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Research article

Lie symmetry analysis, particular solutions and conservation laws for the dissipative (2 + 1)- dimensional AKNS equation

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Abstract: The dissipative (2 + 1)-dimensional AKNS equation is considered in this paper. First, the Lie symmetry analysis method is applied to the dissipative (2 + 1)-dimensional AKNS and six point symmetries are obtained. Symmetry reductions are performed by utilizing these obtained point symmetries and four differential equations are derived, including a fourth-order ordinary differential equation and three partial differential equations. Thereafter, the direct integration approach and the $(G'/G^2)-$expansion method are employed to solve the ordinary differential respectively. As a result, a periodic solution in terms of the Weierstrass elliptic function is obtained via the the direct integration approach, while six kinds of including the hyperbolic function types and the hyperbolic function types are derived via the $(G'/G^2)-$expansion method. The corresponding graphical representation of the obtained solutions are presented by choosing suitable parametric values. Finally, the multiplier technique and the classical Noether's theorem are employed to derive conserved vectors for the dissipative (2 + 1)-dimensional AKNS respectively. Consequently, eight local conservation laws for the dissipative (2 + 1)-dimensional AKNS equation are presented by utilizing the multiplier technique and five local conservation laws are derived by invoking Noether's theorem.

Keywords: Lie symmetry analysis; the dissipative (2 + 1)-dimensional AKNS equation; conservation laws; the multiplier technique; Noether's theorem
Mathematics Subject Classification: 35A25, 35G50, 35Q35, 37K10

1. Introduction

It is well known that nonlinear partial differential equations (NPDEs) have extensive applications in depicting numerous nonlinear appeared in many fields of sciences, such as applied mathematics, plasma physics, biology, hydrodynamics, optics, solid state physics and fluid dynamics etc. In order to truly understand these phenomena described in nature, increased by a large number of scientists to seek the exact solutions of NPDEs. With further research, lots of effective methods have been proposed for exact solutions, including the Lie symmetry analysis method [1–3], the Bäcklund transformation method [4], the Darboux transformation method [5], the Hirota bilinear method [6], the $(G'/G)-$expansion method [7], the $(G'/G^2)-$expansion method [8], the extended homoclinic test method [9] and the Riemann-Hilbert method [10]. Among these methods, the Lie symmetry analysis method can reduce the order of NPDEs, thereby simplifying the equations. It has been proved to be one of the most effective methods for achieving exact solutions of NPDEs. So far, the Lie symmetry analysis method has been widely used in solving many mathematical and physical nonlinear models [11, 12]. It is convenient to acquire similarity solutions and some solitary wave solutions of PDEs [13, 14].

In the study of nonlinear partial differential equation, conservation laws also have played an important part, especially in terms of the reduction of PDEs and their solving process [15, 16]. First, conservation laws have been extensively applied to the existence as well as the stability of solutions of nonlinear PDEs. Conservation laws have been used in the achievements of numerical methods. Furthermore, exact solutions of some classical partial differential equations have been obtained using conserved vectors associated with the Lie point symmetries [17, 18]. Recent years, some methods have been put forward for constructing conservation laws of equations, including Noether's theorem [19, 20], the multiplier method [21], the Ibragimov theorem [22, 23] and so on. Generally speaking, Noether's theorem is proved to be an effective method to construct conservation laws for equations with Lagrangian formulation. While the multiplier method and the Ibragimov theorem can be applied to arbitrary equations whether they have the Lagrangian formulation or not. The multiplier method has wider applications, it may require a lot of complex calculations to obtain the multiplier.

In this paper, we aim to consider the dissipative (2+1)-dimensional AKNS equation [24], which can be expressed in the following form

$$4v_t + v_{xxy} + 8vv_y + 4v_x\partial_x^{-1}v_y + \alpha v_x = 0, \tag{1.1}$$

in which ∂_x^{-1} denotes $\int_{-\infty}^{x} v(x, y, t)\mathrm{d}x$ and α denotes an arbitrary nonzero constant, revealing that there is a dissipative effect. If replacing v with u_x in (1.1), by taking the integral constant to be zero, then Eq. (1.1) can be rewritten as

$$4u_{xt} + u_{xxxy} + 8u_xu_{xy} + 4u_{xx}u_y + \alpha u_{xx} = 0, \tag{1.2}$$

which can be used to describe shallow water waves. The term u_{xt} describes the time evolution of waves, while nonlinear terms such as u_x, u_y specify the steepening of waves, remaining terms such as u_{xx}, u_{xy}, u_{xxxy} depict the spreading of waves. When $\alpha = 0$, the equation (1.2) can be degenerated into the (2+1)-dimensional AKNS equation,

$$4u_{xt} + u_{xxxy} + 8u_xu_{xy} + 4u_{xx}u_y = 0.$$

which has been investigated by Najafi et al. in Ref. [25].

The dissipative (2+1)-dimensional AKNS equation (1.2) has been investigated by a diverse group of researchers. Professor Wazwaz [24] achieved the multiple-soliton solutions by employing the simplified Hirota bilinear method. Liu [26] obtained the travelling wave solutions by utilizing the theory of planar dynamical systems and the undetermined coefficient method. Liu et al. [27] derived the bilinear representation, bilinear Bäcklund transformation with the aid of binary Bell polynomials. Cheng et al. [28] constructed the explicitly periodic wave solutions based on a multi-dimensional

Riemann theta function. Wang [29] verified the CRE solvability and presented the soliton-cnoidal wave interaction solutions using the truncated Painlevé expansion and consistent Riccati expansion method. Professor Ma [30] achieved soliton solutions for all the multi-component AKNS integrable hierarchies by employing the Riemann-Hilbert approach. Ma et al. [31] investigated the dynamical analysis of diversity lump solutions to equation (1.2). Furthermore, Ma et al. [32] obtained the full symmetry group and some exact solutions to equation (1.2) using the Lie symmetry method.

Although partial results have been achieved by Ma [32], their results have certain limitations. We would like to conduct further research on equation (1.2). We aim to deeply consider the Lie symmetry analysis, particular solutions as well as conservation laws of equation (1.2). The rest of this paper is organized as follows: In Section 2, we implement the Lie symmetry analysis method to Eq. (1.2) and obtain four reduced equations. In Section 3, we present the solutions of one reduced equation obtained in Section 2 by invoking direct integration method and the (G'/G^2)-expansion method, respectively. In Section 4, we acquire the conservation laws of the equation by utilizing the multiplier method and Noether's theorem, respectively. Some conclusions are made in the final section.

2. Lie symmetry analysis of Eq. (1.2)

First, we employ the classical Lie group method to derive the symmetry group of the dissipative (2 + 1)-dimensional AKNS equation (1.2). We introduce the following one-parameter Lie group of infinitesimal transformation

$$U = \omega^1(x,y,t,u)\frac{\partial}{\partial x} + \omega^2(x,y,t,u)\frac{\partial}{\partial y} + \omega^3(x,y,t,u)\frac{\partial}{\partial t} + \eta(x,y,t,u)\frac{\partial}{\partial u}, \tag{2.1}$$

in which ω^j, $j = 1,2,3$ and η denote the infinitesimal generators.

Symmetries of Eq. (1.2) can be derived from the following symmetry conditions

$$U^{[4]}[4u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y + \alpha u_{xx}] = 0. \tag{2.2}$$

Here $4u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y + \alpha u_{xx} = 0$ and $U^{[4]}$ represents the fourth prolongation of U, which is specified by

$$U^{[4]} = U + \eta^x\frac{\partial}{\partial u_x} + \eta^y\frac{\partial}{\partial u_y} + \eta^{xx}\frac{\partial}{\partial u_{xx}} + \eta^{xy}\frac{\partial}{\partial u_{xy}} + \eta^{xt}\frac{\partial}{\partial u_{xt}} + \eta^{xxxy}\frac{\partial}{\partial u_{xxxy}}.$$

In terms of Eq. (1.2), by expanding the left hand side of equation (2.2) in detail, equating the coefficients for the terms of $4u_{xt}, u_{xxxy}, 8u_x u_{xy}, 4u_{xx}u_y, \alpha u_{xx}$ to be equal, while the coefficients for other derivatives of u to be zero, we get the following linear partial differential equations:

$$\begin{aligned}
&\eta_{xx} = 0, \eta_{xt} = 0, \eta_{xu} = 0, \eta_{yu} = 0, \omega^1_{xx} = 0, \omega^1_y = 0, \omega^2_x = 0, \omega^3_x = 0, \omega^3_y = 0,\\
&\eta_u - 3\omega^1_x - \omega^2_y = \eta_u - \omega^1_x - \omega^3_t = 2\eta_u - 2\omega^1_x - \omega^2_y = \eta_u - 2\omega^1_x - \frac{1}{2}\omega^2_y - \frac{1}{2}\omega^3_t,\\
&2\eta_x - \omega^2_t = 0, 4\eta_y - 4\omega^1_t + \frac{1}{2}\alpha\omega^2_y + \frac{1}{2}\alpha\omega^3_t = 0, 2\eta_{xy} - \omega^1_{xt} + \eta_{tu} = 0.
\end{aligned}$$

By solving above PDEs with Maple, we can get six symmetries as follows:

$$
\begin{aligned}
U_1 &= f(t)\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + f'(t)\frac{\partial}{\partial u},\\
U_2 &= g(t)\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + g'(t)\frac{\partial}{\partial u},\\
U_3 &= 2t\frac{\partial}{\partial y} + x\frac{\partial}{\partial u},\\
U_4 &= \frac{x}{2}\frac{\partial}{\partial x} + t\frac{\partial}{\partial t} + \left(-\frac{\alpha y}{8} - \frac{u}{2}\right)\frac{\partial}{\partial u},\\
U_5 &= -\frac{x}{2}\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + \left(-\frac{\alpha y}{8} + \frac{u}{2}\right)\frac{\partial}{\partial u},\\
U_6 &= \frac{tx}{4}\frac{\partial}{\partial x} + \frac{ty}{2}\frac{\partial}{\partial y} + \frac{t^2}{2}\frac{\partial}{\partial t} + \left(\frac{(-3t\alpha + 4x)y}{16} - \frac{tu}{4}\right)\frac{\partial}{\partial u}.
\end{aligned}
\tag{2.3}
$$

Based upon the theory of the infinitesimal transformation, the corresponding single parameter transformation groups are:

$$
\begin{aligned}
&G_1 : (x^*, y^*, t^*, u^*) \to \left(x + \int^{t+\varepsilon} f(a)\mathrm{d}a, y, t + \varepsilon, u + f(t + \varepsilon)\right),\\
&G_2 : (x^*, y^*, t^*, u^*) \to \left(x + \int^{t+\varepsilon} g(a)\mathrm{d}a, y + \varepsilon, t, u + g(t + \varepsilon)\right),\\
&G_3 : (x^*, y^*, t^*, u^*) \to (x, y + 2\varepsilon t, t, u + \varepsilon x),\\
&G_4 : (x^*, y^*, t^*, u^*) \to (e^{\frac{1}{2}\varepsilon}x, y, e^{\varepsilon}t, e^{-\frac{1}{2}\varepsilon}u - \frac{1}{8}\varepsilon\alpha y),\\
&G_5 : (x^*, y^*, t^*, u^*) \to (e^{-\frac{1}{2}\varepsilon}x, e^{\varepsilon}y, t, e^{\frac{1}{2}\varepsilon}u - \frac{1}{8}\varepsilon\alpha y),\\
&G_6 : (x^*, y^*, t^*, u^*) \to (\frac{x}{1 - \frac{1}{4}\varepsilon t}, \frac{y}{1 - \frac{1}{2}\varepsilon t}, \frac{t}{1 - \frac{1}{2}\varepsilon t}, \frac{\varepsilon(-3t\alpha + 4x)y}{16\sqrt{1 - \frac{1}{2}\varepsilon t}} + u\sqrt{1 - \frac{1}{2}\varepsilon t}).
\end{aligned}
$$

As a result, if $u = \sigma(x, y, t)$ denotes a solution of the dissipative (2 + 1)-dimensional AKNS equation

(1.2), thus so are

$$
\begin{aligned}
u_1 &= f(t+\varepsilon) + \sigma\left(x - \int^{t+\varepsilon} f(a)\mathrm{d}a, y, t-\varepsilon\right),\\
u_2 &= g(y+\varepsilon) + \sigma\left(x - \int^{y+\varepsilon} g(a)\mathrm{d}a, y-\varepsilon, t\right),\\
u_3 &= \varepsilon x + \sigma\left(x, y-2\varepsilon t, t\right),\\
u_4 &= e^{\frac{1}{2}\varepsilon}(-\frac{1}{8}\varepsilon\alpha + \sigma(e^{-\frac{1}{2}\varepsilon}x, y, e^{-\varepsilon}t),\\
u_5 &= e^{-\frac{1}{2}\varepsilon}(-\frac{1}{8}\varepsilon\alpha + \sigma(e^{\frac{1}{2}\varepsilon}x, e^{-\varepsilon}y, e^{-\varepsilon}t),\\
u_6 &= \frac{\varepsilon(-3t\alpha+4x)y}{16-8\varepsilon t} + \frac{1}{\sqrt{1-\frac{1}{2}\varepsilon t}}\sigma((1-\frac{1}{4}\varepsilon t)x, (1-\frac{1}{2}\varepsilon t)y, (1-\frac{1}{2}\varepsilon t)t).
\end{aligned}
$$

Next, we would like to perform symmetry reductions by utilizing the point symmetries obtained in Eq. (2.3).

(i) For symmetry $U_1 + kU_2$ by taking $f(t) = 0$ and $g(t) = 1$ in Eq. (2.3), it yields three invariants

$$X = x - kt, Y = y - kt, u = Q(X, Y). \tag{2.4}$$

By inserting Eq. (2.4) into Eq. (1.2), we obtain

$$-4kQ_{XX} - 4kQ_{XY} + Q_{XXXY} + 8Q_XQ_{XY} + 4Q_{XX}Q_Y + \alpha Q_{XX} = 0. \tag{2.5}$$

Through symbolic computation, Eq. (2.5) admits three symmetries

$$
\begin{aligned}
\Lambda_1 &= \frac{\partial}{\partial X} + F(Y)\frac{\partial}{\partial Y} - \frac{(\alpha-4k)F(Y)}{4}\frac{\partial}{\partial Q}, \Lambda_2 = G(Y)\frac{\partial}{\partial Y} + (-\frac{(\alpha-4k)G(Y)}{4}+1)\frac{\partial}{\partial Q},\\
\Lambda_3 &= x\frac{\partial}{\partial X} + H(Y)\frac{\partial}{\partial Y} + \left(-\frac{(\alpha-4k)F(Y)}{4} + \frac{(4X+4Y)k}{4} - \frac{\alpha Y}{4} - Q\right)\frac{\partial}{\partial Q},
\end{aligned} \tag{2.6}
$$

Considering the linear combination $\Lambda_1 - \Lambda_2$ in Eq. (2.6), by choosing $F(Y) = -\frac{8}{\alpha-4k}, G(Y) = -\frac{4}{\alpha-4k}$, two invariants for Eq. (2.5) can be derived

$$\chi = \frac{4X + (\alpha-4k)\,Y}{\alpha-4k}, Q = \psi. \tag{2.7}$$

By inserting Eq. (2.7) into Eq. (2.5), we have

$$(1-k)(\alpha-4k)^2\psi_{\chi\chi} + 8(\alpha-4k)\psi_\chi\psi_{\chi\chi} + 4\psi_{\chi\chi\chi\chi} = 0. \tag{2.8}$$

(ii) For symmetry U_3, it admits three invariants

$$X = x, T = t, u = Q(X,T) + \frac{xy}{2t}. \tag{2.9}$$

By substituting Eq. (2.9) into Eq. (1.2), we can obtain the following equation

$$4TQ_{XT} + 4Q_X + 2XQ_{XX} + \alpha TQ_{XX} = 0. \tag{2.10}$$

(iii) For symmetry $U_4 - U_5$, three invariants can be obtained

$$X = xy, T = \frac{t}{x}, u = \frac{Q(X,T)}{x}. \tag{2.11}$$

By substituting Eq. (2.11) into Eq. (1.2), we have

$$\begin{aligned}
&-16XTQ_XQ_{XT} - 8XTQ_{XX}Q_T - 2\alpha XTQ_{XT} - 3X^2TQ_{XXXT} + 3XT^2Q_{XXTT} + 12X^2Q_XQ_{XX}\\
&+8T^2Q_{XT}Q_T - 8XQQ_{XX} + 8TQQ_{XT} + 4T^2Q_XQ_{TT} + 16TQ_XQ_T + \alpha X^2Q_{XX} + \alpha T^2Q_{TT}\\
&+4\alpha TQ_T - 2\alpha XQ_X - 8Q_T - 4TQ_{TT} + 4XQ_{XT} + X^3Q_{XXXX} - 6T^2Q_{XTT} - T^3Q_{XTTT}\\
&+8QQ_X - 8XQ_X^2 - 6TQ_{XT} + 6XTQ_{XXT} + 2\alpha Q = 0.
\end{aligned} \tag{2.12}$$

(iv) For symmetry U_6, it admits three invariants

$$X = \frac{y}{x^2}, T = \frac{t}{x^2}, u = \frac{Q(X,T)}{x} - \frac{\alpha y}{4} + \frac{xy}{2t}. \tag{2.13}$$

By substituting (2.13) into Eq. (1.2), we have

$$\begin{aligned}
&-24X^2TQ_{XXXT} - 144XTQ_{XXT} - 24XT^2Q_{XXTT} + 48X^2Q_XQ_{XX} + 88TQ_XQ_T\\
&+32T^2Q_{XT}Q_T + 16XQQ_{XX} + 16TQQ_{XT} + 16XTQ_XQ_{TT} - 8X^3Q_{XXXX} - 72X^2Q_{XXX}\\
&-72T^2Q_{XTT} - 8T^3Q_{XTTT} - 150XQ_{XX} - 150TQ_{XT} + 88XQ_X^2 + 32QQ_X - 60Q_X\\
&+64XTQ_XQ_{XT} + 32XTQ_{XX}Q_T = 0.
\end{aligned} \tag{2.14}$$

3. Particular solutions for the (2+1)-dimensional dissipative AKNS equation

3.1. *Solutions of Eq.* (2.8) *via direct integration approach*

First of all, Eq. (2.8) can be rewritten as

$$a\psi_{\chi\chi} + 8b\psi_\chi\psi_{\chi\chi} + 4\psi_{\chi\chi\chi\chi} = 0, \tag{3.1}$$

where $a = (1-k)(\alpha - 4k)^2, b = (\alpha - 4k)$. Integrating Eq. (3.1) once with respect to χ, then we get

$$a\psi_\chi + 4b\psi_\chi^2 + 4\psi_{\chi\chi\chi} + C_1 = 0,$$

where C_1 is an integration constant. Multiplying Eq. (3.1) with $\psi_{\chi\chi}$ and integrating again yields

$$\frac{1}{2}a\psi_\chi^2 + \frac{4b}{3}\psi_\chi^3 + 2\psi_{\chi\chi}^2 + C_1\psi_\chi + C_2 = 0, \tag{3.2}$$

where C_2 is also the integration constant. Denoting ψ_χ as Φ, Eq. (3.2) can be rewritten as

$$\Phi'^2 = -\frac{2b}{3}\Phi^3 - \frac{a}{4}\Phi^2 - \frac{C_1}{2}\Phi - \frac{C_2}{2}. \tag{3.3}$$

By utilizing the following transformation

$$\Phi = -\frac{6}{b}\varphi(\chi) - \frac{a}{8b}, \tag{3.4}$$

then Eq. (3.3) can be changed into the well known Weierstrass elliptic function equation

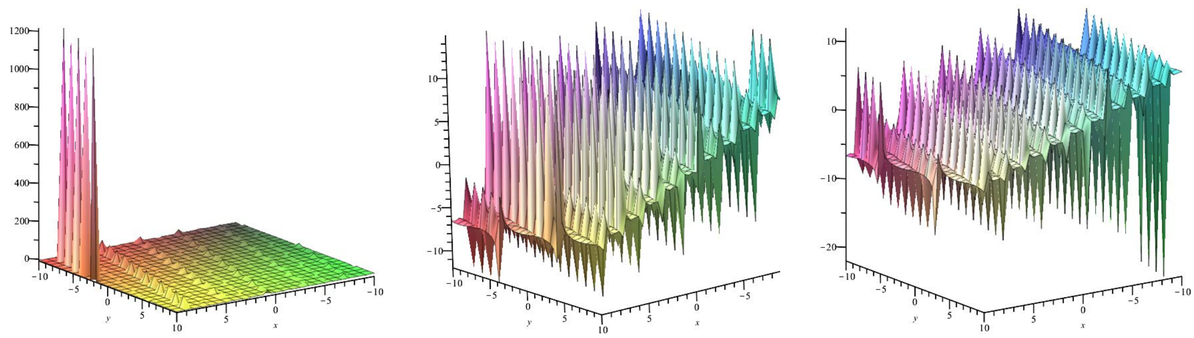
$$\varphi'^2 = 4\varphi^3 - g_2\varphi - g_3,$$

in which $g_2 = -\frac{a^2}{192} + \frac{bC_1}{12}, g_3 = -\frac{1}{13824}a^3 + \frac{1}{576}abC_1 - \frac{1}{72}b^2C_2$. Therefore, integrating Eq. (3.4) with respect to χ and returning to variables t, x, y, we obtain the solution of (2+1)-dimensional AKNS equation (1.2)

$$u = \frac{6}{b}\varphi(\chi; g_2, g_3) - \frac{a}{8b}, \tag{3.5}$$

in which $\varphi(\chi; g_2, g_3)$ denotes the Weierstrass zeta function and $\varphi'(\chi; g_2, g_3) = -\varphi(\chi; g_2, g_3), a = (1 - k)(\alpha - 4k)^2, b = (\alpha - 4k), \chi = -kt - \frac{4kt}{\alpha-4k} + \frac{4x}{\alpha-4k} + y$.

By selecting $\alpha = 1, k = \frac{1}{2}, C_1 = 2, C_2 = 3$ in (3.5), we present the evolution process of solution (3.5) in Figure 1. At $t = 1$, there are several water waves with large peaks in Figure 1(a). When $t = 2$, these water waves exhibit certain periodicity in Figure 1(b). When $t = 3$, the peaks get shorter and the troughs gradually deepen in Figure 1(c).



(a) t=1 (b) t=2 (c) t=3

Figure 1. (Color online) The evolution for the Weierstrass zeta function solution (3.5).

3.2. *Solutions of Eq.* (2.8) *by* (G'/G^2)*-expansion method*

Next, we would like to investigate the solutions of Eq. (2.8) by employing the (G'/G^2)-expansion method. By balancing the highest order derivative $\psi_{\chi\chi\chi\chi}$ and the nonlinear term of the highest order $\psi_\chi\psi_{\chi\chi}$, we obtain $N = 1$, which is the order of the solution, so the expression on the solution of Eq. (2.8) can be written as

$$\psi = a_{-1}\left(\frac{G'}{G^2}\right)^{-1} + a_0 + a_1\left(\frac{G'}{G^2}\right), \tag{3.6}$$

in which G denotes the function of χ and satisfies the following Riccati equation

$$\left(\frac{G'}{G^2}\right)' = \mu + \lambda\left(\frac{G'}{G^2}\right)^2. \tag{3.7}$$

By substituting Eq. (3.6) into Eq. (2.8) together with Eq. (3.7) for simplification, collecting on the same powers of $\left(\frac{G'}{G^2}\right)^{-1}$ and $\left(\frac{G'}{G^2}\right)$, and taking all the resulted coefficients as zero, then six algebraic equations can be derived:

$$\left(\frac{G'}{G^2}\right)^5 : 16a_1^2\alpha\lambda^3 - 64a_1^2k\lambda^3 + 96a_1\lambda^4$$

$$\begin{aligned}\left(\frac{G'}{G^2}\right)^3 :& - 16a_{-1}a_1\alpha\lambda^3 + 64a_{-1}a_1k\lambda^3 + 32a_1^2\alpha\lambda^2\mu - 128a_1^2k\lambda^2\mu - 2a_1\alpha^2k\lambda^2 + 16a_1\alpha k^2\lambda^2 - 32a_1k^3\lambda^2\\ &+ 2a_1\alpha^2\lambda^2 - 16a_1\alpha k\lambda^2 + 32a_1k^2\lambda^2 + 160a_1\lambda^3\mu = 0,\end{aligned}$$

$$\begin{aligned}\left(\frac{G'}{G^2}\right) :& - 16a_{-1}a_1\alpha\lambda^2\mu + 64a_{-1}a_1k\lambda^2\mu + 16a_1^2\alpha\lambda\mu^2 - 64a_1^2k\lambda\mu^2 - 2a_1\alpha^2k\lambda\mu + 16a_1\alpha k^2\lambda\mu\\ &- 32a_1k^3\lambda\mu + 2a_1\alpha^2\lambda\mu - 16a_1\alpha k\lambda\mu + 32a_1k^2\lambda\mu + 64a_1\lambda^2\mu^2 = 0,\end{aligned}$$

$$\begin{aligned}\left(\frac{G'}{G^2}\right)^{-1} :& - 16a_{-1}^2\alpha\lambda^2\mu + 64a_{-1}^2k\lambda^2\mu + 16a_{-1}a_1\alpha\lambda\mu^2 - 64a_{-1}a_1k\lambda\mu^2 - 2a_{-1}\alpha^2k\lambda\mu + 16a_{-1}\alpha k^2\lambda\mu\\ &- 32a_{-1}k^3\lambda\mu + 2a_{-1}\alpha^2\lambda\mu - 16a_{-1}\alpha k\lambda\mu + 32a_{-1}k^2\lambda\mu + 64a_{-1}\lambda^2\mu^2 = 0.\end{aligned}$$

$$\begin{aligned}\left(\frac{G'}{G^2}\right)^{-3} :& - 32a_{-1}^2\alpha\lambda\mu^2 + 128a_{-1}^2k\lambda\mu^2 + 16a_{-1}a_1\alpha\mu^3 - 64a_{-1}a_1k\mu^3 - 2a_{-1}\alpha^2k\mu^2 + 16a_{-1}\alpha k^2\mu^2\\ &- 32a_{-1}k^3\mu^2 + 2a_{-1}\alpha^2\mu^2 - 16a_{-1}\alpha k\mu^2 + 32a_{-1}k^2\mu^2 + 160a_{-1}\lambda\mu^3 = 0,\end{aligned}$$

$$\left(\frac{G'}{G^2}\right)^{-5} : - 16a_{-1}^2\alpha\mu^3 + 64a_{-1}^2k\mu^3 + 96a_{-1}\mu^4 = 0.$$

By solving the above equations with Maple, three kinds of solution sets can be obtained:

Solution set 1.

$$\begin{aligned}&a_{-1} = 0, a_0 = a_0, a_1 = RootOf\Big(16\mu\_Z^3 + (9\alpha\lambda - 36\lambda)\_Z + 54\lambda^2\Big),\\ &k = \frac{RootOf\Big(16\mu\_Z^3 + (9\alpha\lambda - 36\lambda)\_Z + 54\lambda^2\Big)\alpha + 6\lambda}{4RootOf\Big(16\mu\_Z^3 + (9\alpha\lambda - 36\lambda)\_Z + 54\lambda^2\Big)}.\end{aligned} \tag{3.8}$$

Solution set 2.

$$\begin{aligned}&a_{-1} = RootOf\Big(16\lambda\_Z^3 + (9\alpha\mu - 36\mu)\_Z - 54\mu^2\Big), a_0 = a_0, a_1 = 0,\\ &k = \frac{\alpha RootOf\Big(16\lambda\_Z^3 + (9\alpha\mu - 36\mu)\_Z - 54\mu^2\Big) - 6\mu}{4RootOf\Big(16\lambda\_Z^3 + (9\alpha\mu - 36\mu)\_Z - 54\mu^2\Big)}.\end{aligned} \tag{3.9}$$

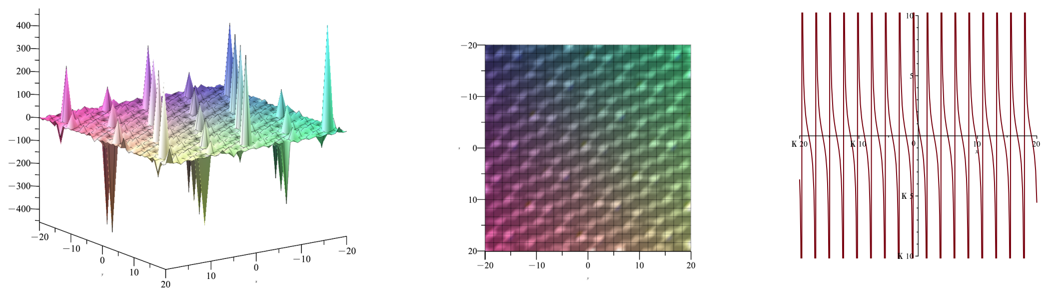
Solution set 3.

$$a_{-1} = RootOf\Big(64\lambda\_Z^3 + (9\alpha\mu - 36\mu)\_Z - 54\mu^2\Big), a_0 = a_0, a_1 =$$
$$\frac{9RootOf\left(64\lambda\_Z^3 + (9\alpha\mu - 36\mu)\_Z - 54\mu^2\right)\alpha - 36RootOf\left(64\lambda\_Z^3 + (9\alpha\mu - 36\mu)\_Z - 54\mu^2\right) - 54\mu}{64RootOf\Big(64\lambda\_Z^3 + (9\alpha\mu - 36\mu)\_Z - 54\mu^2\Big)^2},$$
$$k = \frac{RootOf\Big(64\lambda\_Z^3 + (9\alpha\mu - 36\mu)\_Z - 54\mu^2\Big)\alpha - 6\mu}{4RootOf\Big(64\lambda\_Z^3 + (9\alpha\mu - 36\mu)\_Z - 54\mu^2\Big)}. \tag{3.10}$$

For solution set 1, when $\mu\lambda > 0$, we get the following trigonometric function solution

$$u(x, y, t) = a_0 + a_1\sqrt{\frac{\mu}{\lambda}}\left(\frac{C_1\cos(\sqrt{\mu\lambda}\chi) + C_2\sin(\sqrt{\mu\lambda}\chi)}{C_2\cos(\sqrt{\mu\lambda}\chi) - C_1\sin(\sqrt{\mu\lambda}\chi)}\right), \tag{3.11}$$

where C_1, C_2 are arbitrary nonzero constants and $\chi = -kt - \frac{4kt}{\alpha-4k} + \frac{4x}{\alpha-4k} + y$, while a_0, a_1, k are determined by (3.8). By choosing $\alpha = 2, \lambda = 1, \mu = 3, C_1 = \frac{1}{2}, C_2 = \frac{1}{3}, a_0 = 0$, we present the profiles of solution (3.11) in Figure 2. Obviously, Eq. (3.11) is a rational solution in the form trigonometric functions. Certain periodicity can be seen from Figure 2(a). While according to the changes in Figure 2(b), we can see that the motion of waves is irregular. The characteristics of the cotangent function are shown in Figure 2(c).



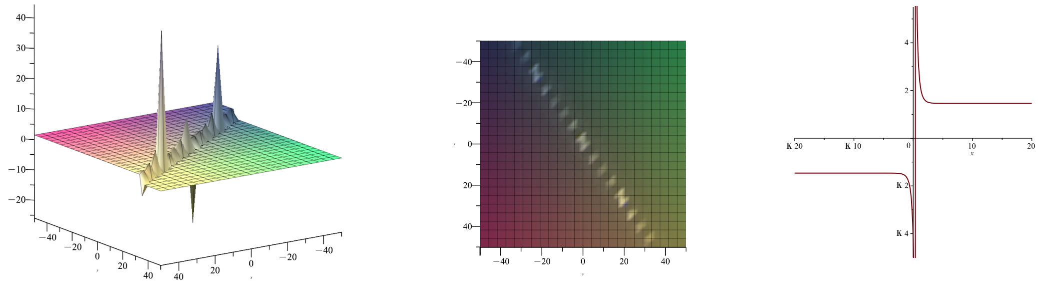
(a) (b) (c)

Figure 2. (Color online) Profiles for the trigonometric function solution (3.11) at $t = 0$. (a) 3D profile from the perspective view; (b) 2D profile from the overhead view; (c) The wave of the propagation style along the x axis for $y = 1$.

For solution set 1, when $\mu\lambda < 0$, we get the following hyperbolic function solution

$$u(x,y,t) = a_0 - \frac{a_1\sqrt{|\lambda\mu|}}{\lambda}\left(\frac{C_1\sinh(2\sqrt{|\lambda\mu|}\chi) + C_1\cosh(2\sqrt{|\lambda\mu|}\chi) + C_2}{C_1\sinh(2\sqrt{|\lambda\mu|}\chi) + C_1\cosh(2\sqrt{|\lambda\mu|}\chi) - C_2}\right), \tag{3.12}$$

where C_1, C_2 are arbitrary nonzero constants and $\chi = -kt - \frac{4kt}{\alpha-4k} + \frac{4x}{\alpha-4k} + y$, while a_0, a_1, k are determined by (3.8). By choosing $\alpha = 2, \lambda = 1, \mu = -2, C_1 = 2, C_2 = 3, a_0 = 0$, the profiles of solution (3.12) are shown in Figure 3. Clearly, Eq. (3.12) is a hyperbolic functions solution. A wave with different peaks is shown in Figure 3(a). The wave mark is clearly visible in Figure 3(b). The characteristics of the hyperbolic cotangent function are shown in Figure 3(c).



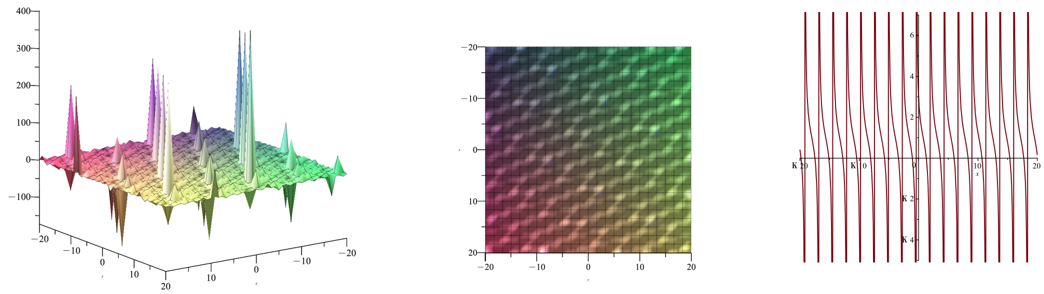
(a) (b) (c)

Figure 3. (Color online) Profile for the hyperbolic function solution (3.12) at $t = 1$. (a) 3D profile from the perspective view; (b) 2D profile from the overhead view; (c) The wave of the propagation style along the x axis for $y = 1$.

For solution set 2, when $\mu\lambda > 0$, we get the following trigonometric function solution

$$u(x,y,t) = a_{-1}\sqrt{\frac{\lambda}{\mu}}\left(\frac{C_2\cos(\sqrt{\mu\lambda}\chi) - C_1\sin(\sqrt{\mu\lambda}\chi)}{C_1\cos(\sqrt{\mu\lambda}\chi) + C_2\sin(\sqrt{\mu\lambda}\chi)}\right) + a_0. \tag{3.13}$$

where C_1, C_2 are arbitrary nonzero constants and $\chi = -kt - \frac{4kt}{\alpha-4k} + \frac{4x}{\alpha-4k} + y$, while a_{-1}, a_0, k are determined by (3.9). By choosing $\alpha = 2, \lambda = 3, \mu = 1, C_1 = \frac{1}{3}, C_2 = \frac{1}{4}, a_0 = 1$, the profiles of solution (3.13) are shown in Figure 4. Although Eq. (3.13) is also a rational solution in the form trigonometric functions, it is seeming like the inverse of solution (3.11) from the perspective of representation. Multi peaks and troughs are exhibited in Figure 4(a). According to the heights of the peaks appeared in Figure 4(a) and the changes in Figure 4(b), we can see that the period of fluctuations is not fixed. The characteristics of the cotangent functions with very small period are shown in Figure 4(c).



(a) (b) (c)

Figure 4. (Color online) Profiles for the trigonometric function solution (3.13) at $t = 1$. (a) 3D profile from the perspective view; (b) 2D profile from the overhead view; (c) The wave of the propagation style along the x axis with $y = 1$.

For solution set 2, when $\mu\lambda < 0$, the following hyperbolic function solution can be achieved

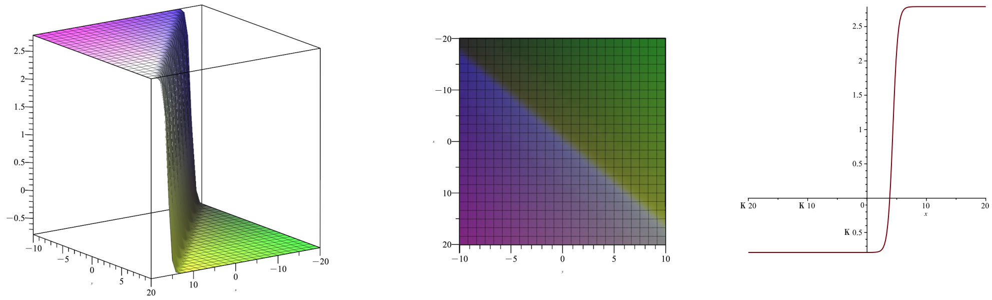
$$u(x,y,t) = -\frac{a_{-1}\lambda}{\sqrt{|\lambda\mu|}}\left(\frac{C_1\sinh(2\sqrt{|\lambda\mu|}\chi) + C_1\cosh(2\sqrt{|\lambda\mu|}\chi) - C_2}{C_1\sinh(2\sqrt{|\lambda\mu|}\chi) + C_1\cosh(2\sqrt{|\lambda\mu|}\chi) + C_2}\right) + a_0, \tag{3.14}$$

where C_1, C_2 are arbitrary nonzero constants and $\chi = -kt - \frac{4kt}{\alpha-4k} + \frac{4x}{\alpha-4k} + y$, while a_{-1}, a_0, k are determined by (3.9). Noticeably, solution (3.14) is seeming like the inverse of solution (3.12). By choosing $\alpha = 3, \lambda = -2, \mu = 2, C_1 = 1, C_2 = 5, a_0 = 1$, the profiles of solution (3.14) are shown in Figure 5. By observation, Eq. (3.14) is a hyperbolic functions solution. A wave seeming like waterfall is shown in Figure 5(a). The dividing line is clearly visible in Figure 5(b). While the features of the hyperbolic tangent function are shown in Figure 5(c).

For solution set 3, when $\mu\lambda > 0$, the following trigonometric function solution can be acquired

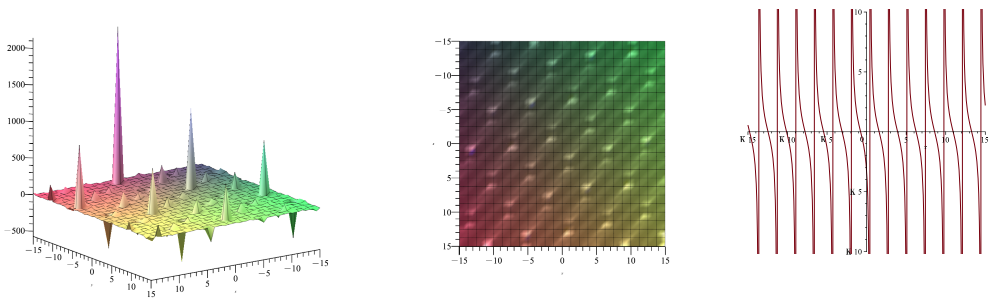
$$u(x,y,t) = a_{-1}\sqrt{\frac{\lambda}{\mu}}\left(\frac{C_2\cos(\sqrt{\mu\lambda}\chi) - C_1\sin(\sqrt{\mu\lambda}\chi)}{C_1\cos(\sqrt{\mu\lambda}\chi) + C_2\sin(\sqrt{\mu\lambda}\chi)}\right) + a_0 + a_1\sqrt{\frac{\mu}{\lambda}}\left(\frac{C_1\cos(\sqrt{\mu\lambda}\chi) + C_2\sin(\sqrt{\mu\lambda}\chi)}{C_2\cos(\sqrt{\mu\lambda}\chi) - C_1\sin(\sqrt{\mu\lambda}\chi)}\right), \tag{3.15}$$

in which C_1, C_2 represents arbitrary nonzero constants and $\chi = -kt - \frac{4kt}{\alpha-4k} + \frac{4x}{\alpha-4k} + y$, while a_{-1}, a_0, a_1, k are determined by(3.10). Noticeably, solution (3.15) is seeming like the superposition of solution (3.11) and (3.13). By choosing $\alpha = 1, \lambda = 1, \mu = 2, C_1 = \frac{1}{2}, C_2 = -\frac{1}{3}, a_0 = 0$, the profiles of solution (3.15) are presented in Figure 6. By analysis, Eq. (3.15) is an intersection solution between solution (3.11) and (3.13). Periodicity is no longer appeared in Figure 6(a). Multi peaks with different heights can be observed in Figure 6(b). Features of the hyperbolic cotangent functions with very small period are shown in Figure 6(c).



(a) (b) (c)

Figure 5. (Color online) Profiles for the hyperbolic function solution (3.14) at $t = 0$. (a) 3D profile from the perspective view; (b) 2D profile from the overhead view ;(c) The wave of the propagation style along the $y = 1$.



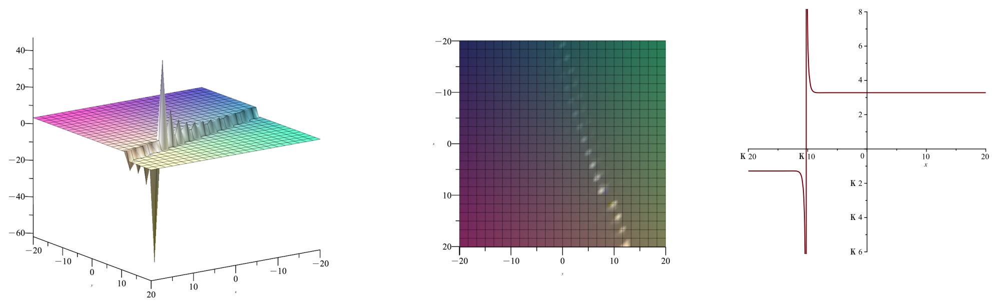
(a) (b) (c)

Figure 6. (Color online) Profiles for the trigonometric function solution (3.15) at $t = 0$. (a) 3D profile from the perspective view; (b) 2D profile from the overhead view; (c) The wave of the propagation style along the x axis with $y = 1$.

For solution set 3, when $\mu\lambda < 0$, the following hyperbolic function solution can be obtained

$$\begin{aligned} u(x,y,t) = &- \frac{a_{-1}\lambda}{\sqrt{|\lambda\mu|}} \left(\frac{C_1 \sinh(2\sqrt{|\lambda\mu|}\chi) + C_1 \cosh(2\sqrt{|\lambda\mu|}\chi) - C_2}{C_1 \sinh(2\sqrt{|\lambda\mu|}\chi) + C_1 \cosh(2\sqrt{|\lambda\mu|}\chi) + C_2} \right) + a_0 \\ &- \frac{a_1\sqrt{|\lambda\mu|}}{\lambda} \left(\frac{C_1 \sinh(2\sqrt{|\lambda\mu|}\chi) + C_1 \cosh(2\sqrt{|\lambda\mu|}\chi) + C_2}{C_1 \sinh(2\sqrt{|\lambda\mu|}\chi) + C_1 \cosh(2\sqrt{|\lambda\mu|}\chi) - C_2} \right), \end{aligned} \tag{3.16}$$

where C_1, C_2 are arbitrary nonzero constants and $\chi = -kt - \frac{4kt}{\alpha-4k} + \frac{4x}{\alpha-4k} + y$, while a_{-1}, a_0, a_1, k are determined by (3.10). Noticeably, solution (3.16) is seeming like the superposition of solution (3.12) and (3.14). By choosing $\alpha = 2, \lambda = -2, \mu = 3, C_1 = 20, C_2 = 30, a_0 = 0$, the profiles of solution (3.16) in are presented in Figure 7. By analysis, Eq. (3.16) is an intersection solution between solution (3.12) and (3.14). A wave with a huge peak and a deep trough is presented in Figure 7(a). The wave mark gradually becomes blurred in Figure 7(b). While the features of the hyperbolic cotangent function are shown in Figure 7(c).



(a) (b) (c)

Figure 7. (Color online) Profiles for the hyperbolic function solution (3.16) at $t = 2$. (a) 3D profile from the perspective view; (b) 2D profile from the overhead view ;(c) The wave of the propagation style along the x axis with $y = 2$.

4. Conservation laws

In what follows, we would like to consider the conservation laws of the (2+1)-dimensional dissipative AKNS equation (1.2) by invoking the multiplier method and Noether's theorem respectively.

4.1. Conservation laws by the multiplier method

In this part, we aim to utilizing the multiplier method to determine the conserved currents of the dissipative AKNS equation (1.2). In order to get the first order multipliers P, namely

$$P = P(x, y, t, u, u_x, u_y, u_t). \tag{4.1}$$

These corresponding multipliers can be derived by

$$\frac{\delta}{\delta u}\left[P(4u_{xt} + u_{xxxy} + 8u_x u_{xy} + 4u_{xx}u_y + \alpha u_{xx})\right] = 0, \tag{4.2}$$

where

$$\frac{\delta}{\delta u} = \frac{\partial}{\partial u} - D_x\frac{\partial}{\partial u_x} - D_y\frac{\partial}{\partial u_y} - D_t\frac{\partial}{\partial u_t} + D_x^2\frac{\partial}{\partial u_{xx}} + D_xD_y\frac{\partial}{\partial u_{xy}} + D_xD_t\frac{\partial}{\partial u_{xt}} + D_x^3D_y\frac{\partial}{\partial u_{xxxy}}$$

represents the Euler operator and D_x, D_y, D_t represent the total derivative operators. By expanding (4.2) in detail, collecting on derivatives of u from second order to fourth order and by equating their coefficients to be zero, we get twenty-one PDEs:

$$\begin{aligned}
&P_{uu} = 0, P_{u_xu_x} = 0, P_{u_yu_y} = 0, P_{u_tu_t} = 0, P_{uu_x} = 0, P_{uu_y} = 0, P_{uu_t} = 0, P_{u_xu_y} = 0, P_{u_xu_t} = 0,\\
&P_{u_yu_t} = 0, P_{xu_y} = 0, P_{xu_t} = 0, P_{yu_x} = 0, P_{yu_t} = 0, P_{yu} = 0, 2P_u + 2P_{xu_x} - P_{tu_t} = 0,\\
&2P_u - P_{yu_y} = 0, P_{xu} + P_{xxu_x} = 0, 8P_x - 8u_xP_{tu_t} + 24u_xP_u + 3P_{xxu} + P_{xxxu_x} + 4P_{tu_y} = 0,\\
&-4u_yP_{yu_y} - 4u_yP_{tu_t} + 12u_yP_u + 4u_yP_{xu_x} - \alpha P_{yu_y} - \alpha P_{tu_t} + 2\alpha P_u + \alpha P_{xu_x} + 4P_{tu_x} + 4P_y = 0,\\
&16u_xu_yP_{xu} + 2\alpha u_xP_{xu} + u_yP_{xxxu} + 4u_yP_{xx} + 4u_tP_{xu} + 4u_xP_{tu} + 8u_xP_{xy} + \alpha P_{xx} + 4P_{xt} + P_{xxxy} = 0.
\end{aligned}$$

By solving above PDEs with the aid of maple, we obtain the following results

$$\begin{aligned}
P = {} & g(t) + f'(t)y - f(t)u_x + C_1\left(-\frac{3}{8}\alpha t^2u_x - 2t^2u_t - xtu_x - 2ytu_y - tu + xy\right)\\
& + C_2\left(\frac{3}{4}\alpha tu_x + 4tu_t + u + 2yu_y + xu_x\right) + C_3(x - 2tu_y) + C_4u_x + C_5u_y + C_6u_t.
\end{aligned}$$

in which C_1, C_2, C_3, C_4, C_5 represent arbitrary constants , while g, f are functions of t. The conserved currents of equation (1.2) can be obtained by utilizing the following divergence expression

$$D_xC^x + D_yC^y + D_tC^t = P(4u_{xt} + u_{xxxy} + 8u_xu_{xy} + 4u_{xx}u_y + \alpha u_{xx}), \tag{4.3}$$

where C^x, C^y are spatial fluxes and C_t is the conserved density. As a result, we obtain eight conserved vectors (C^x, C^y, C^t) according to eight multipliers:

Case 1 Corresponding to $P_1 = -\frac{3}{8}\alpha t^2u_x - 2t^2u_t - xtu_x - 2ytu_y - tu + xy$, we achieve the following conserved vector

$$
\begin{aligned}
C_1^x &= -\frac{3}{8}\alpha t^2 u_x u_{xxy} - \frac{3}{4}\alpha t^2 u_x^2 u_y - \frac{3}{16}\alpha^2 t^2 u_x^2 - 4t^2 u_t^2 - t^2 u_{xxt} u_y + t^2 u_{xy} u_{xt} - t^2 u_t u_{xxy} \\
&+ \frac{8}{3} t^2 u u_{xt} u_y - \frac{8}{3} t^2 u u_{xy} u_t - 8t^2 u_x u_y u_t - 2\alpha t^2 u_x u_t - 2\alpha t^2 u_x u_t - xtu_x u_{xxy} - 2xtu_x^2 u_y \\
&- \frac{1}{2}\alpha xtu_x^2 - 4ytu_y u_t - 2ytu_{xxy} u_y + ytu_{xy}^2 - 4ytu_x u_y^2 - 2\alpha ytu_x u_y - 4tuu_t + tuu_{xxy} + \frac{4}{3} tuu_x u_y \\
&+ 2tu_x^2 u - \alpha tuu_x + 4xyu_x - xu_{xx} + u_x + 4xyu_x u_y + \alpha xyu_x - \alpha yu, \\
C_1^y &= \frac{3}{16}\alpha t^2 u_{xx}^2 - \frac{3}{4}\alpha t^2 u_x^3 + t^2 u u_{xxxt} + \frac{4}{3} t^2 u u_{xx} u_t + \frac{16}{3} t^2 u u_x u_{xt} + \frac{1}{2} xtu_{xx}^2 + 2xtu_x^3 + 4ytu_x u_t \\
&+ \alpha ytu_x^2 - \frac{4}{3} tuu_x^2 + xyu_{xxx} + 2xyu_x^2, \\
C_1^t &= \frac{1}{4}\alpha t^2 u_x^2 - t^2 u u_{xxxy} - \frac{16}{3} t^2 u u_x u_{xy} - \frac{8}{3} t^2 u u_{xx} u_y - 2xtu_x^2 - 4ytu_x u_y.
\end{aligned}
$$

Case 2 Corresponding to $P_2 = \frac{3}{4}\alpha tu_x + 4tu_t + u + 2yu_y + xu_x$, we achieve the following conserved vector

$$
\begin{aligned}
C_2^x &= \frac{3}{4}\alpha tu_x u_{xxy} + \frac{3}{2}\alpha tu_x^2 u_y + \frac{3}{8}\alpha^2 tu_x^2 + 8tu_t^2 + 2tu_{xxy} u_t + 2tu_{xxt} u_y - 2tu_{xy} u_{xt} - \frac{16}{3} tuu_{xt} u_y \\
&+ \frac{16}{3} tuu_{xy} u_t + 16u_x u_y u_t + 4\alpha tu_x u_t + 4uu_t - uu_{xxy} + \frac{2}{3} uu_x u_y + 2u^2 u_{xy} - uu_x + 4yu_y u_t \\
&+ 2yu_{xxy} u_y - yu_{xy}^2 + 8yu_x u_y^2 - 2yuu_{xy} + xu_x u_{xxy} + 2xu_x^2 u_y + \frac{1}{2}\alpha xu_x^2, \\
C_2^y &= -\frac{3}{8}\alpha tu_{xx}^2 + \frac{3}{2}\alpha tu_x^3 - 2tuu_{xxxt} - \frac{16}{3} tuu_{xx} u_t - \frac{32}{3} tuu_x u_{xt} - \frac{8}{3} uu_x^2 - 2u^2 u_{xx} - 4yu_x u_t \\
&+ 2\alpha yuu_{xx} + yu_x^2 - \frac{1}{2} xu_{xx}^2 + 2xu_x^3, \\
C_2^t &= -\frac{1}{2}\alpha tu_x^2 + 2tuu_{xxxy} + \frac{32}{3} tuu_x u_{xy} + \frac{16}{3} tuu_{xx} u_y + 4yu_x u_y + 2xu_x^2.
\end{aligned}
$$

Case 3 Corresponding to $P_3 = x - 2tu_y$, we achieve the following conserved vector

$$
\begin{aligned}
C_3^x &= 2xuu_{xy} + 6xu_x u_y + \alpha xu_x - \alpha u - 4tu_y u_t - 2tu_{xxy} u_y + tu_{xy}^2 - 8tu_x u_y^2 - 2\alpha tu_x u_y, \\
C_3^y &= xu_{xxx} - 2xuu_{xx} - 2uu_x + 4tu_x u_t + \alpha tu_x^2, \\
C_3^t &= 4xu_x - 4tu_x u_y.
\end{aligned}
$$

Case 4 Corresponding to $P_4 = u_x$, we acquire the following conserved vector

$$
\begin{aligned}
C_4^x &= \frac{1}{2}\alpha u_x^2 + u_x u_{xxy} + 2u_x^2 u_y, \\
C_4^y &= 2u_x^3 - \frac{1}{2} u_{xx}^2, \\
C_4^t &= 2u_x^2.
\end{aligned}
$$

Case 5 Corresponding to $P_5 = u_y$, we acquire the following conserved vector

$$\begin{aligned}
C_5^x &= 2u_y u_t + u_y u_{xxy} - \frac{1}{2}u_{xy}^2 + 4u_x u_y^2 + \alpha u_x u_y,\\
C_5^y &= -2u_x u_t - \frac{1}{2}\alpha u_x^2,\\
C_5^t &= 2u_x u_y.
\end{aligned}$$

Case 6 Corresponding to $P_6 = u_t$, we acquire the following conserved vector

$$\begin{aligned}
C_6^x &= \frac{1}{2}u_{xxy}u_t + \frac{1}{2}u_{xxt}u_y - \frac{1}{2}u_{xy}u_{xt} + \frac{4}{3}uu_{xy}u_t - \frac{4}{3}uu_{xt}u_y + 4u_x u_y u_t + \alpha u_x u_t,\\
C_6^y &= -\frac{1}{2}uu_{xxxt} - \frac{4}{3}uu_{xx}u_t - \frac{8}{3}uu_x u_{xt},\\
C_6^t &= 2u_x^2 + \frac{1}{2}uu_{xxxy} + \frac{8}{3}uu_x u_{xy} + \frac{4}{3}uu_{xx}u_y - \frac{1}{2}\alpha u_x^2.
\end{aligned}$$

Case 7 Corresponding to $P_7 = f'(t)y - f(t)u_x$, we acquire the following conserved vector

$$\begin{aligned}
C_7^x &= 4f'(t)yu_t + f'(t)yu_{xxy} + 6f'(t)yu_x u_y + 2f'(t)yuu_{xy} + 2f'(t)uu_x + \alpha f'(t)yu_x - f(t)u_x u_{xxy}\\
&\quad - 4f(t)u_x^2 u_y - 4f(t)uu_x u_{xy} - \frac{1}{2}\alpha f(t)u_x^2,\\
C_7^y &= -2f'(t)yuu_{xx} + \frac{1}{2}f(t)u_{xx}^2 + 4f(t)uu_x u_{xx},\\
C_7^t &= -2f(t)u_x^2.
\end{aligned}$$

Case 8 Corresponding to $P_8 = g(t)$, we achieve the following conserved vector

$$\begin{aligned}
C_8^x &= -4g'(t)u + 4g(t)u_x u_y + \alpha g(t)u_x,\\
C_8^y &= g(t)u_{xxx} + 4g(t)v_x^2 - 2g(t)v_x^2,\\
C_8^t &= 4g(t)u_x.
\end{aligned}$$

4.2. *Conservation laws by Noether's Theorem*

In this part, we would like to derive the conservation laws of Eq. (1.2) by using the classical Noether's theorem [19]. By calculating, we can obtain the second order Lagrangian of equation (1.2)

$$\mathcal{L} = -2u_x u_t + u_{xx}u_{xy} - 2u_x^2 u_y - \frac{1}{2}\alpha u_x^2. \tag{4.4}$$

Consequently, the Noether symmetries

$$S = \xi(x,y,t,u)\frac{\partial}{\partial x} + \phi(x,y,t,u)\frac{\partial}{\partial y} + \tau(x,y,t,u)\frac{\partial}{\partial t} + \eta(x,y,t,u)\frac{\partial}{\partial v}$$

of the (2+1)-dimensional dissipative AKNS equation (1.2) are established by applying the Lagrangian equation (4.4) to the following determining equation

$$S^{[2]}\mathcal{L} + \mathcal{L}\Big(D_x(\xi) + D_y(\phi) + D_t(\tau)\Big) + D_x(T^x) + D_y(T^y) + D_t(T^t) = 0, \tag{4.5}$$

where $S^{[2]}$ is the second prolongation of S , while T^x, T^y, T^t are gauge functions. By expanding equation (4.5) and solving solutions for the resulted partial differential equations, we get the following Noether symmetries as well as their corresponding gauge functions

$$\sigma_1 = \frac{\partial}{\partial x} + \frac{\partial}{\partial t}, T^x = 0, T^y = 0, T^t = 0,$$
$$\sigma_2 = \frac{\partial}{\partial x} + \frac{\partial}{\partial y}, T^x = 0, T^y = 0, T^t = 0,$$
$$\sigma_3 = t\frac{\partial}{\partial y} + \frac{x}{2}\frac{\partial}{\partial u}, T^x = \frac{1}{2}\alpha u, T^y = 0, T^t = u,$$
$$\sigma_4 = \frac{x}{2}\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 2t\frac{\partial}{\partial t} + \left(-\frac{3\alpha y}{8} - \frac{u}{2}\right)\frac{\partial}{\partial u}, T^x = 0, T^y = 0, T^t = 0,$$
$$\sigma_5 = \frac{tx}{4}\frac{\partial}{\partial x} + \frac{ty}{2}\frac{\partial}{\partial y} + \frac{t^2}{2}\frac{\partial}{\partial t} + \left(\frac{(-3t\alpha + 4x)y}{16} - \frac{tu}{4}\right)\frac{\partial}{\partial u}, T^x = -\frac{1}{8}\alpha yu - \frac{1}{4}u_x, T^y = 0, T^t = \frac{1}{2}yu.$$

By employing the following formula [20] along with above Noether symmetries

$$C^k = \mathcal{L}\xi^k + (\eta - u_{x^j}\xi^j)\left(\frac{\partial \mathcal{L}}{\partial u_{x^k}} - \Sigma_{l=1}^k D_{x^l}\left(\frac{\partial \mathcal{L}}{\partial u_{x^l x^k}}\right)\right) + \Sigma_{l=k}^n(\zeta_l - u_{x^l x^j}\xi^j)\frac{\partial \mathcal{L}}{\partial u_{x^k x^l}} - T^k.$$

we derive the corresponding conserved vectors, which are given by

$$\begin{aligned}
C_1^t =& u_{xx}u_{xy} - 2u_x^2 u_y - \frac{1}{2}\alpha u_x^2 + 2u_x^2,\\
C_1^x =& 2u_x^2 u_y + \frac{1}{2}\alpha u_x^2 + u_x u_{xxy} + 2u_t^2 + 4u_x u_y u_t + \alpha u_x u_t + u_t u_{xxy} - u_{xy}u_{xt} - u_{xx}u_{xy} - u_{xx}u_{yt},\\
C_1^y =& 2u_x^3 + u_x u_{xxx} + 2u_x^2 u_t + u_{xxx}u_t.
\end{aligned}$$

$$\begin{aligned}
C_2^t =& 2u_x^2 + 2u_x u_y,\\
C_2^x =& 2u_x^2 u_y + \frac{1}{2}\alpha u_x^2 + u_x u_{xxy} + 2u_y u_t + 4u_x u_y^2 + \alpha u_x u_y + u_{xxy}u_y - u_{xy}^2 - u_{xx}u_{xy} - u_{xx}u_{yt},\\
C_2^y =& -2u_x u_t + u_{xx}u_{xy} - \frac{1}{2}\alpha u_x^2 + 2u_x^3 + u_x u_{xxx} + u_{xxx}u_y.
\end{aligned}$$

$$\begin{aligned}
C_3^t =& -xu_x + 2tu_x u_y - u,\\
C_3^x =& -xu_t - 2xu_x u_y - \frac{1}{2}\alpha x u_x - \frac{1}{2}xu_{xxy} + 2tu_y u_t + 4tu_x u_y^2 + \alpha t u_x u_y + tu_{xxy}u_y + \frac{1}{2}u_{xy} - tu_{xy}^2\\
&- tu_{xx}u_{yy} - \frac{1}{2}\alpha u,\\
C_3^y =& -2tu_x u_t + tu_{xx}u_{xy} - \frac{1}{2}\alpha t u_x^2 - xu_x^2 - \frac{1}{2}xu_{xxx} + tu_{xxx}u_y.
\end{aligned}$$

$$
\begin{aligned}
C_4^t =&2tu_{xx}u_{xy} - 4tu_x^2u_y - \alpha tu_x^2 + \frac{3}{4}\alpha yu_x + uu_x + xu_x^2 + 2yu_xu_y,\\
C_4^x =&xu_x^2u_y + \frac{1}{4}\alpha xu_x^2 + \frac{3}{4}\alpha yu_t + uu_t + 2yu_yu_t + 4tu_t^2 + \frac{5}{2}\alpha yu_xu_y + 2uu_xu_y + 4yu_xu_y^2 + 8tu_xu_yu_t\\
&+ \frac{3}{8}\alpha^2yu_x + \frac{1}{2}\alpha uu_x + 2\alpha tu_xu_t + \frac{3}{8}\alpha yu_{xxy} + \frac{1}{2}uu_{xxy} + \frac{1}{2}xu_xu_{xxy} + yu_{xxy}u_y + 2tu_{xxy}u_t\\
&- \frac{3}{2}u_xu_{xy} - yu_{xy}^2 - 2tu_{xy}u_{xt} - \frac{3}{8}\alpha u_{xx} - \frac{3}{2}u_{xx}u_y - \frac{1}{2}xu_{xx}u_{xy} - yu_{xx}u_{yy} - 2tu_{xx}u_{yt},\\
C_4^y =& - 2yu_xu_t + yu_{xx}u_{xy} + \frac{1}{4}\alpha yu_x^2 + uu_x^2 + xu_x^3 + 4tu_x^2u_t + \frac{3}{8}\alpha yu_{xxx} + \frac{1}{2}uu_{xxx} + \frac{1}{2}xu_xu_{xxx}\\
&+ yu_{xxx}u_y + 2tu_{xxx}u_t.\\
C_5^t =&\frac{1}{2}t^2u_{xx}u_{xy} - t^2u_x^2u_y - \frac{1}{4}\alpha t^2u_x^2 + \frac{3}{8}\alpha tyu_x - \frac{1}{2}xyu_x + \frac{1}{2}tuu_x + \frac{1}{2}txu_x^2 + tyu_xu_y + \frac{3}{16}\alpha tyu_{xxy}\\
&- \frac{1}{4}xyu_{xxy} + \frac{1}{4}tuu_{xxy} + \frac{1}{4}txu_xu_{xxy} + \frac{1}{2}tyu_{xxy}u_y + \frac{1}{2}t^2u_{xxy}u_t - \frac{1}{2}yu,\\
C_5^x =& - \frac{1}{2}txu_x^2u_y + \frac{1}{8}\alpha txu_x^2 + \frac{3}{8}\alpha tyu_t - \frac{1}{2}xyu_t + \frac{1}{2}tuu_t + tyu_yu_t + t^2u_t^2 + \frac{3}{4}\alpha tyu_xu_y - xyu_xu_y\\
&+ tuu_xu_y + txu_x^2u_y + 2tyu_xu_y^2 + 2t^2u_xu_yu_t + \frac{3}{16}\alpha^2tyu_x - \frac{1}{4}\alpha xyu_x + \frac{1}{4}\alpha tuu_x + \frac{1}{2}\alpha tyu_xu_y\\
&+ \frac{1}{2}\alpha t^2u_xu_t + \frac{3}{16}\alpha tyu_{xxxy} - \frac{1}{4}xyu_{xxy} + \frac{1}{4}tuu_{xxy} + \frac{1}{4}txu_xu_{xxy} + \frac{1}{2}tyu_{xxy}u_y + \frac{1}{2}t^2u_{xxy}u_t\\
&+ \frac{1}{4}yu_{xy} - \frac{3}{4}tu_xu_{xy} - \frac{1}{2}tyu_{xy}^2 - \frac{1}{2}t^2u_{xy}u_{xt} - \frac{3}{16}\alpha tu_{xx} + \frac{1}{4}xu_{xx} - \frac{3}{2}tu_{xx}u_y - \frac{1}{4}txu_{xx}u_{xy}\\
&- \frac{1}{2}tyu_{xx}u_{yy} - \frac{1}{2}t^2u_{xx}u_{yt} + \frac{1}{8}\alpha yu + \frac{1}{4}u_x,\\
C_5^y =& - tyu_xu_t + \frac{1}{2}tyu_{xx}u_{xy} + \frac{1}{2}\alpha tyu_x^2 - \frac{1}{2}xyu_x^2 + \frac{1}{2}tuu_x^2 + \frac{1}{2}txu_x^3 + t^2u_x^2u_t + \frac{3}{16}\alpha tyu_{xxx}\\
&- \frac{1}{4}xyu_{xxx} + \frac{1}{4}tuu_{xxx} + \frac{1}{4}txu_xu_{xxx} + \frac{1}{2}tyu_{xxx}u_y + \frac{1}{2}t^2u_{xxx}u_t.
\end{aligned}
$$

5. Conservation laws

In this paper, we implemented the Lie symmetry analysis method to the dissipative (2 + 1)-dimensional AKNS equation and produced six point symmetries, utilized them to perform symmetry reductions and derived four differential equations, including the fourth-order ordinary differential equation (2.8) and three partial differential equations, Eq. (2.10), Eq. (2.12) and Eq. (2.14). Thereafter, we solved Eq. (2.8) by employing the direct integration approach and the $(G'/G^2)-$expansion method, respectively. On one hand, we constructed periodic solution of Eq. (2.8) in terms of the Weierstrass elliptic function. On the other hand, six kinds of including the hyperbolic function types and the hyperbolic function types were obtained via the $(G'/G^2)-$expansion method. The corresponding graphical representation of the obtained solutions were also presented by choosing suitable parametric values. Finally, conserved vectors for (1.2) were derived by invoking the multiplier technique and the classical Noether's theorem, respectively. As a result, eight multipliers were obtained from the multiplier method, thereby eight local conservation laws for the dissipative (2 + 1)-dimensional AKNS equation (1.2) were given. Moreover, five local conservation laws were derived by invoking Noether's theorem. It is necessary to point out that when taking $f(t) = 0, g(t) = 0$ in (2.3), then U_1, U_2 can be reduced to the point symmetries V_1, V_2 in Ref. [32]. Therefore, the symmetry reductions are different from the ones in Ref. [32]. Furthermore, we solved the Eq. (2.8) with the aid of the the $(G'/G^2)-$expansion method. Our results have greater improvements. We presented two kinds of the conservation laws by employing two methods. The difference and relation between these conservation laws is still a puzzle. The solutions for Eq. (2.10), Eq. (2.12) and Eq. (2.14) haven't been solved yet. We will investigate these problems in our future work.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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