Communications in Analysis and Mechanics

https://www.aimspress.com/journal/cam

Communications in Analysis and Mechanics, 16(2): 307–333.
DOI: 10.3934/cam.2024015
Received: 16 October 2023
Revised: 30 January 2024
Accepted: 13 March 2024
Published: 11 April 2024

Research article

Existence and asymptotic behavior for ground state sign-changing solutions of fractional Schrödinger-Poisson system with steep potential well

Xiao Qing Huang[1] **and Jia Feng Liao**[1,2,*]

[1] School of Mathematics and Information, China West Normal University, Nanchong, Sichuan 637009, People's Republic of China

[2] College of Mathematics Education, China West Normal University, Nanchong, Sichuan 637009, People's Republic of China

* **Correspondence:** Email: 2712271568@qq.com; liaojiafeng@163.com, liaojiafeng@cnwu.edu.cn.

Abstract: In this paper, we investigate the existence of ground state sign-changing solutions for the following fractional Schrödinger-Poisson system

$$\begin{cases}(-\Delta)^s u + V_\lambda(x)u + \mu\phi u = f(u), & \text{in } \mathbb{R}^3,\\ (-\Delta)^t \phi = u^2, & \text{in } \mathbb{R}^3,\end{cases}$$

where $\mu > 0$, $s \in (\frac{3}{4}, 1)$, $t \in (0, 1)$ and $V_\lambda(x) = \lambda V(x) + 1$ with $\lambda > 0$. Under suitable conditions on f and V, by using the constraint variational method and quantitative deformation lemma, if $\lambda > 0$ is large enough, we prove that the above problem has one least energy sign-changing solution. Moreover, for any $\mu > 0$, the least energy of the sign-changing solution is strictly more than twice of the energy of the ground state solution. In addition, we discuss the asymptotic behavior of ground state sign-changing solutions as $\lambda \to \infty$ and $\mu \to 0$.

Keywords: fractional Schrödinger-Poisson system; variational method; sign-changing solution; steep potential well
Mathematics Subject Classification: 35A15; 35B40; 35J20; 35J60

1. Introduction and main results

In this work, we consider the existence of ground state sign-changing solutions for the following fractional Schrödinger-Poisson system

$$\begin{cases}(-\Delta)^s u + V_\lambda(x)u + \mu\phi u = f(u), & \text{in } \mathbb{R}^3,\\ (-\Delta)^t\phi = u^2, & \text{in } \mathbb{R}^3,\end{cases} \tag{1.1}$$

where $\mu > 0, s \in (\frac{3}{4}, 1), t \in (0, 1)$ and $V_\lambda(x) = \lambda V(x) + 1$ with $\lambda > 0$. f and V satisfy the following assumptions:
(f_1) $f \in C^1(\mathbb{R}, \mathbb{R})$, $\lim_{t\to 0}\frac{f(t)}{t} = 0$ and $f(t)t > 0$ for all $t \in \mathbb{R} \setminus \{0\}$;
(f_2) for some $4 < p < 2_s^* = \frac{6}{3-2s}$, there exists $C > 0$ such that $|f'(t)| \leq C(1 + |t|^{p-2})$;
(f_3) $\frac{f(t)}{|t|^3}$ is an increasing function of $t \in \mathbb{R} \setminus \{0\}$;
(f_4) $\lim_{t\to\infty}\frac{F(t)}{t^4} = +\infty$, where $F(t) := \int_0^t f(s)ds \geq 0$;
(V_1) $V \in C(\mathbb{R}^3, \mathbb{R})$ and $V(x) \geq 0$ in $\mathbb{R}^3$;
(V_2) there is $b > 0$ such that the set $\{x \in \mathbb{R}^3 : V(x) \leq b\}$ is nonempty and has finite measure;
(V_3) $\Omega := \text{int } V^{-1}(0)$ is nonempty and has a smooth boundary with $\bar{\Omega} = V^{-1}(0)$.
The above conditions imply that V_λ represents a potential well whose depth is controlled by λ. If λ large enough, the potential $\lambda V(x)$ is called a steep potential well which was first proposed by Bartsch and Wang [1].

As we all know, fractional differential equations have become increasingly important over the past few decades due to their different applications in science and engineering. Hence, nonlinear fractional Laplace equations have attracted much attention from many scholars. On the one hand, fractional operators appear in mathematical and physical problems, such as: conformal geometry and minimal surfaces [2], financial modeling [3], fractional quantum mechanics [4, 5], anomalous diffusion [6], obstacle problems [7], etc. On the other hand, compared to the classical Laplacian operator $-\Delta$, the fractional Laplacian $(-\Delta)^s (s \in (0, 1))$ is a non-local, and previous methods may not be directly applicable. Therefore, problems related to fractional equations or systems have attracted a large number of scholars ([8–18]).

In fact, there are many articles about the Schrödinger-Poisson system (see e.g. [8–19]). Among them are studies of the existence of ground state sign-changing solutions or nontrivial solutions under different potentials, such as the vanishing potential ([10, 11]), forced potential ([12–14]), constant potential ([15–17]) and weighted potential [19]. In particular, Wang et al. [10] considered the following nonlinear fractional Schrödinger-Poisson system with the potential vanishing at infinity

$$\begin{cases}(-\Delta)^s u + V(x)u + \phi(x)u = K(x)f(u), & x \in \mathbb{R}^3,\\ (-\Delta)^t\phi = u^2, & x \in \mathbb{R}^3,\end{cases} \tag{1.2}$$

where $s \in (\frac{3}{4}, 1), t \in (0, 1)$ and $V, K : \mathbb{R}^3 \to \mathbb{R}$ are continuous functions and vanish at infinity; f satisfies some growth conditions. They obtained that system (1.2) has a ground state sign-changing solution by using a Nehari manifold and constrained variational methods. Guo [12] considered the existence and asymptotic behavior of ground state sign-changing solutions to the following fractional Schrödinger-Poisson system

$$\begin{cases}(-\Delta)^s u + V(x)u + \lambda\phi(x)u = f(u), & x \in \mathbb{R}^3,\\ (-\Delta)^t\phi = u^2, & x \in \mathbb{R}^3,\end{cases} \tag{1.3}$$

where $s \in (\frac{3}{4}, 1), t \in (0, 1)$, $\lambda > 0$ is a parameter and V satisfies the following conditions:
(V_4) $V \in C(\mathbb{R}^3, \mathbb{R}^+)$ satisfies that $\inf\limits_{x\in\mathbb{R}^3} V(x) \geq V_0 > 0$, where $V_0 > 0$ is a constant;
(V_5) there is $r > 0$ such that $\lim\limits_{|y|\to\infty} \text{meas}(\{x \in B_r(y)|V(x) \leq M\}) = 0$ for any $M > 0$.
f satisfies (f_3) and
(f_5) $f(u) = o(|u|^3)$ as $u \to 0$;
(f_6) for some $q \in (4, 2_s^*)$, $\lim\limits_{|u|\to\infty} \dfrac{f(u)}{|u|^{q-1}} = 0$;
(f_7) $\lim\limits_{|u|\to\infty} \dfrac{f(u)}{|u|^3} = +\infty$.
By using the constrained variational method, the author showed that system (1.3) has a ground state sign-changing solution u_λ and proved that the energy of the sign-changing solution is strictly larger than twice that of the ground state energy. Furthermore, they also studied the asymptotic behavior of the sign-changing solution u_λ as $\lambda \to 0$. Then, Ji [13] considered the existence of the least energy sign-changing solutions for the following system

$$\begin{cases}(-\Delta)^s u + V(x)u + \lambda\phi(x)u = f(x, u), & \text{in } \mathbb{R}^3,\\ (-\Delta)^t\phi = u^2, & \text{in } \mathbb{R}^3,\end{cases} \tag{1.4}$$

where $\lambda > 0$, $s, t \in (0, 1), 4s + 2t > 3$, V satisfies (V_4) and (V_5), and f satisfies the following assumptions:
(f_8) $f : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function and $f(x, u) = o(|u|)$ as $u \to 0$ for $x \in \mathbb{R}^3$ uniformly;
(f_9) for some $1 < p < 2_s^* - 1$, there exists $C > 0$ such that $|f(x, u)| \leq C(1 + |u|^p)$;
(f_{10}) $\lim\limits_{|u|\to\infty} \dfrac{F(x, u)}{u^4} = +\infty$, where $F(x, u) = \int_0^u f(x, s)ds$;
(f_{11}) $\frac{f(x,t)}{|t|^3}$ is an increasing function of t on $\mathbb{R}\backslash\{0\}$ for a.e. $x \in \mathbb{R}^3$.
The author proved that system (1.4) has a least energy sign-changing solution by using the constraint variational method and quantitative deformation lemma. In addition, they also proved that the energy of the least energy sign-changing solutions is strictly more than twice that of the energy of the ground state solution and they studied the convergence of the least energy sign-changing solutions as $\lambda \to 0$. Besides, Chen et al. demonstrated that f exhibits asymptotically cubic or super-cubic growth in [18] . Without assuming the usual Nehari-type monotonic condition on $\frac{f(t)}{t^3}$, they established the existence of one radial ground state sign-changing solution u_λ with precisely two nodal domains. Moreover, they also proved that the energy of any radial sign-changing solution is strictly larger than two times the least energy, and they gave a convergence property of u_λ as $\lambda \to 0$. Moreover, there are many articles about the Schrödinger-Poisson system with steep potential wells (see e.g. [20–27]).

Inspired by the above references, we will study the existence of the ground-state sign-changing solution of system (1.1) and the relationship between the ground-state sign-changing solution and the energy of the ground-state solution. At the same time, we will also study the asymptotic behavior of the ground-state sign-changing solution as $\lambda \to \infty$ and $\mu \to 0$.

Throughout this paper, we define the fractional Sobolev space given by

$$D^{s,2}(\mathbb{R}^3) = \left\{ u \in L^{2_s^*}(\mathbb{R}^3) : \int_{\mathbb{R}^3} |(-\Delta)^{\frac{s}{2}} u(x)|^2 dx < +\infty \right\}.$$

Let us define the Hilbert space

$$H^s(\mathbb{R}^3) = \left\{ u \in L^2(\mathbb{R}^3) : \int_{\mathbb{R}^3} \left(|(-\Delta)^{\frac{s}{2}} u|^2 + |u|^2 \right) dx < +\infty \right\}$$

endowed with the inner product and induced norm

$$(u, v) = \int_{\mathbb{R}^3} \left((-\Delta)^{\frac{s}{2}} u (-\Delta)^{\frac{s}{2}} v + uv \right) dx, \qquad \|u\| = (u, u)^{\frac{1}{2}}.$$

And $L^q(\mathbb{R}^3)$ is a Lebesgue space endowed with the norm $|u|_q = (\int_{\mathbb{R}^3} |u|^q dx)^{\frac{1}{q}}$ for $q \in [1, +\infty)$. For any $\lambda > 0$, we introduce the following working space

$$E_\lambda = \left\{ u \in H^s(\mathbb{R}^3) : \int_{\mathbb{R}^3} \lambda V(x) u^2 dx < +\infty \right\}$$

with a scalar product and norm respectively given by

$$(u, v)_\lambda = \int_{\mathbb{R}^3} \left((-\Delta)^{\frac{s}{2}} u (-\Delta)^{\frac{s}{2}} v + V_\lambda(x) uv \right) dx, \qquad \|u\|_\lambda = (u, u)_\lambda^{\frac{1}{2}}.$$

From (V_1), we can get that $\|u\| \leq \|u\|_\lambda$ for all $u \in E_\lambda$. Then for any $2 \leq q \leq 2_s^*$, the embedding $E_\lambda \hookrightarrow L^q(\mathbb{R}^3)$ is continuous and $S_q > 0$ exists such that $|u|_q \leq S_q \|u\| \leq S_q \|u\|_\lambda$ for all $u \in E_\lambda$. Suppose that $s \in (\frac{3}{4}, 1)$ and $t \in (0, 1)$, we have

$$2 \leq \frac{12}{3 + 2t} < 4 < \frac{6}{3 - 2s} = 2_s^*.$$

Then, by [28], we know that the embedding $H^s(\mathbb{R}^3) \hookrightarrow L^{\frac{12}{3+2t}}(\mathbb{R}^3)$ is continuous. Considering that $u \in H^s(\mathbb{R}^3)$ and $v \in D^{t,2}(\mathbb{R}^3)$, by the Hölder inequality, we have

$$\int_{\mathbb{R}^3} u^2 v \leq \left(\int_{\mathbb{R}^3} |u|^{\frac{12}{3+2t}} dx \right)^{\frac{3+2t}{6}} \left(\int_{\mathbb{R}^3} |v|^{\frac{6}{3-2t}} dx \right)^{\frac{3-2t}{6}} \leq C \|u\|^2 \|v\|_{D^{t,2}}.$$

Thus, thanks to the Lax-Milgram theorem, there exists a unique $\phi_u^t \in D^{t,2}(\mathbb{R}^3)$ such that

$$\int_{\mathbb{R}^3} (-\Delta)^t \phi_u^t v dx = \int_{\mathbb{R}^3} (-\Delta)^{\frac{t}{2}} \phi_u^t (-\Delta)^{\frac{t}{2}} v dx = \int_{\mathbb{R}^3} u^2 v dx.$$

That is, ϕ_u^t satisfies that $(-\Delta)^t \phi_u^t = u^2$ for any $u \in H^s(\mathbb{R}^3)$. Furthermore,

$$\phi_u^t = c_t \int_{\mathbb{R}^3} \frac{u^2(y)}{|x - y|^{3-2t}} dy, \quad x \in \mathbb{R}^3, \tag{1.5}$$

which is called the t-Riesz potential, where

$$c_t = \pi^{-\frac{3}{2}} 2^{-2t} \frac{\Gamma(3-2t)}{\Gamma(t)}.$$

In subsequent work, we often omit the constant c_t. Hence, system (1.1) can be reduced to a single equation with a non-local term

$$(-\Delta)^s u + V_\lambda(x) u + \mu \phi_u^t u = f(u) \quad \text{in } \mathbb{R}^3.$$

We can see that the solutions of system (1.1) are precisely the critical points of the energy functional $J_\lambda^\mu : E_\lambda \to \mathbb{R}$ which is defined by

$$J_\lambda^\mu(u) = \frac{1}{2}\|u\|_\lambda^2 + \frac{\mu}{4}\int_{\mathbb{R}^3} \phi_u^t u^2 dx - \int_{\mathbb{R}^3} F(u) dx, \tag{1.6}$$

where $F(s) = \int_0^s f(t)dt$. It is easy to see that J_λ^μ is well defined and $J_\lambda^\mu \in C^1(E_\lambda, \mathbb{R})$. Moreover, for any $u, \varphi \in E_\lambda$,

$$\langle (J_\lambda^\mu)'(u), \varphi \rangle = (u, \varphi)_\lambda + \mu \int_{\mathbb{R}^3} \phi_u^t u \varphi dx - \int_{\mathbb{R}^3} f(u) \varphi dx. \tag{1.7}$$

Now our main results in this paper can be stated as follows.

Theorem 1.1. Let $(V_1) - (V_3)$ and $(f_1) - (f_4)$ be satisfied, $\lambda > 0$ be sufficiently large and $\mu > 0$; system (1.1) has at least one ground state sign-changing solution which has precisely two nodal domains. Moreover, the energy of the ground state sign-changing solution is strictly larger than twice that of the energy of the ground state solution.

Theorem 1.2. Under the assumptions of Theorem 1.1, for any sequence $\lambda_n \to +\infty$ as $n \to \infty$, the sequence of sign-changing solutions $\{u_{\lambda_n}\}$ for system (1.1) strongly converges to u_* in $H^s(\mathbb{R}^3)$ up to a subsequence, where u_* is a ground state sign-changing solution of the following system

$$\begin{cases} (-\Delta)^s u + u + \dfrac{\mu}{4\pi}\left(\dfrac{1}{|x|} * u^2\right) u = f(u), & \text{in } \Omega, \\ u = 0, & \text{in } \partial\Omega, \end{cases} \tag{1.8}$$

where $\frac{1}{|x|} * u^2 = \int_\Omega \frac{u^2(y)}{|x-y|^{3-2t}} dy$ and there are only two nodal domains.

Theorem 1.3. Under the assumptions of Theorem 1.1, for any $\mu \in (0, 1]$, suppose that u_μ is a ground-state sign-changing solution of system (1.1) that has been obtained according to Theorem 1.1. Then there exists $u_0 \in E_\lambda$ such that $u_\mu \to u_0$ in E_λ as $\mu \to 0$, where u_0 is a ground-state sign-changing solution to the following equation

$$(-\Delta)^s u + V_\lambda(x) u = f(u). \tag{1.9}$$

Moreover, u_0 has two nodal domains.

Remark 1.4. Our results are up to date. On the one hand, similar to [10, 12], we study the fractional Schrödinger-Poisson system with a steep potential well. On the other hand, we generalize the results of [20] to the fractional Laplace operator.

Remark 1.5. It is worth noting that, in [12, 13, 18], they assume that the potential is radially symmetric or forced, which ensures that the Sobolev embedding $H^s(\mathbb{R}^3)$ into $L^p(\mathbb{R}^3)$ with $p \in (2, 2_s^*)$ is compact. However, in our work, our potential is a steep potential well, which makes the Sobolev embedding $H^s(\mathbb{R}^3)$ into $L^p(\mathbb{R}^3)$ with $p \in (2, 2_s^*)$ lack compactness. In order to overcome this difficulty, we use the ideas presented in [20, 29] to find a (PS) sequence of the energy functional of system (1.1) in E_λ, and prove that the local (PS) condition is valid.

We have organized this paper as follows. In Sect. 2, we present some preliminary lemmas which are essential for the proof of the theorems. In Sect. 3, we give the proof of the main results.

We conclude this section by giving some notations, which will be applied later in the work.
•E_λ^* is the dual space of the Banach space of E_λ.
•$B_R(0) := \{x \in \mathbb{R}^3 : |x| \leq R\}$ for any $R \in [0, +\infty)$ and $\Omega^c = \mathbb{R}^3 \setminus \Omega$.
•$u^+(x) := \max\{u, 0\}, u^-(x) := -\min\{u, 0\}$.
•C, C_i denote positive constants that may vary under different conditions.

2. Some preliminary lemmas

On the one hand, we need to prove the existence of the sign-changing solutions of system (1.1); inspired by [30, 31], the following minimization problem is given by

$$m_\lambda^\mu = \inf_{u \in \mathcal{M}_\lambda^\mu} J_\lambda^\mu(u),$$

where

$$\mathcal{M}_\lambda^\mu = \left\{u \in E_\lambda : u^\pm \neq 0, \langle (J_\lambda^\mu)'(u), u^\pm \rangle = 0\right\}.$$

Clearly, $\mathcal{M}_\lambda^\mu$ contains all of the sign-changing solutions for system (1.1). On the other hand, we need to prove the relationship between the energy of the ground state sign-changing solution and that of the ground state solution. Therefore the following Nehari manifold $\mathcal{N}_\lambda^\mu$ is introduced as follows:

$$\mathcal{N}_\lambda^\mu = \left\{u \in E_\lambda \setminus \{0\} : \langle (J_\lambda^\mu)'(u), u \rangle = 0\right\}.$$

Similarly, the following minimization problem is defined by

$$c_\lambda^\mu = \inf_{u \in \mathcal{N}_\lambda^\mu} J_\lambda^\mu(u).$$

By simple calculation, we can also get

$$\int_{\mathbb{R}^3} \phi_u^t u^2 dx = \int_{\mathbb{R}^3} \phi_{u^+}^t |u^+|^2 dx + \int_{\mathbb{R}^3} \phi_{u^-}^t |u^-|^2 dx + 2\int_{\mathbb{R}^3} \phi_{u^+}^t |u^-|^2 dx \tag{2.1}$$

and

$$\int_{\mathbb{R}^3} |(-\Delta)^{\frac{s}{2}}u|^2dx = \int_{\mathbb{R}^3} |(-\Delta)^{\frac{s}{2}}u^+|^2dx + \int_{\mathbb{R}^3} |(-\Delta)^{\frac{s}{2}}u^-|^2dx + 2\int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}}u^+(-\Delta)^{\frac{s}{2}}u^-dx, \tag{2.2}$$

where

$$\int_{\mathbb{R}^3} \phi^t_{u^+}|u^+|^2dx > 0, \qquad \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}}u^+(-\Delta)^{\frac{s}{2}}u^-dx > 0$$

for $u^\pm \neq 0$. Hence,

$$J^\mu_\lambda(u) = J^\mu_\lambda(u^+) + J^\mu_\lambda(u^-) + \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}}u^+(-\Delta)^{\frac{s}{2}}u^-dx + \frac{\mu}{2}\int_{\mathbb{R}^3} \phi^t_{u^+}|u^-|^2dx, \tag{2.3}$$

$$\langle (J^\mu_\lambda)'(u), u^+\rangle = \langle (J^\mu_\lambda)'(u^+), u^+\rangle + \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}}u^+(-\Delta)^{\frac{s}{2}}u^-dx + \mu\int_{\mathbb{R}^3} \phi^t_{u^-}|u^+|^2dx, \tag{2.4}$$

$$\langle (J^\mu_\lambda)'(u), u^-\rangle = \langle (J^\mu_\lambda)'(u^-), u^-\rangle + \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}}u^+(-\Delta)^{\frac{s}{2}}u^-dx + \mu\int_{\mathbb{R}^3} \phi^t_{u^+}|u^-|^2dx. \tag{2.5}$$

In order to prove our results, we give the following propositions and some preliminary lemmas.

Proposition 2.1. (See [32]) For the function ϕ^t_u defined in (1.5), one has
(i) $\phi^t_u \geq 0$ and $\phi^t_{ku} = k^2\phi^t_u$ for all $t \in \mathbb{R}$ and $u \in H^s(\mathbb{R}^3)$;
(ii) there is $C > 0$ such that $\int_{\mathbb{R}^3} \phi^t_u u^2dx \leq C\|u\|^4_{\frac{12}{3+2s}}$.

Proposition 2.2. (See [33], fractional Gagliardo-Nirendo inequality) For any $p \in [2, 2^*_s)$, there exists $C(p) > 0$ such that $|u|^p_p \leq C(p)|(-\Delta)^{\frac{s}{2}}u|_2^{\frac{3p-2^*_s}{2}}|u|_2^{\frac{2^*_s-p}{2}}$ for any $u \in H^s(\mathbb{R}^3)$.

Lemma 2.1. Assume that $(f_1) - (f_4)$ and (V_1) hold; for any $\lambda > 0$ and $u \in E_\lambda$ with $u^\pm \neq 0$, there exists a unique pair of (s_u, t_u) such that $s_u u^+ + t_u u^- \in \mathcal{M}^\mu_\lambda$ and

$$J^\mu_\lambda(s_u u^+ + t_u u^-) = \max_{s,t\geq 0} J^\mu_\lambda(su^+ + tu^-).$$

Proof. We first establish the existence of s_u and t_u. Let

$$\begin{aligned} g_1(s,t) =& \langle (J^\mu_\lambda)'(su^+ + tu^-), su^+\rangle \\ =& s^2\|u^+\|^2_\lambda + st\int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}}u^+(-\Delta)^{\frac{s}{2}}u^-dx + s^4\mu\int_{\mathbb{R}^3} \phi^t_{u^+}|u^+|^2dx \\ &+ s^2t^2\mu\int_{\mathbb{R}^3} \phi^t_{u^+}|u^-|^2dx - \int_{\mathbb{R}^3} f(su^+)su^+dx, \end{aligned} \tag{2.6}$$

$$\begin{aligned} g_2(s,t) =& \langle (J^\mu_\lambda)'(su^+ + tu^-), tu^-\rangle \\ =& t^2\|u^-\|^2_\lambda + st\int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}}u^+(-\Delta)^{\frac{s}{2}}u^-dx + t^4\mu\int_{\mathbb{R}^3} \phi^t_{u^-}|u^-|^2dx \\ &+ s^2t^2\mu\int_{\mathbb{R}^3} \phi^t_{u^+}|u^-|^2dx - \int_{\mathbb{R}^3} f(tu^-)tu^-dx. \end{aligned} \tag{2.7}$$

By (f_1), (f_2) and (f_4), it is not hard to see that $g_1(s,s) > 0$, $g_2(s,s) > 0$ for small $s > 0$, and $g_1(t,t) < 0$, $g_2(t,t) < 0$ for large $t > 0$. Thus, there exists $0 < r < R$ such that

$$g_1(r,r) > 0, \quad g_2(r,r) > 0, \quad g_1(R,R) < 0, \quad g_2(R,R) < 0. \tag{2.8}$$

Thus we can deduce from (2.6)-(2.8) that

$$\begin{aligned} &g_1(r,t) > 0, \quad g_1(R,t) < 0, \quad \forall t \in [r,R]. \\ &g_2(s,r) > 0, \quad g_2(s,R) < 0, \quad \forall s \in [r,R]. \end{aligned} \tag{2.9}$$

By way of Miranda's theorem [34], there exists some point (s_u, t_u) with $r < s_u, t_u < R$ such that $g_1(s_u,t_u) = g_2(s_u,t_u) = 0$. So, $s_u u^+ + t_u u^- \in \mathcal{M}_\lambda^\mu$. Next, we prove that (s_u,t_u) is unique by the following two cases.

Case 1. $u \in \mathcal{M}_\lambda^\mu$.

For any $u \in \mathcal{M}_\lambda^\mu$, it means that

$$\|u^\pm\|_\lambda^2 + \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} u^+ (-\Delta)^{\frac{s}{2}} u^- dx + \mu \int_{\mathbb{R}^3} \phi_u^t |u^\pm|^2 dx = \int_{\mathbb{R}^3} f(u^\pm) u^\pm dx. \tag{2.10}$$

By (2.10), we have that $(s_u,t_u) = (1,1)$. Then, we prove that (s_u,t_u) is the unique. Assume that (s_0,t_0) is another pair of numbers such that $s_0 u^+ + t_0 u^- \in \mathcal{M}_\lambda^\mu$.

$$\begin{aligned} s_0^2\|u^+\|_\lambda^2 &+ s_0 t_0 \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} u^+ (-\Delta)^{\frac{s}{2}} u^- dx + s_0^4 \mu \int_{\mathbb{R}^3} \phi_{u^+}^t |u^+|^2 dx \\ &+ s_0^2 t_0^2 \mu \int_{\mathbb{R}^3} \phi_{u^+}^t |u^-|^2 dx = \int_{\mathbb{R}^3} f(s_0 u^+) s_0 u^+ dx. \end{aligned} \tag{2.11}$$

$$\begin{aligned} t_0^2\|u^-\|_\lambda^2 &+ s_0 t_0 \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} u^+ (-\Delta)^{\frac{s}{2}} u^- dx + t_0^4 \mu \int_{\mathbb{R}^3} \phi_{u^-}^t |u^-|^2 dx \\ &+ s_0^2 t_0^2 \mu \int_{\mathbb{R}^3} \phi_{u^+}^t |u^-|^2 dx = \int_{\mathbb{R}^3} f(t_0 u^-) t_0 u^- dx. \end{aligned} \tag{2.12}$$

It seems that $0 < s_0 \le t_0$; from (2.12), we have

$$\begin{aligned} \frac{1}{t_0^2}\|u^-\|_\lambda^2 &+ \frac{1}{t_0^2} \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} u^+ (-\Delta)^{\frac{s}{2}} u^- dx + \mu \int_{\mathbb{R}^3} \phi_{u^-}^t |u^-|^2 dx \\ &+ \mu \int_{\mathbb{R}^3} \phi_{u^+}^t |u^-|^2 dx \ge \int_{\mathbb{R}^3} \frac{f(t_0 u^-)}{(t_0 u^-)^3} (u^-)^4 dx. \end{aligned} \tag{2.13}$$

From (2.10) and (2.13), we obtain

$$\left(\frac{1}{t_0} - 1\right)\left(\|u^-\|_\lambda^2 + \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} u^+ (-\Delta)^{\frac{s}{2}} u^- dx\right) \ge \int_{\mathbb{R}^3} \left[\frac{f(t_0 u^-)}{(t_0 u^-)^3} - \frac{f(u^-)}{(u^-)^3}\right] (u^-)^4 dx.$$

By (f_3), if $t_0 > 1$, the left-hand side of the inequality is negative and the right-hand side is positive, which leads to a contradiction. Therefore, we obtain that $0 < s_0 \le t_0 \le 1$. Similarly, by (2.10) and (2.11), we get

$$\left(\frac{1}{s_0} - 1\right)\left(\|u^+\|_\lambda^2 + \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} u^+ (-\Delta)^{\frac{s}{2}} u^- dx\right) \le \int_{\mathbb{R}^3} \left[\frac{f(s_0 u^+)}{(s_0 u^+)^3} - \frac{f(u^+)}{(u^+)^3}\right] (u^+)^4 dx.$$

In view of (f_3), we have that $s_0 \geq 1$. Hence, $s_0 = t_0 = 1$.

Case 2. $u \notin \mathcal{M}_\lambda^\mu$

If $u \notin \mathcal{M}_\lambda^\mu$, there exists a pair of positive numbers $(s_u, t_u) \in \mathcal{M}_\lambda^\mu$. Suppose that there exists another pair of positive numbers $(\widetilde{s_u}, \widetilde{t_u})$ such that $\widetilde{s_u}u^+ + \widetilde{t_u}u^- \in \mathcal{M}_\lambda^\mu$. Set $\overline{u}_1 := s_u u^+ + t_u u^- \in \mathcal{M}_\lambda^\mu$ and $\overline{u}_2 := \widetilde{s_u}u^+ + \widetilde{t_u}u^- \in \mathcal{M}_\lambda^\mu$; one has

$$\overline{u}_2 = \left(\frac{\widetilde{s_u}}{s_u}\right) s_u u^+ + \left(\frac{\widetilde{t_u}}{t_u}\right) t_u u^- = \left(\frac{\widetilde{s_u}}{s_u}\right) \overline{u}_1{}^+ + \left(\frac{\widetilde{t_u}}{t_u}\right) \overline{u}_1{}^- \in \mathcal{M}_\lambda^\mu.$$

Since $\overline{u}_1 \in \mathcal{M}_\lambda^\mu$, by Case 1, we get that $\frac{\widetilde{s_u}}{s_u} = \frac{\widetilde{t_u}}{t_u} = 1$, which implies that $\widetilde{s_u} = s_u$ and $\widetilde{t_u} = t_u$ and (s_u, t_u) is the unique pair of numbers such that $s_u u^+ + t_u u^- \in \mathcal{M}_\lambda^\mu$.

Finally, we define $\psi(s,t) := J_\lambda^\mu(su^+ + tu^-)$; it can be seen that $J_\lambda^\mu(su^+ + tu^-) > 0$ as $|(s,t)| \to 0$ and $J_\lambda^\mu(su^+ + tu^-) < 0$ as $|(s,t)| \to \infty$. Then the maximum $\max\limits_{s,t \geq 0} J_\lambda^\mu(su^+ + tu^-)$ is well defined. Now, it is sufficient to check that the maximum point cannot be reached on the boundary of $[0, +\infty) \times [0, +\infty)$. Assume that $(0, t_0)$ is a maximum point of ψ with $t_0 \geq 0$. Then, since

$$\begin{aligned}
\psi(s, t_0) =& J_\lambda^\mu(su^+ + t_0 u^-) \\
=& \frac{s^2}{2}\|u^+\|_\lambda^2 + st_0 \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} u^+ (-\Delta)^{\frac{s}{2}} u^- dx + \frac{\mu s^4}{4} \int_{\mathbb{R}^3} \phi_{u^+}^t |u^+|^2 dx \\
&- \int_{\mathbb{R}^3} F(su^+) dx + \frac{s^2 t_0^2 \mu}{4} \int_{\mathbb{R}^3} \phi_{u^+}^t |u^-|^2 dx + \frac{t_0^2}{2}\|u^-\|_\lambda^2 \\
&+ \frac{\mu t_0^4}{4} \int_{\mathbb{R}^3} \phi_{u^-}^t |u^-|^2 dx - \int_{\mathbb{R}^3} F(t_0 u^-) dx + \frac{s^2 t_0^2 \mu}{4} \int_{\mathbb{R}^3} \phi_{u^-}^t |u^+|^2 dx, \\
(\psi')_s(s, t_0) =& s\|u^+\|_\lambda^2 + t_0 \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} u^+ (-\Delta)^{\frac{s}{2}} u^- dx + s^3 \mu \int_{\mathbb{R}^3} \phi_{u^+}^t |u^+|^2 dx \\
&- \int_{\mathbb{R}^3} f(su^+) u^+ dx + \frac{s t_0^2 \mu}{2} \int_{\mathbb{R}^3} \phi_{u^+}^t |u^-|^2 dx + \frac{s t_0^2 \mu}{2} \int_{\mathbb{R}^3} \phi_{u^-}^t |u^+|^2 dx,
\end{aligned}$$

if s is small enough, $(\psi')_s(s, t_0) > 0$; thus ψ is an increasing function of s and the pair $(0, t_0)$ is not a maximum point of ψ. Similarly, ψ can not achieve its global maximum on $(s_0, 0)$ with $s_0 > 0$. Since (s_u, t_u) is a unique pair of such that $s_u u^+ + t_u u^- \in \mathcal{M}_\lambda^\mu$, it follows that $J_\lambda^\mu(s_u u^+ + t_u u^-) = \max\limits_{s,t \geq 0} J_\lambda^\mu(su^+ + tu^-)$. The proof is now finished. □

Lemma 2.2. $m_\lambda^\mu = \inf\limits_{u \in \mathcal{M}_\lambda^\mu} J_\lambda^\mu(u) > 0$ for any $\lambda, \mu > 0$.

Proof. For every $u \in \mathcal{M}_\lambda^\mu$, we have that $\langle (J_\lambda^\mu)'(u), u \rangle = 0$. By (f_1) and (f_2), for any $\varepsilon > 0$, there is $C_\varepsilon > 0$ such that

$$|f(t)| \leq \varepsilon |t| + C_\varepsilon |t|^{p-1} \quad \text{for all t} \in \mathbb{R}. \tag{2.14}$$

Then, by the Sobolev inequality, we get

$$\begin{aligned}
\|u\|_\lambda^2 \leq& \|u\|_\lambda^2 + \mu \int_{\mathbb{R}^3} \phi_u^t u^2 dx = \int_{\mathbb{R}^3} f(u) u dx \\
\leq& \varepsilon \int_{\mathbb{R}^3} |u|^2 dx + C_\varepsilon \int_{\mathbb{R}^3} |u|^p dx \\
\leq& \varepsilon S_2^{\frac{2}{2}} \|u\|_\lambda^2 + C_\varepsilon S_p^{\frac{p}{p}} \|u\|_\lambda^p.
\end{aligned} \tag{2.15}$$

Taking $\varepsilon = \frac{1}{2S_2^2}$, so there is a constant $\gamma > 0$ such that $\|u\|_\lambda^2 \geq \gamma$. By (f_3), one has

$$\mathcal{F} := \frac{1}{4}f(t)t - F(t) \geq 0, \tag{2.16}$$

consequently,

$$J_\lambda^\mu(u) = J_\lambda^\mu(u) - \frac{1}{4}\langle (J_\lambda^\mu)'(u), u\rangle \geq \frac{1}{4}\|u\|_\lambda^2 \geq \frac{1}{4}\gamma, \tag{2.17}$$

which implies that $m_\lambda^\mu \geq \frac{1}{4}\gamma > 0$. Then the proof is completed. □

Next, we will prove the existence of sign-changing solutions for system (1.1). Given the lack of compactness of the Sobolev embedding $H^s(\mathbb{R}^3)$ into $L^p(\mathbb{R}^3)$, $p \in (2, 2_s^*)$, we need to construct a sign-changing $(PS)_{m_\lambda^\mu}$-sequence. Inspired by [29], we give some definitions. Let P denote the cone of nonnegative functions in E_λ, $Q = [0,1] \times [0,1]$ and Σ be the set of continuous maps σ such that

$$\Sigma = \Bigg\{\sigma \in C(Q, E_\lambda);\ \sigma(s,0) = 0,\ \sigma(0,t) \in P,\ \sigma(1,t) \in -P,\ J_\lambda^\mu(\sigma(s,1)) \leq 0,$$
$$\frac{\displaystyle\int_{\mathbb{R}^3} f(\sigma(s,1))(\sigma(s,1))dx}{\|\sigma(s,1)\|_\lambda^2 + \mu\displaystyle\int_{\mathbb{R}^3}\phi_{\sigma(s,1)}^t|\sigma(s,1)|^2dx} \geq 2,\ \forall s,t \in [0,1]\Bigg\}.$$

For each $u \in E_\lambda$ with $u^\pm \neq 0$, let $\sigma(s,t) = kt(1-s)u^+ + kstu^-$, where $k > 0$ and $s,t \in [0,1]$. It is easy to know that $\sigma(s,t) \in \Sigma$ for $k > 0$ sufficiently large, which means that $\Sigma \neq \emptyset$. Define

$$l(u,v) = \begin{cases} \dfrac{\int_{\mathbb{R}^3} f(u)udx}{\|u\|_\lambda^2 + \int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u(-\Delta)^{\frac{s}{2}}vdx + \mu\int_{\mathbb{R}^3}\phi_u^t u^2dx + \mu\int_{\mathbb{R}^3}\phi_v^t u^2dx}, & \text{if } u \neq 0; \\ 0, & \text{if } u = 0. \end{cases} \tag{2.18}$$

Apparently, $u \in \mathcal{M}_\lambda^\mu$ if and only if $l(u^+,u^-) = l(u^-,u^+) = 1$. Define

$$U_\lambda := \left\{u \in E_\lambda : \frac{1}{2} < l(u^+,u^-) < \frac{3}{2}, \frac{1}{2} < l(u^-,u^+) < \frac{3}{2}\right\}.$$

Lemma 2.3. There exists a sequence $\{u_n\} \subset U_\lambda$ satisfying that $J_\lambda^\mu(u_n) \to m_\lambda^\mu$ and $(J_\lambda^\mu)'(u_n) \to 0$ in E_λ^* as $n \to \infty$.

Proof. We divide three steps to complete the proof. First, we prove the following

$$\inf_{\sigma\in\Sigma}\sup_{u\in\sigma(Q)} J_\lambda^\mu(u) = \inf_{u\in\mathcal{M}_\lambda^\mu} J_\lambda^\mu(u) = m_\lambda^\mu.$$

For each $u \in \mathcal{M}_\lambda^\mu$, there is $\sigma(s,t) = kt(1-s)u^+ + kstu^- \in \Sigma$ for $k > 0$ sufficiently large; by Lemma 2.1, we get

$$J_\lambda^\mu(u) = \max_{s,t\geq 0} J_\lambda^\mu(su^+ + tu^-) \geq \sup_{u\in\sigma(Q)} J_\lambda^\mu(u) \geq \inf_{\sigma\in\Sigma}\sup_{u\in\sigma(Q)} J_\lambda^\mu(u),$$

which implies that

$$\inf_{u\in\mathcal{M}_\lambda^\mu} J_\lambda^\mu(u) \geq \inf_{\sigma\in\Sigma}\sup_{u\in\sigma(Q)} J_\lambda^\mu(u). \tag{2.19}$$

At the same time, we assume that for each $\sigma \in \Sigma$, there exists $u_\sigma \in \sigma(Q) \cap \mathcal{M}^\mu_\lambda$, such that

$$\sup_{u\in\sigma(Q)} J^\mu_\lambda(u) \geq J^\mu_\lambda(u_\sigma) \geq \inf_{u\in\mathcal{M}^\mu_\lambda} J^\mu_\lambda(u).$$

As a matter of fact, on the one hand, for any $\sigma \in \Sigma$ and $t \in [0,1]$, one has

$$l(\sigma^+(0,t),\sigma^-(0,t)) - l(\sigma^-(0,t),\sigma^+(0,t)) = l(\sigma^+(0,t),\sigma^-(0,t)) \geq 0, \tag{2.20}$$

$$l(\sigma^+(1,t),\sigma^-(1,t)) - l(\sigma^-(1,t),\sigma^+(1,t)) = -l(\sigma^-(1,t),\sigma^+(1,t)) \leq 0. \tag{2.21}$$

On the other hand, from the definition of Σ, for any $\sigma \in \Sigma$ and $s \in [0,1]$, by the elementary inequality $\frac{b}{a} + \frac{d}{c} \geq \frac{b+d}{a+c}$ for all $a, b, c, d > 0$, we get

$$\begin{aligned} l(\sigma^+(s,1),\sigma^-(s,1)) + l(\sigma^-(s,1),\sigma^+(s,1)) &\geq \frac{\displaystyle\int_{\mathbb{R}^3} f(\sigma(s,1))(\sigma(s,1))dx}{\|\sigma(s,1)\|^2_\lambda + \mu \displaystyle\int_{\mathbb{R}^3} \phi^t_{\sigma(s,1)}|\sigma(s,1)|^2 dx} \\ &\geq 2. \end{aligned}$$

Therefore,

$$l(\sigma^+(s,1),\sigma^-(s,1)) + l(\sigma^-(s,1),\sigma^+(s,1)) - 2 \ \geq 0, \tag{2.22}$$

$$l(\sigma^+(s,0),\sigma^-(s,0)) + l(\sigma^-(s,0),\sigma^+(s,0) - 2 = -2 < 0. \tag{2.23}$$

According to Miranda's Theorem and (2.20)–(2.23), there exists $(s_\sigma, t_\sigma) \in Q$ such that

$$\begin{aligned} 0 &= l(\sigma^+(s_\sigma,t_\sigma),\sigma^-(s_\sigma,t_\sigma)) - l(\sigma^-(s_\sigma,t_\sigma),\sigma^+(s_\sigma,t_\sigma)) \\ &= l(\sigma^+(s_\sigma,t_\sigma),\sigma^-(s_\sigma,t_\sigma)) + l(\sigma^-(s_\sigma,t_\sigma),\sigma^+(s_\sigma,t_\sigma)) - 2, \end{aligned}$$

then

$$l(\sigma^+(s_\sigma,t_\sigma),\sigma^-(s_\sigma,t_\sigma)) = l(\sigma^-(s_\sigma,t_\sigma),\sigma^+(s_\sigma,t_\sigma)) = 1,$$

which implies that for any $\sigma \in \Sigma$, there exists $u_\sigma = \sigma(s_\sigma, t_\sigma) \in \sigma(Q) \cap \mathcal{M}^\mu_\lambda$. Moreover,

$$\sup_{u\in\sigma(Q)} J^\mu_\lambda(u) \geq J^\mu_\lambda(u_\sigma) \geq \inf_{u\in\mathcal{M}^\mu_\lambda} J^\mu_\lambda(u).$$

Therefore,

$$\inf_{\sigma\in\Sigma} \sup_{u\in\sigma(Q)} J^\mu_\lambda(u) \geq \inf_{u\in\mathcal{M}^\mu_\lambda} J^\mu_\lambda(u). \tag{2.24}$$

So, by (2.19) and (2.24), one obtains

$$\inf_{\sigma\in\Sigma} \sup_{u\in\sigma(Q)} J^\mu_\lambda(u) = \inf_{u\in\mathcal{M}^\mu_\lambda} J^\mu_\lambda(u) = m^\mu_\lambda.$$

Secondly, we look for the $(PS)_{m^\mu_\lambda}$-sequence $\{u_n\} \subset E_\lambda$ for J^μ_λ. Considering a minimizing sequence $\{w_n\} \subset \mathcal{M}^\mu_\lambda$ and $\sigma_n(s,t) = kt(1-s)w^+_n + ktsw^-_n \in \Sigma$ with $(s,t) \in Q$. Then, thanks to Lemma 2.1, we have

$$\lim_{n\to\infty} \max_{w\in\sigma_n(Q)} J^\mu_\lambda(w_n) = \lim_{n\to\infty} J^\mu_\lambda(w_n) = m^\mu_\lambda. \tag{2.25}$$

Using a variant form of the classical deformation lemma, we can deduce that there exists $\{u_n\} \subset \mathcal{M}_\lambda^\mu$ such that

$$J_\lambda^\mu(u_n) \to m_\lambda^\mu,\ \ (J_\lambda^\mu)'(u_n) \to 0,\ \ \text{dist}(u_n, \sigma_n(Q)) \to 0,\ \ \text{as } n \to \infty. \tag{2.26}$$

Assume that this is a contradiction. Then it is possible to find a $\delta > 0$ such that $\sigma_n(Q) \cap D_\delta = \emptyset$ for n sufficiently large, where

$$D_\delta = \left\{u \in E_\lambda : \exists v \in E_\lambda,\ s.t. \|v - u\|_\lambda \le \delta,\ \|(J_\lambda^\mu)'(v)\|_\lambda \le \delta,\ |J_\lambda^\mu(v) - m_\lambda^\mu| \le \delta\right\}.$$

By [35], for some $\epsilon \in (0, \frac{m_\lambda^\mu}{2})$ and all $t \in [0,1]$, there exists a continuous map $\eta : [0,1] \times E_\lambda \to E_\lambda$ satisfying

(i) $\eta(0,u) = u,\ \eta(t,-u) = -\eta(t,u)$;
(ii) $\eta(t,u) = u,\ \forall u \in J_\lambda^{m_\lambda^\mu - \epsilon} \cup (E_\lambda \setminus J_\lambda^{m_\lambda^\mu + \epsilon})$;
(iii) $\eta(1, J_\lambda^{m_\lambda^\mu + \frac{\epsilon}{2}} \setminus D_\delta) \subset J_\lambda^{m_\lambda^\mu - \frac{\epsilon}{2}}$;
(iv) $\eta(1, (J_\lambda^{m_\lambda^\mu + \frac{\epsilon}{2}} \cap P) \setminus D_\delta) \subset J_\lambda^{m_\lambda^\mu - \frac{\epsilon}{2}} \cap P$, where $J_\lambda^d = \{u \in E_\lambda : J_\lambda^\mu(u) \le d\}$.

By (2.25), we can choose n such that

$$\sigma_n(Q) \subset J_\lambda^{m_\lambda^\mu + \frac{\epsilon}{2}},\ \ \sigma_n(Q) \cap D_\delta = \emptyset. \tag{2.27}$$

Let us define $\widetilde{\sigma}_n(s,t) := \eta(1, \sigma_n(s,t))$ for all $(s,t) \in Q$. We need to prove that $\widetilde{\sigma}_n(Q) \in \Sigma$, and thus that $\widetilde{\sigma}_n(Q) \subset J_\lambda^{m_\lambda - \frac{\epsilon}{2}}$ in view of (2.27) and property (iii) of η. This is a contradiction of the inequality below

$$m_\lambda^\mu = \inf_{\sigma \in \Sigma} \sup_{w \in \sigma(Q)} J_\lambda^\mu(w) \le \max_{w \in \widetilde{\sigma}_n(Q)} J_\lambda^\mu(w) \le m_\lambda^\mu - \frac{\epsilon}{2}.$$

By property (ii) of η and $\sigma_n \in \Sigma$, we derive that

$$\widetilde{\sigma}_n(s,0) = \eta(1, \sigma_n(s,0)) = \eta(1,0) = 0.$$

And it is from $\sigma_n(0,t) \in P$, (2.27) and property (iv) of η that $\widetilde{\sigma}_n(0,t) \in P$. Because of $\sigma_n(1,t) \in -P$ and (2.27), we obtain that $-\sigma_n(1,t) \in (J_\lambda^{m_\lambda + \frac{\epsilon}{2}} \cap P) \setminus D_\delta$, which implies that

$$\widetilde{\sigma}_n(1,t) = -\eta(1, -\sigma_n(1,t)) \in -P.$$

Furthermore, by the definition of Σ, we get $J_\lambda^\mu(\sigma_n(l,1)) \le 0$. By property (ii) of η, we can infer that

$$\widetilde{\sigma}_n(s,1) = \eta(1, \sigma_n(s,1)) = \sigma_n(s,1),$$

which implies that

$$J_\lambda^\mu(\widetilde{\sigma}(s,1)) = J_\lambda^\mu(\sigma(s,1)) \le 0$$

and

$$\frac{\displaystyle\int_{\mathbb{R}^3} f(\sigma(s,1))(\sigma(s,1))dx}{\|\sigma(s,1)\|_\lambda^2 + \mu \displaystyle\int_{\mathbb{R}^3} \phi_{\sigma(s,1)}^t |\sigma(s,1)|^2 dx} \ge 2.$$

From the above, we can conclude that $\widetilde{\sigma}_n \in \Sigma$ from the continuity of η and σ_n.

Finally, we claim that $\{u_n\} \subset U_\lambda$ for n sufficiently large. Because $(J_\lambda^\mu)'(u_n) \to 0$, we can see that $\langle (J_\lambda^\mu)'(u_n), u_n^\pm \rangle = o(1)$. Then we only need to prove that $u_n^\pm \neq 0$ because it implies that $l(u_n^+, u_n^-) \to 1$, $l(u_n^-, u_n^+) \to 1$, and thus $\{u_n\} \subset U_\lambda$ for n sufficiently large. From (2.26), there exists a sequence $\{v_n\}$ satisfying

$$v_n = s_n w_n^+ + t_n w_n^- \in \sigma_n(Q), \quad \|v_n - u_n\|_\lambda \to 0. \tag{2.28}$$

In order to prove that $u_n^\pm \neq 0$, we just need to prove that $s_n w_n^+ \neq 0$ and $t_n w_n^- \neq 0$ for n sufficiently large. Since $\{w_n\} \subset \mathcal{M}_\lambda^\mu$, similar to (2.15) and (2.17), we obtain that $C_1 \leq \|w_n^\pm\|_\lambda \leq C_2$. Hence, we only need to prove that $\lim_{n\to\infty} s_n \neq 0$ and $\lim_{n\to\infty} t_n \neq 0$. If $\lim_{n\to\infty} s_n = 0$, by the continuity of J_λ^μ and (2.28), we infer that

$$m_\lambda^\mu = \lim_{n\to\infty} J_\lambda^\mu(v_n) = \lim_{n\to\infty} J_\lambda^\mu(s_n w_n^+ + t_n w_n^-) = \lim_{n\to\infty} J_\lambda^\mu(t_n w_n^-).$$

However, let $\varepsilon = \frac{1}{S_2^2}$; for $s > 0$ small enough, by (2.14) and (2.16), one gets

$$\begin{aligned}
m_\lambda^\mu &= \lim_{n\to\infty} J_\lambda^\mu(w_n) \\
&= \lim_{n\to\infty} \max_{s,t>0} J_\lambda^\mu(s w_n^+ + t w_n^-) \\
&\geq \lim_{n\to\infty} J_\lambda^\mu(s w_n^+ + t_n w_n^-) \\
&= \lim_{n\to\infty} \left(\frac{1}{2}\|s w_n^+ + t_n w_n^-\|_\lambda^2 + \frac{\mu}{4}\int_{\mathbb{R}^3} \phi^t_{s w_n^+ + t_n w_n^-} |s w_n^+ + t_n w_n^-|^2 dx \right. \\
&\qquad \left. - \int_{\mathbb{R}^3} F(s w_n^+ + t_n w_n^-) dx \right) \\
&\geq \lim_{n\to\infty} \left(\frac{s^2}{2}\|w_n^+\|_\lambda^2 - \int_{\mathbb{R}^3} F(s w_n^+) dx \right) + \lim_{n\to\infty} J_\lambda^\mu(t_n w_n^-) \\
&\geq \lim_{n\to\infty} \left(\frac{s^2}{2}\|w_n^+\|_\lambda^2 - \frac{1}{4}\int_{\mathbb{R}^3} f(s w_n^+) s w_n^+ dx \right) + \lim_{n\to\infty} J_\lambda^\mu(t_n w_n^-) \\
&\geq \lim_{n\to\infty} \left(\frac{s^2}{2}\|w_n^+\|_\lambda^2 - \frac{\varepsilon s^2}{4}\int_{\mathbb{R}^3} |w_n^+|^2 dx - \frac{C_\varepsilon s^p}{4}\int_{\mathbb{R}^3} |w_n^+|^p dx \right) + \lim_{n\to\infty} J_\lambda^\mu(t_n w_n^-) \\
&\geq \lim_{n\to\infty} \left(\frac{s^2}{2}\|w_n^+\|_\lambda^2 - \frac{\varepsilon s^2 S_2^2}{4}\|w_n^+\|_\lambda^2 - \frac{C_\varepsilon s^p S_p^p}{4}\|w_n^+\|_\lambda^p \right) + \lim_{n\to\infty} J_\lambda^\mu(t_n w_n^-) \\
&= \lim_{n\to\infty} \left(\frac{s^2}{4}\|w_n^+\|_\lambda^2 - \frac{C_{\frac{1}{S_2^2}} s^p S_p^p}{4}\|w_n^+\|_\lambda^p \right) + \lim_{n\to\infty} J_\lambda^\mu(t_n w_n^-) \\
&\geq C + m_\lambda^\mu \\
&> m_\lambda^\mu,
\end{aligned}$$

which is a contradiction. Therefore, $\{u_n\} \subset U_\lambda$ for n sufficiently large. □

Inspired by [36], with the help of the Nehari manifold, the following results hold. Since the proof is similar, we omit it here.

Lemma 2.4. Assume that (V_1) and $(f_1)-(f_4)$ hold, then, (i) for any $u \in E_\lambda$, there exists a unique $\widetilde{s_u} > 0$ such that $\widetilde{s_u}u \in \mathcal{N}_\lambda^\mu$, and

$$J_\lambda^\mu(\widetilde{s_u}u) = \max_{s\geq 0} J_\lambda^\mu(su);$$

(ii) system (1.1) has a positive ground state solution $\widetilde{u} \in \mathcal{N}_\lambda^\mu$ and $J_\lambda(\widetilde{u}) = c_\lambda^\mu$.

3. The proof of main results

Proof of Theorem 1.1. From Lemma 2.3, there exists a sequence $\{u_n\} \subset U_\lambda$ satisfying that $J_\lambda^\mu(u_n) \to m_\lambda^\mu$ and $(J_\lambda^\mu)'(u_n) \to 0$. Then, we need to prove that $\{u_n\}$ is bounded in E_λ according to Lemma 2.3. From (2.16), one has

$$\begin{aligned} m_\lambda^\mu + o(1) =& J_\lambda^\mu(u_n) - \frac{1}{4}\langle (J_\lambda^\mu)'(u_n), u_n\rangle \\ =& \frac{1}{4}\|u_n\|_\lambda^2 + \int_{\mathbb{R}^3} \mathcal{F}(u_n)dx \\ \geq& \frac{1}{4}\|u_n\|_\lambda^2, \end{aligned} \tag{3.1}$$

that is $\limsup_{n\to\infty} \|u_n\|_\lambda \leq 4m_\lambda^\mu$. Thus, $\{u_n\}$ is bounded in E_λ. Up to a subsequence, still denoted by $\{u_n\}$, there is $u_{\lambda,\mu} \in E_\lambda$ such that, as $n \to \infty$ the following holds:

$$\begin{cases} u_n \rightharpoonup u_{\lambda,\mu}, & \text{in } E_\lambda, \\ u_n \to u_{\lambda,\mu}, & \text{in } L_{loc}^q(\mathbb{R}^3) \ \ (2 \leq q < 2_s^*), \\ u_n(x) \to u_{\lambda,\mu}(x), & \text{a.e. in } \mathbb{R}^3. \end{cases}$$

By Lemma 2.3, we have that $(J_\lambda^\mu)'(u_n) \to 0$ in E_λ^* as $n \to \infty$, which implies that $(J_\lambda^\mu)'(u_{\lambda,\mu}) \to 0$ in E_λ^*. So, $u_{\lambda,\mu}$ is a solution of system (1.1).

Next, we claim that $u_{\lambda,\mu}$ is a ground state solution for system (1.1), that is, $J_\lambda^\mu(u_{\lambda,\mu}) = m_\lambda^\mu$. Since $u_{\lambda,\mu} \in \mathcal{M}_\lambda^\mu$, one obtains that $J_\lambda^\mu(u_{\lambda,\mu}) \geq m_\lambda^\mu$. Then, combining Fatou's Lemma with (2.16), we get

$$\begin{aligned} m_\lambda^\mu = \lim_{n\to\infty} J_\lambda^\mu(u_n) =& \lim_{n\to\infty}\left(J_\lambda^\mu(u_n) - \frac{1}{4}\langle (J_\lambda^\mu)'(u_n), u_n\rangle\right) \\ =& \lim_{n\to\infty}\left(\frac{1}{4}\|u_n\|_\lambda^2 + \int_{\mathbb{R}^3} \mathcal{F}(u_n)dx\right) \\ \geq& \frac{1}{4}\|u_{\lambda,\mu}\|_\lambda^2 + \int_{\mathbb{R}^3} \mathcal{F}(u_{\lambda,\mu})dx \\ =& J_\lambda^\mu(u_{\lambda,\mu}) - \frac{1}{4}\langle (J_\lambda^\mu)'(u_{\lambda,\mu}), u_{\lambda,\mu}\rangle \\ =& J_\lambda^\mu(u_{\lambda,\mu}). \end{aligned}$$

Hence, $J_\lambda^\mu(u_{\lambda,\mu}) = m_\lambda^\mu$. So, $u_{\lambda,\mu}$ is a ground state solution of system (1.1).

Finally, we need to prove that$u^{\pm}_{\lambda,\mu} \neq 0$, that is, $u_{\lambda,\mu}$ is a sign-changing solution of system (1.1). By Lemma 2.3, $\{u_n\} \subset U_\lambda$. It follows from (2.15) with $\varepsilon = \frac{1}{2S_2^2}$ that

$$\begin{aligned}
\|u_n^{\pm}\|_\lambda^2 \leq& \|u_n^{\pm}\|_\lambda^2 + \mu \int_{\mathbb{R}^3} \phi^t_{u_n^{\pm}} (u_n^{\pm})^2 dx + \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} u_n^+ (-\Delta)^{\frac{s}{2}} u_n^- dx = \int_{\mathbb{R}^3} f(u_n^{\pm}) u_n^{\pm} dx \\
\leq& \varepsilon \int_{\mathbb{R}^3} |u_n^{\pm}|^2 dx + C_\varepsilon \int_{\mathbb{R}^3} |u_n^{\pm}|^p dx \\
=& \frac{1}{2S_2^2} \int_{\mathbb{R}^3} |u_n^{\pm}|^2 dx + C_{\frac{1}{2S_2^2}} \int_{\mathbb{R}^3} |u_n^{\pm}|^p dx \\
\leq& \frac{1}{2} \|u_n^{\pm}\|_\lambda^2 + C_{\frac{1}{2S_2^2}} S_p^p \|u_n^{\pm}\|_\lambda^p,
\end{aligned}$$

which means that $\|u_n^{\pm}\|_\lambda \geq \left(\frac{1}{2S_p^p C_{\frac{1}{2S_2^2}}} \right)^{\frac{1}{p-2}}$ and

$$\int_{\mathbb{R}^3} |u_n^{\pm}|^p dx \geq \epsilon := \left(\frac{1}{2S_p^2 C_{\frac{1}{2S_2^2}}} \right)^{\frac{p}{p-2}}. \tag{3.2}$$

Set

$$A_R = \left\{ x \in \mathbb{R}^3 \setminus B_R(0) : V(x) \geq b \right\}, \quad D_R = \left\{ x \in \mathbb{R}^3 \setminus B_R(0) : V(x) < b \right\}.$$

Then, we have

$$\begin{aligned}
\limsup_{n\to\infty} \int_{A_R} |u_n^{\pm}|^2 dx \leq& \frac{1}{\lambda b} \int_{A_R} \lambda V(x) |u_n^{\pm}|^2 dx \\
\leq& \frac{1}{\lambda b} \limsup_{n\to\infty} \|u_n^{\pm}\|_\lambda^2 \\
\leq& \frac{4m_\lambda^\mu}{\lambda b}.
\end{aligned} \tag{3.3}$$

Moreover, we have that $|D_R| \to 0$ as $R \to \infty$ by (V_2). Hence, from the Hölder inequality, as $R \to \infty$,

$$\int_{D_R} |u_n^{\pm}|^2 dx \leq \left(\int_{D_R} |u_n^{\pm}|^s dx \right)^{\frac{2}{s}} \left(\int_{D_R} 1 dx \right)^{\frac{s-2}{s}} \leq C \|u_n^{\pm}\|_\lambda^2 |D_R|^{\frac{s-2}{s}} \to 0, \tag{3.4}$$

where $s \in (2, 2_s^*)$. Moreover, thanks to (3.3), (3.4) and Proposition 2.2, taking $R > 0$ large enough, we get

$$\begin{aligned}
&\limsup_{n\to\infty} \int_{\mathbb{R}^3 \setminus B_R(0)} |u_n^{\pm}|^p dx \\
\leq& C_1(p) \limsup_{n\to\infty} \left(|(-\Delta)^{\frac{s}{2}} u_n^{\pm}|_{\mathbb{R}^3 \setminus B_R(0)}|_2^{\frac{3p-2_s^*}{2}} |u_n^{\pm}|_{\mathbb{R}^3 \setminus B_R(0)}|_2^{\frac{2_s^*-p}{2}} \right) \\
\leq& C_2(p) \limsup_{n\to\infty} \left[\|u_n^{\pm}\|_\lambda^{\frac{3p-2_s^*}{2}} \left(\int_{A_R} |u_n^{\pm}|^2 dx + \int_{D_R} |u_n^{\pm}|^2 dx \right)^{\frac{2_s^*-p}{4}} \right] \\
\leq& C_3(p) \left(\frac{1}{\lambda b} \right)^{\frac{2_s^*-p}{4}} (4m_\lambda^\mu)^{\frac{5p-2_s^*}{4}} + o_R(1).
\end{aligned} \tag{3.5}$$

Let $R_1 > 0$ such that $o_R(1) < \frac{\epsilon}{4}$ for all $R > R_1$. Then, let

$$C_3(p)\left(\frac{1}{\lambda b}\right)^{\frac{2_s^*-p}{4}}(4m_\lambda^\mu)^{\frac{5p-2_s^*}{4}} + o_R(1) \leq \frac{\epsilon}{2},$$

we can deduce that

$$\lambda \geq C(p)b^{-1}(\frac{4}{\epsilon})^{\frac{4}{2_s^*-p}}(4m_\lambda^\mu)^{\frac{5p-2_s^*}{2_s^*-p}} =: \Lambda(\mu). \tag{3.6}$$

So, for any $\lambda \geq \Lambda(\mu)$ and $R \geq R_1$, we have

$$\limsup_{n\to\infty}\int_{\mathbb{R}^3\setminus B_R(0)} |u_n^\pm|^p dx \leq \frac{\epsilon}{2}.$$

Then,

$$\begin{aligned}\limsup_{n\to\infty}\int_{\mathbb{R}^3} |u_n^\pm|^p dx =& \limsup_{n\to\infty}\int_{B_R(0)} |u_n^\pm|^p dx + \limsup_{n\to\infty}\int_{\mathbb{R}^3\setminus B_R(0)} |u_n^\pm|^p dx \\ \leq& \int_{B_R(0)} |u_n^\pm|^p dx + \frac{\epsilon}{2}.\end{aligned} \tag{3.7}$$

By (3.2) and (3.7), one gets that $\limsup_{n\to\infty}\int_{B_R(0)} |u_n^\pm|^p dx \geq \frac{\epsilon}{2} > 0$, that is, $\int_{B_R(0)} |u_{\lambda,\mu}^\pm|^p dx > 0$. Hence, $u_{\lambda,\mu}^\pm \neq 0$. In short, $u_{\lambda,\mu}$ is a ground state sign-changing solution of system (1.1).

Next, we are going to prove that $m_\lambda^\mu > 2c_\lambda^\mu$. From Lemma 2.4 (i), there exists $\widetilde{s}, \widetilde{t} > 0$ such that $\widetilde{s}u_{\lambda,\mu}^+, \widetilde{t}u_{\lambda,\mu}^- \in \mathcal{N}_\lambda^\mu$. Then, it follows from Lemma 2.1 that

$$\begin{aligned} m_\lambda^\mu =& J_\lambda^\mu(u_{\lambda,\mu}) = J_\lambda^\mu(u_{\lambda,\mu}^+ + u_{\lambda,\mu}^-) \geq J_\lambda^\mu(\widetilde{s}u_{\lambda,\mu}^+ + \widetilde{t}u_{\lambda,\mu}^-) \\ =& J_\lambda^\mu(\widetilde{s}u_{\lambda,\mu}^+) + J_\lambda^\mu(\widetilde{t}u_{\lambda,\mu}^-) + \widetilde{s}\widetilde{t}\int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}}u_{\lambda,\mu}^+(-\Delta)^{\frac{s}{2}}u_{\lambda,\mu}^- dx \\ &+ \frac{\mu\widetilde{s}^2\widetilde{t}^2}{4}\int_{\mathbb{R}^3} \phi_{u_{\lambda,\mu}^-}^t (u_{\lambda,\mu}^+)^2 dx + \frac{\mu\widetilde{s}^2\widetilde{t}^2}{4}\int_{\mathbb{R}^3} \phi_{u_{\lambda,\mu}^+}^t (u_{\lambda,\mu}^-)^2 dx \\ >& J_\lambda^\mu(\widetilde{s}u_{\lambda,\mu}^+) + J_\lambda^\mu(\widetilde{t}u_{\lambda,\mu}^-) \geq 2c_\lambda^\mu. \end{aligned}$$

Lastly, we prove that $u_{\lambda,\mu}$ changes sign only once, that is, $u_{\lambda,\mu}$ has two nodal domains. By contradiction, we assume that $u_{\lambda,\mu} = u_1 + u_2 + u_3$ with

$$u_i \neq 0, \quad u_1 \geq 0, \quad u_2 \leq 0, \quad u_3 \geq 0,$$

$$\text{supp}(u_i) \cap \text{supp}(u_j) = \emptyset, \quad i \neq j (i, j = 1, 2, 3).$$

Then, let $v = u_1 + u_2$, $v^+ = u_1$ and $v^- = u_2$; by Lemma 2.1, there exists a unique pair of $(s_v, t_v) \in (0, 1] \times (0, 1]$ such that

$$s_v^+ + t_v^- = s_v u_1 + t_v u_2 \in \mathcal{M}_\lambda^\mu, \qquad J_\lambda^\mu(s_v u_1 + t_v u_2) \geq m_\lambda^\mu.$$

By $\langle (J_\lambda^\mu)'(u_{\lambda,\mu}), u_i\rangle = 0$ $(i = 1, 2, 3)$, it follows that $\langle (J_\lambda^\mu)'(v), v^\pm\rangle < 0$ since

$$0 = \frac{1}{4}\langle (J_\lambda^\mu)'(u_{\lambda,\mu}), u_3\rangle$$

$$
\begin{aligned}
&=\frac{1}{4}\|u_3\|_\lambda^2+\frac{1}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_1(-\Delta)^{\frac{s}{2}}u_3dx+\frac{1}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_2(-\Delta)^{\frac{s}{2}}u_3dx\\
&\quad+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_1}^t u_3^2dx+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_2}^t u_3^2dx+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_3}^t u_3^2dx-\frac{1}{4}\int_{\mathbb{R}^3}f(u_3)u_3dx\\
&\le\frac{1}{4}\|u_3\|_\lambda^2+\frac{1}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_1(-\Delta)^{\frac{s}{2}}u_3dx+\frac{1}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_2(-\Delta)^{\frac{s}{2}}u_3dx\\
&\quad+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_1}^t u_3^2dx+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_2}^t u_3^2dx+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_3}^t u_3^2dx-\int_{\mathbb{R}^3}F(u_3)dx\\
&<J_\lambda^\mu(u_3)+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_1}^t u_3^2dx+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_2}^t u_3^2dx.
\end{aligned}
$$

From (2.16), we have

$$
\begin{aligned}
m_\lambda^\mu &\le J_\lambda^\mu(s_vu_1+t_vu_2)\\
&=J_\lambda^\mu(s_vu_1+t_vu_2)-\frac{1}{4}\langle (J_\lambda^\mu)'(s_vu_1+t_vu_2),s_vu_1+t_vu_2\rangle\\
&=\frac{s_v^2}{4}\|u_1\|_\lambda^2+\int_{\mathbb{R}^3}\mathcal{F}(s_vu_1)dx+\frac{t_v^2}{4}\|u_2\|_\lambda^2+\int_{\mathbb{R}^3}\mathcal{F}(t_vu_2)dx+\frac{s_vt_v}{2}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_1(-\Delta)^{\frac{s}{2}}u_2dx\\
&\le\frac{1}{4}\|u_1\|_\lambda^2+\int_{\mathbb{R}^3}\mathcal{F}(u_1)dx+\frac{1}{4}\|u_2\|_\lambda^2+\int_{\mathbb{R}^3}\mathcal{F}(u_2)dx+\frac{1}{2}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_1(-\Delta)^{\frac{s}{2}}u_2dx\\
&\le J_\lambda^\mu(u_1)+J_\lambda^\mu(u_2)+\frac{\mu}{2}\int_{\mathbb{R}^3}\phi_{u_1}^t u_2^2dx+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_1}^t u_3^2dx+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_2}^t u_3^2dx\\
&\quad+\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_1(-\Delta)^{\frac{s}{2}}u_2dx+\frac{1}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_3(-\Delta)^{\frac{s}{2}}u_1dx+\frac{1}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_3(-\Delta)^{\frac{s}{2}}u_2dx\\
&<J_\lambda^\mu(u_1)+J_\lambda^\mu(u_2)+J_\lambda^\mu(u_3)+\frac{\mu}{2}\int_{\mathbb{R}^3}\phi_{u_1}^t u_2^2dx+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_1}^t u_3^2dx+\frac{\mu}{4}\int_{\mathbb{R}^3}\phi_{u_2}^t u_3^2dx\\
&\quad+\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_1(-\Delta)^{\frac{s}{2}}u_2dx+\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_3(-\Delta)^{\frac{s}{2}}u_1dx+\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}u_3(-\Delta)^{\frac{s}{2}}u_2dx\\
&=J_\lambda^\mu(u_{\lambda,\mu})=m_\lambda^\mu.
\end{aligned}
$$

which is impossible, so $u_{\lambda,\mu}$ has exactly two nodal domains. □

In what follows, we will give the asymptotic behavior of the ground state sign-changing solution. We define J_∞^μ as the energy functional of system (1.8):

$$
J_\infty^\mu=\frac{1}{2}\int_\Omega |(-\Delta)^{\frac{s}{2}}u|^2+u^2dx+\frac{\mu}{4}\int_\Omega\left(\int_\Omega\frac{u^2(y)}{4\pi|x-y|^{3+2s}}dy\right)u^2dx-\int_\Omega F(u)dx.
$$

It is not difficult to obtain that $J_\infty^\mu\in C^1$. Define

$$
\mathcal{M}_\infty^\mu=\{u\in H_0^s(\Omega):u^\pm\neq 0,\langle (J_\infty^\mu)'(u),u^\pm\rangle=0\}\ \text{ and }\ m_\infty^\mu=\inf_{u\in\mathcal{M}_\infty^\mu}J_\infty^\mu(u).
$$

It is easy to get that $\mathcal{M}_\infty^\mu\subset\mathcal{M}_\lambda^\mu$ and $J_\lambda^\mu(u)=J_\infty^\mu(u)$ for $\lambda>0$. Thus, we have that $m_\lambda^\mu\le m_\infty^\mu$.

Proof of Theorem 1.2. For any sequence $\lambda_n \to \infty$ as $n \to \infty$, $\{u_{\lambda_n}\}$ is a sequence of sign-changing solutions for system (1.1) with $J^{\mu}_{\lambda_n}(u_{\lambda_n}) = m^{\mu}_{\lambda_n} \le m^{\mu}_{\infty}$ and $(J^{\mu}_{\lambda_n})'(u_{\lambda_n}) = 0$. By (2.16), we conclude that

$$\begin{aligned} m^{\mu}_{\infty} \ge m^{\mu}_{\lambda_n} &= J^{\mu}_{\lambda_n}(u_{\lambda_n}) - \frac{1}{4}\langle (J^{\mu}_{\lambda_n})'(u_{\lambda_n}), u_{\lambda_n}\rangle \\ &= \frac{1}{4}\|u_{\lambda_n}\|^2_{\lambda_n} + \int_{\Omega} \mathcal{F}(u_{\lambda_n})dx \\ &\ge \frac{1}{4}\|u_{\lambda_n}\|^2_{\lambda_n}. \end{aligned} \tag{3.8}$$

Hence, $\{u_{\lambda_n}\}$ is bounded in $H^s(\mathbb{R}^3)$. Passing to a subsequence, there is $u_* \in H^s(\mathbb{R}^3)$ such that

$$\begin{cases} u_{\lambda_n} \rightharpoonup u_*, & \text{in } H^s(\mathbb{R}^3), \\ u_{\lambda_n} \to u_*, & \text{in } L^q_{loc}(\mathbb{R}^3)\ (q \in [2, 2^*_s)), \\ u_{\lambda_n}(x) \to u_*(x), & \text{a.e. in } \mathbb{R}^3. \end{cases}$$

Step 1: We will prove that u_* is a solution of system (1.8). By (V_1) and Fatou's lemma, one gets

$$0 \le \int_{\mathbb{R}^3} V(x)u_*^2 dx \le \liminf_{n\to\infty} \int_{\mathbb{R}^3} V(x)u_{\lambda_n}^2 dx \le \liminf_{n\to\infty} \frac{\|u_{\lambda_n}\|^2_{\lambda_n}}{\lambda_n} = 0.$$

By (V_3), we can deduce that $u_*|_{\Omega^c} = 0$. Hence, it follows that $u_* \in H^s_0(\Omega)$ from the boundary of Ω which is smooth. Because $(J^{\mu}_{\lambda_n})'(u_{\lambda_n}) = 0$, we can deduce that $\langle (J^{\mu}_{\infty})'(u_*), v\rangle = 0$ for any $v \in H^s_0(\Omega)$, which means that u_* is a solution of system (1.8).

Step 2: We need to prove that $u_{\lambda_n} \to u_*$ in $H^s(\mathbb{R}^3)$. Then, similar to (3.3) and (3.4), we have that

$$\begin{aligned} \lim_{n\to\infty} \int_{\mathbb{R}^3} |u_{\lambda_n} - u_*|^2 dx &= \lim_{n\to\infty} \left(\int_{B_R(0)} |u_{\lambda_n} - u_*|^2 dx + \int_{\mathbb{R}^3\setminus B_R(0)} |u_{\lambda_n} - u_*|^2 dx \right) \\ &= \lim_{n\to\infty} \left(\int_{A_R} |u_{\lambda_n} - u_*|^2 dx + \int_{D_R} |u_{\lambda_n} - u_*|^2 dx \right) \\ &\le \lim_{n\to\infty} \frac{\|u_{\lambda_n} - u_*\|^2_{\lambda_n}}{\lambda_n b} = 0. \end{aligned}$$

Hence, $\lim_{n\to\infty} \int_{\mathbb{R}^3} |u_{\lambda_n} - u_*|^q dx = 0$ with $q \in [2, 2^*_s)$. That is, $u_{\lambda_n} \to u_*$ in $L^q(\mathbb{R}^3)$ with $q \in [2, 2^*_s)$. Then,

$$\begin{aligned} \|u_{\lambda_n} - u_*\|^2_{\lambda} =& \langle (J^{\mu}_{\lambda_n})'(u_{\lambda_n} - u_*), u_{\lambda_n} - u_*\rangle - \mu \int_{\mathbb{R}^3} (\phi^t_{u_{\lambda_n}} u_{\lambda_n} - \phi^t_{u_*} u_*)(u_{\lambda_n} - u_*)dx \\ &+ \int_{\mathbb{R}^3} (f(u_{\lambda_n}) - f(u_*))(u_{\lambda_n} - u_*)dx. \end{aligned}$$

Obviously, we can draw the conclusion that $\langle (J^{\mu}_{\lambda_n})'(u_{\lambda_n} - u_*), u_{\lambda_n} - u_*\rangle = 0$. Applying an argument similar to that in Lemma 2.1 in [37], we can get

$$\mu \int_{\mathbb{R}^3} (\phi^t_{u_{\lambda_n}} u_{\lambda_n} - \phi^t_{u_*} u_*)(u_{\lambda_n} - u_*)dx \to 0$$

as $n \to \infty$. By the Hölder inequality and (2.14), we have

$$
\begin{aligned}
&\int_{\mathbb{R}^3} [f(u_{\lambda_n}) - f(u_*)](u_{\lambda_n} - u_*)dx \\
\le &\int_{\mathbb{R}^3} \left[\varepsilon(|u_{\lambda_n}| + |u_*|) + C_\varepsilon(|u_{\lambda_n}|^{p-1} + |u_*|^{p-1})\right] |u_{\lambda_n} - u_*| dx \\
\le &\varepsilon(|u_{\lambda_n}|_2^2 + |u_*|_2^2)|u_{\lambda_n} - u_*|_2^2 + C_\varepsilon(|u_{\lambda_n}|_p^{p-1} + |u_*|_p^{p-1})|u_{\lambda_n} - u_*|_p.
\end{aligned}
$$

Since $u_{\lambda_n} \to u_*$ in $L^q(\mathbb{R}^3)$ for $q \in (2, 2_s^*)$, we get that $\int_{\mathbb{R}^3}[f(u_{\lambda_n}) - f(u_*)](u_{\lambda_n} - u_*)dx \to 0$ as $n \to \infty$. Hence, $\|u_{\lambda_n} - u_*\|_\lambda^2 = 0$, that is, $u_{\lambda_n} \to u_*$ in $H^s(\mathbb{R}^3)$ as $n \to \infty$.

Step 3: We claim that u_* is a ground state sign-changing solution of system (1.8), that is, $J_\infty^\mu(u_*) = m_\infty^\mu$ and $u_{\lambda_n}^\pm \neq 0$. On the one hand, for $m_{\lambda_n}^\mu \le m_\infty^\mu$ and $m_{\lambda_n}^\mu \to J_\infty^\mu(u_*)$, we get that $J_\infty^\mu(u_*) \le m_\infty^\mu$. On the other hand, since $u_* \in \mathcal{M}_\infty^\mu$, by (2.16), we have

$$
\begin{aligned}
m_{\lambda_n}^\mu &= J_{\lambda_n}^\mu(u_{\lambda_n}) \\
&= \lim_{n\to\infty} \left[J_{\lambda_n}^\mu(u_{\lambda_n}) - \frac{1}{4}\left\langle (J_{\lambda_n}^\mu)'(u_{\lambda_n}), u_{\lambda_n} \right\rangle \right] \\
&= \lim_{n\to\infty} \left(\frac{1}{4}\|u_{\lambda_n}\|_{\lambda_n}^2 + \int_{\mathbb{R}^3} \mathcal{F}(u_{\lambda_n})dx \right) \\
&\ge \frac{1}{4}\|u_*\|_\lambda^2 + \int_\Omega \mathcal{F}(u_*)dx \\
&= J_\infty^\mu(u_*) - \frac{1}{4}\langle (J_\infty^\mu)'(u_*), u_* \rangle \\
&= J_\infty^\mu(u_*) \\
&\ge m_\infty^\mu.
\end{aligned}
$$

Thus, $J_\infty^\mu(u_*) = m_\infty^\mu$, that is, u_* is a ground state sign-changing solution of system (1.8) and $u_{\lambda_n} \to u_*$ in $H^s(\mathbb{R}^3)$ up to a subsequence. Then, analogous to the proof of Theorem 1.1, we can get that u_* has two nodal domains. Hence, we have completed the proof of Theorem 1.2. □

Next, we will prove the asymptotic properties of sign-changing solutions given in Theorem 1.1 as $\mu \to 0$. For convenience, we let $u_\mu := u_{\lambda,\mu}$, $J_\mu := J_\lambda^\mu$ and $m_\mu := m_\lambda^\mu$. In addition, we set the energy functional and constraint set of (1.9) as $J_0(u) = J_\lambda^0(u)$ and $\mathcal{M}_0 = \mathcal{M}_\lambda^0$; similarly, $m_0 = \inf_{u\in\mathcal{M}_0} J_0(u)$.

Proof of Theorem 1.3. For any $\{\mu_n\} \subset (0, 1)$ with $\mu_n \to 0$ as $n \to \infty$, u_{μ_n} is a ground state solution of system (1.1) with $\mu = \mu_n$ which has been obtained in Theorem 1.1. In other words, $J_{\mu_n}(u_{\mu_n}) = m_{\mu_n}$ and $J'_{\mu_n}(u_{\mu_n}) = 0$. Similar to Theorem 1.1, we have that $\{u_{\mu_n}\}$ is bounded in E_λ. Up to a subsequence, we can assume the following:

$$
\begin{cases}
u_{\mu_n} \rightharpoonup u_0, & \text{in } E_\lambda, \\
u_{\mu_n} \to u_0, & \text{in } L_{loc}^q(\mathbb{R}^3)\ (q \in (2, 2_s^*)), \\
u_{\mu_n}(x) \to u_0(x), & \text{a.e. in } \mathbb{R}^3.
\end{cases}
$$

Step 1: We need to prove that u_0 is a weak solution of (1.9).

For any $\varphi \in E_\lambda$, thanks to Proposition 2.1 (ii), we have

$$\int_{\mathbb{R}^3} \phi^t_{u_{\mu_n}} u_{\mu_n} \varphi dx \le C$$

and

$$\int_{\mathbb{R}^3} \left((-\Delta)^{\frac{s}{2}} u_{\mu_n} (-\Delta)^{\frac{s}{2}} \varphi + V_\lambda(x) u_{\mu_n} \varphi\right) dx + \mu_n \int_{\mathbb{R}^3} \phi^t_{u_{\mu_n}} u_{\mu_n} \varphi dx - \int_{\mathbb{R}^3} f(u_{\mu_n}) \varphi dx = 0. \tag{3.9}$$

Then, let $n \to \infty$; we get

$$\int_{\mathbb{R}^3} \left((-\Delta)^{\frac{s}{2}} u_0 (-\Delta)^{\frac{s}{2}} \varphi + V_\lambda(x) u_0 \varphi\right) dx - \int_{\mathbb{R}^3} f(u_0) \varphi dx = 0. \tag{3.10}$$

Hence, u_0 is a weak solution of (1.9).

Step 2: We will prove that $u_{\mu_n} \to u_0$ in E_λ as $n \to \infty$.

First, we need to prove that $u_{\mu_n} \to u_0$ in $L^q(\mathbb{R}^3)$ with $q \in (2, 2^*_s)$ as $n \to \infty$. Thus, for $r > 0$, let $\xi_r \in C^\infty(\mathbb{R}^3)$ such that

$$\xi_r(x) = \begin{cases} 1, & |x| > \frac{r}{2}, \\ 0, & |x| < \frac{r}{4}, \end{cases} \tag{3.11}$$

with $\int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} \xi_r dx \le \frac{8}{r}$. Let $u \in E_\lambda$ such that $\|u_{\mu_n}\|_\infty \le L$, for some $L > 0$. Then, for any $\eta \in C^1(\mathbb{R}^3)$ with $\eta \ge 0$, we obtain

$$\int_{\mathbb{R}^3} \left((-\Delta)^{\frac{s}{2}} u_{\mu_n} (-\Delta)^{\frac{s}{2}} (u_{\mu_n} \eta) + V_\lambda(x) u^2_{\mu_n} \eta\right) dx + \mu_n \int_{\mathbb{R}^3} \phi^t_{u_{\mu_n}} u^2_{\mu_n} \eta dx = \int_{\mathbb{R}^3} f(u_{\mu_n}) u_{\mu_n} \eta dx.$$

Taking $\eta = \xi_r$ and $\varepsilon = \frac{1}{2}$, by (2.14), it follows that

$$\begin{aligned}
&\int_{\mathbb{R}^3} \left(|(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 + V_\lambda(x) u^2_{\mu_n}\right) \xi_r dx + \mu_n \int_{\mathbb{R}^3} \phi^t_{u_{\mu_n}} u^2_{\mu_n} \xi_r dx \\
= &\int_{\mathbb{R}^3} f(u_{\mu_n}) u_{\mu_n} \xi_r dx - \int_{\mathbb{R}^3} u_{\mu_n} (-\Delta)^{\frac{s}{2}} u_{\mu_n} (-\Delta)^{\frac{s}{2}} \xi_r dx \\
\le &\varepsilon \int_{\mathbb{R}^3} u^2_{\mu_n} \xi_r dx + C_\varepsilon \int_{\mathbb{R}^3} u^p_{\mu_n} \xi_r dx + \frac{8}{r} \int_{\mathbb{R}^3} \left(|(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 + u^2_{\mu_n}\right) dx \\
\le &\frac{1}{2} \int_{\mathbb{R}^3} u^2_{\mu_n} \xi_r dx + C_{\frac{1}{2}} L^{p-2} \int_{\mathbb{R}^3} u^2_{\mu_n} \xi_r dx + \frac{8}{r} \int_{\mathbb{R}^3} \left(|(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 + u^2_{\mu_n}\right) dx,
\end{aligned}$$

that is

$$\begin{aligned}
&\int_{\mathbb{R}^3} \left(|(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 + \left[\lambda V(x) - C_{\frac{1}{2}} L^{p-2}\right] u^2_{\mu_n}\right) \xi_r dx \\
\le &\frac{8}{r} \int_{\mathbb{R}^3} \left(|(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 + u^2_{\mu_n}\right) dx.
\end{aligned} \tag{3.12}$$

Besides, for $R > 0$, we set

$$\widetilde{A}_R := \{x \in \mathbb{R}^3 \setminus B_R(0) : V(x) \le b\} \quad \text{and} \quad \widetilde{D}_R := \{x \in \mathbb{R}^3 \setminus B_R(0) : V(x) > b\}.$$

In fact, by (V_2), we have that $|\widetilde{A}_R| \le \varepsilon$ as $R \to \infty$; then, $\lambda V(x) > M$ in $\widetilde{D}_R$ from $\lambda > \frac{M}{b}$, where $M = C_{\frac{1}{2}} L^{p-2}$. Let $r = R$; by (3.12), one has

$$\int_{|x|>R} |(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 + [\lambda V(x) - M]\, u_{\mu_n}^2 dx \le \frac{8}{R} \int_{\mathbb{R}^3} \left(|(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 + u_{\mu_n}^2 \right) dx \le \frac{T}{R}, \tag{3.13}$$

where $T = 8 \sup \|u_{\mu_n}\|_\lambda$. Since

$$\begin{aligned} &\int_{|x|>R} |(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 + [\lambda V(x) - M]\, u_{\mu_n}^2 dx \\ \ge &\int_{\widetilde{A}_R} |(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 + [\lambda V(x) - M]\, u_{\mu_n}^2 dx + \int_{\widetilde{D}_R} |(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 dx \\ \ge & - M \int_{\widetilde{A}_R} u_n^2 dx + \int_{\widetilde{D}_R} |(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 dx \\ \ge & - C \|u_{\mu_n}\|_\lambda^2 |\widetilde{A}_R|^{\frac{2}{3}} + \int_{\widetilde{D}_R} |(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 dx, \end{aligned} \tag{3.14}$$

thanks to (3.13) and (3.14), one gets

$$\int_{\widetilde{D}_R} |(-\Delta)^{\frac{s}{2}} u_{\mu_n}|^2 dx \le \frac{T}{R} + C \|u_{\mu_n}\|_\lambda^2 |\widetilde{A}_R|^{\frac{2}{3}}. \tag{3.15}$$

We have that $H^1(B_R(0)) \hookrightarrow L^q(B_R(0))$ is compact for $2 < q < 2_s^*$, that is, $u_n \to u$ in $L^q(B_R(0))$ with $2 < q < 2_s^*$. For any R large enough, according to (3.15), Proposition 2.2 and the boundedness of $\{u_n\}$, we have

$$\begin{aligned} &|u_n - u|_q^q \\ = &\int_{B_R(0)} |u_n - u|^q dx + \int_{\mathbb{R}^3 \setminus B_R(0)} |u_n - u|^q dx \\ = &\int_{B_R(0)} |u_n - u|^q dx + \int_{\widetilde{A}_R} |u_n - u|^q dx + \int_{\widetilde{D}_R} |u_n - u|^q dx \\ \le &\varepsilon + C \|u_n - u\|_\lambda^q |\widetilde{A}_R|^{\frac{2_s^* - q}{2_s^*}} + C(q) |(-\Delta)^{\frac{s}{2}} (u_n - u)|_{L^2(\widetilde{D}_R)}^{\frac{3q - 2_s^*}{2}} |u_n - u|_{L^2(\widetilde{D}_R)}^{\frac{2_s^* - q}{2}} \\ \le &C_\varepsilon + C(q) \|u_n - u\|_\lambda^{\frac{2_s^* - q}{2}} \left(|(-\Delta)^{\frac{s}{2}} u_n|_{L^2(\widetilde{D}_R)}^{\frac{3q - 2_s^*}{2}} + |(-\Delta)^{\frac{s}{2}} u|_{L^2(\widetilde{D}_R)}^{\frac{3q - 2_s^*}{2}} \right) \\ \le &C_\varepsilon + C(q) \|u_n - u\|_\lambda^{\frac{2_s^* - q}{2}} \left(\frac{T}{R} + C \|u_n\|_\lambda^2 |\widetilde{A}_R|^{\frac{2}{3}} \right)^{\frac{3q - 2_s^*}{2}} \\ \le &C_\varepsilon. \end{aligned} \tag{3.16}$$

Thus, $u_{\mu_n} \to u_0$ in $L^q(\mathbb{R}^3)$ with $q \in (2, 2_s^*)$ as $n \to \infty$. Then, by Lebesgue's dominated convergence theorem, we get

$$\int_{\mathbb{R}^3} f(u_{\mu_n}) u_{\mu_n} dx \to \int_{\mathbb{R}^3} f(u_0) u_0 dx \quad \text{as } n \to \infty.$$

Let $\varphi = u_{\mu_n}$ apply capitalization (3.7) and $\varphi = u_0$ in (3.8), we have that $u_{\mu_n} \to u_0$ in E_λ as $n \to \infty$.

Step 3: we claim that u_0 is a ground state sign-changing solution. That is, $u_0^{\pm} \neq 0$ and $J_0(u_0) = m_0$.

Similar to (2.15), from $\langle J'_{\mu_n}(u_{\mu_n}), u^{\pm}_{\mu_n}\rangle = 0$, we can deduce that $\|u_0^{\pm}\|_\lambda^2 > 0$. So $u_0^{\pm} \neq 0$, that is, u_0 is a sign-changing solution for (1.9).

Next, we will prove that u_0 is also a ground state solution for (1.9). Similar to the discussion of Theorem 1.1, we can obtain that (1.9) has a ground state sign-changing solution when $\mu = 0$. That is to say, we have that $v_0 \in \mathcal{M}_0$ such that $J'_0(v_0) = 0$ and $J_0(v_0) = m_0$. Thanks to Lemma 2.1, there exists only a pair of positive numbers (s_{μ_n}, t_{μ_n}) such that $s_{\mu_n} v_0^+ + t_{\mu_n} v_0^- \in \mathcal{M}_{\mu_n}$. Then, we need to prove that $\{s_{\mu_n}\}$ and $\{t_{\mu_n}\}$ are bounded. Indeed, we assume that $\lim\limits_{n\to\infty} s_{\mu_n} = \infty$. According to (f_1) and (f_4), for any $a > 0$, there is $b > 0$ such that

$$F(t) \geq at^4 - bt^2 \text{ for all } t \in \mathbb{R}. \tag{3.17}$$

Then, let $a > 0$ sufficiently enough, thanks to (3.17), Lemma 2.2 and the Young inequality, we have

$$\begin{aligned}
0 <& J_{\mu_n}(s_{\mu_n} v_0^+ + t_{\mu_n} v_0^-)\\
=& \frac{s_{\mu_n}^2}{2}\|v_0^+\|_\lambda^2 + \frac{t_{\mu_n}^2}{2}\|v_0^-\|_\lambda^2 + s_{\mu_n} t_{\mu_n}\int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx\\
&+ \frac{s_{\mu_n}^4}{4}\mu_n \int_{\mathbb{R}^3} \phi^t_{v_0^+}|v_0^+|^2 dx + \frac{t_{\mu_n}^4}{4}\mu_n \int_{\mathbb{R}^3} \phi^t_{v_0^-}|v_0^-|^2 dx - \int_{\mathbb{R}^3} F(s_{\mu_n} v_0^+) dx\\
&+ \frac{s_{\mu_n}^2 t_{\mu_n}^2}{2}\mu_n \int_{\mathbb{R}^3} \phi^t_{v_0^+}|v_0^-|^2 dx - \int_{\mathbb{R}^3} F(t_{\mu_n} v_0^-) dx\\
\leq& \left(\frac{1}{2} + bS_2^2\right) s_{\mu_n}^2 \|v_0^+\|_\lambda^2 + \frac{s_{\mu_n}^4}{4}\mu_n \int_{\mathbb{R}^3} \phi^t_{v_0^+}|v_0^+|^2 dx - a s_{\mu_n}^4 \int_{\mathbb{R}^3} |v_0^+|^4 dx\\
&+ \left(\frac{1}{2} + bS_2^2\right) t_{\mu_n}^2 \|v_0^-\|_\lambda^2 + \frac{t_{\mu_n}^4}{4}\mu_n \int_{\mathbb{R}^3} \phi^t_{v_0^-}|v_0^-|^2 dx - a t_{\mu_n}^4 \int_{\mathbb{R}^3} |v_0^-|^4 dx\\
&+ \frac{s_{\mu_n}^2 t_{\mu_n}^2}{2}\mu_n \int_{\mathbb{R}^3} \phi^t_{v_0^+}|v_0^-|^2 dx + s_{\mu_n} t_{\mu_n}\int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx\\
\leq& \left(\frac{1}{2} + bS_2^2\right) s_{\mu_n}^2 \|v_0^+\|_\lambda^2 + \frac{s_{\mu_n}^4}{4}\mu_n \int_{\mathbb{R}^3} \phi^t_{v_0}|v_0^+|^2 dx - a s_{\mu_n}^4 \int_{\mathbb{R}^3} |v_0^+|^4 dx\\
&+ \left(\frac{1}{2} + bS_2^2\right) t_{\mu_n}^2 \|v_0^-\|_\lambda^2 + \frac{t_{\mu_n}^4}{4}\mu_n \int_{\mathbb{R}^3} \phi^t_{v_0}|v_0^-|^2 dx - a t_{\mu_n}^4 \int_{\mathbb{R}^3} |v_0^-|^4 dx\\
&+ s_{\mu_n} t_{\mu_n}\int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx\\
\leq& \left(\frac{1}{2} + bS_2^2\right) s_{\mu_n}^2 \|v_0^+\|_\lambda^2 + \frac{s_{\mu_n}^4}{4} \int_{\mathbb{R}^3} \phi^t_{v_0}|v_0^+|^2 dx - a s_{\mu_n}^4 \int_{\mathbb{R}^3} |v_0^+|^4 dx\\
&+ \left(\frac{1}{2} + bS_2^2\right) t_{\mu_n}^2 \|v_0^-\|_\lambda^2 + \frac{t_{\mu_n}^4}{4} \int_{\mathbb{R}^3} \phi^t_{v_0}|v_0^-|^2 dx - a t_{\mu_n}^4 \int_{\mathbb{R}^3} |v_0^-|^4 dx\\
&+ s_{\mu_n} t_{\mu_n}\int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx\\
<& 0.
\end{aligned}$$

This is a contradiction. Hence, $\{s_{\mu_n}\}$ is bounded in $\mathbb{R}$. Analogously, $\{t_{\mu_n}\}$ is bounded in $\mathbb{R}$. Then, by

(f_3), we obtain

$$\begin{aligned}\int_t^1 \left[\frac{f(\xi)}{\xi^3} - \frac{f(s\xi)}{(s\xi)^3}\right] s^3\xi^4 ds =& \int_t^1 \left[f(\xi)s^3\xi - f(s\xi)\xi\right] ds\\ =&\xi f(\xi)\frac{1-t^4}{4} - F(\xi) + F(t\xi)\\ \geq& 0.\end{aligned} \tag{3.18}$$

Consequently, thanks to (3.18), we get

$$\begin{aligned}J_{\mu_n}(v_0) =& J_{\mu_n}(s_{\mu_n}v_0^+ + t_{\mu_n}v_0^-) + \frac{1-s_{\mu_n}^4}{4}\langle J'_{\mu_n}(v_0), v_0^+\rangle + \frac{1-t_{\mu_n}^4}{4}\langle J'_{\mu_n}(v_0), v_0^-\rangle\\ &+ \frac{(s_{\mu_n}^2-1)^2}{4}\|v_0^+\|_\lambda^2 + \frac{(t_{\mu_n}^2-1)^2}{4}\|v_0^-\|_\lambda^2 + \frac{\mu}{4}(s_{\mu_n}^2 - t_{\mu_n}^2)^2\int_{\mathbb{R}^3}\phi_{v_0^+}^t|v_0^-|^2dx\\ &+ \int_{\mathbb{R}^3}\left[\frac{1-s^4}{4}f(v_0^+)v_0^+ - F(v_0^+) + F(tv_0^+)\right]dx\\ &+ \int_{\mathbb{R}^3}\left[\frac{1-t^4}{4}f(v_0^-)v_0^- - F(v_0^-) + F(tv_0^-)\right]dx\\ &- s_{\mu_n}t_{\mu_n}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}v_0^+(-\Delta)^{\frac{s}{2}}v_0^-dx - \frac{1-s_{\mu_n}^4}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}v_0^+(-\Delta)^{\frac{s}{2}}v_0^-dx\\ &- \frac{1-t_{\mu_n}^4}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}v_0^+(-\Delta)^{\frac{s}{2}}v_0^-dx\\ \geq& J_{\mu_n}(s_{\mu_n}v_0^+ + t_{\mu_n}v_0^-) + \frac{1-s_{\mu_n}^4}{4}\langle J'_{\mu_n}(v_0), v_0^+\rangle + \frac{1-t_{\mu_n}^4}{4}\langle J'_{\mu_n}(v_0), v_0^-\rangle\\ &- s_{\mu_n}t_{\mu_n}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}v_0^+(-\Delta)^{\frac{s}{2}}v_0^-dx - \frac{1-s_{\mu_n}^4}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}v_0^+(-\Delta)^{\frac{s}{2}}v_0^-dx\\ &- \frac{1-t_{\mu_n}^4}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}v_0^+(-\Delta)^{\frac{s}{2}}v_0^-dx.\end{aligned}$$

Hence, by $\langle J'_0(v_0), v_0^\pm\rangle = 0$, we infer that

$$\begin{aligned}m_0 =& J_0(v_0)\\ =& J_{\mu_n}(v_0) - \frac{\mu_n}{4}\int_{\mathbb{R}^3}\phi_{v_0}^t v_0^2dx\\ \geq& J_{\mu_n}(s_{\mu_n}v_0^+ + t_{\mu_n}v_0^-) + \frac{1-s_{\mu_n}^4}{4}\langle J'_{\mu_n}(v_0), v_0^+\rangle + \frac{1-t_{\mu_n}^4}{4}\langle J'_{\mu_n}(v_0), v_0^-\rangle\\ &- s_{\mu_n}t_{\mu_n}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}v_0^+(-\Delta)^{\frac{s}{2}}v_0^-dx - \frac{1-s_{\mu_n}^4}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}v_0^+(-\Delta)^{\frac{s}{2}}v_0^-dx\\ &- \frac{1-t_{\mu_n}^4}{4}\int_{\mathbb{R}^3}(-\Delta)^{\frac{s}{2}}v_0^+(-\Delta)^{\frac{s}{2}}v_0^-dx - \frac{\mu_n}{4}\int_{\mathbb{R}^3}\phi_{v_0}^t v_0^2dx\\ \geq& m_{\mu_n} + \frac{1-s_{\mu_n}^4}{4}\mu_n\int_{\mathbb{R}^3}\phi_{v_0}^t|v_0^+|^2dx + \frac{1-t_{\mu_n}^4}{4}\mu_n\int_{\mathbb{R}^3}\phi_{v_0}^t|v_0^-|^2dx\end{aligned}$$

$$
\begin{aligned}
&- s_{\mu_n} t_{\mu_n} \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx - \frac{1 - s_{\mu_n}^4}{4} \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx \\
&- \frac{1 - t_{\mu_n}^4}{4} \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx - \frac{\mu_n}{4} \int_{\mathbb{R}^3} \phi_{v_0}^t v_0^2 dx \\
=& m_{\mu_n} - \frac{s_{\mu_n}^4}{4} \mu_n \int_{\mathbb{R}^3} \phi_{v_0}^t |v_0^+|^2 dx - \frac{t_{\mu_n}^4}{4} \mu_n \int_{\mathbb{R}^3} \phi_{v_0}^t |v_0^-|^2 dx \\
&- s_{\mu_n} t_{\mu_n} \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx - \frac{1 - s_{\mu_n}^4}{4} \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx \\
&- \frac{1 - t_{\mu_n}^4}{4} \int_{\mathbb{R}^3} (-\Delta)^{\frac{s}{2}} v_0^+ (-\Delta)^{\frac{s}{2}} v_0^- dx.
\end{aligned}
$$

which implies that $\limsup_{n\to\infty} m_{\mu_n} \le m_0$. Then, thanks to (2.16) and the Fatou Lemma, one has

$$
\begin{aligned}
m_0 = J_0(v_0) \le & J_0(u_0) = J_0(u_0) - \frac{1}{4}\langle J_0'(u_0), u_0\rangle \\
=& \frac{1}{4}\|u_0\|_\lambda^2 + \int_{\mathbb{R}^3} \mathcal{F}(u_0) dx \\
\le & \lim_{n\to\infty} \left[\frac{1}{4}\|u_{\mu_n}\|_\lambda^2 + \int_{\mathbb{R}^3} \mathcal{F}(u_{\mu_n}) dx \right] \\
=& \lim_{n\to\infty} \left[J_{\mu_n}(u_{\mu_n}) - \frac{1}{4}\langle J_{\mu_n}'(u_{\mu_n}), u_{\mu_n}\rangle \right] \\
=& \lim_{n\to\infty} J_{\mu_n}(u_{\mu_n}) \\
=& \lim_{n\to\infty} m_{\mu_n} \\
\le & m_0.
\end{aligned}
$$

Hence, $J_0(u_0) = m_0$. In conclusion, u_0 is a ground state sign-changing solution of equation (1.9). By the same proof method as in Theorem 1.1, we can obtain that u_0 has two nodal domains. Hence, we complete the proof of Theorem 1.3. □

Use of AI tools declaration

The authors declare that they have not used artificial intelligence tools in the creation of this article.

Acknowledgements

The authors express their gratitude to the reviewers for their careful reading and helpful suggestions which led to an improvement of the original manuscript. This research was supported by the Natural Science Foundation of Sichuan [2022NSFSC1847].

Huang Xiao-Qing wrote the main manuscript, and Liao Jia-Feng wrote and revised the manuscript.

Conflict of interest

The authors declare no conflict of interest.

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