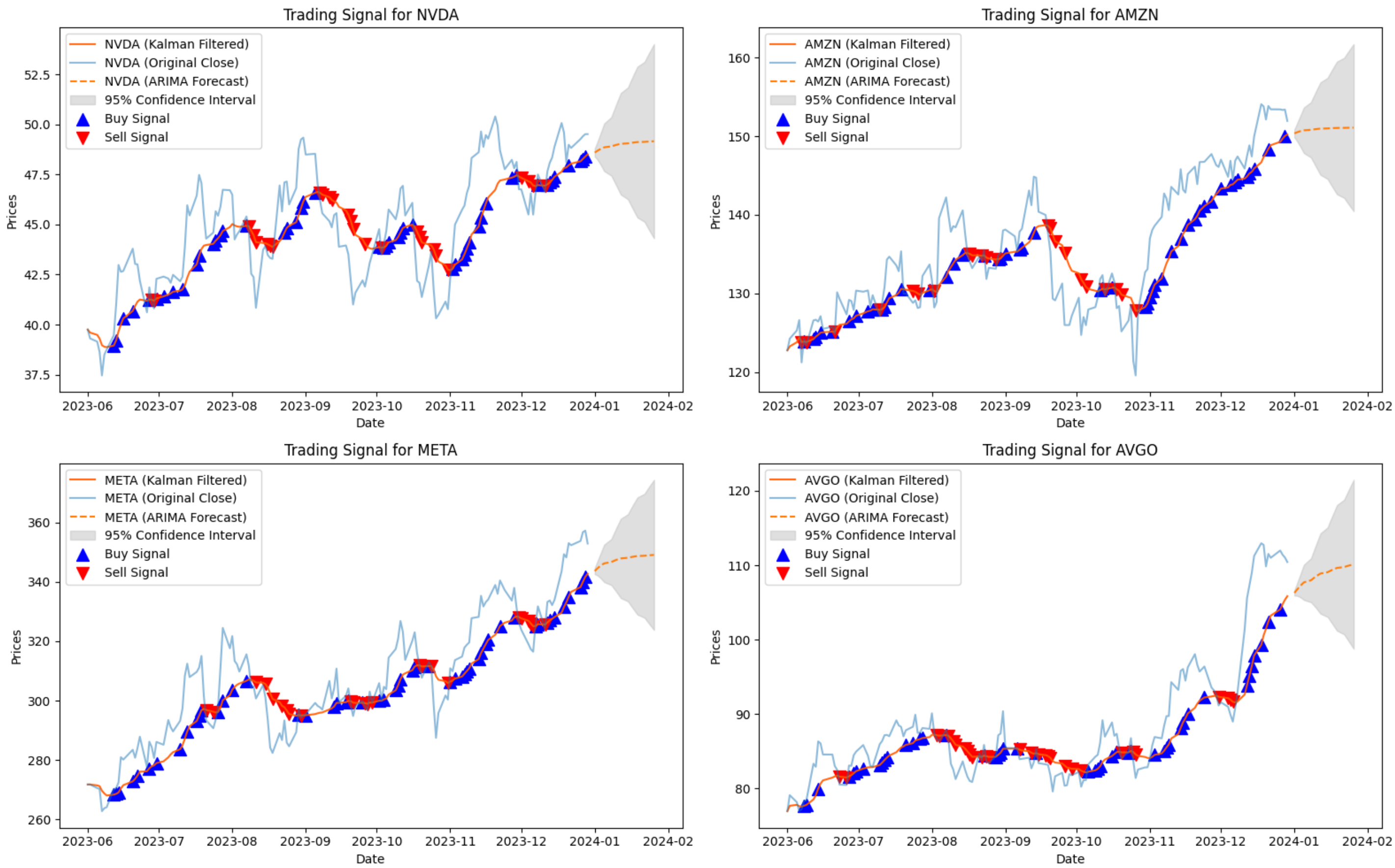

Figure 3. Trading signals for the first four stocks within the portfolio.

- The blue line shows the historical daily prices over the period of 6 months starting from 01 June 2023 to 31 December 2023 for the first four stocks within the portfolio on Table 1.
- The Kalman filter reduces the short-term fluctuations of the historical daily data, minimizes noise, and provides a better view of the historical trends. The trend generated by the Kalman filter is used as an indicator for the buy and sell signal.
- The Kalman filtered historical daily data is used for the ARIMA forecast (forecast period: 20 days).
- The 95% confidence interval is estimated for the forecast, and this increases as the forecast period increases.
- The four stocks are currently showing an increasing upward trend, signaling an increase in the momentum score.
- The buy signal is generated when the Kalman filter trend line is below the historical prices and forecast.
- The sell signal is generated when the Kalman filter trend line is above the historical prices and forecast.

6. Transaction costs

There are a number of transaction costs that can be detrimental to portfolio returns and could be the main difference between the outperformance and underperformance of a factor timing strategy (Wang and Siu, 2024). An active factor timing strategy depends heavily on the costs involved during the implementation process. An equilibrium of transaction costs is essential, especially when trading occurs frequently and liquidity premium becomes an added cost (Isaenko, 2023). Figure 4 shows a breakdown of transaction costs.

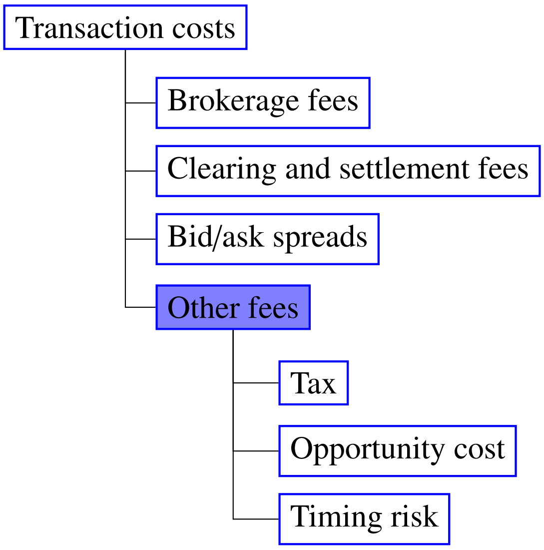

Figure 4. Transaction costs breakdown.

A brokerage fee is a commissions that a broker will charge on a trade; clearing and settlement fees are charged by the exchange for the execution of a trade; and bid/ask spreads are the price difference of the stock that is being traded (Galati, 2024). The breakdown of transaction costs can be broader and might include other costs depending on the market and country.
Figure 5 shows a comparison between the buy-and-hold strategy with the monthly, quarterly and annual rotations, illustrating the impact of transaction costs on each strategy. The transaction costs here are considered on a total basis.

The transaction costs make short-term timing rotation strategy more expensive and diminishes cumulative returns. A momentum portfolio is vulnerable to market downturns, which can negatively affect the portfolio's performance. The portfolio has large concentration of technology stocks, which could contribute to low returns when that industry is underperforming.

- A 0% transaction cost in reality is not possible, as there are unavoidable costs when trading on the market. However, from the simulated results, the monthly rotation has the highest cumulative returns, and significantly outperforms other strategies.
- A total transaction costs of 0.1% is low. The monthly rotation closely matches that of the buy-and-hold strategy.
- At 0.5% total transaction cost adjustment, the buy-and-hold strategy has higher cumulative returns, followed by the monthly rotation.

- The 1% total transaction costs affects the returns significantly, and the rotation strategies have diminished cumulative returns compared to the lower 0.1% and 0.5% transaction costs.
- At 2% total transaction costs, the cumulative returns are still increasing, however, at a much lower rate. The buy-and-hold strategy is not affected as there are no significant trades occurring.
- The transaction costs can in some cases, be as high as 4% and the strategy with the least amount of trades, will have the highest cumulative returns. The buy-and-hold strategy significantly outperforms other rotation strategies, which have decreasing cumulative returns, closer to 0.

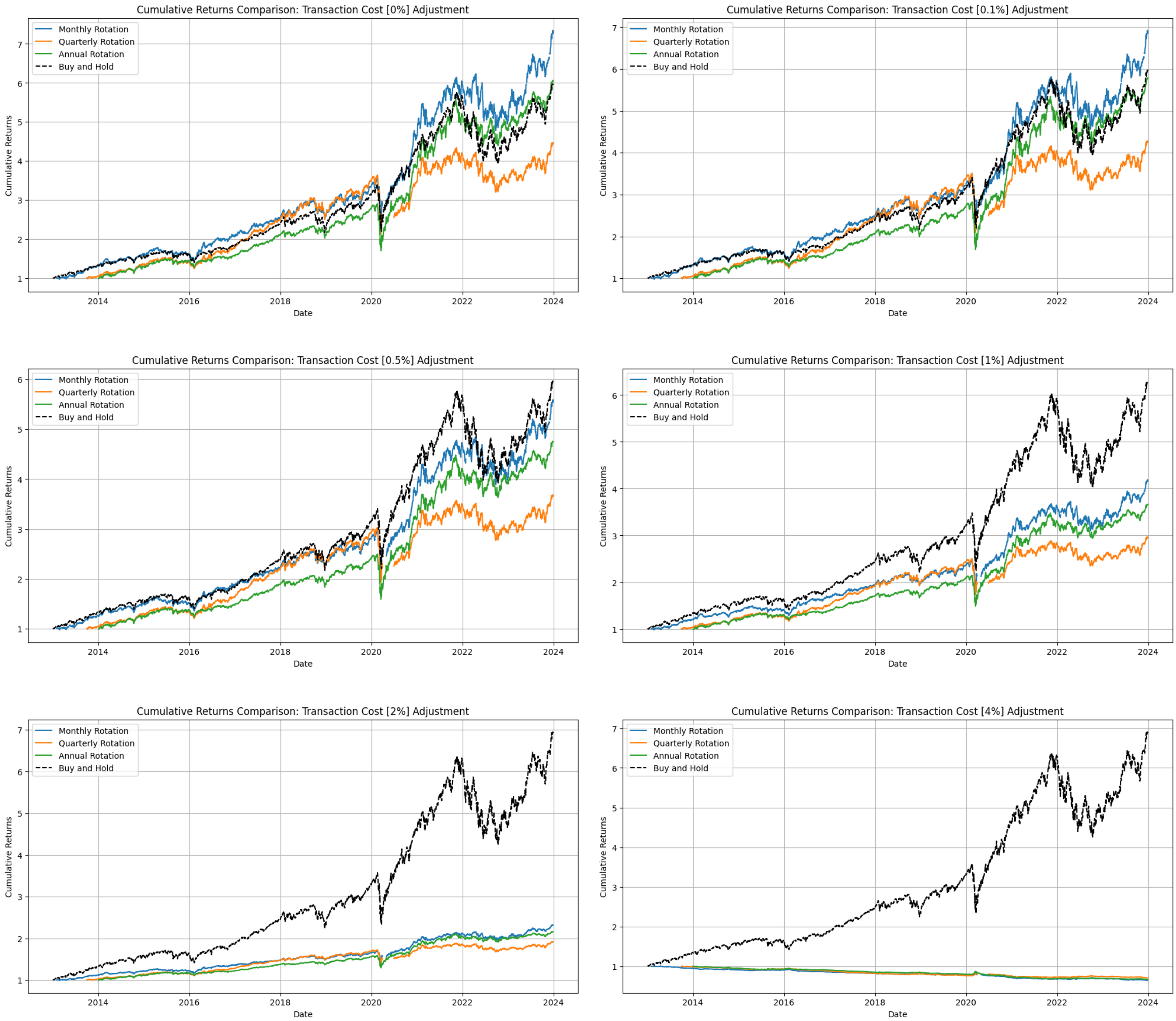

Figure 5. Buy-and-hold strategy, monthly, quarterly, and annual rotations' cumulative returns comparison with transaction costs [0%, 0.1%, 0.5%, 1%, 2% and 4%] adjustments.

Relatively, low transaction costs should have a limited effect on the rotation strategy. The monthly rotation process will have the best results, followed by the annual and lastly quarterly rotation, when the costs are at 0% and 0.1%. Surprisingly, the rotation that is below the buy-and-hold strategy at 0% costs is the quarterly rotation. 3 months momentum is subject to reversals and, as such, at the 3 month point this reversal might affect the bid/ask spread, and make the stocks more costly. 12 months momentum in some cases will be higher than 3 months momentum and, as a result, the annual rotation performed better than the quarterly rotation. The monthly rotation takes advantage of the short-term increases, and this has resulted in the highest cumulative returns when transaction costs are 0%.

7. Conclusions

This article presents a practical pure momentum factor timing strategy. summarized below:

- Investment objectives and portfolio strategy defined.
- Portfolio construction process using momentum scores, Sharpe ratio, and standard deviation.
- Implementation of the Kalman filter for signal processing.
- Implementation of the ARIMA and confidence intervals.
- Signal assessments of both historical trends and the forecasts trend.
- Rebalancing (adjusting portfolio exposure) procedure based on the results of the Kalman filter, ARIMA and confidence intervals.

A dynamic portfolio construction process, together with the Kalman filter, was implemented for the state dynamic system estimation using daily historical data. Additionally, the ARIMA forecasting and confidence interval enhanced the forecast accuracy. Momentum strategies can be susceptible to sharp downturns, reversals, market volatility, and sentiment arising from unforeseen events. However, returns are somewhat predictable, and this predictability plays a pivotal role in the way a pure momentum strategy is implemented.

A factor timing strategy does rely on this predictability and the forecasts of future returns, even though past returns do not guarantee future returns. A Kalman filter approach addresses this past returns limitation by iteratively incorporating new information as it becomes available. The momentum definition does assert that if a fund or stock has been on a recent increasing trajectory, it tends to continue unless a significant event disrupts the trend.

A high pure momentum only equity factor timing framework presented in this article, offers a comprehensive approach starting with the selection of stocks, dynamic monitoring, and rebalancing, together with the forecasting procedure to achieve superior returns. The costs involved with implementing this strategy can be minimized by choosing a rebalancing period that limits the trading frequency.

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Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

All authors declare no conflicts of interest in this paper.

References

Aked M (2021) Factor Timing: Keep It Simple. Research Affiliates.

Aiken AL, Kang M (2023) Hedge fund manager timing and selectivity skill over time, A holdings-based estimate. *Financ Res Lett* 58.

Aggarwal D (2022) Defining and measuring market sentiments: a review of the literature. *Qual Res Financ Mark* 14: 270–288. https://doi.org/10.1108/QRFM-03-2018-0033

Asness C, Chandra S, Ilmanen A, et al. (2018) Contrarian Factor Timing Is Deceptively Difficult. *J Portfoli Manage Forthcoming* 43.

Asness C, Ilmanen A, Maloney T (2017) Market Timing: Sin a Little, Resolving the Valuation Timing Puzzle. *J Invest Manag* 15: 23–40.

Carhart MM (1997) On Persistence in Mutual Fund Performance. *J Financ* 52: 57–82.

Chin A, Gupta P (2020) Timing is Not Everything—Assessing Manager Skill in Factor Timing. *J Invest Manage* 18: 34–51.

Clare A, Sherman M, O'Sullivan N, et al. (2022) Manager characteristics: Predicting fund performance. *Int Rev Financ Anal* 80: 102049. https://doi.org/10.1016/j.irfa.2022.102049

Daniel K, Grinblatt M, Titman S, et al. (1997) Measuring Mutual Fund Performance with Characteristic-Based Benchmarks. *J Financ* 52: 1035–1058. https://doi.org/10.1111/j.1540-6261.1997.tb02724.x

Davies J, Gibbon D, Shores S, et al. (2019) Implementation Matters: Relaxing Constraints Can Improve the Potential Returns of Factor Strategies. *J Portfoli Manage Quant* 45: 101–114. https://doi.org/10.3905/jpm.2019.45.3.101

Dichtl H, Drobetz W, Lohre H, et al. (2019) Optimal Timing and Tilting of Equity Factors. *Financ Anal J* 75: 84–102. https://doi.org/10.1080/0015198X.2019.1645478

Drew ME, Veeraraghavan M, Wilson V (2005) Market Timing, Selectivity and Alpha Generation: Evidence from Australian Equity Superannuation Funds. *Invest Manage Financ Innov* 2: 111-127.

Fama EF, French KR (1992) The cross-section of expected stock returns. *J Financ* 47: 427–465.

Fergis K, Gallagher K, Hodges P, et al. (2019) Defensive Factor Timing. *J Portfoli Manage Quant* 45: 50–68. https://doi.org/10.3905/jpm.2019.45.3.050

Fischer AM, Greminger RP, Grisse C, et al. (2021) Portfolio rebalancing in times of stress. *J Int Money Financ* 113. https://doi.org/10.1016/j.jimonfin.2021.102360

Galati L (2024) Exchange market share, market makers, and murky behavior: The impact of no-fee trading on cryptocurrency market quality. *J Bank Financ* 165.https://doi.org/10.1016/j.jbankfin.2024.107222

George TJ, Hwang C (2004) The 52-Week High and Momentum Investing. *J Financ* 59: 2145–2176. https://doi.org/10.1111/j.1540-6261.2004.00695.x

Gupta T, Kelly B (2019) Factor Momentum Everywhere. *J Portfoli Manage Quant* 45: 13–36. https://doi.org/10.3905/jpm.2019.45.3.013

Huang J, Zhang P, Zhang J (2024) Understanding Momentum and Reversal Investing Strategies. *J Econ Financ Account Stud* 5: 106–112. https://doi.org/10.32996/jefas.2023.5.1.8

Huberman G, Wang Z (2005) Arbitrage Pricing Theory. Federal Reserve Bank of New York Staff Reports. Available from: https://www.econstor.eu/handle/10419/60653.

Hong H, Stein JC (1999) A Unified Theory of Underreaction, momentum trading, and overreaction in Asset markets. *J Financ* 54: 2143–2184. https://doi.org/10.1111/0022-1082.00184

Isaenko S (2023) Transaction costs, frequent trading, and stock prices. *J Financ Mark* 64: 100775. https://doi.org/10.1016/j.finmar.2022.100775

Jegadeesh N, Titman S (1993) Returns to buying winners and selling losers: Implications for stock market efficiency. *J Financ* 48: 65–91.

Kálmán RE (1960) A New Approach to Linear Filtering and Prediction Problems. *J Basic Eng* 82: 35–45. https://doi.org/10.1115/1.3662552

Karki D, Khadka PB (2023) Momentum Investment Strategies across Time and Trends: A Review and Preview. *Nepal J Multidiscip Res* 7: 62–83. https://ssrn.com/abstract=4837507

Keisler J, Linkov I (2011) Managing a portfolio of risks. Management Science and Information Systems Faculty Publication Series, 33.

Kimball MS, Shapiro MD, Shumway T, et al. (2020) Portfolio rebalancing in general equilibrium. *J Financ Econ* 135: 816–834. https://doi.org/10.3386/w24722

Kwon D (2022) Dynamic Factor Rotation Strategy: A Business Cycle Approach. *Int J Financ Stud* 10: 46. https://doi.org/10.3390/ijfs10020046

Lintner J (1965) The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Rev Econ Stat* 47: 13–37.

Markowitz H (1952) Portfolio Selection. *J Financ* 7: 77–91.

Moskowitz TJ, Grinblatt M (1999) Do industries explain momentum? *J Financ* 54: 1249–1290.

Osinga B, Schauten M, Zwinkels RCJ (2020) Timing is Money: The Factor Timing Ability of Hedge Fund Managers. *Available at SSRN*, 2811163.

Plastira S (2014) Performance evaluation of size, book-to-market and momentum portfolios. *Procedia Econ Financ* 14: 481–490. https://doi.org/10.1016/S2212-5671(14)00737-0

Ross SA (1976) The Arbitrage Theory of Asset Pricing. *J Econ Theory* 13: 341–360.

Sharpe WF (1964) Capital asset prices: A theory of market equilibrium under conditions of risk. *J Financ* 19: 425–442

Shumway RH, Stoffer DS (2017) Time Series Analysis and Its Applications: With R Examples (4th ed.). Springer.

Souza TO (2020) Macro-Finance and Factor Timing: Time-Varying Factor Risk and Price of Risk Premiums. *Available at SSRN*.

Treynor J, Mazuy K (1966) Can Mutual Funds Outguess the Market? *Harvard Bus Rev* 44: 131–136.

Wang N, Siu TK (2024) Investment-consumption optimization with transaction cost and learning about return predictability. *Eur J Oper Res* 318: 877–891. https://doi.org/10.1016/j.ejor.2024.06.024

Zheng Y, Osmer E, Zu D (2024) Timing sentiment with style: Evidence from mutual funds. *J Bank Financ* 164. https://doi.org/10.1016/j.jbankfin.2024.107197