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Research article

Shear crack control for a reinforced concrete T-beam using coupled stochastic-multi-objective optimization methods

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Abstract: We use multi-objective optimization and numerical simulations to optimize the shear strength of a reinforced concrete T-beam. The optimization process involves four factors that are related to the Young's modulus of elasticity and density of both the steel reinforcement and the concrete. The factors, which have a limited range, are utilized in the construction of the regression equation that forecasts the reinforced concrete T-beam's ductility and elastic shear strain. Using ABAQUS finite element programs, 27 models were prepared for numerical analysis and simulation using the well-known sampling technique Box-Behnken design. To find the coefficients that correspond to the regression equations, MATLAB codes are utilized to solve complex matrices using the least squares method. Checking the regression equation's reliability to compare the outcomes of the numerical simulations and the regression equations, a reliability check for the regression equation has been implemented. Due to the simultaneous R^2 values of 1 and 1 for ductility and elastic shear strain, the reliability check was 100%. The optimization of the reinforced concrete T-beam's shear strength capacity can be easily determined, according to multi-objective optimization results, and the design of this structural system is highly controllable.

Keywords: elastic shear strain; ductility; box-Behnken design; regression; multi-criteria

1. Introduction

Frequently, the reinforced concrete T-beams frequently appear in industrial construction, such as floors of building, walls retaining and decks of bridges. More generally, all projects concerning construction of reinforced concrete [1]. Multiple methods can be used to predict the shear strength of reinforced concrete (RC) beams. Because of the complex nature and the numbers of the parameters like aggregate interlock and concrete in compression regions, the precision remains limited. Also, there are some other causes affecting the precision, the axial load of the beam and the cross section [2]. Many methods supporting the cracking particle method were implemented, which are an alternative to the extended finite element method used to study and analyze the crack propagation in materials [3–5]. To design any structure, it considers that the design strength and shear behavior of members are among the most central concern. Besides that, there are different types of failure, and among them shear failure is considered the most critical because of the fragility of concrete systems. It is difficult to accurately predict shear failure. Over the years, many experiments and interpretations have been done, but shear failure is poorly understood. In spite of that, the design continues, depending on the formula derived from experimental data [6]. Abd [7] applied the artificial neural network (ANN) method to predict the shear strength of T-beams. He created the model of ANN to precede a parametric analysis for studying the impact of the many parameters on the shear strength of T-beams. He discovered an excellent agreement between the theoretical results and the experimental data. Also, he realized that the ANN model has better accuracy compared to the equations in the guidelines. Shahbazian et al. [8] employed models of ANN to assess the feasibility of modeling the shear strength of reinforced concrete beams coupled with the tabu search training (TST) algorithm. They implemented 248 experimental results from the literature to predict the shear strength of the reinforced concrete beam in addition to multiple regression equations. They compared the shear design equations of ACI-318-2019 with Tabu Search Trained ANN model. They discovered the efficiency of Tabu Search Trained ANNs compared to the other suggested models in literature and the design code ACI-318-2019.

Based on the theory of linear elastic fracture mechanics (LEFM), a fairly high coefficient is applied to the stress in the vicinity of the crack tip. This coefficient is called the stress intensity factor. The LEFM converts stress to a unique form of distribution. The stress intensity factor depends on the material properties, on the size of the crack, on the load and on the geometry of the structure. This factor presents a relationship between the material and the reaction of the structure. Numerous researchers have aimed to calculate stress intensity factor in concrete structure using experimental, analytical and numerical methods [9]. The first mode based on the fracture mechanics theory in concrete material was presented by Hillerborg. It was demonstrated that there is a fracture zone ahead of the real crack that has an ability to transfer stress. This fracture zone is known as fracture process zone (FPZ). The modeling of the FPZ in the beam-column joint is an important topic to predict crack propagation [10]. The mechanism of shear failure in the support zone of RC elements is determined by many factors: Sliding and rotation of both parts of the element crossed by the diagonal shear crack accompanied by the aggregate interlock action in concrete, dowel action of the longitudinal reinforcement, transfer of the shear force by the un-cracked concrete in the compression zone and direct strut action for point load close to the support. The percentage of each component in the shear capacity of steel RC beams without shear reinforcement is determined as: 33%–50% (effect of aggregate interlock), 20%–40% (compressive concrete zone) and 15%–25% (dowel action effect) [11,12]. Cracking, in particular, is caused by different factors, conditioning both the stiffness and durability of structures. Cracking is normal in reinforced concrete structures subject to bending, shear, torsion or tension resulting from either direct loading or restraint or imposed deformations. Cracks may also arise from other causes such as plastic shrinkage or expansive chemical reactions within the hardened concrete [13].

In this paper, the multi-criteria optimization method is dedicated to control the shear crack of a reinforced concrete T-beam. Four factors (concrete density, concrete modulus of elasticity, steel density and steel modulus of elasticity) are considered for the optimization process which are the most important and vital involving factors for any type of reinforced concrete structures that withstand the dynamic loading applied for crack and failure purposes. The Box-Behnken design sampling method is used to create 27 numerical models in ABAQUS finite element program. Regression equations are constructed for the prediction of the shear strength of the structural system. It is worth mentioning that this design sampling method is so effective and easy to use for most prediction cases and the literature provides strong evidences that this method results in great resemblances of the actual behavior of the structural system, especially for numerical analysis.

2. Materials and methods

The regression equation is constructed by dedicating four factors, which are concrete density, Young's modulus of elasticity of concrete, steel density and Young's modulus of elasticity of steel (see Table 1). The regression equations for the elastic shear strain and the ductility in the reinforced concrete T-beam are represented by the following Eq 1:

$$y = f(x)\alpha + \epsilon \tag{1}$$

where x is a vector of x from $i = 1, \ldots, k$ with a function $f(x)$ of k elements. α is a regression coefficients vector, and ϵ is with zero mean which is random error. The regression equation needs to calculate the regression coefficients which are represented by α and can be calculated as shown in Eq 2:

$$\alpha = (X'X)^{-1}X' \tag{2}$$

where X' is the transpose of X, and $(X'X)^{-1}$ is the inverse of $X'X$ [14].

The function $f(x)$ for both the elastic shear strain and the ductility in the reinforced concrete T-beam consists of multiple terms such as linear, quadratic and interaction terms for the four considered factors.

2.1. Multi-criteria optimization method

The multi-criteria optimization method is deployed to optimize the involved four factors, which are bounded by certain range data as mentioned in Table 1. The regression equations are having bounded or limit values where the equations of the elastic shear strain and the ductility in the reinforced concrete T-beam are further processed to determine the minimum and maximum values of both outputs. The optimized values of each factor are then checked and it is a condition to be in the imposed range limits with a condition of being less than the minimum maximum values for each regression equation to insure the optimization process for the design of shear strength of the structural system. For this purpose, it is difficult to optimize more than one objective, especially in the conflicting case. The treatment is to find a compromise solution that achieves all the objectives simultaneously. Different methods have been proposed for solving multi-objective optimization problems by converting these objectives to a single one and keeping the same constraints available. Among the methods, we introduce the harmonic mean for this purpose (see section 3.5.)

2.2. Finite element method

The finite element model of the reinforced concrete T-beam has a length of 2.8 m. The flange is 0.3 m width and 0.06 m thickness. The web is 0.08 m width and 0.22 m height. The flange is reinforced with 4 longitudinal steel bars with 0.012 m diameter and 19 transverse steel bars of 0.012 m diameter with regular space between each two steel bars. The web has two longitudinal steel bars with 0.012 m diameter positioned at the bottom of the web. The stirrups are provided in a regular distribution with 19 numbers of 0.012 m diameters also. The elastic model, which is Young's modulus and Poisson's ration data, has been considered for the steel. Also, plastic model data, which consists of yield stress and plastic strain, have been assigned for the steel. Furthermore, elastic models, which are Young's modulus and Poisson's ration data, and concrete damaged plasticity model data, which are the plasticity, compressive behavior and tensile behavior, have been added for the concrete.

The T-beam is loaded with a concentrated force 70000 N at a distance of 0.65 m from the center of the left support so that to analyze it for shear capacity (see Figure 1). Amplitude is applied for the concentrated load regularly with a loading rate of 260 N/s for 100 s. The load value has been selected in order to stay in the elastic stage. The elastic stage lets to prepare the end of each model simulation for the purpose of comparison for 27 models all. The boundary conditions of the T-beam model are in such a way to provide freedom in longitudinal and transverse directions at both supports. The supports are constrained only in the vertical direction. The T-beam model is meshed using 828 linear hexahedral elements of type C3D8R for the concrete and 1280 linear line elements of type T3D2 for the steel bars both for the longitudinal steel bars and the stirrups.

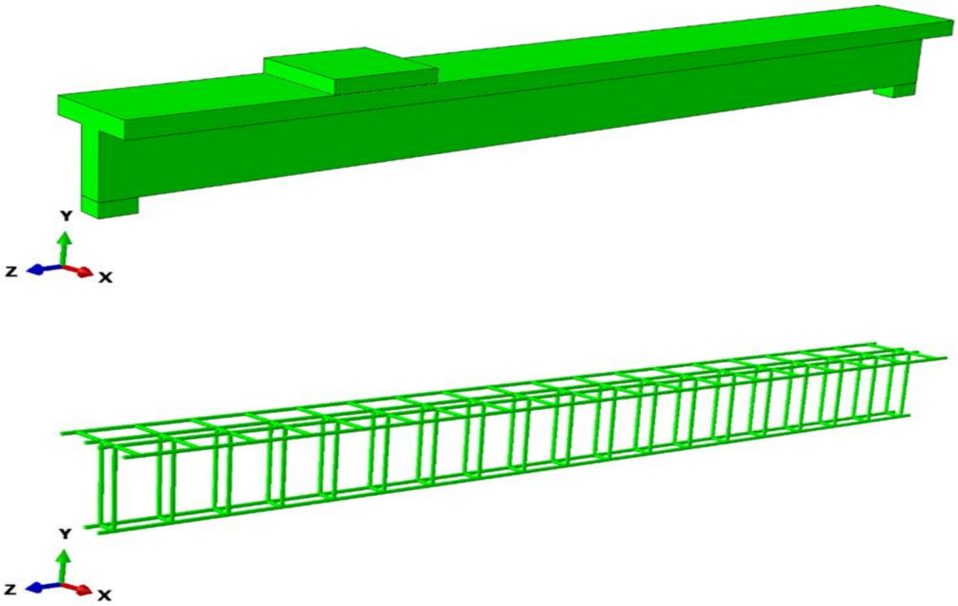

Figure 1. Finite element model (reinforced concrete T-beam).

2.3. Material range data

The regression equations are created supporting the four factors using the Box-Behnken design sampling method by dedicating 27 models of the reinforced concrete T-beam in ABAQUS finite element program. The models are numerically simulated, and both the elastic shear strain and the deflection data are collected after running the models. Table 1 shows the adopted factors and their range values.

Table 1. Factors and range data.

Factor Symbol	Factor	Range Data
X_1	Concrete density (ρc) kg/m^3	2200–2600
X_2	Concrete Young's modulus (Ec) GPa	25–35
X_3	Steel density (ρs) kg/m^3	7800–8000
X_4	Steel Young's modulus (Es) GPa	190–230

2.4. Box-Behnken sampling method

Box-Behnken designs are used to generate higher order response surfaces using fewer required runs than a normal factorial technique. This and the central composite techniques essentially suppress selected runs in an attempt to maintain the higher order surface definition. The Box-Behnken design uses the twelve middle edge nodes and three centre nodes to fit a 2nd order equation. The central composite plus Box-Behnken becomes a full factorial with three extra samples taken at the centre. Box-Behnken designs place points on the midpoints of the edges of the cubical design region, as well as points at the centre. The Box-Behnken designs of experiments provide modeling of the response surface. These designs are not based on full or fractional factorial designs. The design points are positioned in the middle of the subareas of the dimension k-1. In the case of three factors, for instance, the points are located in the middle of the edges of the experimental domain (see Figure 2).

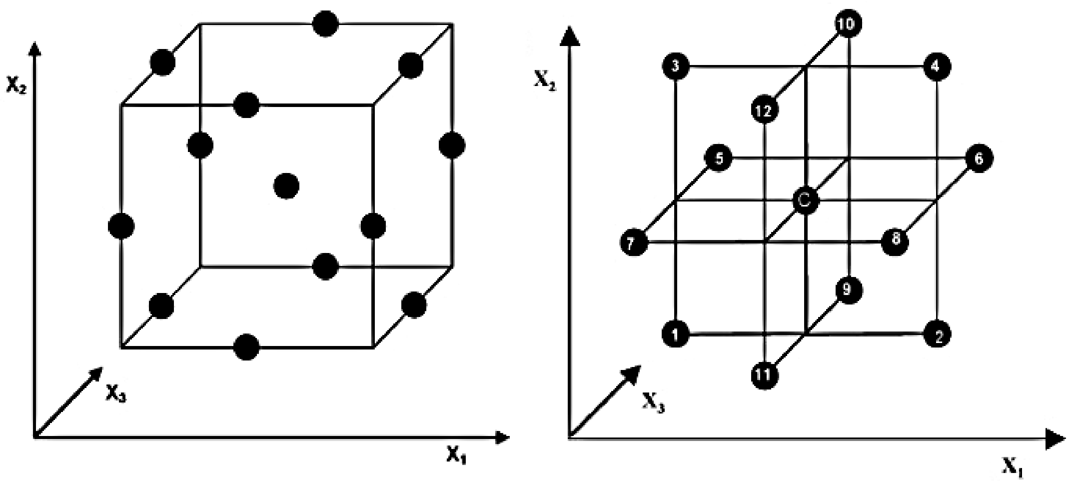

Figure 2. Box-Behnken sampling method (three factors) (Reproduced from Ref. [15] with permission).

3. Results and discussion

3.1. Elastic shear strain

The results of the numerical simulation of the 27 models in ABAQUS for the maximum elastic shear strain (EE23) in ABAQUS have been collected. The maximum shear strain in each model has been recorded as a critical response case for the design optimization of the reinforced concrete T-beam for shear strength capacity. Different results have been realized due to difference in the model data arrangement up-to Box-Behnken design method. The maximum elastic shear strain between the 27 models was in the model 21 that was 0.0003274499031 and the minimum response was 0.0002360623912 seen in model 24 (see Figure 3). The nature of the responses is due to certain patterns of factor arrangement in each model, which produces random responses without any regular relation between them. The important point is to identify the maximum elastic shear strain in each model so that to identify the maximum of this response between all models because it is the base for shear strength capacity for the design of the T-beam. The response of each model displays the role of each design factor and its effect on the overall elastic shear strain response in structural system, which is a primary indication for the design optimization process of the shear strength capacity.

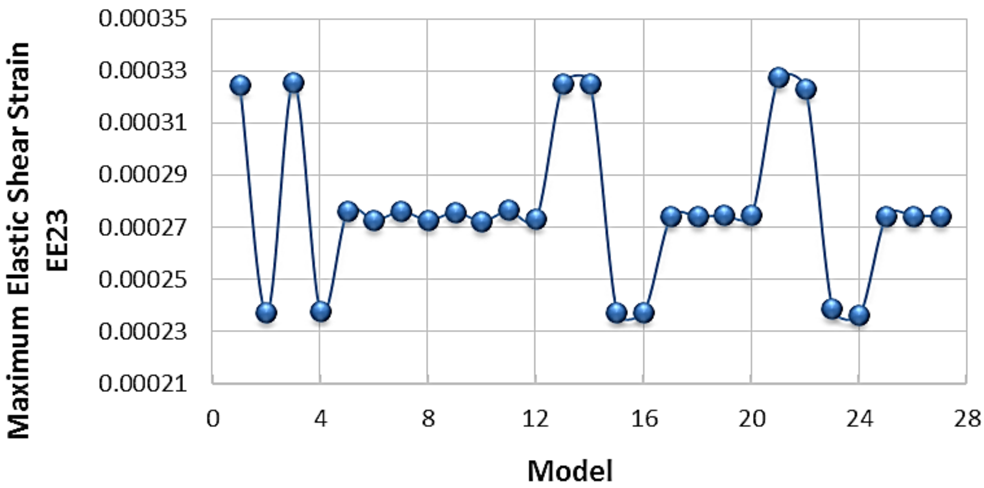

Figure 3. Maximum elastic shear strain (EE23)-27 models.

The position and magnitude of the elastic shear strain in each numerical simulation for the reinforced concrete T-beam can be seen very clearly through the color and the legend data available in each model. The position of the maximum of maximum elastic shear strain that was mentioned above was in model 21 and was 0.0003274499031. This can be seen in the web of the T-beam in red color in the left side which is occurring in the right place near the left support and propagating from the place near the support towards the web and then toward the flange with a slope (see Figure 4). In the same way, the position of minimum of the maximum elastic shear strain that occurred in model 24 with a magnitude of 0.0002360623912 is in the same place with the same pattern and propagation but with a different value. These two outputs are used for the construction of the regression equation, which is the base of the prediction of the shear behavior of the T-beam (see Figure 5).

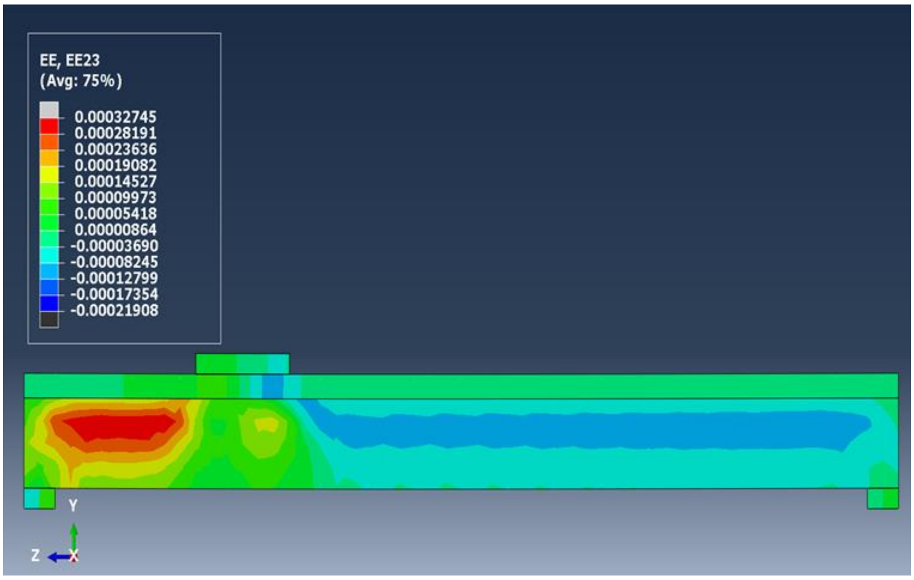

Figure 4. Maximum elastic shear strain (EE23)-model 21.

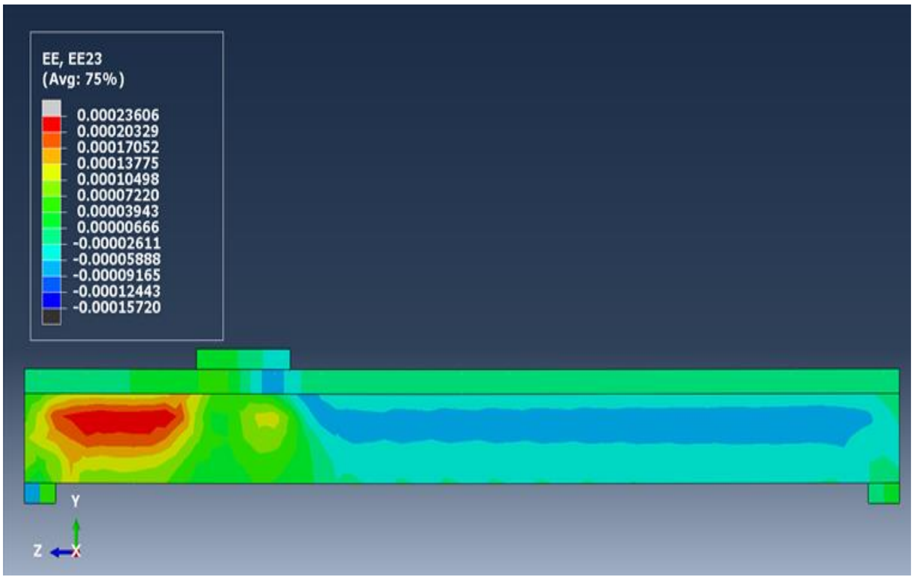

Figure 5. Maximum elastic shear strain (EE23)-model 24.

The elastic shear strain is safe until the propagation of cracks in the concrete. When the cracks appear, shear strain is no longer elastic but plastic and it is the first step for the failure stage if the loading is increased. It is worth mentioning that the stirrups are positioned in this location of the T-beam to withstand the shear stresses to prevent or delay the occurrence of the shear cracks.

3.2. Ductility

The ductility results for all 27 models show that the maximum of maximum deflection (U2) occurred in model 21 with a value of 0.002570510609 m. The minimum of maximum deflection occurred in model 24, which was 0.001898202347 m. It is obvious that the maximum and minimum values of deflection are occurring in the same models for the maximum and minimum values of elastic shear strain. The deflection of the T-beam is an index of the ductility and consequently it represents the shear strength because the generated energy from the applied dynamic load would be dissipated by the deflection response and this behavior is increasing the shear strength. There is a random relation between the maximum deflection responses of the 27 models like the maximum elastic shear strain responses as described above. The irregular distribution of the model data between the 27 models is the origin of this pattern of behavior that is clearly seen in Figure 6.

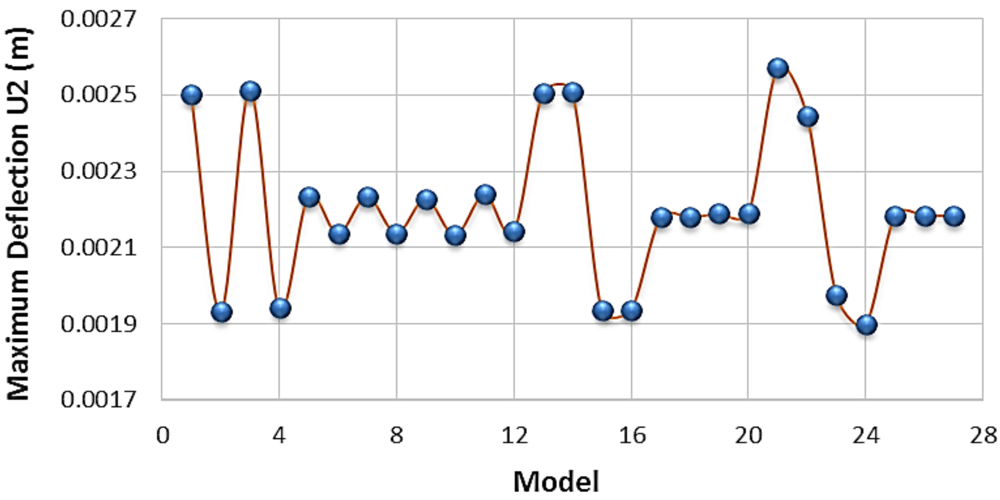

Figure 6. Maximum deflection (U2)-27 models.

The positions of the both maximum and minimum of maximum deflections that occurred in model 21, which was 0.002570510609 m, and model 24, which was 0.001898202347 m, simultaneously are clear and exist at the middle of the reinforced concrete T-beam close to the left support which is in blue. These two responses of the reinforced concrete T-beam are dedicated for further analysis to construct the regression equation for the deflection of the beam, which is an indication of the behavior under dynamic loading for ductility design control (see Figures 7 and 8).

3.3. Regression equations

Both regression equations' results were obtained using MATLAB codes and the Box-Behnken design sampling method. The regression coefficients have been found by least square method. The shear strain is denoted by EE23 and the ductility is denoted by U2 as they have the same symbol in ABAQUS. These equations are predicting the shear strength and the ductility of the reinforced concrete T-beam loaded by a dynamic concentrated load as mentioned in the section of the finite element model. The regression equations should be checked for the reliability process to calculate the coefficient of determination for both regression equations, which is denoted by R^2 (see supplementary file).

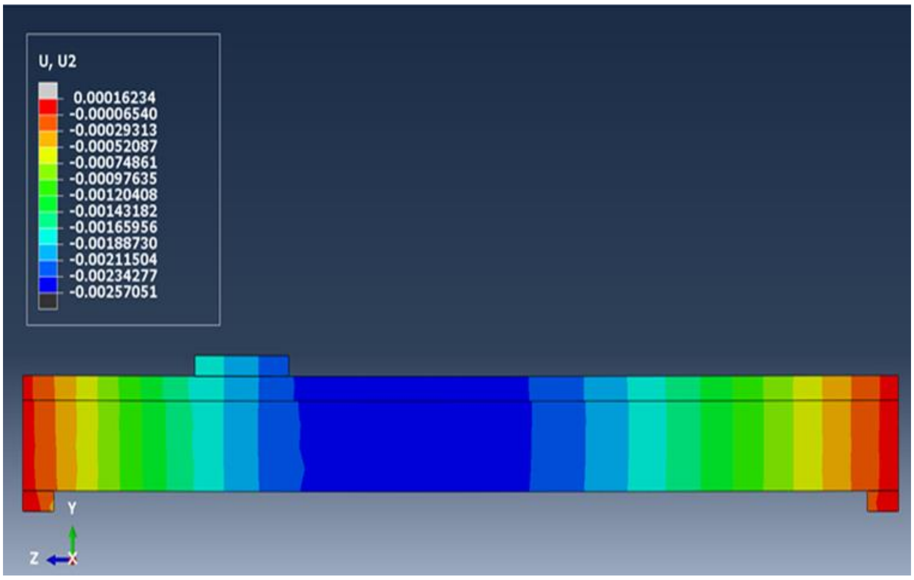

Figure 7. Maximum deflection (U2)-model 21.

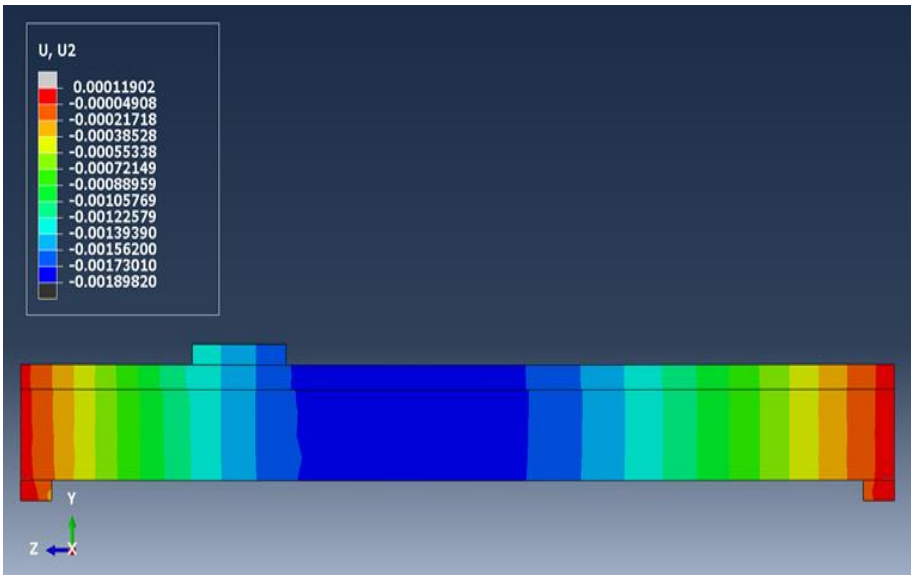

Figure 8. Maximum deflection (U2)-model 24.

3.4. Determination the coefficient

It is necessary to build reliable regression equations, and this depends on determining the coefficient of determination R^2. A comparison between the results of the numerical analysis and the regression equation will display the value of the coefficient of determination. The determination of the coefficient for the elastic shear strain was $R^2 = 1$ (see Figure 9). This is an indication that the regression equation for the elastic shear strain is a great representation in predicting the structural system response with 100% efficiency and it is totally considered a reliable regression equation.

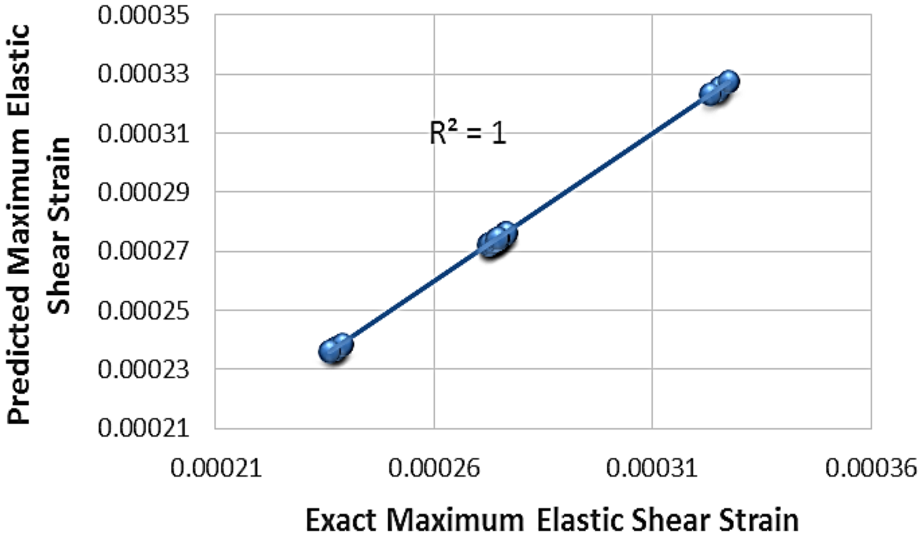

Figure 9. Coefficient of determination-Elastic shear strain.

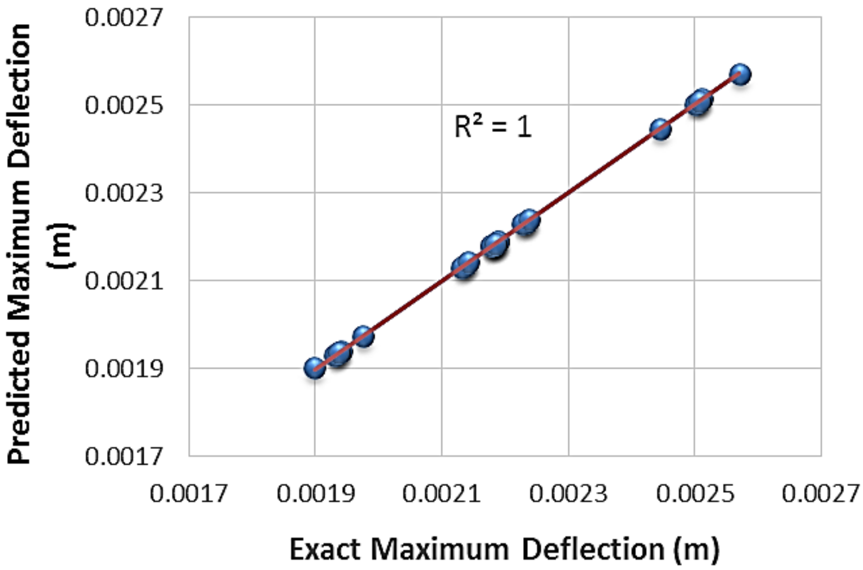

Figure 10. Coefficient of determination-Maximum deflection.

The determination of coefficient for the ductility was also $R^2 = 1$ (see Figure 10). This is evidence that this regression equation is representing the ductility of the reinforced concrete T-beam under the loading up-to efficiency of 100%, which is also considered a reliable regression equation.

3.5. Multi-criteria optimization problems

In [16], a method is suggested to solve a multi-criteria programming problem (MOPP). He considered that the individual optimal value was greater than zero. Later, many techniques were introduced to solve these types of problems [17–20]. Among these techniques, we used the Harmonic mean for values of function to transform the problem into a single one [21]. Our objective function is quadratic with a special type of constraint. Mathematically, usually this model is optimizing objectives that are simultaneously subject to some constraints. Thus, the multi-criteria quadratic programming problem (MOQPP) can be defined as in Eq 3:

$$Max.\ f_i = c_i^T X + 0.5X^T G_i X,\ i = 1, 2, \ldots, r$$
$$Min.\ f_i = c_i^T X + 0.5X^T G_i X,\ i = r+1, 2, \ldots, s \tag{3}$$
$$lb_i \leq x_i \leq ub_i,\ X \geq 0$$

where X is with n-dimensional vector of decision variables, c is n-dimensional vector of constants, r is the number of objective functions to be maximized, s the number of criteria to maximized plus minimized and $(s\text{-}r)$ is the number of criteria that is to be minimized. G is $(n \times n)$ a matrix of coefficients with G as a symmetric matrix. *lb* and *ub* are lower bound and upper bound of the variables, respectively. Harmonic means that H of a set of data is defined as the reciprocal of the arithmetic average of the reciprocal of the given values as in Eq 4. If $(x_1, x_2, \ldots, x_n)$ are n observations, then:

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} \tag{4}$$

To combine the objective functions, we determine the common set of the variables from the following combined objective function. Let $Max.\ f_i = m_i,\ i = 1, \ldots, r$ and $Min.\ f_i = m_i,\ i = 1 + 1, \ldots, s$. To formulate the problem to single objective and by using harmonic mean we have Eq 5:

$$Max.\,g = \sum_{k=1}^{r} \frac{Max.f_k}{H} - \sum_{k=r+1}^{s} \frac{Min.f_k}{H_1} \tag{5}$$

where H and H_1 are the harmonic mean for maximized and minimized objectives, respectively. *Max. g* is the combined criteria as shown in Eq 6:

$$Max.\,g = \sum_{k=1}^{r} \frac{Max.f_k}{H} \tag{6}$$

The algorithm is constructed as follows: Solving the objective function $Max.\ f_k$ by the simplex method, then check the feasibility. Go to the next step; otherwise, use dual simplex method to remove infeasibility. m_i is the optimum value for $Max.\ f_i$, H the harmonic mean for $Max.\ f_i$. Optimize the Eq 5 under the same constraints. Substitute the optimal value to the individual objective to get optimal solution for each one. Finally, stop.

$Max.\ F_1 = 1.737983739999329 \times 10^9$, $Max.\ F_2 = 0.004465897119022$. So, the harmonic mean is 0.0089. Now, divide the coefficients of each objective functions by 0.0089, and then sum them. The optimal point of *Max. g* is $x^* = (2600, 3.500000000000000 \times 10, 800, 2.099999999886206 \times 10^{11})$, which is in the range of the feasible solution. To get an optimal solution for each objective individually, substitute x^* and we get $f_1 = 2.326594836907840 \times 10^{-4}$ and $f_2 = 0.001424567916517$.

4. Conclusions

The only flaw in the research study's successful and trustworthy outcomes is that they are dependent on ABAQUS program-supported numerical simulation results. In order to confirm the optimization results, it is recommended to expand the research study to include lab specimens of the T-beam as a check for the numerical simulations and the analysis of the Box-Behnken design sampling method. Also, to confirm the findings and determine which approach is most successful, additional sampling techniques, such as LP-TAU, MONTE CARLO and Latin Hypercube sampling techniques, can be used for the same research project. Based on the outcomes:

1. The method predicts the reinforced concrete T-beam's ductility and elastic shear strain outputs very well. Because the coefficient of determination for both outputs of the regression equations is 100% reliable, they show excellent representation of the structural system's responses under loading.

2. Using ABAQUS software, numerical simulations are used to identify the four factors' roles.

3. By creating numerous effective design solutions in a very quick and inexpensive process, the regression analysis process is an effective tool that can be used to predict and test the structural responses during the design stage and after construction.

4. The goal is to introduce a new algorithm that uses the harmonic mean of the values of the objective functions to convert a multi-criteria quadratic programming problem to a single quadratic programming problem. The new method involves using MATLAB to code and run the process, yielding an acceptable feasible value and the best possible solution for the reinforced concrete T-beam's shear strength capacity.

Use of AI tools declaration

The author declares that no Artificial Intelligence (AI) tools were used in the creation of this article.

Conflict of interest

The author declares no conflict of interest.

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