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Research article

Expanding insurability through exploiting linear partial information

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Abstract: This contribution aims to expand insurability using partial linear information (LPI) on probabilities of the type, $r_1 \geq r_3$ (with $r_1 + r_2 + r_3 + \cdots r_n = 1$). LPI theory permits to exploit such weak information for systematic decision-making provided the decision-maker is willing to apply the maxEmin criterion in a game against Nature. The maxEmin rule is a natural generalization of expected profit (probabilities are known) and the maximin rule (probabilities are unknown). LPI theory is used to find out whether a crypto assets portfolio offered to an insurance company is insurable. In an example, an unfavorable future development of losses causes maximum expected loss to exceed the present value of premiums, rendering the portfolio uninsurable according to maxEmin. However, this changes when LPI concerning this development is available, while the integration of uncertain returns from investing the extra premium fails to achieve insurability in this example. Evidently, LPI theory enables insurers to accept risks that otherwise are deemed uninsurable.

Key words: insurability; linear partial information; LPI theory; maxEmin criterion

JEL Codes: C14, C44, C72, D81, D83, G11, G17, G22, G34

1. Introduction

Every few years, a new risk emerges which pushes insurers to the limits of insurability. Nuclear accidents, AIDS, climate change, crypto assets, cyber crime, and most recently pandemics such as Covid-19 are cases in point. It would be highly beneficial for society if individuals and businesses were able to transfer these risks to an entity that is better capable of bearing them. While there are calls for the government to step in as an insurer of last resort, on the long run insurance provided by the government is inefficient. Notably, it often fails to charge premiums scaled to risk, undermining the insureds' incentive to invest in prevention. For instance, the US National Flood Insurance Program created in 1968 has been charging subsidized premiums in an attempt to motivate the purchase of flood insurance but failed to adjust premiums to surging losses. This not only encouraged homeowners to settle in flood plains but prevented communities from zoning out tracts at risk (Hanscom, 2014). Already in their review of governmental coverage of catastrophes, Jaffee and Russell (1997) noted important shortcomings, in particular insufficient funding.

Ever since Berliner (1982, 1985) the conditions for a risk to be insurable have been those listed in Table 1 below. The most important addition is criterion no. 7, which takes into account that an insurance company derives profit from two sources, risk underwriting and capital investment. It calls for a nonnegative correlation of losses with returns on the capital market, extended by Gründl et al. (2021) to the concept of positive co-skewness. Given positive co-skewness, a risk with a long tail in its loss distribution can be hedged with a positively skewed distribution of returns on the capital market. Whereas the original list of criteria in Table 1 focused exclusively on underwriting activity, it is important to note that high losses to be paid are potentially hedged by high returns on capital investment given a nonnegative correlation (co-skewness, respectively) between losses and returns on the capital market.

For concreteness, let the decision at hand be whether a portfolio of policies covering crypto assets in a given country should be acquired. Munich Re (2019), for one, emphasized the limited insurability of crypto assets in view of the surge of cyber crime, once again applying the criteria of Berliner (1982, 1985). Assume that a portfolio of this type is offered on the capital market in the guise of an alternative risk transfer (ART). The issue here is to evaluate the insurability of the portfolio for the acquiring company rather than for the insurance industry in general, in keeping with Karten's (1997) argument that a risk is insurable if at least one company is willing to underwrite it. Therefore, the fact that a portfolio is for sale does not mean that it is generally uninsurable. The ceding company may e.g. have noted that while the losses within the portfolio offered occur independently (criterion no. 2 of Table 1), they are positively correlated with its remaining risk portfolio. For instance, theft of crypto assets may go along with the blocking of access to information bases that are vital for the operation of an enterprise, thus causing losses also in business interruption insurance. However, the acquiring insurer may be active in a number of countries where businesses have sufficiently invested in the protection of their data to make the simultaneous occurrence of the two types of risk improbable and the offered portfolio insurable.

Importantly, most of the criteria listed in Table 1 cannot be ticked off as "satisfied" or "not satisfied". For example, when is the maximum loss "moderate" (criterion no. 4 of Table 1)? One way to deal with this ambiguity is to introduce probabilities of criteria being satisfied. However, such probabilities are no more than rough estimates subject to considerable uncertainty. In the context of a risk transfer through ART, it is usually impossible to discern the reason why the ceding company decides to offer the portfolio in question. In the present context, it may not be the positive correlation with other types of risk but the fear of an unfavorable future development in the size of losses affecting crypto assets (which may materialize or not).

In view of these ambiguities, the objective of this contribution is to apply the theory of linear partial information (LPI) to the concept of insurability. Developed by Kofler and Menges (1977), LPI theory permits to exploit very weak information for systematic decision-making. For instance, the restriction, $0.2 \leq r_1 \leq 0.4$ over three probabilities $\{r_1, r_{2,} r_3\}$, while insufficient for conventional maximization of expected profit or utility, constitutes useful information in LPI theory. The only condition is that the decision-maker is willing to apply the so-called maxEmin criterion, which will be stated in detail in Section 2.1 below. It reflects the assumption that humans are involved in a game against Nature, who is adversarial to the extent that she presents them with an urn (i.e. a probability distribution) with the minimum *expected* payoff whereas a true adversary would choose the action presenting them with the minimum payoff for certain.

Table 1. Insurability criteria.

	Category	Criterion	Description
1		Risk, uncertainty	Measurable
2		Occurrence of losses	Independent
3	Actuarial	Maximum loss	Within capacity
4	criteria	Average loss	Moderate
5		Frequency of loss	High
6		Moral hazard, antiselection	Controllable
7		Correlation of loss with return on investment	Nonnegative
8		Insurance premium	Adequate, reasonable
9	Market	Guarantee limits	Acceptable
10	criteria	Underwriting capacity	Sufficient
11		Public policy	No strict premium regulation
	Societal		
12	criteria	Judicial system	Insurable interest given

Source: Adapted from Berliner (1982, 1985)

The remainder of this paper is structured as follows. Section 2.1 contains a literature review regarding decision-making under uncertainty and the contribution of LPI theory. In Section 2.2, LPI theory is applied to the issue of whether substantial uncertainty regarding the future losses to crypto assets (criterion no. 1 of Table 1) renders the portfolio uninsurable to the acquiring insurer, meaning that it would fail to generate a long-run profit on expectation. However, insurability may attain if management uses its past experience with acquisitions to formulate an LPI concerning the future development of losses. In Section 2.3, management draws upon the advice of a consultancy, who furnishes its own partial information regarding the development of future losses which however may be erroneous. This calls for the combination of two sources of partial information; depending on the credibility of the advice provided, management will be in a position to decide the insurability issue. Section 2.4 is devoted to the added criterion no. 7 of Table 1, i.e. the uncertain returns to the premium income invested that may make the risk portfolio insurable after all. This results in a convolution of two probability distributions each characterized by an LPI, which will have the form of an LPI as well. The final Section 3 contains a summary and an outlook on additional possibilities of applying LPI theory to evaluating insurability for decision-making by the management of insurance companies.

2. Materials and methods

2.1. Literature review

Ever since Knight (1921), the concept of risk (where probabilities are known) has been distinguished from uncertainty (where they are unknown). Strictly speaking, the criteria for insurability listed in Table 1 are not satisfied unless the associated probabilities are fully known. Otherwise, the uncertainty surrounding a loss is not measurable (criterion no. 1), the independence of losses cannot be verified (criterion no. 2), the maximum (probable) loss cannot be established and compared with underwriting capacity (criterion no. 3), and the frequency of loss is cannot be measured (criterion no. 5). Similar problems affect the other criteria as well, severely limiting the domain of insurability.

In the case of complete uncertainty, the prudent decision-maker has to adopt the view of Wald (1945). Here, Nature is an adversary to humans, who are involved in a zero-sum game against her and therefore constrained to apply the pessimistic maximin decision rule. As a consequence, an insurance company would have to deem all risks non-insurable for which it lacks years of experience (in principle under unchanging circumstances yielding repeated observations). To avoid this impasse, management might adopt the Hurwicz criterion (Hurwicz, 1951), using a linear combination of the maximin and the maximax value (the highest of all possible maxima, whose ponderation reflects management's degree of optimism, a purely subjective parameter). In addition, the Hurwicz criterion requires the intervals of payoffs to be known with certainty, a condition which is satisfied for a single insurance policy with a limit on payment in case of loss but may not hold when the frequency of losses covered is random.

There are several additional decision-making criteria in a situation characterized by what is commonly called ambiguity [see Gilboa (2013), in particular chs. 7 and 15-17 for a survey]. A natural idea to overcome ambiguity is to define probabilities over probabilities, e.g. over r_1 such as $\text{Prob}(r_1 = 0.9) = 0.1$, $\text{Prob}(r_1 = 0.8) = 0.1$, $\text{Prob}(r_1 = 0.7) = 0.2$, etc. The resulting compound probability distribution could then be used to calculate expected profit (expected utility, respectively). However, actual decision-makers were found to evaluate the original compound lottery in a different way from the reduced-form one containing final outcomes and probabilities only (Bar-Hillel, 1973). This finding led to an extended literature on the axioms necessary to preserve the equivalence between the two settings and the importance of ambiguity aversion in the event these axioms are not satisfied [see Klibanoff et al. (2005) and the interchange between Epstein (2010) and Klibanoff et al. (2012)]. As will become evident in Section 2.4 below, LPI theory offers a much simpler practical alternative.

Indeed, Kofler and Menges (1977) were able to show that a piece of information regarding probabilities of possible outcomes such as $r_1 \geq r_3$ (with $r_1 + r_2 + r_3 + \ldots r_n = 1$) can be systematically exploited for decision-making. LPI theory has led to a series of publications, notably by Kofler and Menges (1979), Kofler (1988), Kofler and Zweifel (1981, 1988, 1991, 1993), Zimmermann et al. (1985), and Zweifel (2021).

2.2. Uncertain development of future losses and insurability

Let the portfolio of policies covering crypto assets mentioned in Section 1 be offered for USD 300 mn, while the premium volume can be estimated at USD 1 bn in present value (PV) terms (reflecting a planning horizon of five years, say). These values are assumed to be fixed, not subject to uncertainty in order to focus on the uncertain loss distribution. Let there be three loss categories; their average and aggregate values are displayed in Table 2, along with LPI_1 and LPI_2 regarding the associated current and future probability values.

The simpler case of future values is examined first because only one restriction on probabilities is available for them, the LPI_2: $0.1 \leq r_1' \leq 0.3$. It is depicted in Figure 1 below. On the left-hand side, the probability distributions satisfying the summation restriction, $r_1' + r_2' + r_3' = 1$ are displayed, represented by points on the triangular plane ABC. Note that along BC, $r_1' = 0$. The same plane is shown as the triangle ABC on the right-hand side of Figure 1, with the single LPI_2: $0.1 \leq r_1' \leq 0.3$ added. It is represented by two straight lines; the one depicting $r_1' = 0.1$ lies at one tenth of the distance from CB to A; the one for $r_3' = 0.3$, at three tenths of the distance. The set of distributions satisfying the LPI_2: $0.1 \leq r_1' \leq 0.3$ is shown as the shaded area. The linearity of the restrictions guarantees the convexity of this area, a requirement that will prove important below.

Table 2. Loss categories for crypto assets portfolio.

Category	Average loss, USD	Aggregate loss, USD mn	LPI regarding probabilities
Low, current values	10,000	100	$0.2 \leq r_1 \leq 0.4$
Medium, current values	300,000	300	LPI_1: $0.3 \leq r_2 \leq 0.7$
High, current values	10,000,000	1,000	$0.1 \leq r_3 \leq 0.2$
Low, future values	15,000	150	$0.1 \leq r_1' \leq 0.3$
Medium, future values	500,000	500	LPI_2: n.a.
High, future values	13,000,000	1,400	n.a.

In addition, the indifference curves reflecting Nature' s preferences are drawn as dashed straight lines. Her survival being assured, she can act in a risk-neutral manner while her preferences by assumption are in terms of expected values which are linear functions of probabilities. Since she is assumed to present the decision-maker with the most unfavorable distribution, her preferred one would presumably be vertex C with (0,0,1), which however is out of reach in view of the LPI restriction. Her optimum cannot be in the interior of ABC either but must lie at one of the shaded area's vertices in Figure 1. This follows from the fundamental equivalence between a zero-sum game and a linear program [Chiang (1984), Section 21.4]. The optimum solution to a linear program is given by one of the extreme points of the feasible region [Chiang (1984), Theorem II in Section 18.3][1]. These extreme points (vertices) satisfy the condition that for a *k*-dimensional feasible space, the number of restrictions satisfied as exact equalities (including equality to zero and the summation restriction) is also *k* [Chiang (1984), Section 18.4]. Whichever the direction of Nature's preferences, it is therefore sufficient to examine the vertices, corresponding to a set of extremal distributions.

[1] Since Nature's preferences are linear, the decision maker's solution to the linear program corresponds to Nature's solution of the dual due to the equivalence of a zero-sum game and a linear program. Therefore, Nature necessarily selects a vertex point [Intriligator (1971, p. 75)].

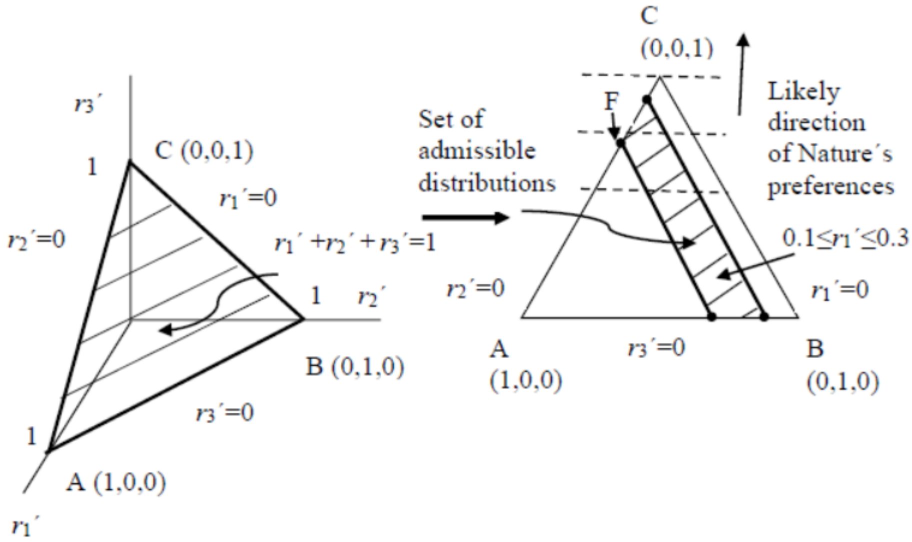

Figure 1. Probability distributions in three- and two-dimensional space with LPI_2 (future values in Table 2).

These extremal distributions are assembled in the matrix $V[LPI_2]$below, starting with vertex F in Figure 1 and proceeding in clockwise manner,

$$V[LPI_2] = \begin{bmatrix} 0.3 & 0.1 & 0.1 & 0 \\ 0 & 0 & 0.9 & 0.7 \\ 0.7 & 0.9 & 0 & 0.3 \end{bmatrix}. \quad (1)$$

In view of Table 2, this gives rise to the following expected payoffs (with the minimum underlined),

$$EP[LPI_2] = [-150 \quad -500 \quad -1{,}400] \begin{bmatrix} 0.3 & 0.1 & 0.1 & 0 \\ 0 & 0 & 0.9 & 0.7 \\ 0.7 & 0.9 & 0 & 0.3 \end{bmatrix}$$
$$= \left[-1{,}025 \quad -\underline{1{,}275} \quad -465 \quad -770\right]. \quad (2)$$

Under the maxEmin criterion and accepting an unfavorable future development of losses as certain, management has to brace itself with a loss amounting to USD 1,275 mn in expected PV, which exceeds the premium volume of USD 700 mn (after deduction of USD 300 mn for the acquisition). Therefore, it would have to deem the offered portfolio of crypto asset risks as non-insurable.

However, the feared development of losses might not materialize. In that event, the current values entered in Table 2 are relevant. The set of distributions satisfying the pertinent LPI_1 is depicted in Figure 2 below. It can be assembled in the matrix $V[LPI_1]$, again proceeding in clockwise manner,

$$V[LPI_1] = \begin{bmatrix} 0.4 & 0.2 & 0.2 & 0.4 \\ 0.4 & 0.6 & 0.7 & 0.5 \\ 0.2 & 0.2 & 0.1 & 0.1 \end{bmatrix}. \tag{3}$$

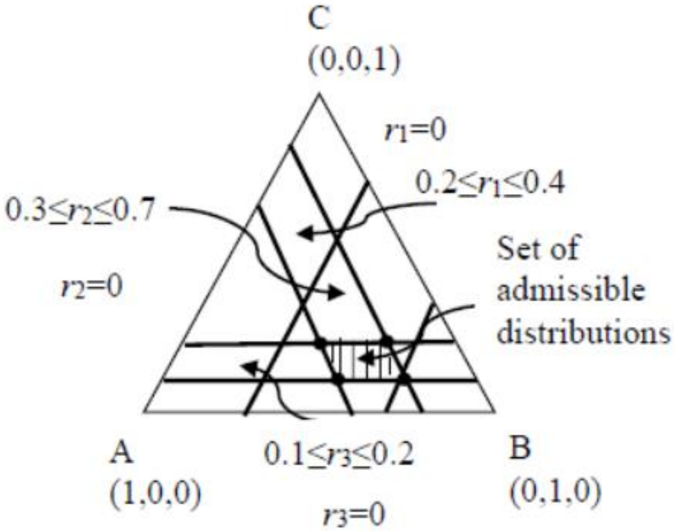

Figure 2. Probability distributions in two-dimensional space with LPI1 (current values in Table 2).

Therefore, expected payoffs are given by

$$EP[LPI_1] = [-100 \quad -300 \quad -1{,}000] \begin{bmatrix} 0.4 & 0.2 & 0.2 & 0.4 \\ 0.4 & 0.6 & 0.7 & 0.5 \\ 0.2 & 0.2 & 0.1 & 0.1 \end{bmatrix}$$

$$= \left[-360 \quad \underline{-400} \quad -330 \quad -290\right]. \tag{4}$$

With a maximum expected loss of USD 400 mn in PV and a premium volume of USD 700 mn, net of acquisition cost, the portfolio now appears to be eminently insurable.

To resolve this apparent contradiction [see Equation 2 again], management may draw on past experience with portfolio acquisitions. Let its estimate be that the non-occurrence of the surge in future losses q_2 is at least two times as likely as its occurrence, thus LPI3: $q_2 \geq 2q_1,\ q_1 + q_2 = 1$. The pertinent matrix of vertices is given by

$$V[LPI_3] = \begin{bmatrix} 0.333 & 0 \\ 0.667 & 1 \end{bmatrix}. \tag{5}$$

Since Nature is assumed to always choose the distribution with the minimum expected payoff to the decision-maker, these vertices must be applied to the minimum expected payoffs in Equations 2 and 4, resulting in

$$EP[LPI_3] = [-1{,}275 \quad -400\ ]\begin{bmatrix}0.333 & 0\\0.667 & 1\end{bmatrix} = \left[\underline{-678} \quad -400\right]. \qquad (6)$$

As the portfolio generates a net premium volume of USD 700 mn in present value, its insurability is now given for this insurance company. The verdict would possibly have to be qualified if the future premium volume were also subject to uncertainty, calling for another application of the maxEmin criterion, an extension which is not pursued here.

2.3. Combining two sources of partial information

The difference between the maxEmin value of USD −678 mn in Equation 6 and the premium volume of USD 700 mn (after deduction of acquisition cost) is rather small. In this situation, it may be worthwhile to commission a management consultancy specializing in insurance markets to provide additional information concerning the future development of losses to crypto assets. The objective is to sharpen the LPI3: $q_2 \geq 2q_1,\ q_1 + q_2 = 1$concerning the likelihood $q_1$of a future surge in losses as entered in the lower half of Table 2. Indeed, let the management consultancy come up with the more optimistic LPI4: $q_1' \leq 0.2, q_1' + q_2' = 1$, where q_1' symbolizes the likelihood of the feared surge of future losses. The task at hand is to determine a weighting in the combination of the two sources of partial information, LPI3 and LPI4. The complication is that previous experience with the consultancy suggests that its projections sometimes fail to materialize. Let this less than perfect credibility be reflected by the

LPI5:$0.8 \leq m_1 \leq 0.9, m_1 + m_2 = 1$,thus $min(m_2) = 0.1$ and $max(m_2) = 0.2$ for later use. (7)

with m_1 denoting the probability that the information provided will turn out to be true.

Nature now presents the decision-maker with two urns, one characterized by the probability distributions compatible with LPI3, the other, compatible with LPI4. The assumption regarding Nature's behavior calls for retaining the extremal distributions resulting in the respective minimum payoffs, i.e.

$$\bar{q}_1^e = [0.333\ \ 0.667]' \text{ and } \bar{q}_1^{e'} = [0.2\ \ 0.8]' \qquad (8)$$

[see Equations 5 and 6 again]. The best available estimates of the two urns being in place are $m_1$and m_2, reflecting the credibility of the information provided by the consultancy. Therefore, the total probability e of drawing one of the extremal distributions is given by the bilinear form [see Kofler and Zweifel (1988), Appendix 3 for details],

$$e = m_1 q_1^{e'} + m_2 q_1^e. \qquad (9)$$

The probability of drawing the first extremal distribution is given by

$$e_1 = \frac{q_1^{e'} m_1}{q_1^{e'} m_1 + q_1^e m_2}. \qquad (10)$$

To obtain the LPI6 pertaining to e_1, one needs to determine its minimum and maximum value. This is most easily done by rewriting Equation 10 in the following way,

$$e_1 = \frac{q_1^{e'} m_1}{q_1^{e'} m_1 + q_1^e m_2} = \frac{1}{1+\frac{q_1^e m_2}{q_1^{e'} m_1}} = \frac{1}{1+\frac{m_2}{m_1}\cdot\frac{q_1^e}{q_1^{e'}}} = \left[1 + \frac{m_2}{m_1} \cdot \frac{q_1^e}{q_1^{e'}}\right]^{-1}. \qquad (11)$$

Evidently, the minimum value of e_1 is attained when both m_2 / m_1 and $q_1^e / q_1^{e'}$ are at their maximum. Now according to Equation 7, $max(m_2/m_1) = 0.2/0.8 = 0.25$, while according to Equation 8 $max\left(q_1^e/q_1^{e'}\right) = 0.667/0.2 = 3.335$. Therefore,

$$min(e_1) = [1 + 0.25 \cdot 3.335]^{-1} = 0.545. \qquad (12)$$

Conversely, e_1 is maximum when both $m_2/m_1$and $q_1^e/q_1^{e'}$ are at their minimum. According again to Equation 7, $min(m_2/m_1) = 0.1/0.9 = 0.111$, and $min\left(q_1^e/q_1^{e'}\right) = 0.333/0.8 = 0.416$. Thus

$$max(e_1) = [1 + 0.111 \cdot 0.416]^{-1} = [1 + 0.046]^{-1} = 0.956. \qquad (13)$$

One therefore obtains
LPI6: $0.545 \le e_1 \le 0.956$, giving rise to the extremal distributions

$$V[LPI_6] = \begin{bmatrix} 0.545 & 0.956 \\ 0.455 & 0.044 \end{bmatrix} \qquad (14)$$

and hence expected payoffs

$$EP[LPI_6] = [-678 \quad -400] \begin{bmatrix} 0.231 & 0.706 \\ 0.769 & 0.294 \end{bmatrix} = [-464.2 \quad -\underline{596.3}]. \qquad (15)$$

in view of Equations 6 and 4. Finally, one needs to determine the LPI pertaining to e_2, the probability of drawing the second extremal distribution. This calls for the evaluation of

$$e_2 = \frac{q_1^e m_2}{q_1^{e'} m_1 + q_1^e m_2} = \frac{1}{\frac{q_1^{e'} m_1}{q_1^e m_2}+1} = \frac{1}{1+\frac{m_1}{m_2}\cdot\frac{q_1^{e'}}{q_1^e}} = \left[1 + \frac{m_1}{m_2} \cdot \frac{q_1^{e'}}{q_1^e}\right]^{-1}. \qquad (16)$$

This time, the minimum value of e_2 is attained when both m_1 / m_2 and $q_1^e / q_1^{e'}$ are at their maximum. According to Equation 7, $max(m_1/m_2) = 0.9/0.1 = 9$, while according to Equation 8, $max\left(q_1^{e'}/q_1^e\right) = 0.333/0.2 = 1.665$. Therefore,

$$min(e_1) = [1 + 9 \cdot 1.665]^{-1} = 0.063. \qquad (17)$$

Conversely, e_2 is maximum when both m_1 / m_2 and $q_1^e/q_1^{e'}$ are at their minimum. According again to Equation 7,

$$min(m_1/m_2) = 0.8/0.2 = 4, \text{ and} \tag{18}$$

$$min(q_1^{e'}/q_1^e) = 0.2/0.667 = 0.3. \tag{19}$$

Thus

$$max(e_1) = [1 + 0.4 \cdot 0.3]^{-1} = [1 + 0.12]^{-1} = 0.893. \tag{20}$$

One therefore obtains from Equations 17 and 20 LPI_7: $0.063 \leq e_2 \leq 0.893$, giving rise to

$$V[LPI_7] = \begin{bmatrix} 0.063 & 0.893 \\ 0.937 & 0.107 \end{bmatrix} \text{ and in view of Equations 6 and 4} \tag{21}$$

$$EP[LPI_7] = [-678 \quad -400] \begin{bmatrix} 0.063 & 0.893 \\ 0.937 & 0.107 \end{bmatrix} = [-417.5 \quad -\underline{648.3}]. \tag{22}$$

With the minimum guaranteed expected payoff of USD −648.3 mn and a net premium income of USD 700 mn, the portfolio can be judged insurable with some confidence thanks to the additional information obtained from the consultancy. It is noteworthy that LPI theory also provides an estimate of the value of this information, respectively). Indeed, the transition from LPI_3 to LPI_7 raises the maxEmin value of losses from USD −678 mn to USD –648.3 mn, a gain of USD 29.7 mn in expected value.

2.4. Uncertain future losses, uncertain returns on investment, and insurability

As pointed out in the Introduction, examining the underwriting side of the insurance business falls short of a full evaluation. Premiums earned can be invested to generate additional revenue. However, returns on investment are subject to uncertainty as well. For instance, during the decade from 2010 to 2020, the rate of return on US bonds (an asset that insurers are mandated to hold) varied between a low of −3.6 percent in 2013 to a high of 6.3 percent in 2019 (https://www.visualcapitalist.com/historical-returns-by-asset-class/, accessed on 8 Jan. 2022).

This section is devoted to the question of whether in spite of uncertainty, a insurer's investment activity can contribute to the insurability of a risk portfolio. The point of departure is the negative verdict based on Equation 2. As to the returns on investment of the USD 700 mn extra premium income, let management distinguish three possible outcomes, again in PV terms (see Table 3).

Table 3. Returns on investment of the USD 700 mn premium.

PV of returns from investing USD 700 mn		**LPI_8 regarding probability**
Below average	−50	$p_1 \leq 0.2$
Average	50	$0.4 \leq p_2 \leq 0.8$
Above average	80	$p_3 \leq 0.1$

PV: Present value

The LPI entered reflects the experience that a negative return of USD −50 mn on USD 700 mn. is unlikely, as indicated by $p_1 \leq 0.2$. With $0.4 \leq p_2 \leq 0.8$, most of the probability mass pertains to an average outcome, while there is also a small chance ($p_3 \leq 0.1$) of an above average outcome.

In Figure 3, the convex polyhedron reflecting the distributions satisfying this LPI shrinks to two points. Therefore, the two extremal distributions are given by

$$V[LPI_8] = \begin{bmatrix} 0.2 & 0.2 \\ 0.7 & 0.8 \\ 0.1 & 0 \end{bmatrix}, \tag{23}$$

and the expected payoffs, by

$$EP(R) = [-50 \quad 50 \quad 80] \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \\ 0.1 & 0 \end{bmatrix} = \begin{bmatrix} 33 & \underline{30} \end{bmatrix}. \tag{24}$$

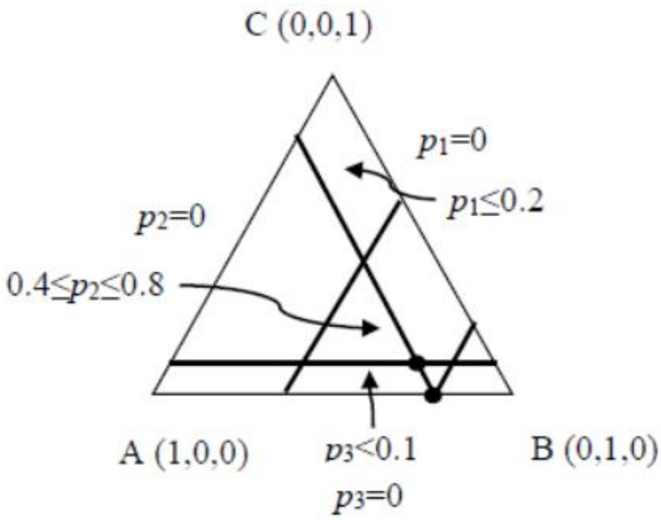

Figure 3. Probability distributions in two-dimensional space with LPI restrictions according to Table 3

For evaluating the combined uncertainties characterizing losses and returns, the convolution of the two extremal distributions needs to be formed. A reasonable assumption is that losses and returns are independent of each other. Indeed, between 2012 and 2019 in the United States, the correlation coefficient between commercial losses paid and the rate of return on T-bonds was only 0.405 (https://www.statista.com/statistics/429012/incurred-losses-for-commercial-insurance-usa/; https://www.calculators.org/math/correlation.php, accessed on 15 Jan. 2022). Another simplification is to neglect that the PV of losses depends on the discount factor and hence ultimately on the rates of return on the capital market, permitting to use the values of Equation 2.

For obtaining the possible combinations of probability values associated with the payoffs, one needs to form the Cartesian product of the two extremal distributions (Kofler and Zweifel, 1991),

$$V(LPI_2) \times V(LPI_8) = \begin{bmatrix} (0.3, 0, 0.7; 0.2, 0.7, 0.1) & (0.3, 0, 0.7; 0.2, 0.8, 0) ; \\ (0.1, 0, 0.9; 0.2, 0.7, 0.1) & (0.1, 0, 0.9; 0.2, 0.8, 0) ; \\ (0.1, 0.9, 0; 0.2, 0.7, 0.1) & (0.1, 0.9, 0; 0.2, 0.8, 0) ; \\ (0, 0.7, 0.3; 0.2, 0.7, 0.1) & (0, 0.7, 0.3; 0.2, 0.8, 0) \end{bmatrix}. \quad (25)$$

Table 4. Calculation of probabilities from Table 3 and Equation 25.

Vector combination[a]	Associated probability	Associated expected payoff
$\overline{r}_1' \; \overline{p}_1$	$(0.3, 0, 0.7)'(0.2, 0.7, 0.1) = 0.13$	−1,025+33=−992
$\overline{r}_1' \; \overline{p}_2$	$(0.3, 0, 0.7)'(0.2, 0.8, 0) = 0.06$	−1,025+30=−995
$\overline{r}_2' \; \overline{p}_1$	$(0.1, 0, 0.9)'(0.2, 0.7, 0.1) = 0.11$	−1,275+33=−1,242
$\overline{r}_2' \; \overline{p}_2$	$(0.1, 0, 0.9)'(0.2, 0.8, 0) = 0.02$	−1,275+30 = −1,245
$\overline{r}_3' \; \overline{p}_1$	$(0.1, 0.9, 0)'(0.2, 0.7, 0.1) = 0.65$	−465+33=−432
$\overline{r}_3' \; \overline{p}_2$	$(0.1, 0.9, 0)'(0.2, 0.8, 0) = 0.74$	−465+30=−435
$\overline{r}_4' \; \overline{p}_1$	$(0, 0.7, 0.3)'(0.2, 0.7, 0.1) = 0.493$	−770+33=−737
$\overline{r}_4' \; \overline{p}_2$	$(0, 0.7, 0.3)'(0.2, 0.8, 0)= 0.56$	−770+30=−740

[a] $\overline{r}_i'$, $i = 1,2,3,4$: Vector of probabilities taken from the $V(LPI_2)$ matrix of extremal distributions

p_j^M, $j = 1,2$: Vector of probabilities taken from the $V(LPI_8)$ matrix of extremal distributions

It is noteworthy that all the expected payoffs entered in Table 4 can materialize only in a single way, obviating the aggregation of probabilities. For the application of the maxEmin criterion, one obtains the following LPI regarding the convoluted distributions from Table 3 (Kofler and Zweifel, 1991),

$$\text{LPI}_9\text{: } 0.02 \le V(LPI_2) \times V(LPI_8) \le 0.74 \quad (26)$$

which gives rise to the extremal distributions

$$V(LPI_2) \times V(LPI_8) = \begin{bmatrix} 0.02 & 0.74 \\ 0.98 & 0.26 \end{bmatrix}. \quad (27)$$

One obtains the expected payoffs

$$EP[LPI_2 \times LPI_8] = [-1{,}245 \;\; -435 \,] \begin{bmatrix} 0.02 & 0.74 \\ 0.98 & 0.26 \end{bmatrix} = \begin{bmatrix} -451.2 & -\underline{1{,}034.4} \end{bmatrix}. \quad (28)$$

While the integration of investment income does increase the minimum expected payoff from USD −1,275 mn according to Equation 2 to USD −1,034.4 mn, this value still falls short of the pemium income of USD 700 mn net of acquisition cost. Under the maxEmin criterion, management is therefore well advised to judge the offered risk portfolio as non-insurable unless it can somehow change the minimum expected payoff of USD −1,275 mn in underwriting.

There are several ways to achieve this. One is to layer the risk portfolio, as advised by Karten (1997). For instance, it might be possible to cede USD 400 mn to an enterprise with excellent diversification possibilities at a premium of USD 100 (say), thus limiting exposure to USD 1,000 mn in Table 1. Assuming unchanged probabilities, Equation 28 then becomes in view of Table 3

$$EP[LPI_2 \times LPI_8] = [-845 \quad -435 \ ] \begin{bmatrix} 0.02 & 0.74 \\ 0.98 & 0.26 \end{bmatrix} = \begin{bmatrix} -443.2 & -\underline{738.4} \end{bmatrix}. \quad (29)$$

Evidently, with net premium income of now USD 600 mn, insurability fails to be attained.

Alternatively, additional information concerning the probabilities pertaining to the three categories of future losses may achieve this. While LPI concerning $r_3^{'}$ (the probability of high losses) may not be available, a look at Figure 1 reveals that shifting probability mass to the lowest category would be effective. Since small and medium-sized losses typically occur with high frequency, gaining experience with them in the aim of modifying the estimated loss distribution may be feasible. For instance, attaining the LPI9: $r_1' \geq 0.55$ is just about sufficient. It leads to the matrix of extremal distributions

$$V[LPI_9] = \begin{bmatrix} 0.55 & 0.55 & 1 \\ 0 & 0.45 & 0 \\ 0.45 & 0 & 0 \end{bmatrix} \quad (30)$$

and hence

$$EP[LPI_9] = [-150 \quad -500 \quad -1{,}400] \begin{bmatrix} 0.55 & 0.55 & 1 \\ 0 & 0.45 & 0 \\ 0.45 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\underline{712.5} & -307.5 & -150 \end{bmatrix}, \quad (31)$$

rendering the risk portfolio almost insurable even before taking investment income into account.

3. Conclusions and outlook

The objective of this contribution is to expand the limits of insurability. Its point of departure is Berliner's (1982, 1985) well-known criteria of insurability complemented by the requirement that income from premium revenue invested should serve as a hedge against losses. However, these criteria typically are not satisfied with certainty because e.g. future aggregate losses follow a probability distribution that is partially known at best. The existing literature has sought to solve this problem by defining probabilities over probability distributions [Klibanoff et al. (2005); Epstein (2010); Klibanoff et al. (2012)] or by applying fuzzy set theory, which however requires the specification of a function defining the exact likelihood that a given observation is a member of the set of interest [Zadeh (1965); Yager and Basson (2007)].

As a practical alternative, the theory of partial linear information (LPI theory) is proposed. If the decision-maker is willing to assume to be engaged in a game against Nature, the maxEmin decision criterion becomes the natural choice. This criterion is a generalization of the maximin rule (where the adversary selects the course of action imparting maximum loss with certainty) on the one hand and maximizing expected utility (which cannot be applied unless the probability distribution is fully known) on the other. The maxEmin rule enables the decision-maker to focus on a finite set of so-called extremal probability distributions, calculate the minimum expected payoff, and select the course of action that maximizes his or her expected payoff over these minima.

LPI theory is illustrated using the decision problem of an insurance company who, being offered through ART a portfolio of policies covering crypto assets exposed to cyber crime, needs to find out whether the portfolio is insurable. This question is answered in three ways. First, when the future development of losses is known only very partially, the maximum expected loss exceeds turns out to exceed the present value of the future premium income net of the cost of acquisition, rendering the portfolio non-insurable under the maxEmin rule. The second step is the recognition that an unfavorable future development of losses may not be certain but have a maximum probability of occurring. This renders the risk portfolio just about insurable. Therefore, additional information provided by a management consultancy could enhance insurability. This is indeed the outcome, resulting in a maxEmin value of the consultancy's advice even though it fails to be always correct. In a third step, a partially known probability distribution concerning the future income earned from investing the extra premium volume is introduced. However, given an unfavorable future development of losses, the crypto asset portfolio in question continues to be non-insurable. Interestingly, changing this verdict does not necessarily call for probability information regarding the high-loss category; a sufficiently modified LPI regarding the low-loss category may prove sufficient.

Evidently, LPI theory can be useful for examining the insurability of other novel risks. An early contribution designed to expand the limits of insurability was by Karten (1997). In contrast to Berliner (1982, 1985) and in the spirit of this paper, the author emphasizes that it is the individual insurance company who needs to decide whether or not it deems a risk insurable or not. He also cites a number of internal risk management possibilities ranging from the design of policies to farming out (parts of) a risk portfolio to a large firm outside the insurance industry. This firm might be exposed to other risks which are uncorrelated (or even negatively correlated) with those of the portfolio. Obviously, both partners to such a risk transfer would benefit from applying LPI theory, permitting them to use all types of information available.

Recently, Schanz (2020) concludes based on available premium and loss data and own estimates that business interruptions caused by the Covid-19 pandemic are uninsurable for the private property-casualty (P&C) insurance industry. In particular, criteria no. 1, 2, and 10 listed in Table 1 are violated since lockdowns mandated by governments are unpredictable, governments tend to act in concertation causing positive correlation, and the magnitude of losses easily exceeds underwriting capacity. Yet partial information concerning the likelihood and duration of lockdowns in selected countries could be exploited. Moreover, a company's Covid-19 pandemic portfolio can possibly be hedged using e.g. a real estate portfolio and coverage be limited or layered for partial cession, e.g. through ART (Hochrainer-Stigler and Reiter, 2021). Again, any partial information would be valuable to both parties.

Hartwig and Gordon (2020) argue in favor of a general exclusion of communicable diseases in the aim of securing the stability of the P&C insurance industry. They emphasize criterion no. 12 of Table 1, which fails to be satisfied because of the risk of litigation seeking to obtain payment for losses caused by risks that were not explicitly excluded and the unwillingness of US regulatory authorities to file more restrictive policies. Their analysis concerns the industry as a whole, while some P&C insurers may well be in a different situation. An important consideration is the fact that US insurance is regulated at the state level, and some authorities may be more likely than others to accept the filing of such policies. Therefore, an insurance company who has the majority of its business in such states (which may in addition be less affected by the pandemic) could be in a position to benefit from partial information concerning the likelihood of obtaining permission to limit its risk exposure. As to the unwillingness of reinsurers to underwrite risks caused by communicable disease cited by Hartwig and Gordon (2020), at least some of them are engaged in business worldwide, allowing them to glean some partial information concerning possible correlations between a communicable disease portfolio in the United States and an auto reinsurance portfolio in Europe (say).

Of course, this analysis is subject to a number of limitations. First, the market environment may not be correctly represented by a Nature who is satisfied to present the decision-maker with an urn that offers a minimum payoff on expectation. There might be a particular competitor who will select the strategy that minimizes the decision-maker’s payoff with certainty, rendering the maxEmin criterion too optimistic. On the other hand, Wald’s (1945) minimax rule would presumably condemn the management of an enterprise to complete inactivity. The second limitation is the requirement that all possible states of the world are known; there must be no “unkown unknowns” (former US Secretary of Defense Ronald Rumsfeld, 2 Sept. 2007). However, this requirement is common to all conventional variants of decision theory, while LPI theory permits to reach a decision even in the case where there is no probability information regarding a possibly neglected scenario (as in the lower half of Table 2 in the text). Finally, insurability traditionally has been thought of as a concept applying to all companies active in a market [with the notable exception of Karten (1997)] whereas LPI theory makes it applicable to an individual company as well. Its management may evaluate objective information available to all insurers with a view to firm-specific (partial) information. In the case of the acquisition of a risk portfolio at least, the issue becomes of whether the new risks provide a hedging with the existing ones or whether they exacerbate the correlation between losses. In sum, LPI theory arguably provides a practical way forward to expand the limits of insurability in concrete situations.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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