



Research article

Dynamics of stability of the world economic system

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Abstract: The current global economic crisis has actualized the problem of studying non-equilibrium processes in the economy. To approach the problem, a discrete dynamic model (DDM) was developed that describes the dynamics of the rate of change in the global GDP for the period 1961-2021. This paper publishes the results of studies of two types of stability. The first is connected with the global economic system stability, which can be described by a DDM in different time periods. The second studies sensitivity of the DDM to the annual GDP growth values that were used to construct it (sensitivity analysis). The conducted research depicted the various periods of development of the global economic system, characterized by varying degrees of stability. The DDM determined that during periods of crises, the stability of the global economic system decreases. Our studies have shown that at present the world economic system is on the verge of a complete loss of stability. To assess the sensitivity of the DDM to the input data, a sensitivity indicator was introduced. It was found that the DDM is insensitive to input data during periods of crisis, i.e., the results obtained are reliable. In turn, periods of stable development of the global economic system are characterized by less reliability in the DDM results.

Keywords: discrete dynamic model; world GDP growth rate; stability of the global economic system; sensitivity to variability of initial data

JEL Codes: C51, C53, C62, C65, E32, E37

Abbreviations: DDM: discrete dynamic model

1. Introduction

The current global economic crisis has actualized the study of non-equilibrium processes in the economy. Numerous works explore the cyclical development of the global economic system, but they are mainly devoted to the consideration of Kondratieff cycles (Ayres, 2006; Dator, 2006; Dickson, 1983; Freeman, 1987; Glazyev, 1993; Hirooka, 2006; Mayevsky, 1997; Modelski, 2001, 2006; Papenhausen, 2008). Despite the differences in the approaches of the researchers mentioned above, they agree on the fundamental point, the nature of K-waves is the change of technological modes.

Separate works are devoted to the mechanisms of the origin and development of economic crises. One of the most famous theories explaining the nature of cyclical processes and their role in economic crises is the theory of overproduction, which is attributed to the laws of economic development aimed at making a profit (Marx, 1867). A further theory (Khazin, 2019) proposes that the growth of a closed economic system is achieved by deepening the division of labor, which can only develop up to a certain level. After reaching this level, innovation stops paying off, scientific and technological progress slows down, and then stops.

However, some issues that, in our opinion, are important for understanding the stability of the global economic system, are not given enough attention. These include the relationship of well-known economic cycles: Kitchin (1923), Juglar (1862), Kuznets rhythms (Kuznets, 1930) and Kondratieff waves (Kondratieff, 1922, 1925, 1926, 1928, 1935). Also, in our opinion, scientific literature has not properly evaluated the results of the work (Korotayev and Tsirel, 2010), which revealed a complex frequency spectrum of the rates of change in global GDP. Until now, integral indicators of the stability of the global economic system, which could be used for a visual assessment of its state, have not been developed.

To study the dynamics of the development of the world economic system, we previously proposed a model that describes the dynamics of the global GDP growth rates (Chaldaeva and Kilyachkov, 2012). This model is based on some assumptions (Kilyachkov et al., 2022):

1) The global economy depends on many factors that are of different nature. Consequently, global GDP growth rates (X) accumulates information about the influence of these factors.

2) The influence of factors on the growth rate of the global GDP has a complex and, often, non-obvious nature.

3) The global economic system is characterized by inertia, i.e. economic parameters in the n -th year “remember” their value in the $(n - 1)$ year. This can be formally recorded as: $X_n = F(X_{n-1})$, although in the general case it is impossible to specify the form of such a dependence.

The assumption made allows X_n to be represented in the form of a power series:

$$X_n = F(X_{n-1}) = \sum_{k=0}^m a_k \cdot X_{n-1}^k, \quad (1)$$

where a_k , are the expansion coefficients of the function $F(X_{n-1})$ in a Taylor series up to the polynomial of degree m .

Note that the coefficients a_k depend on the parameters of the global economic system. If there is insignificant change in these parameters over the considered time interval, then the representation of the rate of change in global GDP in the form of Equation 1 will be sufficiently accurate, and one can speak of a stable development of the global economic system. Otherwise, when the parameters change significantly from year to year, the global economic system is in an unstable state.

Representation of the function (Equation 1) by a polynomial of the second degree ($m = 2$) allowed (Chaldaeva and Kilyachkov, 2012; Kilyachkov and Chaldaeva, 2013) for a qualitative description of the known economic cycles of Kitchin (1923) and Juglar (1862), Kuznets rhythms (Kuznets, 1930) and

Kondratiev waves (Kondratiev, 1922, 1925, 1926, 1928, 1935) as a bi-furcation of the basic cycle with a period of about 3 years. In addition, the proposed model describes a cycle with a period of 25 years, as well as a complex spectrum of GDP change rates (Korotaev and Tsirel, 2010).

The expansion of the function (Equation 1) in a series up to third-degree polynomials ($m = 3$) is the most suitable for obtaining summarized quantitative estimates of the dynamics of the global economic system (Chaldaeva and Kilyachkov, 2014). The coefficients (a_0, a_1, a_2, a_3) of the approximating polynomial (Equation 1) are calculated using the least squares method by approximation in coordinates (X_n, X_{n-1}) of five consecutive values of the rate of annual global GDP growth. The polynomial thus obtained describes the average state of the global economic system over the selected five-year interval. It looks like this:

$$X_n = F(T, X_{n-1}) = a_0(T) + a_1(T) \cdot X_{n-1} + a_2(T) \cdot X_{n-1}^2 + a_3(T) \cdot X_{n-1}^3, \quad (2)$$

where T is the year corresponding to the end of the approximation interval.

A set of approximating polynomials built for different T form discrete dynamic model (DDM). In research (Kilyachkov et al., 2015a, 2015b, 2017, 2018) it was found that stable states of the global economic system described by the DDM have a form of either stable fixed point, periodic points, or a strange attractor. They are characterized by a convergence radius (Julia set), which has a unique structure for attractors of each type (Kilyachkov et al., 2019). Similarly, unique structures of the approximating polynomials are not only Julia sets, but also Mandelbrot sets, which describe the ranges of coefficients of approximating polynomials where these polynomials converge (Kilyachkov et al., 2022). Thus, using the DDM, it is possible to describe the state of the global economic system in a form of a generalized picture, a “pictogram”.

However, as study has shown (Kilyachkov et al., 2019), in many cases, the global economic system described by the DDM is inherently unstable, i.e., the expression (Equation 2) starts to diverge as n exceeds a certain value. Therefore, it is necessary to consider the possibility of using the DDM to study unstable states. In addition, the rates of change in global GDP growth as published by the World Bank are constantly being updated¹. Therefore, it is also necessary to check the sensitivity of the DDM to the variability of the initial data upon which it is based. In this paper, we publish the results of both types of studies: those of stability of the global economic system, as described by the DDM; and those of the sensitivity of the DDM to variance in annual GDP growth per annum as used for its construction.

2. Materials and methods

The stability of the global economy as described by the DDM can be estimated using a polynomial (Equation 2). For this purpose an initial value X_0 in the expression $F(T, X_0)$ is used to evaluate X_1 , (the first iteration). The resulting value X_1 is then applied to calculate X_2 (the second iteration) and so on until either the value of X_n or the number of iterations n exceeds the specified threshold value. Practice shows that the expression (Equation 2):

- loses stability at X_n value greater than 0.1;
- remains stable at $n \rightarrow \infty$ if it is stable for $n = 25$.

This makes it possible to determine the maximum number of iterations (N), at which the DDM converges for the considered five-year approximation interval (T) and the given initial state (X_0), i.e., draw a conclusion about the stability of the DDM.

¹ World Bank website: <https://data.Mirbank.org/indicator/NewYork.GDP.MKTP.KD.ZG?view=diagram>

2.1. Stability of the global economic system described by the DDM

To assess the stability of the global economic system as described by the DDM, a stability indicator $k(N)$ was introduced that depends on the maximum number of iterations (N), before the DDM converges. This function must meet the following conditions: as $N \rightarrow \infty$ it should converge to 1 (stable state), and for $N=0$ it should equal to 0 as well (unstable state). The simplest expression meeting the above conditions is a function of the form:

$$k(N) = \frac{N}{(N+\alpha)}, \quad (3)$$

where N is the maximum number of iterations at which DDM converges; α is a coefficient characterizing the duration of adaptive processes in the economy. Our studies show, the most convenient value of α is equal to three.

The above function is quite simple, however, it inadequately characterizes stable and unstable states. E.g. when $N=1$ (practically unstable state), the value of $k(1) = 0.25$, and for $N = 100$ (practically stable state), the value of $k(100) = 0.97$. As it can be seen, these values differ by four times only, which does not show adequately their stability in relation to each other. Therefore, another function is required to better characterize the stability of the global economy:

$$k(N) = \frac{N}{(N + \alpha \cdot e^{(\alpha-N)})}, \quad (4)$$

for which $k(1) = 0.03$, and $k(100) \approx 1$, which is commensurate with the relative stability of these extreme limits. Thus, the advantage of Equation 4 over Equation 3 is a more adequate and visual representation of the degree of stability of the global economy.

When constructing a graph of the dependence of the indicator $k(N)$ of the economic system on time (T), N in Equation 4 was the average value of the maximum number of iterations, upon which the DDM converges, for the five values of the rate of change in GDP volume (X_t), which were used for calculating the coefficients of the corresponding approximating polynomial (Equation 2).

2.2. Sensitivity of the DDM to the variability of initial data

As GDP growth rate data is constantly changing, it is necessary to determine the sensitivity of the DDM to changes in underlying data. We have considered this issue using the World Bank global GDP growth rate data as published on its website. Information about the date of obtaining the datasets and their main characteristics are presented in Table 1.

For each set of initial data, its own DDM was built, i.e., for each five-year interval (T), the coefficients of the approximating polynomials were calculated (Equation 2), and their convergence was checked. Note that to assess the sensitivity of the DDM to the initial data in the interval 1961 - 2012, all 8 data sets from Table 1 were used; in the interval 2013–2016 seven data sets; for 2017 - five, for 2018 - four, for 2019–2020 - two, and for 2021 - one.

Table 1. Main characteristics of the datasets on the rate of the global GDP change obtained on the World Bank website at different times.

Dataset number	Date of receipt of data	Data interval	Number of approximating DDM polynomials
1	09.01.2014	1961 - 2012	47
2	02.10.2017	1961 - 2016	51
3	19.03.2018	1961 - 2016	51
4	17.10.2018	1961 - 2017	52
5	21.02.2019	1961 - 2017	52
6	08.05.2020	1961 - 2018	53
7	22.03.2022	1961 - 2020	55
8	02.09.2022	1961 - 2021	56

To assess the sensitivity of the DDM to the original data, a sensitivity parameter $s(T)$ was introduced. This function for each five-year approximation interval (T) must meet the following requirements:

- be equal to 1 if the nature of the difference in the convergence of the approximating polynomials for different datasets (d) is the highest. For our case of 8 datasets, this is realized when for 4 datasets the approximating polynomials diverge, and for the 4 other datasets they remain stable (any types of stability do: stable point, stable cycle or strange attractor). In this case, the DDM sensitivity to the underlying data hits the maximum;
- be equal to 0 if the nature of the convergence of the approximating polynomials is the same for all datasets (d), i.e., they all either diverge or remain stable. In this case, the sensitivity of DDM to the original data is the lowest;
- in other cases, the value of $s(T)$ must be between 0 and 1.

In order to obtain the expression meeting the conditions above, we introduce the convergence indicator $ind(T, d)$ for each approximating polynomial (T) of the initial dataset (d). Its value for unstable states will be equal to minus one ($ind(T, d) = -1$), and for stable states – plus one ($ind(T, d) = +1$). Then an index of the DDM's sensitivity to the initial data can be calculated using the formula:

$$s(T) = 1 - \frac{ABS(\sum_d ind(T,d))}{nd}, \quad (5)$$

where $s(T)$ – index of the DDM sensitivity to initial data; T – year, the last one of the five-year interval of approximation; ABS is the modulus of the number in brackets; d is the number of the initial dataset; \sum_d - summation over datasets d ; $ind(T, d)$ is an indicator of the convergence of the approximating polynomial for the dataset with number d corresponding to year T ; nd is the number of input datasets that are used to evaluate the sensitivity. In our case, $nd = 8$ for T between 1966 and 2012. Then, as T rises, the quantity of initial datasets used for determining sensitivity decreases, reaching one at $T = 2021$.

3. Results

The stability of the global economic system as described by the DDM can be visualized in the form of a Heatmap (Figure 1). This graph reflects the dependence of the maximum number of iterations $N(T, X_0)$, upon which the approximating polynomial (Equation 2) remains stable, on the initial value of

X_0 and on different time T . To build the graph, we used the World Bank data, obtained on 03.09.2022, which allowed us to build a DDM for the interval $T \in [1966, 2021]$.

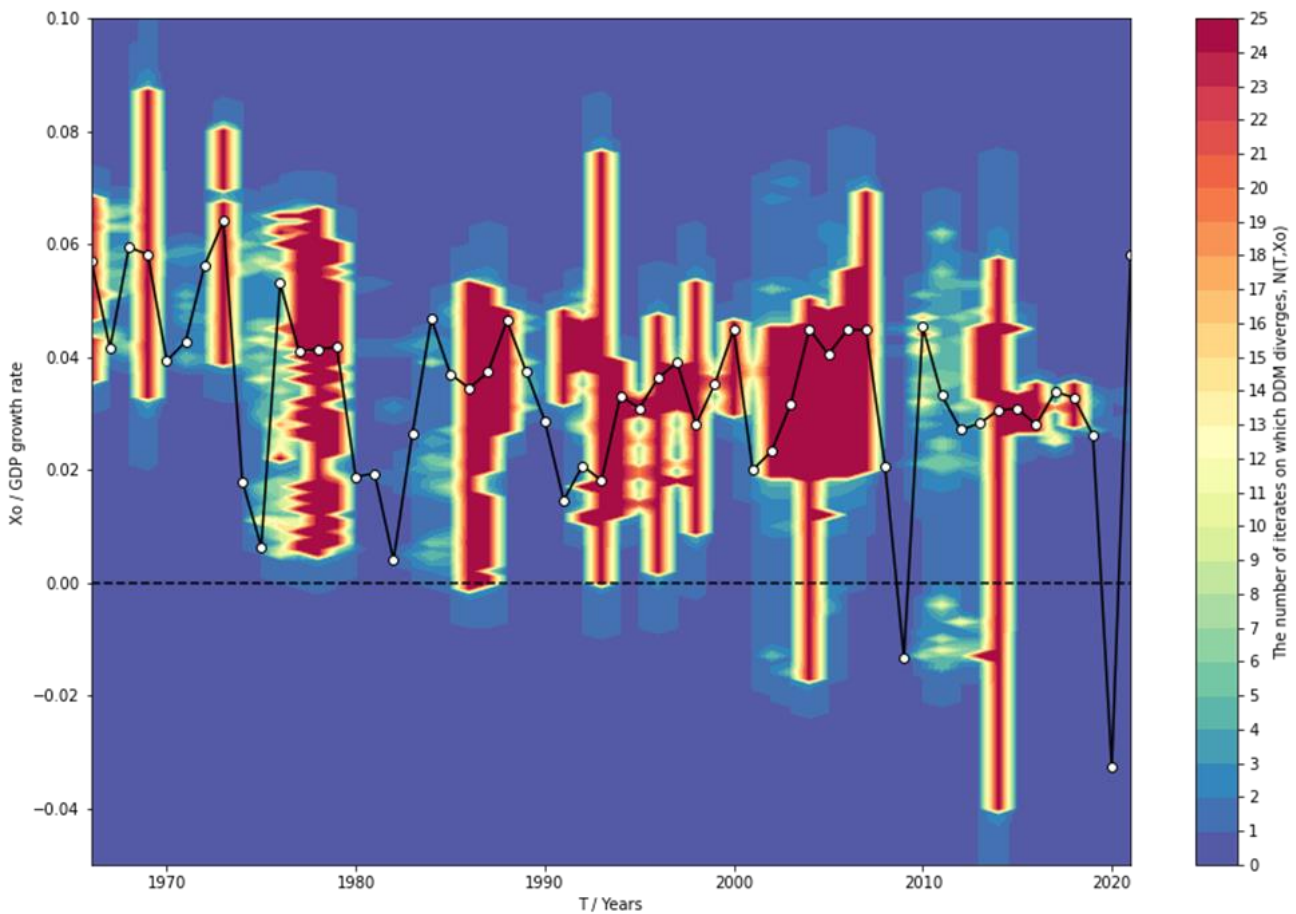


Figure 1. Dependence on time (T) and on the initial value of X_0 of the maximum number of iterations $N(T, X_0)$, upon which the global economic system as described by DDM remains stable. \bullet - the rate of change of the global GDP in the corresponding years.

In Figure 1 you can see periods of stable and unstable development of the global economic system, as represented by the DDM, over the interval from 1966 to 2021. Steady states correspond to red zones, unstable states to blue, and transitional states to a color change from green through yellow to orange.

The beginning of the graph (the second half of the 1960s - the middle of the 1970s), when stable and unstable states alternated, corresponds to the transition from one economic system to another. At this time, the crisis of the Bretton Woods system broke out, which ended with the transition to the Jamaican monetary system. In 1971, capital efficiency began to fall in the American Technological Zone (Khazin, 2019).

From the late 1970s to the mid-1980s, there was a long period of unstable development of the global economic system. During this time, the United States has developed a model for overcoming the crisis by stimulating consumer demand. It's called Reaganomics.

From the second half of the 1980s until 2008, there has been a long period of stable development of the global economic system, which is explained by the results of "Reaganomics" (at the beginning

of the period) and the development in the opened markets of Eastern Europe and post-Soviet states (in the middle and end of the period). Short instability in the area (near) 1990 is explained by the change in the parameters of the global economic system in the process of expanding markets globally.

From the end of 2008 to 2013, there has been a sharp drop in the stability of the global economic system, associated with the financial crisis in the United States, which developed into a global economic crisis. Recovery of stability in the period 2013 to 2018 is replaced by a sharp drop in 2019, which continues to the present.

However, the degree of reliability of the outcomes obtained can be calculated by comparing them with the sensitivity of the DDM to the original data, upon which the $s(T)$ was built. Combining on the same graph the indicator of the global economic system stability $k(N(T))$ represented by the DDM and the index of DDM sensitivity to initial data $s(T)$, we obtain Figure 2. To build the stability indicator graph we used the information of the World Bank obtained on 03.09.2022, and for the graph of the sensitivity index, the World Bank datasets listed in Table 1 were applied.

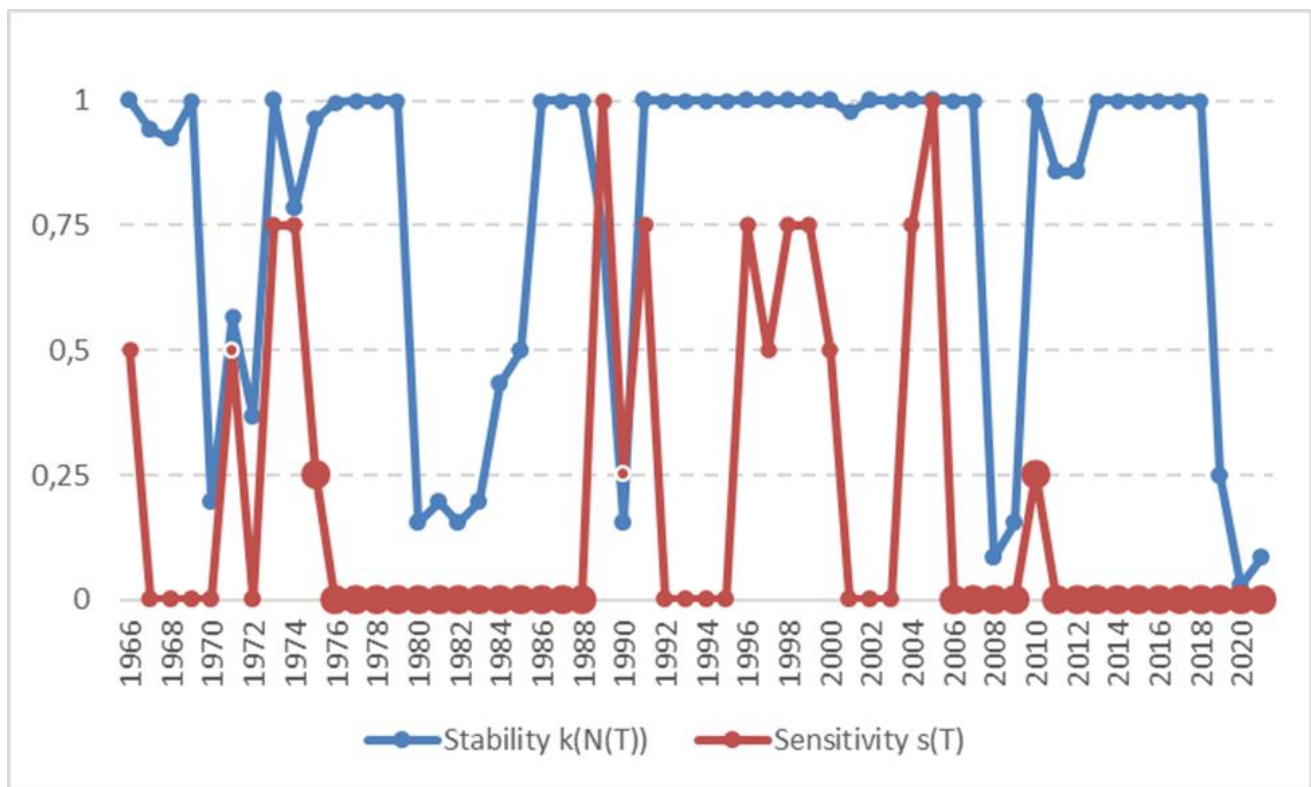


Figure 2. Dependence on time (T) of the indicator of the global economic system stability $k(N(T))$ as described by the DDM and the indicator of DDM sensitivity $s(T)$ to the initial data (T).

It can be seen from the above figure that the current crisis (commencing in 2019) is characterized by the lowest level of stability of the global economic system during the analyzed period. At the same time, the DDM describing this crisis is characterized by low sensitivity to the input data.

An interesting feature of Figure 2 is that the longest periods of insignificant sensitivity of the DDM to the input data (periods from 1975 to 1988 and 2006 to 2021) coincide with periods of instability in the global economic system. In turn, the longest period of stable development of the global economic

system from 1991 to 2007 is the most sensitive to the change in input data. In other words, during crises, the DDM is less sensitive to the input data, which means that the results obtained for crisis periods are more reliable. This is especially evident from Table 2.

Table 2. Number of approximating polynomials, values of stability and sensitivity indicators for which correspond to the pre-set ranges of values.

Sensitivity $s(T)$:	Stability $k(N(T))$:	
	low ($k < 0.5$)	high ($k \geq 0.5$)
low ($s < 0.5$)	13	30
high ($s \geq 0.5$)	0	13

From Table 2 it can be seen that for all years that were characterized by low stability, the DDM was insensitive to the input data, i.e., gave the most reliable results. For years with high stability, almost a quarter of them were highly sensitive to the original data, thus the DDM results are less reliable. The information related to the current crisis looks especially alarming (Table 3).

Table 3. The maximum number of iterations $N(T, X_0)$, upon which the DDM polynomial retains stability for the period from 2019 to 2021.

Year*	End year of a five-year approximation interval (T)		
	2019	2020	2021
2015	3		
2016	2	1	
2017	2	1	2
2018	3	1	2
2019	1	1	2
2020		0	1
2021			0

* The year, whose GDP value was used as the initial value (X_0) to determine the convergence of the approximating polynomial of the discrete dynamic model.

Table 3 shows the stability characteristics of three successive approximating DDM polynomials (Equation 2) corresponding to the years from 2019 to 2021. The coefficients of the first polynomial (2019) were calculated from the values of the rate of change in global GDP, corresponding to the years from 2015 to 2019. The coefficients of the second approximating polynomial (2020) were calculated from the values of GDP corresponding to the years from 2016 to 2020. To determine the coefficients of the third polynomial (2021), data for the period from 2017 to 2021 were used. As noted earlier, the coefficients of the approximating DDM polynomial (a_0, a_1, a_2, a_3) depend on the parameters of the global economic system. If these parameters change insignificantly over the considered time interval, then the approximating DDM polynomial converges for all initial values (X_0) corresponding to the values of the global GDP change rates over this time interval. In this case, we can talk about the stable development of the global economic system. When the parameters of the global economic system change over time, we can talk about the appearance of signs of instability in the global economic system. In other words, if we substitute the real values of GDP corresponding to the corresponding

years as the initial values of X_0 , then the DDM polynomial, starting from a certain iteration, will diverge. The stronger the parameters characterizing the global economic system change over time, then the less iterations need the approximating polynomial to lose stability. Table 3 shows that the approximating polynomial corresponding to 2019 remains stable for 1–3 iterations. The approximating polynomials corresponding to 2020 and 2021 are even less stable. From this we can conclude that at present the parameters of the global economic system are changing very quickly, which indicates its unstable state.

4. Conclusions

The paper analyzes the stability of the global economic system using a discrete dynamic model for the period from 1961 to 2021. It was shown that periods of crises in the global economy are accompanied by a decrease in its stability. Moreover, the conducted studies indicate that the current state of the global economic system, as described by the DDM is close to a complete loss of stability. The result for the current period may appear to be as expected, for the global economy was hit by a number of external shocks in the recent years, like the coronavirus pandemic (starting at the very end of 2019) or the Russian-Ukrainian conflict and related sanctions (2022). But it is the exact timing that counts. Calculations made with the help of the DDM show a fall in stability right before the shock happens. This leads us to the conclusion that either the shock itself (e.g., a military conflict) or a reaction to it may originate from the internal processes in the economy. These processes may be small, numerous, and hard to detect by fundamental analysis, so the DDM that captures the overall situation could serve as a valuable predictive tool.

This application of the DDM is greatly supported by the fact that in the periods of economic instability it turns to be virtually insensitive to input data. Accurate and timely statistics is not a thing to be expected on a verge of a crisis, so error-proof predictive tools are strongly needed. However, the decision-makers are likely to be short of time and would prefer to get forecasts promptly based on preliminary and incomplete data. So, we need to seek a possibility, when further developing the DDM, to apply it to periods shorter than a year and to country or regional data.

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Conflict of interest

The authors declare no conflict of interest.

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