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Research article

Quantification of the stock market value at risk by using FIAPARCH, HYGARCH and FIGARCH models

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Abstract: The South African financial market is developing with periods of high and low volatility. Employing an adequate volatility model is essential to manage market risk. This research study was designed to investigate the effectiveness of the fractionally integrated asymmetric power autoregressive conditional heteroskedasticity contrasted with long-memory GARCH-type models, such as the fractionally integrated generalized autoregressive conditional heteroskedasticity and the hyperbolic generalized autoregressive conditional heteroskedasticity for producing the measure of market risk known as the value at risk. These long-memory GARCH-type models assume that the distributions of the index returns follow normal, student-*t*, skewed student-*t* and generalized error distributions. The historical closing price time series of the Johannesburg Stock Exchange all share, the mining and the banking indices are considered. The value at risk and its backtesting for short and long trading positions on the different confident levels are computed and they correspond to the right and left quantiles of the return distributions, respectively. The results reveal that FIAPARCH with a standard student-*t* distribution is an appropriate model for producing a robust value at risk in the context of mining and banking indices. Alternatively, FIGARCH with the assumed skewed student-*t* distribution model is a good fit to produce a value at risk for the Johannesburg Stock Exchange All Share Index.

Keywords: backtesting; FIAGARCH; FIGARCH; HYGARCH; parametric distribution; value at risk; volatility

JEL Codes: C12, C22, C46, C58, G11

1. Introduction

The historical financial crisis experienced across the world has prompted extensive research on financial risk management and its relative quantitative model usage. As such, practitioners and researchers across the world from interdisciplinary sciences are working together to investigate and understand key aspects and important characteristics of the long memory in the time series analysis and modeling. Ma (2019) has alluded to the fact that the research testing for long memory impact on modeling time series may be traced back to the 1990's. Indeed, several studies published discoveries that the long memory can be observed in the autocorrelation of the absolute or square returns of different financial stock markets. It is, in this regard, that the fractionally integrated generalized autoregressive conditional heteroskedasticity (GARCH) model was introduced (Baillie, 1996). The South African financial markets, specifically the Johannesburg Stock Exchange (JSE) major indices, were susceptible to suffering losses which spilled over to their financial market participants. This is attributed to the lack of assessment of the stylized facts described by Cont (2001). First, there is slow decay of the autocorrelation function, which denotes the presence of long memory. Second, there is volatility clustering, which describes that large changes tend to be followed by large changes over time. Third, there is profit and loss asymmetry and heavy tails, which indicate that the financial stock or index prices exhibit large downward changes as opposed to upward changes. The analysis and assessment of these properties play an important role in financial market applications and assessment of risk management through the value at risk (VaR) modeling and optimal portfolio selection stemming from their developed diversification strategies.

The financial market returns data exhibit persistence or long-range dependence. According to Klar et al. (2012), the traditional time series models are unable to incorporate the presence of long memory into their quantitative analysis and modeling. Thupayagale (2011) investigated the existence of long-range dependence in the South African fixed income market and evaluated its performance against the basic GARCH model. The authors' findings suggest that the basic GARCH model is accurate for forecasting long-range dependence volatility and managing financial risk. The presence of long-range dependence was explored for Brazil, Russia, India, China and South Africa index market returns and volatility, with 19 years of history of time series implemented in both symmetric and asymmetric models (Tripathy, 2022). Specifically, for the South African financial stock market index that considered the JSE, it was discovered that the asymmetric-type model exhibits a high ability to forecast the index price due to high levels of volatility persistence shocks. As such, the autoregressive fractionally integrated moving average and fractionally integrated GARCH (FIGARCH) model performances were better than that of the traditional GARCH models in terms of forecasting the volatility of the South African stock market. The conditional volatility of equity returns in Bangladesh was modeled to assess the presence of risk-return trade-off, asymmetric effects and long memory property from 2004 to the end of 2020 (Haque and Farzana, 2021). The authors found that fractionally integrated exponential GARCH (FIEGARCH) and fractionally integrated asymmetric power autoregressive conditional heteroskedasticity (FIAPARCH) models outperform the other FIGARCH specifications in terms of modeling the conditional volatility of the Dhaka Stock Exchange and Shariah index returns.

The long-memory GARCH-type models have become a generally better option for modeling the volatility of many financial assets than the short-memory GARCH-type models (Baillie, 1996; Wu and Shieh, 2007; Arouri et al., 2012; Chkili et al., 2014; Bentes, 2015). As a result, this study gives greater emphasis on modeling alternative risk measurements, such as the VaR, using fractionally integrated

GARCH-type models to better capture long memory, forecast conditional volatility and accurately assess the behavior of South African financial market index. VaR is among the most significant market risk indicators that are frequently used by institutions, such as regulators and portfolio managers, to manage financial market risk (Jorion, 2007). This market risk measure is mostly utilized to determine the maximum limit or trigger levels of portfolio profit or loss within financial institutions. However, there are some drawbacks to it because of the return distribution that yields heavier tails than normal. This can cause a significant bias in the VaR estimation because it affects the return distribution's tail properties. Recent empirical studies have uncovered that several financial return series exhibit a long memory of market volatility over an extended period (Cheung, 1993; So, 2000). According to Tripathy (2022), the trade between South Africa's financial market and other global participants has always been on the rise. Hence, the related studies have revealed asymmetric stock market tendencies, such as negative shocks, which have a greater impact than positive shocks of equal magnitude and high volatility persistence over time. This is an indication that the financial stock market return volatility is a significant variable when long memory is realized. Thus, it is of paramount importance that the modeling of return and volatility be carried out for South African financial market indices to determine the risks related to different sectors of the market and give global investors appropriate information for investment decision-making. There are limited previous studies that have researched the existence of long-range memory in the South African stock return and volatility. This is especially true when taking into account the risk measures (i.e., VaR) and the corresponding backtesting for risk management. Therefore, the research study presented here serves as the foundation to fill this gap.

The question posed for this research study is as follows: *Does considering the long memory, fat tails, heteroskedasticity, volatility clustering and asymmetry in the financial time series allow for a better VaR estimation?* The suggested models to handle the aforementioned characteristics include the hyperbolic GARCH (HYGARCH), FIGARCH and FIAPARCH. These models are combined with various parametric distributions, such as the normal distribution, student-*t* distribution (STD), generalized error distribution (GED) and skewed STD (SSTD). This research study investigates the best-fit model of the long-memory GARCH-type in terms of the stylized facts, as they are applied for risk management. The background of the long-memory GARCH-type models from the perspective of the South African financial market, is introduced in this section. Section 2 outlines the prior research work and concludes with the motivation of the study based on a literature review. Section 3 describes the methodology followed to produce the results. Section 4 presents and discusses the results, while Section 5 concludes the investigation of the study.

2. Literature review

Baillie (1996) introduced a FIGARCH model to advance some flexible processes for conditional variance by describing and illustrating the observed temporal dependencies in financial market volatility. Asymmetric responses of volatility to positive and negative shocks are captured by the FIAPARCH model, which was developed by Tse (1998) as an improvement on the FIGARCH model. This allows for the determination of the power on the returns for which the predictable structure in the volatility is the strongest. To address the weaknesses of the FIGARCH model, Davidson (2004) further proposed the HYGARCH model, which includes the hyperbolic decaying response coefficients. To model the VaR on the JSE-ALSI, McMillan and Thupayagale (2010) used GARCH, exponential GARCH, threshold

GARCH, integrated GARCH (IGARCH), FIGARCH and FIEGARCH. The results showed that models incorporating both asymmetric and long memory attributes generally outperform all other methods in terms of estimating the VaR across all three considered percentiles. However, the VaR for the FIAPARCH model was not assessed.

Tang and Shieh (2006) investigated the long memory characteristics of the three stock indices futures markets and found that the HYGARCH model with SSTD performed better. Wu and Shieh (2007) analysed various GARCH-type VaR models for treasury bond interest rates and found that long-memory GARCH-type models outperform short-memory GARCH-type models in terms of performance. Härdle and Mungo (2008) estimated the VaR and expected shortfall for the two long-memory GARCH-type models known as the FIGARCH and HYGARCH models, with various error distribution's assumptions. They found that models assuming asymmetric distributions in the volatility specifications and fractional integration in the volatility process perform better one day in advance. The Turkish futures market's long memory property was examined by Kasman (2009), who applied the GARCH and FIGARCH models. Assuming that the returns on these models follow normal, STD and SSTD distributions, the VaR values were estimated. The estimation of FIGARCH with the SSTD and related VaR yielded more effective results for the Istanbul Stock Index (ISE-30) futures returns.

In their analysis of the Stock Exchange of Thailand, Sethapramote et al. (2014) found that the VaR estimation produced by the FIGARCH model under the assumption of a normal distribution is more accurate than those produced by the short-memory GARCH models. Furthermore, to estimate the VaR, Li et al. (2015) used the Hang Seng Index and South Korean Won to compare the performance of three innovations, i.e., the HYGARCH, the new generalized autoregressive conditional heteroskedasticity and FIGARCH models. The HYGARCH model with a normal distribution turned out to be a reliable model for estimating VaR forecasts. In order to find the most suitable GARCH model for the Nigerian All Share Index, Yaya (2013) used the models HYGARCH$(1, d, 1)$, IGARCH$(1, d, 1)$ and FIEGARCH$(1, d, 1)$ for the normal distribution, STD and GED, respectively. The most effective of the three models was found to be HYGARCH$(1, d, 1)$. The stationary GARCH model does not account for the observed persistence in the conditional variance of financial time series because it assumes an exponentially decaying autocorrelation for the squared returns (Bollerslev, 1986). As a result, long-memory GARCH-type models have attracted a lot of interest. This study was purposed to determine how well the FIAPARCH model performs as compared to FIGARCH and HYGARCH long-memory GARCH-type models. That is, we assumed that the returns on the South African JSE indices follow the normal distribution, STD, SSTD and GED. This was carried out by modeling the VaR of the major indices in South Africa's financial stock market, i.e., ALSI, MNGI and BNKI. All of these distributions, except for the normal distribution, have heavy tails, suggesting that the SSTD may be appropriate to capture the asymmetry parameter that other distributions lack. We conducted a further backtest of the results and calculated the VaR for short and long trading positions at 0.3%, 1%, 5%, 95%, 99% and 99.7%, which correspond to the right and left quantiles of the return distributions, respectively.

3. Methodology

The interesting feature of the volatility models is that GARCH models have the parameter $d = 0$, IGARCH has $d = 1$ and volatility-purposed GARCH-type models have $0 < d < 1$, which captures long memory behavior. The time series have a short-memory if $\sum_{i=1}^{n} |\rho(h)| < \infty$, where $\rho(h)$ is the autocorrelation at lag h. And, the time series is said to exhibit long memory range dependence if $\sum_{i=1}^{n} |\rho(h)| = \infty$, where n is the total number of time series realizations.

Let $y_1, y_2, \ldots, y_n$ be a stochastic time series. Thus, for a short-memory time series, $\mathrm{Var}(\bar{y}_n)$ goes to 0 as the size of the sample increases at the rate $(\frac{\sigma^2}{n})$. But, with different multipliers the fractionally integrated white noise process is given as

$$(1-B)^d y_t = w_t, \ 0 < d < 0.5, \tag{1}$$

where the fractional difference operator $(1-B)^d$ is defined by a binomial series as

$$(1-B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k = 1 - dB - \frac{d(1-d)B^2}{2} - \frac{d(1-d)(2-d)B^3}{6} - \ldots, \tag{2}$$

$$(1-B)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)z^k}{\Gamma(-d)\Gamma(k+1)}. \tag{3}$$

The autocorrelation of the fractionally integrated series is defined as

$$\rho(h) = \frac{\Gamma(h+d)\Gamma(1-d)}{\Gamma(h-d+1)\Gamma(d)} \sim h^{2d-1}. \tag{4}$$

So, suppose that $0 < d < 0.5$ and $\sum_{h=-\infty}^{\infty} |\rho(h)| = \infty$. $\mathrm{Var}(\bar{y}_n)$ decays like h^{2d-1} such that $(\frac{1+\alpha}{2})$, gives a rough empirical estimate of d, where α is the slope of the variance-time graph. The IGARCH process, which considers infinite memory, is not appropriate given that the situation of a long memory process is very unlikely to happen in the real world. An asymmetric power autoregressive conditional heteroskedasticity (APARCH) model of volatility differs from existing GARCH-type volatility models because it included the power term *delta*, which must be evaluated. The power term estimation is an effort to account for the true distribution underlying volatility. Higher volatility is caused by the leverage impact in the model, and vice versa. Since the assumption of normality, which limits *delta* to either the value 1 or 2, is typically impractical when modeling financial time series because of high skewness and kurtosis, the idea of introducing a power factor emerged. The following subsections briefly outline the FIAPARCH, HYGARCH and FIGARCH models.

3.1. The FIGARCH model

According to Barkoulas et al. (2000), the presence of long memory term persistence in the time series returns provided evidence against a weak type of financial market efficiency. Furthermore, research done by Fama (1965) proved that financial markets are only considered efficient when asset prices completely reflect the available information. As a result, the FIGARCH volatility model was designed to reflect all available information of the time series. It is defined as

$$\sigma_t^2 = \omega[1 - \beta(L) - \phi(L)(1-L)^d]\varepsilon_t^2 + \beta(L)\sigma_t^2, \tag{5}$$

with the lag operator represented by L; the parameters of the model are ω, β, ϕ and d, where d lies within $0 \leq d \leq 1$. When $d = 1$, the FIGARCH(p, d, q) model is reduced to IGARCH(p, q) and the GARCH(p, q) model provides the parameter $d = 0$.

The necessary and sufficient criteria for the non-negativity of the conditional variance in the FIGARCH(p, d, q) model was derived by Conrad and Haag (2006), along with the sufficient conditions for the general model. These constraints use an infinity order autoregressive conditional heteroskedasticity (ARCH) representation of the FIGARCH$(1, d, 1)$ model. According to Tayefi and Ramanathan (2012), the persistence of the conditional variance is simply characterized in terms of the impulse response coefficients, and only when the conditional variance is parametrized as a linear function of the past squared residuals. As a result, the FIGARCH model considers an intermediate level of long memory captured by the parameter d. Thus, the FIGARCH model has the benefit of nesting both the GARCH ($d = 0$) and IGARCH ($d = 1$) models as special instances, but it does not provide a covariance stationary process.

3.2. The HYGARCH model

In order to accurately represent the long memory range dependency observed in the geometric or hyperbolic decay of the coefficients in the ARCH(∞) model, Davidson (2004) recommended using the HYGARCH model. The FIGARCH model's conditional variance was increased by adding weights to its difference operator. The HYGARCH model is defined as

$$\sigma_t^2 = \frac{\omega}{(1 - \beta(L))} + \left\{1 - \frac{\phi(L)(1 - L)^d}{(1 - \beta(L))}\right\}\varepsilon_t^2, \tag{6}$$

where L is the lag operator, $0 \leq d \leq 1$, $\omega > 0$ and $\beta < 1$.

The time series with leptokurtic, long memory and volatility clustering are properties of the aforementioned model. However, it ignores asymmetry and the fact that non-integer powers of the absolute value of the observations are the best representations of conditional volatility. When $\alpha = 1$, the HYGARCH model simplifies to a FIGARCH model. Conrad (2010) provided detailed inequality constraints for the HYGARCH(p, d, q) model, sufficient conditions for the HYGARCH$(1, d, 1)$ model and more detailed conditions for higher-order HYGARCH models. Furthermore, Conrad (2010) established that the estimated parameters of the HYGARCH model are similar in structure to the FIGARCH parameter estimators. The only difference been the two models is a result of the additional parameter that allows the HYGARCH model to constitute a covariance stationary process. Hence, the model is considered to capture the latter behavior in the context of this study.

3.3. The FIAPARCH model

The FIAPARCH model is an extension of the FIGARCH model that incorporates the APARCH model (Ding et al., 1993). The FIAPARCH (p, d, q) model is defined as follows:

$$\sigma^\delta = \omega + [1 - (1 - \beta(L))^{-1}(1 - \phi(L))(1 - L)^d](|\varepsilon_t| - \gamma\varepsilon_t)^\delta, \tag{7}$$

where $\omega > 0$, $\delta > 0$, $-1 < \gamma < 1$ and $0 < d < 1$. When $\gamma > 0$, negative shock increases volatility more than positive shock, and vice versa. The FIAPARCH model becomes a FIGARCH model when $\delta = 2$ and $\gamma = 0$. Additionally, $(1 - L)^d$ is the difference operator in terms of a hypergeometric function (Bentes, 2015). When $d = 0$, the HYGARCH process reduces to the APARCH$(1, 1)$ model. The model can capture both long memory and asymmetry in the conditional variance.

3.4. Heavy tailed distributions

The heavy tailed distributions employed in the study are the STD, SSTD and GED. The STD resembles the normal distribution in that it is symmetrical and bell-shaped, but it varies from normal distribution because it can capture the heavier tails of the series. However, due to the symmetric nature of the STD, it is still unable to deal with time series observations that are asymmetric. The SSTD deals with an asymmetry conferred by heavy tailed long memory time series. The SSTD distribution includes a random variable with a location parameter of zero and a scale parameter of one; it tends toward the STD if and given that the shape parameter is zero. Through the generalization of the parent STD, to capture asymmetry, one acquires the SSTD by ranging the skewness parameter from -1 to 1 and applying the positive parameters that control the kurtosis of the series. Lastly, the GED was considered because of its more flexible generalizations of the normal distribution (Cerqueti et al., 2019). The maximum likelihood estimation of the distributions was obtained through the application of numerical optimization methods. The log-likelihood functions of the STD and SSTD have been defined by Peters (2001). The approach to determine the estimates of the GED through maximum likelihood estimation was given by Purczyński and Bednarz-Okrzyńska (2014).

3.5. Long memory tests

The long memory tests were used to detect the long-range dependence behavior in the daily log returns. Here, we used the stationary process with long memory and asymptotic definitions provided by Beran (2017). The long-range dependence tests used in the study are Geweke and Porter-Hudak (GPH), Whittle estimation and Hurst exponent-type R/S tests.

- *GPH test*: The GPH estimator is widely used to distinguish between long memory and short-memory effects, and is called the spectral regression method (Geweke and Porter-Hudak, 1983). It is specifically used as an alternative method for estimating the fractional differencing parameter d given in Equation (1). Geweke and Porter-Hudak showed that the least squares estimate $\hat{d}$, using regression, is normally distributed in large samples if $n_f(T) = T^{\alpha}$, with $0 < \alpha < 1$.
- *Whittle estimation*: The long memory process is also characterized in the frequency domain with the aid of a spectral density function proportional to λ^{-2d} as the frequency λ approaches zero at a rate dictated by the memory parameter d. The Whittle estimator is akin to the maximum likelihood estimator in the frequency domain. The Whittle estimation method has become famous due to its likelihood interpretation and asymptotic characteristics.
- *R/S test*: The Hurst (1951) method estimates H through spectral regression in the exploitation of the relationship between β and the Hurst coefficient. The short-memory process has $H = 0.5$, and the autocorrelation function also decays faster; however, when it is completely related to the long memory process, it is characterized by the Hurst exponent in the interval $0.5 < d < 1$. It can be interpreted as follows: if $H = 0.5$, the time series assumes a random walk and it is independent. If $0 < H < 0.5$, the time series are anti-persistent and the process occupies only a small distance as compared to a random walk. Else, if $0.5 < H < 1$, the time series is persistent and the process covers a larger distance than a random walk (i.e., a long memory process). The Hurst parameter is understood to be a self-similarity parameter, and it is defined as $H = d + 0.5$.

3.6. Model selection criteria

The best statistical model among all of the candidate models was selected by using only the Akaike information criteria (AIC) and Bayes information criterion (BIC). The AIC applied the diagnostic test to residuals to deduce which model is most preferable. They propose the measure of the model's goodness-of-fit and parsimony by balancing the error of fit against the number of parameters in the model (Cavanaugh and Neath, 2019). The AIC is defined as $AIC = 2q - 2\ln(L)$, where L is the likelihood of the model and q is the number of parameters. The other measure similar to the AIC is the BIC, which is defined as $BIC = q\ln\mathrm{T} - 2\ln(L)$, where T is the sample size. Notice that the models related to the lower AIC and BIC are more adequate.

3.7. VaR and Kupiec's LR test

The VaR is a measure of the maximum potential change in the value of an index or stock over a predefined duration. Since it is a percentile of the distribution of the fitted values (y) from the model, with reference to Mighri and Jaziri (2023) and Basel Committee on Banking Supervision (2013), we define VaR_p for a given probability as

$$VaR_p = \inf\{y \in \mathbb{R} : F(y) \geq p\}. \tag{8}$$

The VaR of the STD is given as $VaR_t(1-\alpha) = \left(\hat{\mu}_t + t_\nu^{-1}(\alpha)\sqrt{\frac{\nu-2}{\nu}}\sigma\right)$, where ν denotes the degrees of freedom and t_ν^{-1} is the α-quantile of the standard STD with ν degrees of freedom.

The VaRs of the SSTD at the αth quantile for long and short positions, respectively, are $VaR_L = \hat{\mu}_t + sstd_{\alpha,\nu,\xi}\hat{\sigma}_t$ and $VaR_S = \hat{\mu}_t + sstd_{1-\alpha,\nu,\xi}\hat{\sigma}_t$. Here, $sstd_{\alpha,\nu,\xi}\hat{\sigma}_t$ is the left quantile at $\alpha\%$ for the SSTD and $sstd_{1-\alpha,\nu,\xi}\hat{\sigma}_t$ is the long position, with ν being the degrees of freedom, ξ an asymmetry coefficient and μ_t the conditional mean process.

The VaR of the GED is $VaR_L = \hat{\mu}_t + \phi_p^{-1}(\varepsilon_t)\hat{\sigma}_t$, where $\phi_p^{-1}(\varepsilon_t)$ is the left quantile of the GED at the p level. Once $\nu = 2$, the normal distribution could be a special case of the GED. If $\nu < 2$, then the GED has fatter tails than the ordinary normal distribution.

Backtesting is a technique for simulating a model or strategy on a historical time series to gauge its accuracy and effectiveness. In this study, we chose to use the Kupiec LR test to backtest VaR (Kaya Soylu et al., 2020). The failure rate is the number of times the return series exceeds the forecasted VaR. The assumption is that the range of exceedances over time follows the binomial distribution, and the purpose of Kupiec's test is to establish the consistency of these violations with the given confidence level. If the range of exceedances differs drastically from what is expected, the risk model's adequacy is questionable. To perform the test, the wide variety of actual violations E, the number of observations N and the VaR probability level (p) are needed. Assuming that E is distributed as $\mathrm{Bin}(N, p)$ the null hypothesis is $H_0 : p = p_0$, and the alternative hypothesis is $H_1 : p \neq p_0$, where p is estimated by $\frac{E}{N}$. As Kupiec (1995) proposed this test is based on the likelihood ratio test statistic given as

$$LR = 2\log\left[\frac{\left(1-\frac{E}{N}\right)^{N-E}\frac{E^E}{N}}{\left(1-p_0\right)^{N-E}p_0^E}\right] \sim \chi^2_{(1)}, \tag{9}$$

where N is the number of observations used to forecast VaR values and E is the observed number of actual exceedances.

4. Numerical results and discussion

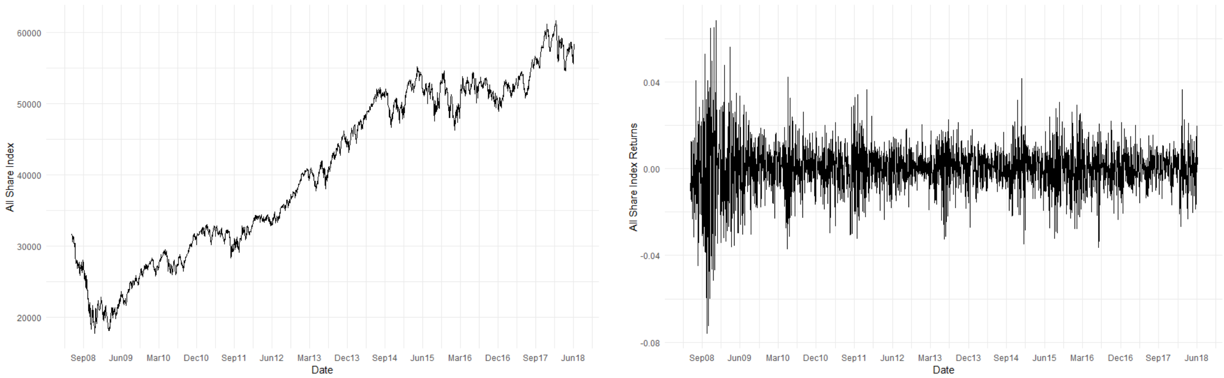
4.1. Data selection and transformation

The daily closing prices of the South African JSE ALSI, mining index (MNGI) and banking index (BNKI) were used for the quantification of the VaR by using long-memory GARCH-type models. The historical time series was obtained from INET Bridge and Bloomberg for the period June 7, 2008 to June 7, 2018 with a total of 2,500 observations for each index. The daily natural logarithmic returns were calculated, comprising a total of 2,449 return series for each index. This implies that the daily returns for the three indices were expressed as a logarithmic first order difference between successive business days. The natural logarithmic return (R_t) of the index is defined as

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(P_t) - \ln(P_{t-1}), \tag{10}$$

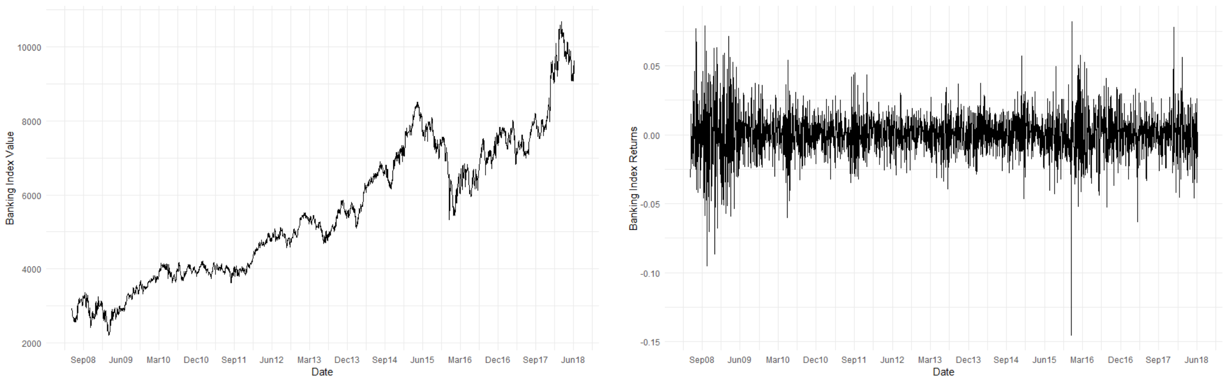
with P_t and P_{t-1} as the current and one lagged index price on day t and $t-1$, respectively.

The statistical computations were executed by using R-Studio and Ox-Matrix packages.



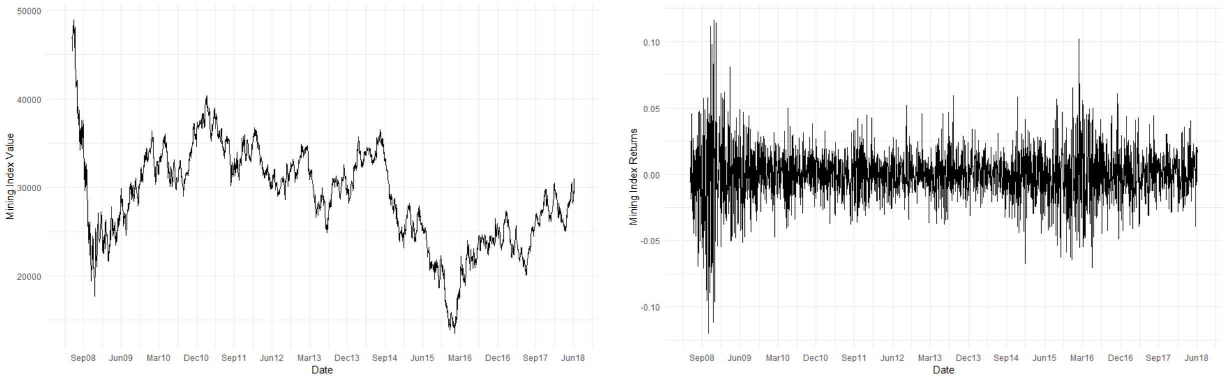
(a) Daily closing price **(b)** Returns

Figure 1. The time series plots for the ALSI.



(a) Daily closing price **(b)** Returns

Figure 2. The time series plots for the BNKI.



(a) Daily closing price **(b)** Returns

Figure 3. The time series plots for the MNGI.

Figures 1 to 3, present the time series prices and returns for the three indices. All of the index returns show periods of high and low volatility, indicating that volatility clustering and heteroskedasticity are present. The time series plot for the MNGI, generally shows decreasing tendencies. This can be attributed to the slowdown in domestic economic conditions, including factors related to labor unrest, electricity constraints and the political landscape in South Africa, which contributed to a downward shock that took place during the period of March 2015. The ALSI and BNKI were observed to increase in price over time.

4.2. Descriptive statistical analysis

The descriptive statistics for the log returns of the ALSI, BNKI and MNGI indices are revealed in Tables 1 to 3. The BNKI contained 2,449 daily log returns ranging from −0.14521 to 0.08189, with the highest mean return of 0.0005. The ALSI had a mean of 0.00025 with a range from −0.0758 to 0.0683; the MNGI had a mean of −0.00017, and it ranged from −0.11966 to 0.11616. The ALSI and BNKI skewness values for the returns were negative, given as −0.1379 and −0.1757, respectively. The negatively skewed returns indicate a greater likelihood of large decreases in the index price returns during the sample period. All of the indices had a kurtosis larger than three, indicating heavy-tailed distributions. A kurtosis of more than three indicates that volatile price movements occurred frequently during the sampling period. Furthermore, the standard deviations of the log return series were higher than the standard errors of their means, indicating the presence of volatility.

Table 1. Descriptive statistics of the log returns for the three indices.

	Min	Max	Mean	Std.Dev	Skewness	Kurtosis
ALSI	−0.0758	0.0683	0.0002	0.0119	−0.1379	7.2814
BNKI	−0.1452	0.0819	0.0005	0.0169	−0.1757	7.6664
MNGI	−0.1197	0.1162	−0.0002	0.0203	0.0469	6.8978

Table 2. Serial correlation, normality and heteroskedasticity results for the log returns.

Index	$Q(5)$	$Q(10)$	JB	SW	$Q^2(5)$	$Q^2(10)$	ARCH-LM
ALSI	23.35	34.67	1917.2	0.9509	1085.0	2417.4	142.87
	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)
BNKI	44.87	55.18	2277.2	0.9586	344.15	543.92	48.292
	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)
MNGI	22.34	44.85	1582.9	0.9590	894.55	1986.8	115.49
	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)	(<0.0001)

Table 2 shows the tests of serial correlation, normality and heteroskedasticity for the log returns. The Jarque-Bera and Shapiro-Wilk normality tests for all three indices' log returns revealed p-values less than 0.0001, rejecting the normality assumption at all levels of significance. The findings imply that the heavy-tailed models should be considered when analyzing and predicting the given log return series. Furthermore, the ARCH Lagrange multiplier (ARCH-LM) test statistic revealed a p-value less than 0.0001 for all three daily log returns. Thus, we reject the null hypothesis of the absence of potential time-varying volatility (no ARCH effect) for at most lag 10. The serial correlation was analyzed by using the Ljung-Box test statistic, which led to the rejection of the null hypothesis of no serial correlation for both the log returns and squared returns.

Table 3. Unit root and stationary test results for the log returns.

Statistic	Model Type	*ALSI*	*p-value*	*BNKI*	*p-value*	*MNGI*	*p-value*
	No constant	−48.5	<0.0001	−48.9	<0.0001	−47.5	<0.0001
ADF	No Trend	−48.6	<0.0001	−48.9	<0.0001	−47.5	<0.0001
	With Trend	−48.6	<0.0001	−48.9	<0.0001	−47.5	<0.0001
	No constant	−2151	<0.0001	−2079	<0.0001	−2145	<0.0001
PP	No Trend	−2149	<0.0001	−2074	<0.0001	−2145	<0.0001
	With Trend	-2148	<0.0001	−2074	<0.0001	−2145	<0.0001
	No constant	0.5130	>0.1	0.9970	>0.1	0.3340	>0.1
KPSS	No Trend	0.1230	>0.1	0.0197	>0.1	0.1340	>0.1
	With Trend	0.1100	>0.1	0.0815	>0.1	0.0674	>0.1

Table 3 presents the stationarity test results by using the augmented Dickey–Fuller (ADF) test statistic, Phillips–Perron (PP) unit root test and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) for all the three indices returns. The null hypothesis implies that the daily log return series is non-stationary, and the alternative hypothesis implies that the daily log return series is stationary. All three daily log returns rejected the null hypothesis of the unit root test. The p-values of the ADF and PP statistics for the three indices were less than 0.0001, rejecting the null hypothesis at all significance levels. The KPSS test also shows that we cannot reject the stationary null hypothesis for daily log returns. Therefore, this attests that all three indices' daily log returns are stationary. These findings led to the choice and usage of the three integrated GARCH-type models with fat-tailed distributions, as discussed in Section 3.

4.3. Long memory statistical test

Tables 4 to 7 reflect the tests for long memory. The Whittle estimator was utilized to determine the existence of the long memory process for both log-returns and squared log returns (Table 4). The Hurst index values for the three indices for the daily log-returns were close to 0.5, and the p-values were less than 0.05, implying the absence of long memory features. The Hurst indices for the squared log returns were considerably more than 0.5, with p-values less than 0.0001, indicating the presence of long memory. The finding attests to some studies done on stocks and other portfolios of the financial market analysis (Chinhamu et al., 2022; Mighri and Jaziri, 2023).

Table 4. Whittle estimator (fG_n) long memory test results.

	Indices	Hurst	Standard Error	t-value	p-value
Returns	ALSI	0.4899	0.0124	39.4828	<0.0001
	BNKI	0.4677	0.0123	38.0386	<0.0001
	MNGI	0.5076	0.0125	40.6302	<0.0001
Squared Returns	ALSI	0.6615	0.0130	50.7236	<0.0001
	BNKI	0.6373	0.0129	49.1413	<0.0001
	MNGI	0.6564	0.0130	50.3995	<0.0001

Table 5. GPH's long memory test results for returns.

Returns	Bandwidths	d	Standard Error	t-value	p-value
ALSI	$m = T^{0.5}$	0.0307	0.0945	−10.2155	<0.0001
	$m = T^{0.6}$	0.0232	0.0731	−13.3656	<0.0001
	$m = T^{0.7}$	−0.0682	0.0469	−22.7351	<0.0001
BNKI	$m = T^{0.5}$	−0.1923	0.1008	−11.8332	<0.0001
	$m = T^{0.6}$	−0.1357	0.0608	−18.6839	<0.0001
	$m = T^{0.7}$	−0.1455	0.0403	−28.4297	<0.0001
MNGI	$m = T^{0.5}$	−0.0409	0.0681	−15.2742	<0.0001
	$m = T^{0.6}$	−0.0063	0.0492	−20.3630	<0.0001
	$m = T^{0.7}$	−0.0153	0.0414	−24.4951	<0.0001

The results of the GPH and Hurst tests for the returns and squared log-returns are shown in Tables 5 and 6. The bandwidths used are as follows: $m = T^{0.5}$, $m = T^{0.6}$ and $m = T^{0.7}$. The presence of a long memory process is indicated by the squared log-returns, which implies that long memory is mean-reverting but is not covariance stationary. The p-values less than 0.0001 display the presence of the long memory. The Hurst exponent test results are shown in Table 7, and the findings reveal the presence of the long memory for the squared log-returns.

4.4. Estimating long memory models with heavy-tailed distributions

The parameter estimation results for the FIGARCH, HYGARCH and FIAPARCH models with a standard normal distribution as a base and the STD, SSTD and GED were generated; the selection

Table 6. GPH's long memory test results for squared returns.

Returns	Bandwidths	d	Standard Error	t-value	p-value
ALSI	$m = T^{0.5}$	0.8497	0.0796	−1.8860	0.0297
	$m = T^{0.6}$	0.7379	0.0511	−5.1375	<0.0001
	$m = T^{0.7}$	0.6058	0.0412	−9.5639	<0.0001
BNKI	$m = T^{0.5}$	0.4150	0.0927	−6.3128	<0.0001
	$m = T^{0.6}$	0.3293	0.0616	−10.8941	<0.0001
	$m = T^{0.7}$	0.2210	0.0372	−20.9142	<0.0001
MNGI	$m = T^{0.5}$	0.7674	0.0731	−3.1839	0.0007
	$m = T^{0.6}$	0.6913	0.0533	−5.7867	<0.0001
	$m = T^{0.7}$	0.5605	0.0419	−10.4649	<0.0001

Table 7. Hurst exponent's long memory test results.

Simple R/S Hurst Estimation		
	Returns	Squared Log-Returns
ALSI	0.52956	0.73689
BNKI	0.46249	0.71008
MNGI	0.49765	0.72270

criteria for the model are presented in Table 8. The long-range dependence parameter of the GARCH-type models was within $0.5 < d_v < 1$ for all models. This points to a strong case for long memory of the time series. It also means that historical shocks appear to influence current shocks (Arouri et al., 2012). The Q-statistic for squared standardized residuals was assessed; it revealed that, for all models, the Ljung-Box test p-value exceeded 1% level of significance. This means that the null hypothesis of independently and identically distributed standardized residuals cannot be rejected. As a result, the volatility equations pass the Ljung-Box test for squared standardized residuals. The ARCH-LM test revealed that the residuals are not heteroskedastic. Finally, we used the AIC and BIC to find the optimum model for conditional dependency in the volatility process (Table 8). The FIGARCH-SSTD model has been proven by both the AIC and BIC to be the best model for ALSI returns that represent the conditional variance dependence. Both the AIC and BIC agree that the FIAPARCH-STD model is the best model for BNKI returns. The AIC and BIC from the FIAPARCH-STD model indicates that the model is appropriate for the MNGI returns.

As a result, our findings point to a few common facts in the volatility processes of stock index markets. First, the empirical findings reveal that all of the three stock markets under investigation exhibit long memory behavior to different degrees as a result of using FIGARCH-type models. Second, asymmetric response parameter estimates indicate that index stock market volatility responds asymmetrically to both good and bad news. When comparing the models, we find that the FIAPARCH process under STD and SSTD offer a better fit than the other models in terms of the minimum values of the AIC and BIC calculated by using the maximum log likelihood values for all of the markets displaying heavy tails, volatility clustering, high kurtosis and skewness. Overall, Mabrouk and Saadi (2012), Gaye Gencer and

Demiralay (2016) and Abuzayed et al. (2018) have all published studies in the realm of financial stock markets that are consistent with our findings.

Table 8. Model selection criteria for the three index returns. N: normal distribution.

Returns	Model	Selection criteria					
		FIGARCH		HYGARCH		FIAPARCH	
		AIC	BIC	AIC	BIC	AIC	BIC
ALSI	N	−6.3790	−6.3673	−6.3804	−6.3665		
	STD	−6.3886	−6.3747	−6.3885	−6.3722		
	SSTD	**−6.3956**	**−6.3793**	−6.3956	−6.3769		
	GED	−6.3872	−6.3732	−6.3877	−6.3714		
BNKI	N	−5.5443	−5.5327	−5.5451	−5.5312	−5.5530	−5.5367
	STD	−5.5860	−5.5720	−5.5861	−5.5698	**−5.5881**	**−5.5695**
	SSTD	−5.5854	−5.5691	−5.5855	−5.5668	−5.5879	−5.5669
	GED	−5.5748	−5.5608	−5.5752	−5.5588	−5.5782	−5.5596
MNGI	N	2.2936	2.3053	2.2919	2.3059	2.2832	2.2995
	STD	2.2865	2.3005	2.2858	2.3021	**2.2778**	**2.2964**
	SSTD	2.2872	2.3035	2.2864	2.3051	2.2783	2.2992
	GED	2.2868	2.3007	2.2857	2.3020	2.2781	2.2968

4.5. VaR estimation and backtesting models

The VaR method is essential for market risk management in most institutions and regulatory bodies (Basel Committee on Banking Supervision, 2013). It plays an important role in ensuring that financial institutions are able to meet the necessary capital requirements to cover the market risk that they incur in the process of their daily financial stock market trading. The VaR forecasts for both long and short trading positions at different significance levels using the FIGACH, HYGARCH and FIAPARCH models were computed. The VaR estimates were then backtested by using the Kupiec LR test at the significant levels of 0.3%, 1%, 5%, 95%, 99% and 99.7%; related results are presented in Tables 9 to 11. The most effective and robust model is the one with the highest *p*-values at a given significant level.

The FIGARCH-N model for the ALSI returns was not adequate at 0.3%, 1% and 95% VaR levels. While the FIGARCH-SSTD model was acceptable at all VaR levels of the long and short positions. The VaR estimates for the FIGARCH-STD model produced the lowest *p*-values at the five levels and yielded an adequacy level at 99.7%. The FIGARCH-GED model produced similar results to the FIGARCH-STD model, with similar levels of lowest *p*-values and adequate level at 99.7%. The FIGARCH-STD, FIGARCH-SSTD and FIGARCH-GED models for the BNKI returns produced adequate results at all levels. Additionally, the FIGARCH-STD model was adequate at all VaR levels, while the FIGARCH-N model was not adequate at the 0.3% and 95% levels. The BNKI returns conclude that all three models, i.e., FIGARCH-SSTD, FIGARCH-GED and FIGARCH-STD, produced adequate results. The FIGARCH-GED is adequate for the MNGI returns at all levels, except for level 95%. Alternatively, the FIGARCH-SSTD model produced the highest *p*-values at all levels, except the 95% level. The FIGARCH-N model was found to be inappropriate at the 95% and 99% VaR levels.

Table 9. In-sample VaR backtesting: FIGARCH with different distributions for the three indices.

Returns	Model	*p*-values of Kupiec LR test					
		Long positions			Short positions		
		0.3%	1%	5%	99.7%	99%	95%
ALSI	FIGARCH-N	0.0011	0.0003	0.1050	0.9205	0.0529	0.0001
	FIGARCH-STD	0.0411	0.0242	0.0077	0.3349	0.0159	0.0003
	FIGARCH-SSTD	0.7675	0.8401	0.7803	0.9205	0.1391	0.8915
	FIGARCH-GED	0.0183	0.0378	0.0724	0.3349	0.0300	0.0001
BNKI	FIGARCH-N	0.0077	0.0580	0.5169	0.1674	0.2450	0.0229
	FIGARCH-STD	0.6039	0.6836	0.9269	0.1473	0.6836	0.4034
	FIGARCH-SSTD	0.6039	0.5382	0.6442	0.2961	0.8396	0.5789
	FIGARCH-GED	0.9197	0.8396	0.4582	0.3444	0.5382	0.0737
MNGI	FIGARCH-N	0.7682	0.0580	0.2066	0.0861	0.0093	0.0295
	FIGARCH-STD	0.0469	1.0000	0.5847	0.9197	0.6836	0.0134
	FIGARCH-SSTD	0.0469	0.2079	0.7839	0.7682	0.6915	0.0176
	FIGARCH-GED	0.1473	1.0000	0.7839	0.5019	0.8396	0.0056

Table 10. In-sample VaR backtesting: HYGARCH with different distributions for the three indices.

Returns	Model	*p*-values of Kupiec LR test					
		Long positions			Short positions		
		0.3%	1%	5%	99.7%	99%	95%
ALSI	HYGARCH-N	0.0004	0.0001	0.0077	0.5012	0.0300	0.0003
	HYGARCH-STD	0.0411	0.0151	0.0127	0.3349	0.0159	0.0007
	HYGARCH-SSTD	0.3012	0.5527	0.2758	0.5012	0.2986	0.2267
	HYGARCH-GED	0.0183	0.0242	0.0162	0.6045	0.0159	0.0002
BNKI	HYGARCH-N	0.0184	0.0282	0.7122	0.1674	0.0580	0.0907
	HYGARCH-STD	0.3344	0.8417	0.7839	0.3344	0.8396	0.5169
	HYGARCH-SSTD	0.3344	0.6836	0.7269	0.3344	0.8396	0.4442
	HYGARCH-GED	0.9197	0.6836	0.6442	0.4039	0.8417	0.1106
MNGI	HYGARCH-N	0.7683	0.0580	0.2066	0.0861	0.0093	0.0295
	HYGARCH-STD	0.1473	0.3299	0.1060	0.9197	0.6915	0.0376
	HYGARCH-SSTD	0.1473	0.5541	0.2403	0.5019	0.5514	0.1106
	HYGARCH-GED	0.1473	0.3299	0.5242	0.3017	0.6915	0.0229

The Kupiec LR test findings are shown in Table 10; for ALSI returns, the HYGARCH-SSTD model had the highest *p*-values at all VaR levels. The HYGARCH-STD model had the lowest *p*-values at all VaR levels, indicating that it is not a suitable model. The MNGI returns has the highest *p-values* at all VaR levels, whereas the HYGARCH-STD model was insignificant at the 95% VaR level. At all VaR levels, the HYGARCH-SSTD and HYGARCH-GED yielded the highest *p*-values for BNKI returns. At all VaR levels, the HYGARCH-STD model had the highest *p*-values.

Table 11. In-sample VaR backtesting: FIAPARCH with different distributions for the three indices.

Returns	Model	*p*-values of Kupiec LR test					
		Long positions			Short positions		
		0.3%	1%	5%	99.7%	99%	95%
BNKI	FIAPARCH-N	0.0030	0.0093	0.5169	0.0030	0.0580	0.1603
	FIAPARCH-STD	0.9197	0.8915	0.7839	0.8473	0.8417	0.3058
	FIAPARCH-SSTD	0.6039	0.8396	0.7823	0.1473	0.6915	0.3442
	FIAPARCH-GED	0.5019	0.6915	0.6442	0.1473	0.8322	0.1106
MNGI	FIAPARCH-N	0.6039	0.5541	0.4140	0.0412	0.2450	0.0176
	FIAPARCH-STD	0.8473	0.8977	0.3193	0.7019	0.8417	0.9029
	FIAPARCH-SSTD	0.1473	0.2079	0.5243	0.5019	0.8417	0.0737
	FIAPARCH-GED	0.1473	0.0942	0.5243	0.5019	0.8417	0.0134

The Table 11 results show that, for MNGI returns, the FIAPARCH-N model offers the largest *p*-values at all VaR levels, except at the 95% and 99.7% levels. The FIAPARCH-STD and FIAPARCH-SSTD models yielded the highest *p-values* at all VaR levels. The FIAPARCH-GED model yielded the largest *p*-values at all VaR levels, except the 95% level. For the BNKI returns, the FIAPARCH-STD, FIAPARCH-SSTD and FIAPARCH-GED are adequate at all VaR levels. While, on the other hand, the FIAPARCH-N model was not adequate at the 5% and 95% VaR levels. Overall, the long-memory GARCH models combined with well-chosen parametric distributions were adequate in terms of estimating the VaR of the three index returns at the levels of 0.3%, 1%, 5%, 95%, 99% and 99.7%.

The finding in the study draws attention to important practical implications for modeling long-memory financial time series. First, the selected daily closing prices of the three South African market indices exhibit positive serial-correlation behavior that is characterized by large changes that tend to be followed by successive large changes over time. This phenomenon was depicted by Cont (2001) and referred to as volatility clustering. Figures 1 to 3 show a very clear picture of the volatility clustering in the daily log returns of the three indices during the period of 2008 to 2009 (i.e., the global financial crisis), as well as during the year 2016 (uncertainty about several changes of the Minister of Finance in the South African government). Enow (2023) has described some causes and implications as being related to investor behavior, market sentiment, new information, market reactions, risk management, derivatives pricing, trading strategies and financial stability. Hence, financial market participants and stakeholders such as portfolio managers and the central banks have to ensure that appropriate measures are implemented. That is, well understood and robust modeling are in place to enhance market liquidity and high trading activities. Second, the log-return probability distributions for the three indices show a departure from normal (i.e., leptokurtosis) distribution, which is due to the skewness and the excess kurtosis beyond the threshold of 3, as shown in Tables 1 and 2. Szczygielski and Chipeta (2023) have stated that the violation of normality assumption when the model is estimated should not be a concern. Indeed, the other types of probability distributions, such as the SSTD may be assumed to capture the tail behavior of the returns. Lastly, the indication of leptokurtosis might mean that there exists varying variations of the return series overtime, which further implies non-stationarity and heteroskedasticity. In the study, the log-return series were stationary, but conditional heteroskedasticity prevailed. Hence, the long-memory GARCH-type models were selected as the modeling techniques. According to Haque and Farzana (2021), all the FIGARCH-type models are not susceptible to the assumption violation of non-constant variance of the return series. The same results were observed in this study, as these model specifications were already built to capture conditional volatility. However, other implications may be model misspecification through the choice of the long-memory modeling technique, coupled with the probability distribution assumed to calculate the VaR. The application of the VaR model is centered around the probability distribution theory. If the model and its properties are not carefully assessed or chosen, then the probability estimation might not always produce an accurate forecast of the risk measure. Patra and Padhi (2015) suggest that the mitigating steps include regular backtesting of the model and evaluating the findings. Therefore, it is important to exercise proper caution and examine alternate VaR models with the assumption of the appropriate distributions.

5. Summary

Empirical characteristics of the three indices, such as heavy tails, volatility clustering, and long memory (squared returns), were uncovered. As a result, the study utilized long-memory GARCH-type models, which included FIGARCH, HYGARCH and FIAPARCH models. The chosen heavy-tailed distributions consisted of STD, SSTD, and GED. To evaluate the suitability of these models, the Kupiec LR test was employed in sample backtesting. Among the models, the FIGARCH with the SSTD model demonstrated overall appropriateness for JSE ALSI returns in both long and short positions, producing the highest p-values at different VaR levels (0.3%, 1%, 5%, 95%, 99% and 99.7%). This indicated that the FIGARCH with the SSTD model effectively estimated the VaR for both long and short positions. The FIAPARCH with STD model yielded the highest p-values for JSE MNGI and BNKI returns at various VaR levels. Consequently, the FIAPARCH with the STD model emerged as an appropriate and robust choice of a modeling technique for both short and long trading positions.

Accuracy and performance take precedence when selecting a risk model nowadays. There is a growing recognition of long-memory hybrid models as promising approaches to overcome the constraints of single long-memory models to estimate extreme risk measures. The hybrid models and Bayesian structural time series models demonstrate excellence in capturing various patterns and relationships in time series, leading to improved accuracy in risk modeling and management. However, it is of paramount importance that the model should be well understood, and that the decisions and predictions made by a model should be well explained. Future research will utilize long-memory hybrid models that incorporate parametric and semi-parametric distributions like the generalized Pareto, lambda, generalized lambda and Pearson type-IV distributions to estimate the extreme risk market measures.

Use of AI tools declaration

The authors declare that they have not used artificial intelligence tools in the creation of this article.

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Conflict of interest

All authors declare no conflict of interest that could influence the publication of this paper.

Supplementary

The time series supporting this study's findings are available from the corresponding author upon reasonable request.

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