



Research article

Integrated wavelet-machine learning approach for effective identification of dynamic parameters in damped dynamical systems

Hao Dong*, Jiale Linghu and Yiheng Lou

School of Mathematics and Statistics, Xidian University, Xi'an 710071, China

* **Correspondence:** Email: donghaoxd@xidian.edu.cn.

Abstract: In this paper, a novel integrated wavelet-machine learning approach is presented to identify critical dynamic parameters of damped dynamical systems based on wavelet transform method (WTM) and support vector regression (SVR). First, a novel data preprocessing technique based on WTM was introduced to extract important features from the raw data of time series with high-dimensional and highly nonlinear features. Moreover, a computational tool based on SVR was developed to excavate the predictive models from dimension-reduced data of the time series generated from damped dynamical systems. The proposed integrated wavelet-SVR method is capable of learning data from given damped dynamical systems with known dynamic parameters and then making parametric identification for the unknown damped dynamical systems. With the help of data preprocessing by WTM, the computational efficiency and identification accuracy of the integrated wavelet-SVR method could be improved greatly. Finally, extensive numerical experiments were conducted to validate the computational performance of the proposed wavelet-SVR approach on randomly generated data. Furthermore, this new approach has the best performance for resisting noise compared with other methods.

Keywords: damped dynamical system; parametric identification; support vector regression; wavelet transform method

1. Introduction

In real-world engineering applications, the extraction of model parameters from observed data is a task of great engineering value and theoretical significance in practical applications, such as pattern recognition, data analysis, and data prediction, etc [1]. In the aerospace industry, effective flight test data analysis is crucial for the success of flight tests, and successful flight tests are, in turn, vital to the stability and control of an aircraft. For a successful flight test, it is not only necessary to accurately extract aerodynamic parameters from flight data, but also to be able to predict their values in a real-time environment [2–4].

Until now, scientists and engineers have developed various methods for aerodynamic parameter estimation, including the least-squares curve fitting method, maximum likelihood technique, Kalman filter method, and the filter error method, etc. Detailed discussions on these methods are given in [2]. In these techniques, mathematical formulations are needed to perform mathematical manipulations for estimating the required aerodynamic parameters (such as damping and frequency values) from given data. These methods generally will cost much computing time to predict the values of parameters. In [2], Wong et al. systematically developed a new machine learning method based on the artificial neural network (ANN) and wavelet multi-resolution analysis. This proposed wavelet-neural network model performs well as long as the modal frequencies are not very close. The only limitation is due to the failure in using wavelet transform to decompose the multi-mode signals into a sequence of single-mode signals when the modal frequency values are very close.

As far as we know, support vector regression is a machine learning method developed from support vector machines, which is widely applied in data analysis and data prediction. The SVR method is characterized by high precision and high efficiency when an appropriate kernel function is selected [5, 6]. Furthermore, the wavelet transform method is a mathematical tool widely used in fields, such as signal processing, image processing, and pattern recognition, and it possesses the capability to compress data and filter noise [7–11]. Therefore, by combining the respective advantages of the SVR method and the wavelet transform method, this study develops a novel integrated wavelet-SVR approach. This new approach does not require explicit mathematical formulation unlike the conventional methods discussed in [2]. Moreover, this integrated machine learning approach does not need to decompose the multi-mode signals into a sequence of single-mode signals and naturally leads to a parallel process. In a word, this new integrated machine learning approach is very suitable to extract and estimate the parameters in a real time environment.

The outline of this paper is organized as follows. In Section 2, the detailed description of data analysis for simulated flutter signal is introduced, which essentially is a damped dynamical system. In Section 3, SVR and WTM are introduced in detail, respectively. Based on these two methods, a novel integrated machine learning approach is developed, which combines the respective advantages of SVR and WTM together. In Section 4, some numerical experiments are conducted, which strongly support our method. Finally, some conclusions are given in Section 5.

2. The description and setting of model problem

In this study, we focus on the typical flutter test model in [2, 15], whose flutter signal can be represented by a time series resulting from a linear superposition of a number of exponentially decaying sine waves, as follows:

$$W(t) = \sum_{i=1}^N A_i e^{-\alpha_i t} \sin(\omega_i t + \phi_i), \quad (2.1)$$

where N denotes the number of modes. A_i , α_i , ω_i , and ϕ_i represent the amplitude, damping ratio, modal frequency and phase angle associated with the i -th mode. The signal $W(t)$ given in (2.1) represents a clean data without noise pollution. In practice, all signals are subject to noise pollution. Thus, a

realistic flutter signal is expressed in the form

$$Y(t) = \sum_{i=1}^N A_i e^{-\alpha_i t} \sin(\omega_i t + \phi_i) + n(t), \quad (2.2)$$

where $n(t)$ represents the noise signal. The equation given in (1) provides a multimode coupled vibratory time series that is often used to model simulated flutter signals generated by an impulsive input (please refer to [2]). The simulated data represents the decaying portion of the response signals from sine dwell or sine sweep excitations of the aircraft. The accurate analysis of this multimode coupled signals is vital to complete flutter data analysis.

In the flutter data analysis, we typically aim to extract the values of α_i and ω_i from a real-world flutter signal $Y(t)$ that is contaminated by noise. Furthermore, it should be noted that damping ratio α_i is the most critical parameter, and its values are usually difficult to estimate accurately, especially when the modes are very close in frequency. For a real-world flight flutter data, each signal is represented as a time series, and the number of modes N depends on the number of significant structural modes. The main target of this study is to develop an effective machine learning method to accurately extract the values of α_i and ω_i of damped dynamical system (2.2).

3. Novel wavelet-machine learning approach

3.1. Support vector regression

Unlike the ANN method, the SVR method is another machine learning method, which is developed based on support vector machine (SVM) in [6]. Next, the mathematical model of SVR is presented in detail. Given training set $D = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \dots, (\mathbf{x}_m, y_m)\}$ with $\mathbf{x}_i \in \mathbb{R}^n$ and $y_i \in \mathbb{R}$, the aim of SVR is to determine a function $f(\mathbf{x}) = \boldsymbol{\theta}^T \mathbf{x} + b$, which satisfies

$$\min_{\boldsymbol{\theta}, b} \frac{1}{2} \|\boldsymbol{\theta}\|^2 + C \sum_{i=1}^m \ell_\varepsilon(f(\mathbf{x}_i) - y_i), \quad (3.1)$$

where $\boldsymbol{\theta}$ and b are the unknown parameters of SVR model, and C is a regularization constant. ℓ_ε is an ε -insensitive loss function with the following definition:

$$\ell_\varepsilon(z) = \begin{cases} 0, & \text{if } |z| < \varepsilon, \\ |z| - \varepsilon, & \text{otherwise.} \end{cases} \quad (3.2)$$

In model (3.1), we assume the training samples are linearly separable. However, in real-world applications, there are many problems that cannot be linearly separated, as shown in Figure 1(a). We address these problems using a kernel function to map the original, non-linearly separable sample space into a new, high-dimensional sample space in [6]. In this new space, it is possible to obtain a linearly separable sample space, a result demonstrated in Figure 1(b). Fortunately, mathematicians have proven that, for any finite-dimensional sample space, a higher-dimensional feature space exists, in which the samples can be separated in [6].

Denote $\phi(\mathbf{x})$ as a feature vector of original vector \mathbf{x} after mapping, the aim of the new SVR model is to determine a function $F(\mathbf{x}) = \boldsymbol{\theta}^T \phi(\mathbf{x}) + b$, which satisfies

$$\min_{\boldsymbol{\theta}, b} \frac{1}{2} \|\boldsymbol{\theta}\|^2 + C \sum_{i=1}^m \ell_\varepsilon(F(\mathbf{x}_i) - y_i), \quad (3.3)$$

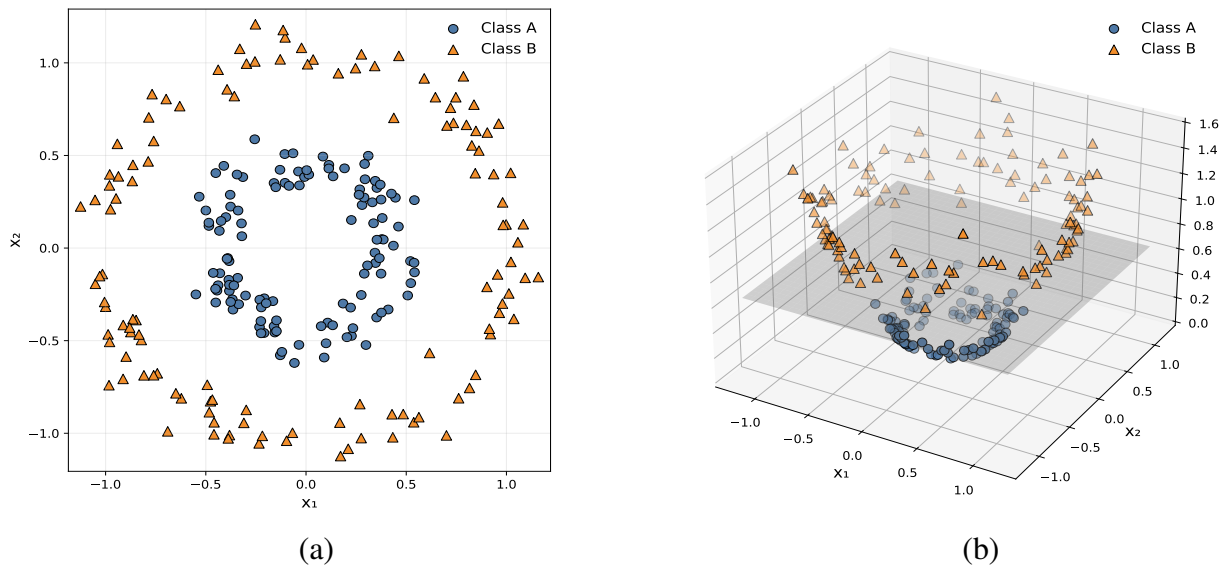


Figure 1. Visualization of SVR method: (a) Non-linearly separable data in original 2D space; and (b) linearly separable data in high-dimensional feature space.

where C is regularization constant and ℓ_ε is ε -insensitive loss function with the same definition as SVR model (3.1). Furthermore, the kernel function $\kappa(\bullet, \bullet)$ is defined as follows:

$$\kappa(\mathbf{x}_i, \mathbf{x}_j) = \langle \phi(\mathbf{x}_i), \phi(\mathbf{x}_j) \rangle = \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j), \quad (3.4)$$

where $\langle \bullet, \bullet \rangle$ represents inner product in the higher dimensional feature space. The kernel function is always used in solving the dual problem of the SVR model (3.3). Commonly used kernel functions include the linear kernel, polynomial kernel, Gaussian kernel, Laplacian kernel, and Sigmoid kernel, among others [6]. With regard to SVR model (3.3), it should be underlined that selecting appropriate kernel functions mainly contributes to the numerical accuracy of the established SVR model. Generally, we use an exhaustive search to try all known kernel functions to find the optimal one. In this study, we utilize the SVR method as the fundamental algorithm of our proposed integrated approach for parameter identification.

3.2. The wavelet transform method (WTM)

As is known, the training process is a crucial component to ensure the success of a learning machine. To a certain extent, large input data in the training set will affect the architecture of machine learning methods. Using SVR as an example, the size of the undetermined vector θ depends on the number of data points of input data \mathbf{x} . Therefore, taking a large data set as input is not a trivial task for a learning machine. We now propose a novel idea to deal with large input data by using WTM. WTM has been successfully demonstrated as a powerful tool for data compression and feature extraction in signal and image processing in [7, 8]. One of the advantages of WTM lies in its ability to extract multiscale information from the input data. By recursively applying WTM, it leads to multi-level wavelet decomposition (WD) [16]. The procedure for a three-level wavelet decomposition is illustrated in Figure 2, where the raw time series are represented by RTS. In the first-level of WD, the original RTS is decomposed into two vectors, CA_1 and CD_1 , representing the approximate and detail coefficients,

respectively. In the second-level of decomposition, the WD is applied again to CA_1 , resulting in two vectors, CA_2 and CD_2 . This process is then continued recursively, decomposing CA_2 into CA_3 and CD_3 , to finally obtain the three-level WD of RTS.

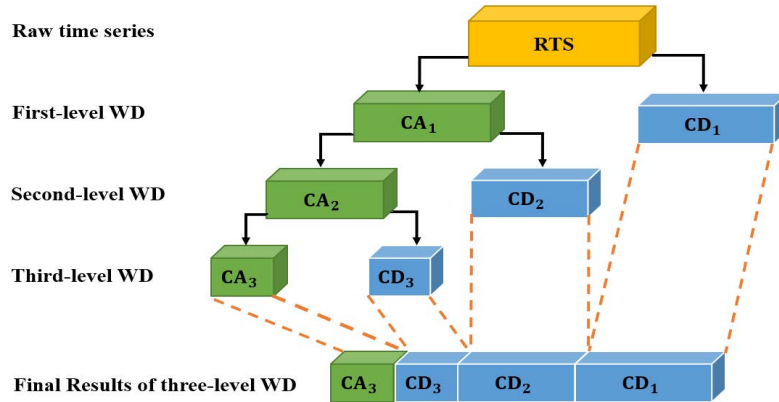


Figure 2. The diagram of three-level wavelet decomposition.

Based on the preceding analysis, using wavelet coefficients as the input data for the machine learning model can significantly reduce the dimensionality of the original data. This, in turn, greatly lowers the training complexity of the predictive models. Additionally, the wavelet transform helps to filter out noise, as illustrated in Figure 3. For these reasons, it is employed as the data preprocessing technique in our proposed integrated approach.

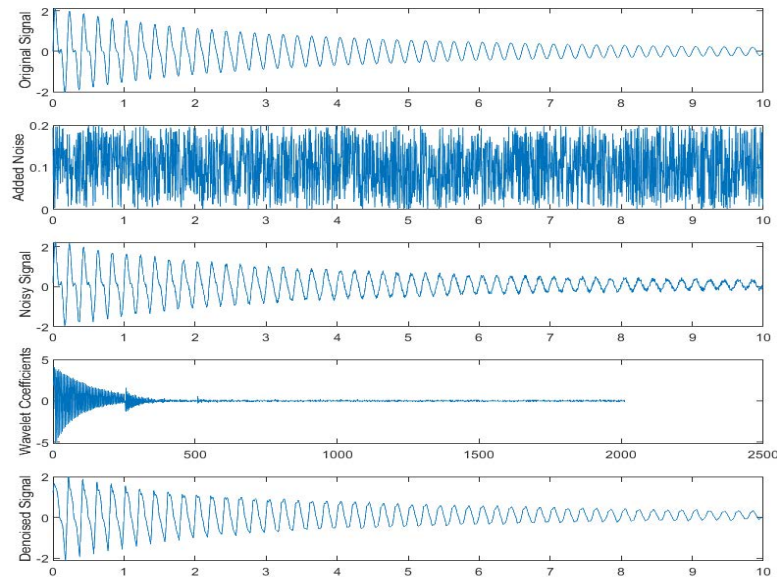


Figure 3. The diagram of wavelet decomposition, noise reduction, and reconstruction.

3.3. The integrated wavelet-SVR method

In the traditional SVR method, the RTS is used as input. The input features are formed by the values of different time series $Y(t)$ at corresponding time points, while the output features consist of

the individual model parameters α_i and ω_i from each time series. Comparatively, the new integrated method first leverages the data compression and noise filtering capabilities of the wavelet transform to perform a multi-level wavelet decomposition on the RTS. The resulting small-scale wavelet coefficients are then used as the new input data. With this data preprocessing, the original problem in a high-dimensional space is solved in the corresponding low-dimensional wavelet space. The wavelet-SVR method proposed in this paper is essentially an integrated approach that uses the wavelet transform as a data preprocessing technique and the SVR method as the fundamental regression analysis method, as illustrated in Figure 4. Additionally, this method can independently extract the dynamic parameters of the damped dynamic system (2.1) one by one, possessing a natural parallel mechanism. Compared with the wavelet-neural network approach proposed in [2, 12, 13], our novel method does not have the limitation, which is due to the failure in using WTM to decompose the multi-mode signals into a sequence of single-mode signals when the modal frequency values are very close. For our integrated method, we should choose suitable kernel functions for different dynamic parameters by trying all kernel functions in [13, 14]. The major advantages of our proposed new wavelet-SVR method are its ability to significantly reduce the scale of the input data and its superior capability to resist noise.

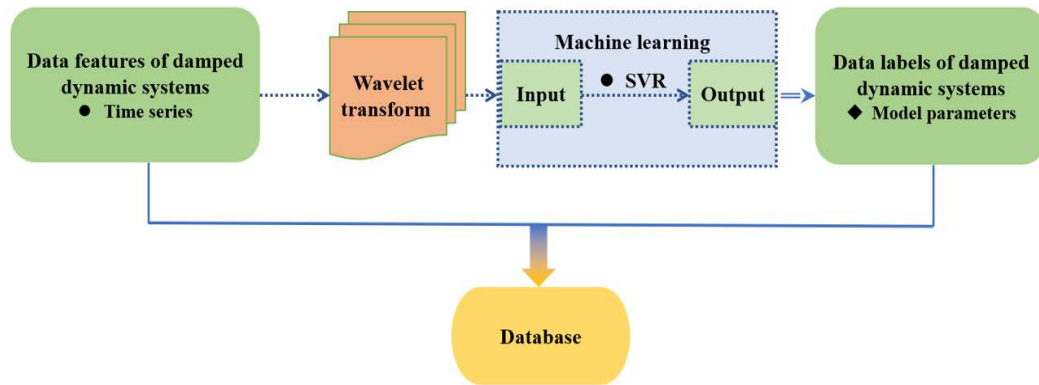


Figure 4. The schematic of the integrated wavelet-SVR method.

4. Numerical experiments

To illustrate the validation of our novel machine learning approach, without loss of generality, we consider the following damped dynamical system consisting of two modes:

$$\begin{cases} Y(t) = A_1 e^{-\alpha_1 t} \sin(\omega_1 t) + A_2 e^{-\alpha_2 t} \sin(\omega_2 t) + n(t), \\ A_1 = 1.0, A_2 = 1.5, \\ \omega_1 = 2\pi f_1, \omega_2 = 2\pi f_2, \\ 0.6 < \alpha_1 < 0.8, 0.2 < \alpha_2 < 0.4, \\ 10 < f_1 < 15, 5 < f_2 < 8. \end{cases} \quad (4.1)$$

Here, the amplitudes A_i and the phase angles ϕ_i , as defined in (4.1), are kept as constants $i = 1, 2$. Moreover, the phase angles ϕ_i are defined as 0. The target of our work is to accurately identify the model parameters α_1, α_2, f_1 , and f_2 of dynamical system (4.1) by machine learning methods.

Before starting numerical computation, we give four machine learning approaches as our research methods adopted in this paper. The first method is to use the ANN method to extract dynamical

parameters based on raw data, which is defined as method 1. The second method is to use the ANN method to build a machine learning model on wavelet coefficients of raw data. The third method is to use the SVR method to directly identify dynamical parameters on raw data. The fourth method is to use SVR method to build a machine learning model on wavelet coefficients, which is proposed in this paper.

In this section, to train the predictive models, the dataset is divided into two components: the training set (for training the predictive model) and the test set (for performance assessment). Moreover, the numerical experiments are conducted on the damped dynamic system (4.1) over the time interval $[0, 5.12]$. Next, we set that the dataset contains 550 random samples in total. Each sample is a time series of damped dynamical system (4.1), and each time series has 512 uniformly distributed sampling points. Hence, the raw data can be stored as a 512×550 matrix. The training set is built using the initial consecutive 500 samples. For method 1 and method 2, after a randomization, half the remaining samples (25 samples) are chosen to act as the validation set. All other samples (25 samples) are used as the test set. For method 3 and method 4, there is no validation set, and all remaining samples (50 samples) are used as the test set.

To improve the computational accuracy and efficiency of the machine learning model, the three-level WD is applied to raw data. In this paper, we use the Daubechies wavelet “db3” with vanishing moment 3 to perform a three-level wavelet decomposition. The zero-padding mode is adopted for the extension in the discrete wavelet transform (DWT). The final decomposition results of raw data after three-level WD are shown as below:

$$\text{raw data} - 512 \times 550 \text{ matrix} \xrightarrow{\text{WD}} \begin{cases} \text{CD}_1 - 258 \times 550 \text{ matrix,} \\ \text{CD}_2 - 131 \times 550 \text{ matrix,} \\ \text{CD}_3 - 68 \times 550 \text{ matrix,} \\ \text{CA}_3 - 68 \times 550 \text{ matrix.} \end{cases} \quad (4.2)$$

From (4.2), we can find that the original big matrix is decomposed as four small matrix after three-level WD. If we choose wavelet coefficients CA_3 or the combination of different kinds of wavelet coefficients as new input data, the scale of original machine learning models can be reduced greatly.

4.1. Example 1: identifying parameters of damped dynamical systems without noise

In this example, the aforementioned four machine learning methods are adopted to identify the parameters of dynamical system (4.1) without noise.

For method 1, we pursue a 1×10^{-6} mean square error (MSE) on the training set of the ANN model. For method 2, the approximation coefficients CA_3 are used as input data for establishing the wavelet-ANN model. For this ANN model, we pursue a 1×10^{-5} MSE on the training set because the amount of new input data is far less than that of raw input data. For method 1 and method 2, we define the maximum number of training epochs equal to 1000 in the training process. The number of validity checks is set to 100. For these two ANN models, each model parameter is defined as one attribute of output data in the output layer. Furthermore, a two-layer feedforward network is adopted to build the ANN model for extracting parameters. The learning function uses the default gradient descent with momentum weight and bias learning function in MATLAB. The number of neurons in single hidden layer choose the optimal parameter $\lceil \sqrt{m_1 + m_2} \rceil$, where m_1 and m_2 stand for the number of neurons in the input layer and output layer, respectively.

For method 3, the half width of ε -insensitive band in the SVR model is specified as default $iqr(Y)/13.49$ in MATLAB, which is an estimate of a tenth of the standard deviation using the interquartile range of the response variable Y . In this paper, Y represents the undetermined parameters of damped dynamical system (4.1). The kernel function of this SVR model uses the default linear kernel function. Moreover, the optimization routine uses the sequential minimal optimization (SMO) and the maximum number of numerical optimization iterations is set as 1×10^6 in this SVR model. For method 4 proposed in this paper, wavelet coefficients are used as input data to build a new wavelet-SVR model to further improve computational efficiency. After trying different wavelet coefficients and kernel functions of the SVR model, we obtain the optimal SVR model for parametric identification as follows: (I) For parameter α_1 , we use wavelet coefficients $CA_3 + CD_2$ as input data of the new SVR model. The half width of ε -insensitive band in the SVR model is specified as default $iqr(Y)/13.49$ in MATLAB. The kernel function of our SVR model uses the default linear kernel function. (II) For parameter α_2 , we use wavelet coefficients CA_3 as input data. The half width of ε -insensitive band in the SVR model is specified as 1×10^{-6} . The kernel function of our SVR model uses the cubic polynomial. (III) To identify parameter f_1 , wavelet coefficients CA_3 are used as input data. The half width of ε -insensitive band in the SVR model is specified as $iqr(Y)/13.49$. The kernel function of our SVR model uses the default linear kernel function. (IV) For identifying parameter f_2 , we use the same SVR model as that of identifying parameter f_1 .

In order to reduce the influence of randomness, we repeat the training and performance estimation cycle 50 times. After numerical computation, the final experimental results of four approaches are presented in Table 1.

Table 1. The numerical results for damped dynamical system without noise.

Methods	Parameters	Training error	Test error	Training time	Test time
method 1	α_1	5.50%	8.64%	10616 s	0.0781 s
	α_2	11.76%	15.18%		
	f_1	1.05%	1.91%		
	f_2	1.52%	2.36%		
method 2	α_1	2.24%	3.68%	53.5303 s	0.0209 s
	α_2	2.15%	2.81%		
	f_1	0.0196%	0.0286%		
	f_2	0.0276%	0.0341%		
method 3	α_1	2.38%	3.15%	34.6466 s	0.0228 s
	α_2	2.45%	3.04%		
	f_1	1.14%	1.15%		
	f_2	1.42%	1.43%		
method 4	α_1	2.90%	3.72%	32.3631 s	0.0166 s
	α_2	1.72%	3.54%		
	f_1	0.82%	0.82%		
	f_2	0.84%	0.85%		

Remark 1. The training and test errors in Table 1 are defined as the mean relative absolute error (MAE). The training and testing errors in the subsequent tables are consistent with those in Table 1.

According to the final results in Table 1, it can be found that method 2, method 3, and method 4 can be trained to produce testing results within 5% accuracy. However, the identification accuracy of method 1 is not ideal even when it is wrong. Besides, it costs a tremendous amount of computational time when we use raw data to build an ANN model directly. It should be noted that there is no obvious improvement on computational efficiency of method 4 compared with method 3. The reason is that the SVR method is one kind of regression method, and the scale of undetermined parameters θ is linearly increasing along the expansion of input data. Only when the scale of original dataset is huge can we see the obvious improvement of computational efficiency of method 4 compared with method 3.

4.2. Example 2: identifying parameters of a damped dynamical system with noise

In this example, we study the parametric identification of damped dynamical system (4.1) with noise pollution. The optimal ANN and SVR models built in Example 1 are adopted to identify the parameters of damped dynamical system (4.1) with noise at different intensity. Four groups of noise with different intensities $N(0, 0.0001^2)$, $N(0, 0.001^2)$, $N(0, 0.01^2)$ and $N(0, 0.1^2)$ are added into original damped dynamical system as a new dataset, respectively.

To fully train the ANN model for noise-polluted signals, we increase the maximum number of training epochs to 10000 in the training process. Furthermore, the number of validation check is set as 1000, and we want to pursue a 1×10^{-5} mean square error on the training set. After repeating the training and performance estimation cycle 50 times, we exhibit the final numerical results for different noise in Tables 2–5, respectively.

Table 2. The numerical results for a damped dynamical system with noise $N(0, 0.0001^2)$.

Methods	Parameters	Training error	Test error	Training time	Test time
method 2	α_1	0.73%	1.46%	71.1431 s	0.0247 s
	α_2	0.99%	1.58%		
	f_1	0.0123%	0.0147%		
	f_2	0.0096%	0.0128%		
method 3	α_1	2.39%	3.16%	34.7147 s	0.0222 s
	α_2	2.50%	3.02%		
	f_1	1.15%	1.16%		
	f_2	1.45%	1.45%		
method 4	α_1	2.90%	3.76%	32.3825 s	0.0122 s
	α_2	1.74%	3.37%		
	f_1	0.82%	0.82%		
	f_2	0.84%	0.85%		

According to the results in Tables 2–5, we can conclude that our models all are not sensitive to low intensity noise. However, high intensity noise has a great impact on the performance of the ANN and SVR models, especially for the SVR model using raw data as input data. Furthermore, according to the results in Table 5, it can be clearly seen that the SVR model building on wavelet coefficients exhibits the best performance for resisting noise within 10% accuracy. The numerical accuracy for method 2 is too low to be accepted for flutter data analysis. As for method 3 in Table 5, the over-fitting phenomenon may have occurred when processing signals with high-intensity noise. This

Table 3. The numerical results for a damped dynamical system with noise $N(0, 0.001^2)$.

Methods	Parameters	Training error	Test error	Training time	Test time
method 2	α_1	1.94%	3.95%	302.8988 s	0.0181 s
	α_2	1.89%	3.34%		
	f_1	0.0327%	0.0445%		
	f_2	0.0241%	0.0291%		
method 3	α_1	2.70%	3.84%	34.2950 s	0.0300 s
	α_2	3.02%	4.02%		
	f_1	1.16%	1.16%		
	f_2	1.44%	1.46%		
method 4	α_1	3.18%	4.31%	34.5781 s	0.0063 s
	α_2	1.80%	3.44%		
	f_1	0.80%	0.82%		
	f_2	0.85%	0.87%		

Table 4. The numerical results for a damped dynamical system with noise $N(0, 0.01^2)$.

Methods	Parameters	Training error	Test error	Training time	Test time
method 2	α_1	6.13%	7.15%	218.5406 s	0.0153 s
	α_2	4.42%	6.89%		
	f_1	0.11%	0.18%		
	f_2	0.0821%	0.0993%		
method 3	α_1	2.16%	8.24%	33.0219 s	0.0306 s
	α_2	3.63%	13.44%		
	f_1	1.05%	1.07%		
	f_2	1.37%	1.38%		
method 4	α_1	4.22%	7.09%	32.1591 s	0.0053 s
	α_2	1.68%	4.25%		
	f_1	0.75%	0.78%		
	f_2	0.81%	0.80%		

is indicated by the significant discrepancy between its low training error and high testing error. In conclusion, the proposed integrated wavelet-SVR method 4 is an effective and robust approach for extracting parameters from damped dynamic systems.

5. Conclusions

In this paper, we present an innovative integrated wavelet-SVR framework for parametric identification. This method is very accurate and efficient for extracting the parameters of a damped dynamical system, which has wide application in flutter data analysis. In contrast to conventional mathematical approaches, our proposed hybrid machine learning method eliminates the need for explicit mathematical formulas. Furthermore, through the exploitation of the data compression and feature extraction capabilities inherent in the wavelet transform, this method achieves a considerable

Table 5. The numerical results for a damped dynamical system with noise $N(0, 0.1^2)$.

Methods	Parameters	Training error	Test error	Training time	Test time
method 2	α_1	7.12%	7.46%	197.8844 s	0.0234 s
	α_2	16.97%	17.11%		
	f_1	0.79%	1.89%		
	f_2	0.63%	0.78%		
method 3	α_1	1.14%	22.34%	33.0666 s	0.0366 s
	α_2	2.71%	48.88%		
	f_1	0.86%	0.98%		
	f_2	1.05%	1.14%		
method 4	α_1	4.80%	9.66%	34.6819 s	0.0163 s
	α_2	1.07%	7.48%		
	f_1	1.80%	2.11%		
	f_2	1.29%	1.45%		

reduction in the computational complexity of the parametric identification problem. Besides, our new approach exhibits the best performance for resisting noise compared to ANN model based on wavelet coefficients and the SVR model based on raw data. The numerical results clearly validate the computational performance of the proposed wavelet-SVR method. Additionally, the methodology is inherently amenable to parallel computation, owing to the independent extraction of distinct model parameters. To summarize, the novel integrated wavelet-learning approach developed herein achieves high-precision and high-efficiency parameter extraction and prediction for damped dynamic systems in real-time settings, demonstrating extensive applicability to the analysis of flutter data within the aerospace sector. In the future, we will further develop a parallel wavelet-SVR method based on the wavelet-SVR framework proposed in this study and add a denoising technique into this integrated framework to accurately analyze the noise-polluted signals.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there is no conflict of interest.

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