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Review

A review on analytical failure criteria for composite materials

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Abstract: Fiber-reinforced composite materials have found increasing industrial applications in the last decades, especially in the aerospace and ground transport fields, due to their high specific strength. However, even though this property allows having a lightweight and strong structure, some critical aspects still limit their use. Most of these depend on several types of undetectable defects and damages, which could affect the residual strength of the composite structures. The paper deals with failure mechanisms involving composite materials under several critic loading conditions, with the aim to assess the limits of currently used failure criteria for composite materials and to show the actual request of developing new failure criteria in order to increase the effectiveness of current design practices. Nowadays, such design practice is based on a damage tolerance philosophy, which allows a structure to tolerate the presence of damages during its in service life, only if the residual strength is kept higher than specific threshold values depending on the damage severity. Hence, the main goal of such paper is to assess the failure criteria's capability to predict the life of composite components under several quasi-static and dynamic loading conditions. Intra-laminar and inter-laminar failure criteria have been investigated and considerations have been provided about the possibility to model the post-failure phase and to implement them within numerical predictive tools, based on finite element method.

Keywords: failure criteria; composite materials; damage tolerance; CFRP; delamination

1. Design Strategy for Composites

Fiber-reinforced composite materials have found increasing industrial applications in the last decades, especially in the aerospace and ground transport fields, due to their high specific strength. However, even though this property allows having a lightweight and strong structure, some critical aspects still limit their use. Most of these depend on several types of undetectable defects and damages, which could affect the residual strength of the composite structures. Mainly, defects and damages affecting composite materials can be classified in two groups:

- manufacturing defects and uncertainness,
- in-service/accidental damages.

Belonging to the former domain there are such defects as porosities, delaminations, matrix cracks and fiber breakage, curing stresses, fiber misalignment, cure and processing tolerances etc. Such defects are covered during the design phase by means of safety factors, which could be related to the results carried out from the quality controls. The latter domain consists of damages produced by accidental/in-service loads. One of the most dangerous aspects related to such damage type is that a composite structure after such in-service loads may return to its original shape without any detectable damage (Barely Visible Damage (BVD)), while it has suffered massive internal damage [1,2]. As a result, such damages must be tolerated in the structure under the in-service loading conditions. It means that, if such damages affect a composite component, its structural health must not be compromised, within a damage tolerance design philosophy. Such defects and damages can result in catastrophic failures if not detected and treated in time. In fact, although damage occurs at material level (micro-meso scale), the overall structural performance (macro level) will be influenced.

Nowadays, to allow composite structures tolerating the presence of such damages, several safety factors are applied to the ultimate design load, by lowering it up to 70%, by leading to an oversizing of the components. In fact, during the current design practice, the main idea is to make a structural component able to tolerate damages by keeping the residual strength higher than specific threshold values, which depend on the damage severity. For civil aviation, for example, according to the FAR25/CS25, BVDs are contained in the damage tolerance domain expressed by the green rectangle showed in Figure 1 and their presence must not reduce the residual strength of the damaged structure below the limit load (LL) [1]. Concerning the readily-detectable damages, the requirements allow the structure to work with a residual strength lower than the LL threshold value, up to the 85% of the LL [1].

For immediately detectable damages, the threshold value is equal to the 70% of the LL criteria. Then, according to the regulation requirements, different safety factors are applied during the design phase as a function of the damage severity.

As more difficult is the damage detection as higher is the residual sustainable load threshold value and, then, the safety factors which must be used during the design practice.

The application of composite materials in the aerospace, as well as in the whole transport field, is pushing the research community to find the best solution to allow the structure to tolerate the presence of damages. Such need led to the born of new composite materials which are inertly damage tolerant and, then, less damage prone than traditional ones. Tsai et al. [3], in their book, proposed a new family of composite laminates, named double-double (with two sets of angle-ply laminates), coupled with thin plies, that can mitigate the concern of BVID and other inter-and intra-laminar damages. In their book [3], Tsai et al. proposed a new method, named Stanford method, providing search engines that can select the best laminates for multiple-load sets based either on smooth or notched coupon. In this way, a less total of uniaxial tests, with and without hole, can be performed and used to understand the structural behavior in presence of damage, allowing reducing the total of wasted specimens, time and costs. Nowadays, in order to achieve a damage tolerance structure, in fact, thousands of different specimens are tested under uniaxial experimental tests, such as Open Hole Tension (OHT), Open Hole Compression (OHC), Filled Hole Tension (FHT), Filled Hole Compression (FHC), etc., which give no clue to how stress rises do under bi-axial loading. So, the authors suggest a new method to select efficient laminates aimed to endure under multiple loads, before any tests.

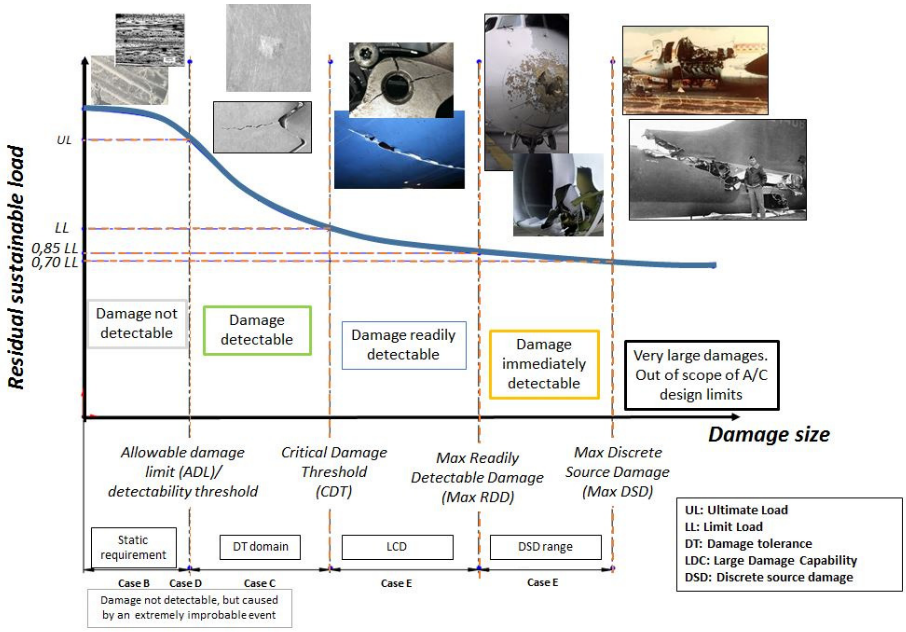

Figure 1. Residual sustainable load vs. damage size.

This paper deals with failure mechanisms involving composite materials under several critic loading conditions, with the aim to assess the limits of currently used failure criteria for composite materials and to show the actual request of developing new failure criteria in order to increase the effectiveness of current design practices.

2. Failure Mechanisms Overview

Actually, failures involving composite structures are characterized by two phases: the first one (elastic phase), where damage does not appear in the structure and the second one, where, at specific stress levels some damages occur [4,5].

Concerning multi-layered composite laminate, damages can be classified in two types: “intra-laminar failures” such as the breaking of the fibers, the debonding between fiber and matrix (Figure 2.a) [4] and the progressive damage in the matrix (Figure 2.b) [5]; the “inter-laminar” failures such as delamination damages (Figure 2.c) [6], which occur at the interface between adjacent layers. Such damages can occur either separately or simultaneously.

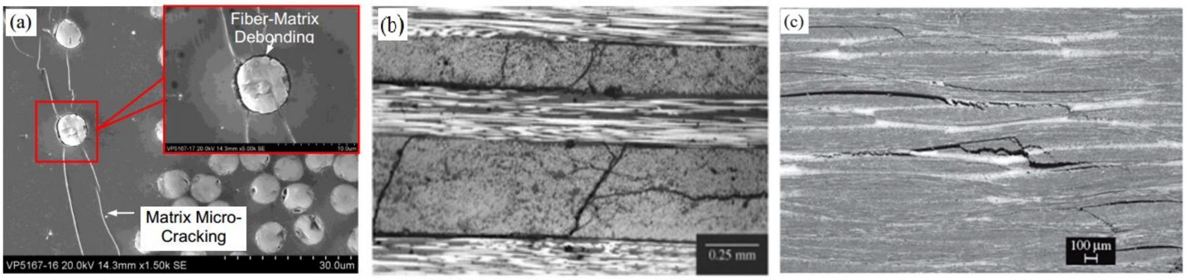

Figure 2. a) matrix-fiber debonding [4]; b) progressive damage in the matrix [5]; c) delamination damages [6].

Since their anisotropy, no equations can be found in literature to link failure modes, material properties, loading conditions and the stress/strain levels. However, supported by the experimental evidence, some assertions about the failure mechanisms can be made.

For example, the first failures involving a unidirectional lamina under tensile load involve few fiber s, in correspondence of the weakest section. As the load increases, as more fiber failures occur.

By the experimental evidence it has been seen that fiber failures increase exponentially up to the material collapse. Because of the fiber s progressive failure, stresses can increase locally by leading, rather to the brittle fiber failure, to the fiber pull-out, shear matrix failure and debonding. Such failure mechanisms are also dependent on the fiber volume ratio (Vf).

Other failure mechanisms occur when a unidirectional lamina is subjected to compression load. Such loading condition, in fact, produce fiber micro-buckling phenomena, which increase with Vf decreasing. With usual value of Vf, (0.4/0.7) such buckling phenomena are preceded by matrix plasticity, matrix micro-cracks and fiber debonding phenomena. Other failure mechanisms can occur under different loading as well as boundary conditions. For example, under bending loads, the simultaneous presence of laminae characterized by different fiber orientations produces the onset of such different failure mechanisms as delamination one because of the different bending property.

Speaking about failure mechanism does not make sense if the loading conditions are not specified. For example, the failures involving a composite component under Low Velocity Impact (LVI) can be different by failure mechanisms under other such loading conditions as fatigue. At this proposal, it is stated that failures under LVI phenomena can be classified in two main groups: “normal fractures”, which involve laminae placed nearby the bottom (not-impacted laminate surface) of the plate; “shear fractures” which involve the upper (impacted laminate surface) laminae. Both kinds of fractures are so named due to the different opening mode affecting them. Normal fracture is caused by planar stresses which depend on the flexural laminate deformation; shear fracture is due to the shear load due to the contact force. Usually, such fractures evolve through the lamina thickness and stop between two laminae with different fiber s orientation [7,8,9]. Then, they propagate like a delamination. Moreover, as thinner is the laminate, its behavior is more affected by bending phenomena and the failure modes involved are mainly those due to longitudinal in plane stresses. As thicker is the laminate and as smaller is the drop mass diameter, more shear failures can occur inside the laminate [7,8,9].

Nowadays, many researchers are paying their attention on such matter, which influences significantly the design phase.

For examples, Caputo et al. [10,11,12] deeply investigated the structural behavior of composite component under impact loading conditions and several experimental and numerical procedures are presented in their papers. Moreover, the residual strength of impacted laminate has been assessed under compression load. Compression tests have been experimentally performed on composite panels, which have been previously subjected to low velocity impact phenomena, considering impact energies of 6 J, 10 J and 13 J respectively. FE models able to simulate in a single analysis both Low Velocity Impact (LVI) and Compression After Impact (CAI) tests onto CFRP panels is proposed. In such a way, it is possible to import automatically the impact damage distribution into the compression step.

Lopresto et al. [13,14], in some of their study, carried out low velocity impact tests at three different impact energy values and three different temperatures on glass fiber composite laminates made by infusion technology. Two different resins, epoxy and vinylester, were considered to impregnate the fiber s: the first is mainly of aeronautical interest, whereas the second one is mainly applied in naval field. Delaminated areas were investigated by the Ultra Sound technique and the results obtained on the different materials at different temperatures were compared.

S.X. Wang et al. [15] investigated on LVI behavior of CFRP laminate. More in detail, the residual tensile strength has been investigated by means of both experimental tests and numerical method. A good agreement between numerical and experimental results has been achieved; it was created and used a subroutine to enhance the damage simulation, which included Hashin and Yeh failure criteria [16,17]. Two different stacking sequences were investigated and the numerical degradation of residual tensile strength was compared with experimental results.

Caprino [18] arranged experimental tests to evaluate the residual tensile strength of AS4/3501 (0/±45)2S composite panels. The results were utilized to develop a semi-empirical expression to predict the residual tensile strength of similar panels. All specimens are visually inspected to evaluate the appearance of damage and identify the most relevant impact energy threshold. It was detected that delamination at low impact energies had little effect on the tensile strength but significantly reduced the compressive strength. It has been found out that the residual compressive strength is influenced by the delaminated area which is a function of the impact energy but it is also dependent on the distance between the delaminated area and the lateral borders of the specimens.

W. Cantwell et al. [19] presented a study on the residual strength lowering of impacted laminates under fatigue and static loads. Woven and non-woven carbon fiber laminates were subjected to drop weight impact at energies of up to 7 Joules. The damaged laminates were tested at differing periods of O-tension and O-compression fatigue and then static residual strength tests were performed. The work shows that non-woven composites are characterized by greater losses in static strength after impact than the mixed-woven composite, particularly when tested in compression. After tensile fatigue, significant improvements in residual strength were recorded, which coincided with the observation of micro-delaminations throughout the laminate.

Fardin et al. [20] showed an analytical approach to estimate the compression after impact strength of quasi-static composite laminates. In such approach the damages are modelled by means of ellipses, each with different stiffness and strength properties. Once a portion of the damaged region fails (failures are obtained when local stresses exceed the load carrying capability of the elliptical inclusions) the load is redistributed to adjacent elliptical inclusion. In such way progressive failure analysis is simulated and the compressive residual strength is predicted. However, the implemented failure criteria introduced many approximations and laminae buckling is not considered. Hence, such work shows the difficulty in predict all phenomena which involve composite laminates under such loading conditions.

Cestino et al. [21] showed a procedure to investigate on the effects of LVI damages on the residual tensile and buckling behavior of composite structures. In this work a simplified analytical method is used to enhance the damage simulation and a degradation model is introduced in the subsequent finite element analysis.

Two different types of analysis, based on the FE method, are performed to estimate the residual tensile strength and the critical uniaxial and shear buckling loads after impact. Tensile and buckling experiments are used to validate the present methodology and a good agreement has been reached.

S.S. Saez et al. [22] studied the residual compressive strength of different layup of laminated composites both numerically and experimentally. They obtained the trend of the average compression strength as a function of the impact energy, compared to non-impacted specimens. All the tested laminated panels showed a similar trend: a fairly sharp reduction at low impact energy and less reduction when the impact energies increase. Failure of damaged laminates under uniaxial compression load was caused by local buckling of the sub-laminates originated in the impact. Among all the specimens, quasi-isotropic laminates showed better damage tolerance, since their normalized strength reduction was smaller at all the impact energies tested.

N. Li et al. [23] investigated on the effects of low velocity edge impact damages on composite stiffened panel with both T-shaped and I-shaped stiffeners under compression load, in order to develop further information about the damage tolerance philosophy.

Before CAI tests, LVIs have been performed on the skin and free edge of the stiffener. A comparison between the structural behavior of both damaged panel with T and I-stiffeners has been shown. Under identical edge impact levels, the damage tolerance behavior of T-stiffened composite panel is distinctly superior to that of I-stiffened composite panel.

As aforementioned, other failure mechanisms occur when composite materials are affected by fatigue loading conditions. Composite material as well as conventional ones such as metal can fail at stress levels lower than the ultimate one, if fatigue occurs.

Hashin [16], showed that there are two major failure processes under fatigue loads: intra-lamina and inter-laminar failure mechanisms.

The former are characterized by the spread of intra-laminar cracks, accumulated in both fiber s and matrix, which propagate along the fiber s direction and the latter, consists in delaminations growth. The main issue as a result of such loading condition is the establishment of the fatigue life.

Fong [24], for example, uses four damage parameters to identify the damage severity. Among such parameters, the residual strength, the maximum damage size, the number of the debonded fiber s and the total of resin crack length. As a result of such phenomenon, debonding between fiber and matrix, the progressive failure of the matrix, fiber failure and delamination damages can occur. However, contrary to isotropic material where the damage onset is usually followed by an evolution phase characterized by a relatively high-velocity growth, in a non-isotropic material the damage propagation can be even stopped allowing the structure to live in presence of such damage.

A damage, in fact, can be stopped at the interface between matrix and fiber. The experimental evidence showed that the damages produced by fatigue loads start in correspondence of the laminae oriented at 90° respect to the applied load direction and do not propagate inside the adjacent laminae. Such damages evolve through the lamina thickness and then propagate like delaminations [25] (Figure 3).

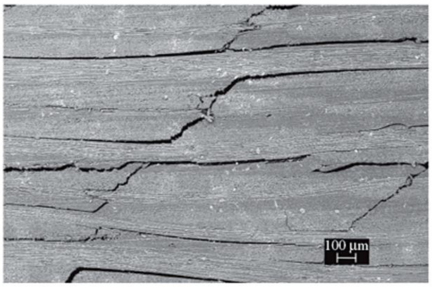

Figure 3. Matrix cracks propagating like delaminations [25].

Such delaminations usually evolve with the increase of load cycle up to the complete separation of the laminae. At the end of fatigue life, new cracks appear also inside the lamina oriented at 0°.

As a result of such overview, damage mechanisms in composite material are still an open topic for researcher.

Hojo et al. [26,27], in their study investigated on the mode I and II delamination fatigue crack growth for carbon fiber (CF)/epoxy laminates. The fracture toughness was investigated.

Tests under mode I loading were carried out using double cantilever beam specimens. For tests under mode II loading, three-point end notched flexure specimens were used for inter-laminar fracture toughness tests, while four-point end notched flexure specimens were used for delamination fatigue tests.

Tseper et al. [28] investigated on the fatigue damage accumulation and residual strength of CFRP laminates under fully reversed cyclic loading ($R = \sigma min/\sigma max = -1$) by employing the of progressive damage modelling method. The accumulation of different damage modes has been assessed, as a function of number of cycles, using a three-dimensional fatigue progressive damage model (FPDM). The residual strength of the CFRP laminates has been assessed through the combined use of the FPDM with a static three-dimensional progressive damage model (PDM). By simulating the experimental procedure, the FPDM has been applied up to certain number of cycles, to estimate the accumulated fatigue damage and then, the static PDM has been applied (quasi-static tensile loading) to predict final tensile failure of the laminates, which corresponds to the residual strength of the laminate, after it has been exposed at the specific cycles. The analysis has been validated experimentally (a) by assuming a laminate free of internal defects, and (b) by considering the initial defects, which were determined experimentally for certain laminates.

Hochard et al [29] developed a non-linear cumulative damage model for woven ply laminates under static and fatigue loads. The validity scope of this model depends on the 'diffuse damage' phase up to the first intra-laminar macro-crack only (first ply failure model). The model, which is based on a continuum damage approach (CDM) and a non-local fiber rupture criteria, was implemented in a FEM code. Finally, full field strain measurements are used to compare experiments and computations for a plate with an open hole submitted to a fatigue tensile load.

Miyano et al. [30] investigated on the tensile fatigue strength in unidirectional CFRP. A prediction method of fatigue strength proposed for polymer composites meeting certain conditions and confirmed for flexural fatigue strength of satin-woven CFRP laminates is applied to the tensile fatigue life of unidirectional CFRP. The method is based upon the four hypotheses: same failure mechanisms for constant strain-rate (CSR), creep, and fatigue failure, same time-temperature super-position principle for all failure strengths, linear cumulative damage law for monotonic loading, and linear dependence of fatigue strength upon stress ratio. Data are provided for tensile CSR, creep, and fatigue tests at various temperatures in the longitudinal direction of unidirectional CFRP. Experimental verification of the prediction method for the tensile fatigue strength of the unidirectional CFRP is presented.

Hosoi et al. [31] carried out a study on the fatigue behavior of quasi-isotropic carbon fiber reinforced plastic (CFRP) laminates. In the relationship between a transverse crack density and initiation and growth of edge delamination, it was found that fatigue damage growth behavior varied depending on applied stress. It was observed that edge delamination initiated and grew at parts where transverse cracks were dense at ordinary applied stress, whereas it was observed that edge delamination grew before or simultaneously with transverse crack propagation at a low applied stress and high-cycle loading. In addition, the critical transverse crack density where delamination begins growing was calculated to evaluate the interaction between transverse crack and edge delamination growth.

3. Failure Mechanisms Criteria

As aforementioned, failures involving composite structures are characterized by three phases: the first one (elastic phase), where no damage appears in the structure, the second one, where, at specific stress levels the damages born and the last one in which the damage propagate inside the structure (post-failure phase).

The main attempt is to try to develop failure criteria which allow designers to understand as better as possible the structural behavior of a composite component and to predict its durability under different loading conditions. Currently it does not exist any criterion universally accepted by designers. However, as matter of the fact, the main used criteria can be divided in two groups: dependent and not dependent failure modes.

It is also important to highlight that some of these allow modelling the post-failure phase too, by describing the degradation of the stiffener matrix which occurs when a damage propagates.

3.1. Not dependent failure modes criteria

Usually, according to such criteria, failures are predicted by means of equations defined as a function of the material strength. For example, the Tensor Polynomial Criterion one proposed by Tsai and Wu [32] (Equation 1).

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j + F_{ijk}\sigma_i\sigma_j\sigma_k \geq 1 \tag{1}$$

where i, j, k = 1, ... , 6 for a 3-D case. The parameters Fi, Fij and Fijk represent the material strengths of the lamina in the principal directions. Because of the large number of material constants, Fijk is usually neglected by leading Equation 1 to a general quadratic expression expressed by Equation 2 [33]:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j \geq 1 \tag{2}$$

where i, j = 1, ... , 6. Assuming that the failures are insensitive to the sign of shear stresses, all terms containing a shear stress to first power vanish: F4 = F5 = F6 = 0. Then, the explicit form of the general expression is given by Equation 3:

$$F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + 2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + +2F_{23}\sigma_2\sigma_3 + F_{11}{\sigma_1}^2 + F_{22}{\sigma_2}^2 + F_{33}{\sigma_3}^2 + F_{44}{\sigma_4}^2 + F_{55}{\sigma_5}^2 + F_{66}{\sigma_6}^2 \geq 1 \quad (3)$$

Other criteria, such as Tsai-Hill [34], Aziz-Tsai [35], Hoffman [36] and Chamois [37] can be found in literature in the form of Tsai-Wu criterion, by using different Fi and Fiji parameters in order to fit the experimental results achieved by other tests.

For example the parameters used in Tsai-Wu criterion are shown in Equations 4:

$$\begin{gathered}
\mathrm{F}_1 = \frac{1}{\sigma^u_{1t} - \sigma^u_{1c}}; \quad \mathrm{F}_2 = \frac{1}{\sigma^u_{2t} - \sigma^u_{2c}}; \quad \mathrm{F}_3 = \frac{1}{\sigma^u_{3t} - \sigma^u_{3c}}; \\
\mathrm{F}_{12} = -\frac{1}{2\sqrt{\sigma^u_{1t}\sigma^u_{1c}\sigma^u_{2t}\sigma^u_{2c}}}; \ \mathrm{F}_{13} = -\frac{1}{2\sqrt{\sigma^u_{1t}\sigma^u_{1c}\sigma^u_{3t}\sigma^u_{3c}}}; \ \mathrm{F}_{23} = -\frac{1}{2\sqrt{\sigma^u_{2t}\sigma^u_{2c}\sigma^u_{3t}\sigma^u_{3c}}}; \\
\mathrm{F}_{11} = \frac{1}{\sigma^u_{1t}\sigma^u_{1c}}; \ \mathrm{F}_{22} = \frac{1}{\sigma^u_{2t}\sigma^u_{2c}}; \ \mathrm{F}_{33} = \frac{1}{\sigma^u_{3t}\sigma^u_{3c}}; \ \mathrm{F}_{44} = \frac{1}{{\sigma^u_{23}}^2}; \\
\mathrm{F}_{55} = \frac{1}{{\sigma^u_{13}}^2}; \quad \mathrm{F}_{66} = \frac{1}{{\sigma^u_{12}}^2}.
\end{gathered} \quad (4)$$

The parameters used in Tsai-Hill criterion are shown in Equations 5 [34]:

$$\begin{gathered}
\mathrm{F}_1 = \mathrm{F}_2 = \mathrm{F}_3 = 0; \qquad \mathrm{F}_{12} = -\frac{1}{2}\left(\frac{1}{\sigma^{u2}_1} + \frac{1}{\sigma^{u2}_2} - \frac{1}{\sigma^{u2}_3}\right); \\
\mathrm{F}_{13} = -\frac{1}{2}\left(\frac{1}{\sigma^{u2}_3} + \frac{1}{\sigma^{u2}_1} - \frac{1}{\sigma^{u2}_2}\right); \qquad \mathrm{F}_{23} = -\frac{1}{2}\left(\frac{1}{\sigma^{u2}_2} + \frac{1}{\sigma^{u2}_3} - \frac{1}{\sigma^{u2}_1}\right); \\
\mathrm{F}_{11} = \frac{1}{\sigma^{u2}_1}; \ \mathrm{F}_{22} = \frac{1}{\sigma^{u2}_2}; \ \mathrm{F}_{33} = \frac{1}{\sigma^{u2}_3}; \ \mathrm{F}_{44} = \frac{1}{{\sigma^u_{23}}^2}; \\
\mathrm{F}_{55} = \frac{1}{{\sigma^u_{13}}^2}; \qquad \mathrm{F}_{66} = \frac{1}{{\sigma^u_{12}}^2}.
\end{gathered} \quad (5)$$

The parameters used in Azzi-Tsai criterion are shown in Equations 6 [35]:

$$\begin{gathered}
\mathrm{F}_1 = \mathrm{F}_2 = \mathrm{F}_3 = \mathrm{F}_{13} = \mathrm{F}_{23} = \mathrm{F}_{33} = \mathrm{F}_{44} = \mathrm{F}_{55} = 0; \\
\mathrm{F}_{12} = -\frac{1}{\sigma^{u2}_1}; \ \mathrm{F}_{11} = \frac{1}{\sigma^{u2}_1}; \ \mathrm{F}_{22} = \frac{1}{\sigma^{u2}_2}; \quad \mathrm{F}_{66} = \frac{1}{{\sigma^u_{12}}^2}.
\end{gathered} \quad (6)$$

The parameters used in Hoffman criterion are shown in Equations 7 [36]:

$$\begin{gathered}
F_1 = \frac{1}{\sigma_{1t}^u - \sigma_{1c}^u}; \quad F_2 = \frac{1}{\sigma_{2t}^u - \sigma_{2c}^u}; \quad F_3 = \frac{1}{\sigma_{3t}^u - \sigma_{3c}^u}; \\
F_{12} = -\frac{1}{2}\left(\frac{1}{\sigma_{1t}^u \sigma_{1c}^u} + \frac{1}{\sigma_{2t}^u \sigma_{2c}^u} - \frac{1}{\sigma_{3t}^u \sigma_{3c}^u}\right); \quad F_{13} = -\frac{1}{2}\left(\frac{1}{\sigma_{3t}^u \sigma_{3c}^u} + \frac{1}{\sigma_{1t}^u \sigma_{1c}^u} - \frac{1}{\sigma_{2t}^u \sigma_{2c}^u}\right); \\
F_{23} = -\frac{1}{2}\left(\frac{1}{\sigma_{2t}^u \sigma_{2c}^u} + \frac{1}{\sigma_{3t}^u \sigma_{3c}^u} - \frac{1}{\sigma_{1t}^u \sigma_{1c}^u}\right); \quad F_{11} = \frac{1}{\sigma_{1t}^u \sigma_{1c}^u}; \quad F_{22} = \frac{1}{\sigma_{2t}^u \sigma_{2c}^u}; \\
F_{33} = \frac{1}{\sigma_{3t}^u \sigma_{3c}^u}; \quad F_{44} = \frac{1}{{\sigma_{23}^u}^2}; \quad F_{55} = \frac{1}{{\sigma_{13}^u}^2}; \quad F_{66} = \frac{1}{{\sigma_{12}^u}^2}.
\end{gathered} \tag{7}$$

The parameters used in Chamis criterion are shown in Equations 8 [37]:

$$\begin{gathered}
F_1 = F_2 = F_3 = 0; \quad F_{12} = -\frac{K_{12}}{\sigma_1^u \sigma_2^u}; \quad F_{13} = -\frac{K_{13}}{\sigma_1^u \sigma_3^u}; \quad F_{23} = -\frac{K_{23}}{\sigma_2^u \sigma_3^u}; \\
F_{11} = \frac{1}{{\sigma_1^u}^2}; \quad F_{22} = \frac{1}{{\sigma_2^u}^2}; \quad F_{33} = \frac{1}{{\sigma_3^u}^2}; \quad F_{44} = \frac{1}{{\sigma_{23}^u}^2}; \quad F_{55} = \frac{1}{{\sigma_{13}^u}^2}; \quad F_{66} = \frac{1}{{\sigma_{12}^u}^2}.
\end{gathered} \tag{8}$$

where, for all criteria the same nomenclature has been used as in the following:

$\sigma_1^u, \sigma_2^u, \sigma_3^u$: normal strength of the lamina in the 1, 2 and 3 directions;

$\sigma_{23}^u, \sigma_{13}^u, \sigma_{12}^u$: shear strength of the material in the 2-3, 1-3 and 1-2 planes;

$\sigma_1^u, \sigma_2^u, \sigma_3^u$ for Tsai-Hill, Azzi-Tsai and Chamis criteria depend on the sign of $\sigma_1^u, \sigma_2^u, \sigma_3^u$, respectively.

$\sigma_{1t}^u, \sigma_{2t}^u, \sigma_{3t}^u$ and $\sigma_{1c}^u, \sigma_{2c}^u, \sigma_{3c}^u$: normal strength of the lamina in the 1, 2 and 3 directions under tensile and compression loading conditions, respectively;

It must be highlight that the nature of such criteria introduces several approximations because it does not consider the material inhomogeneity which governs failure mechanisms.

Contrary to such criteria the second group of failure criteria considers the lack of homogeneity by introducing the different failure modes.

3.2. Dependent failure modes criteria

Contrary to the aforementioned criteria, there are some which take into account the non-homogeneity of the composite materials, by considering the effects of the different failure modes. Such criteria can be used also for Progressive Failure Analysis—PFA purposes.

Currently, the State of the Art can rely on several criteria covering the following failure modes:

- fiber failure;
- matrix Failure;
- delamination damages.

Such failure modes can be further classified in two other sub-groups:

a) non-interactive failure criteria, which do not consider the interaction between stresses/strains acting on lamina. Among such criteria, the main used are:

- Maximum Strain Criterion: according to such criterion, a composite component fails when the strain exceeds the allowable strain related to the failure mode (Equations 9–11).

- Fiber : $\varepsilon_1 \geq \varepsilon^f_{1t}$ $|\varepsilon_1| \geq \varepsilon^f_{1c}$; (9)

- Matrix: $\varepsilon_2 \geq \varepsilon^f_{2t}$ $|\varepsilon_2| \geq \varepsilon^f_{2c}$; (10)

- Shear: $|\varepsilon_{12}| \geq \varepsilon^f_{12}$. (11)

- Maximum Stress criterion: according to such criterion, when the stress exceeds the respective allowable stress, the material fails (Equations 12–14).

- Fiber: $\sigma_1 \geq X_t$ $|\sigma_1| \geq X_c$; (12)

- Matrix: $\sigma_2 \geq Y_t$ $|\sigma_2| \geq Y_c$; (13)

- Shear: $|\sigma_{12}| \geq S_l$. (14)

b) interactive failure criteria, which consider the interaction between stress/strains acting on each lamina. Belonging to such domain there are the following criteria:

- Hashin [38]: according to such criterion, failure mechanisms associated to both matrix and fiber failures are considered. As matter of the fact, such failures are assessed by distinguishing them in: failures under tensile and compression load (Equations 15–18):

Fiber failure under tension (σ11 > 0):

$$\left(\frac{\sigma_{11}}{X_t}\right)^2 + \alpha\left(\frac{\tau_{12}}{S_l}\right)^2 = 1; \tag{15}$$

Fiber failure under compression (σ11 < 0):

$$\left(\frac{\sigma_{11}}{X_c}\right)^2 = 1; \tag{16}$$

Matrix failure under tension (σ22 > 0):

$$\left(\frac{\sigma_{22}}{Y_t}\right)^2 + \left(\frac{\tau_{12}}{S_l}\right)^2 = 1; \tag{17}$$

Matrix failure under compression (σ22 < 0):

$$\left(\frac{\sigma_{22}}{2S_t}\right)^2 + \left[\left(\frac{Y_t}{2S_t}\right)^2 - 1\right]\left(\frac{\sigma_{22}}{Y_t}\right)^2 + \left(\frac{\tau_{12}}{S_l}\right)^2 = 1; \tag{18}$$

Hashin [39] developed also a failure criterion for composite components under a three-dimensional stress-state (Equations 19–22).

Fiber failure under tension (σ11 > 0): (19)

$$\left(\frac{\sigma_{11}}{X_t}\right)^2 + \frac{\tau_{12}{}^2+\tau_{13}{}^2}{S_l{}^2} = 1;$$

Fiber failure under compression (σ11 < 0):

$$-\sigma_{11} = X_c; \quad (20)$$

Matrix failure under tension (σ22 > 0):

$$\left(\frac{\sigma_{22}+\sigma_{33}}{Y_t}\right)^2 + \frac{\tau_{23}{}^2-\sigma_{33}\sigma_{33}}{S_t{}^2} + \frac{\tau_{12}{}^2+\tau_{13}{}^2}{S_l{}^2} = 1; \quad (21)$$

Matrix failure under compression (σ22 < 0):

$$\left[\left(\frac{Y_C}{2S_t}\right)^2 - 1\right]\frac{\sigma_{22}+\sigma_{33}}{Y_C} + \left(\frac{\sigma_{22}+\sigma_{33}}{2S_t}\right)^2 + \frac{\tau_{23}{}^2-\sigma_{33}\sigma_{33}}{S_t{}^2} + \frac{\tau_{12}{}^2+\tau_{13}{}^2}{S_l{}^2} = 1; \quad (22)$$

In the above equations:
Xt denotes the longitudinal tensile strength;
Xc denotes the longitudinal compressive strength;
Yt denotes the transverse tensile strength;
Yc denotes the transverse compressive strength;
Sl denotes the longitudinal shear strength;
St denotes the transverse shear strength;
α is a coefficient that determines the contribution of the shear stress to the fiber tensile initiation criterion;

σ11, σ22, σ33, τ12, τ13, τ23 are the components of the effective stress tensor, {σ}.

Concerning delamination [40-45], such damage is one of the most dangerous involving composite laminates. Such damage can decrease significantly the residual strength of composite components, especially under compression and buckling loading conditions. Concerning the prediction of such damage, the most efficient method involves finite element techniques, since analytical method cannot take into account all variables which delamination are dependent on. In the last years, several techniques have been developed and special-purpose elements have been implemented in the most commercial code based on FEM theory [10,11]. A useful representation of the cohesive element can be achieved by thinking it as an element composed by a two surfaces whose relative motion measured along the thickness direction represents either opening or closing of the interface as well as the transverse shear behavior of the cohesive element. The constitutive model of cohesive elements is based on a bi-linear traction-separation law. By increasing the separation displacement, tractions across the interface increase linearly, with a prefixed slope (penalty stiffness kp), up to reach a maximum value (σc) in correspondence of the effective displacement at the damage initiation (δ0i); then, it decreases linearly up to vanish if the complete de-cohesion at the inter-laminar interface occurs, that is δmax value is reached (Figure 4).

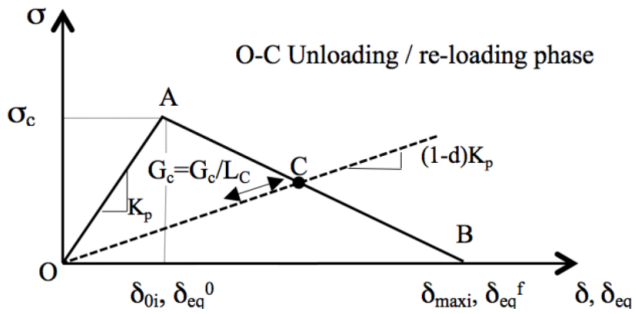

Figure 4. Bilinear cohesive constitutive law.

In Figure 4, the point A the stress-strain values corresponding to damage initiation: when the element stress exceeds this limit value, the element is considered partially damaged and the damage propagation starts.

Damage initiation refers to the onset of degradation at a material point. More in detail, the process of degradation begins when the stresses and/or strains satisfy the specified damage initiation criteria. A part from such numerical procedure, some analytical equations can be found in literature to predict approximatively the delamination area in composite plate as a result of an impact event, such as:

Davies and Zang equation which provides the delamination area Ad as a function of the contact force P, the plate thickness t and the inter-laminar strength St, as shown in Equation 23 [46]:

$$A_d = \frac{9}{16\pi t^2}\left(\frac{P}{{S_t}^2}\right); \tag{23}$$

According to the experimental evidence, such equation seems to be more efficient for thinner laminates and lower loads.

Jakson and Poe equation which allows assessing the envelope of the delamination area along the thickness by means of Equation 24 [47]:

$$A_d = \frac{9}{4\pi t^2}\left(\frac{P}{{S_t}^2}\right); \tag{24}$$

According to the experimental evidence, such equation seems to be more efficient for thinner laminates and lower loads.

3.3. Strain-rate-dependent failure criteria

Nowadays, an important open topic concerns the strain-rate dependent failure criteria. Composite materials, because of the large application in transport field, are usually exposed to severe loading conditions, such as high velocity dynamic loads. Under such conditions, composite materials show a rate-dependent behavior.

Among several attempts proposed by the literature to describe strain rate effects of composite materials, the first strategy adopted by researchers consisted in considering the constitutive laws applied for metallic materials.

Cowper-Symonds law [48], for example, has been considered to model composite laminate tensile and compressive strengths variation under high strain rates. According to such model, the dynamic tensile/compressive strength, σ_d, corresponding to the generic strain rate, $\dot{\varepsilon}$, is given by Equation 25:

$$\frac{\sigma_d}{\sigma_s} = 1 + (\frac{\dot{\varepsilon}}{D})^{\frac{1}{q}}; \tag{25}$$

where, σ_s is the tensile strength achieved under quasi-static loading conditions and D and q are two parameters carried out by analyzing results from experimental tests. In particular, D is calculated as the strain rate value, $\dot{\varepsilon}$, which produces a dynamic stress, σ_d, twice of the static one, σ_s. The parameter q ranges between 4 and 5 for conventional materials and it might assume values also lower than one [49] for composite laminates under tensile loading conditions.

Other authors, as for example Yen-Caiazzo [50,51], attempted developing predicting specific laws for composite materials. According to Yen-Caiazzo law, the dynamic stress, σ_d, is given by Equation 26:

$$\sigma_d = \sigma_s \beta \ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} + \sigma_s \; ; \tag{26}$$

where, β is a parameter carried out from experimental tests and $\dot{\varepsilon}_0$ is the reference strain rate (1 s^{-1}).

Other authors [52, 53] proposed some empirical laws able to fit their own experimental results from tests. Their laws can be considered only for the specific test cases for which they have been pulled out

According to De Luca et al. [54], it can be asserted that Cowper Simonds law is an efficient law to model the strain rate effects for composite materials. The tuning phase, as well as for conventional materials, can be carried out easily by setting D parameter as the strain rate value which produces a dynamic stress twice of the static one, whilst, q parameter can assume different values according to the experimental results trend. In particular regarding the q parameter, it has been estimated to be lower than 1, when the trend of experimental dynamic failure stresses can be expressed by means of an exponential law, and considering values greater than 3, when the trend of dynamic failure stresses can be expressed by a power law.

As a result, in order to design more efficiently such materials, it is important to update the aforementioned failure criteria in a way to consider the strain-rate effects.

In a recent study, a new failure theory named NU-Daniel theory [55], developed at North-western University, has been presented. Such theory allows predicting failures in composite components under multi-axial stress states at different strain rates.

NU-Daniel theory allows predicting micromechanical failure mechanisms even if it is expressed as a function of macro-mechanical properties. More in detail, such theory considers three failure modes consisting in fiber and inter-fiber matrix failures under compression shear and tension modes.

Under the compression dominated case, the composite component is assumed to be loaded primarily in transverse compression with a non-dominant shear component. Under such loading condition, failure is assumed to be governed by the maximum elastic shear strain in the inter-fiber matrix, while the strain along the fiber is constrained to be zero. Under the second mode the component is loaded in in-plane shear with a non-dominant compression component.

Failure is assumed to be governed by the maximum elastic tensile strain in the inter-fiber matrix while constraining the strain component along the fiber s. In the tension dominated case, the composite element is loaded primarily in tension with a non-dominant shear component. Failure is assumed to be governed by the maximum elastic tensile strain in the inter-fiber matrix, while constraining the strain component along the fibers.

These failure modes are expressed by the following failure criteria (Equations 27–29):

Compression dominated failure:

$$\left(\frac{\sigma_{22}}{Y_c}\right)^2 + \alpha^2\left(\frac{\tau_{12}}{Y_c}\right)^2 = 1; \tag{27}$$

Shear dominated failure:

$$\left(\frac{\tau_{12}}{S_l}\right)^2 + \frac{2}{\alpha}\frac{\sigma_{22}}{S_l} = 1; \tag{28}$$

Tension dominated failure:

$$\frac{\sigma_{22}}{Y_t} + \left(\frac{\alpha}{2}\right)^2\left(\frac{\tau_{12}}{Y_t}\right)^2 = 1; \tag{29}$$

where α = E2/G12 is the ratio of the transverse Young's to the in-plane shear modulus.

As matter of the fact, the effect of strain-rate is taken into account by developing an equation of the matrix strength (Yt, Yc and Sl) as a function of the strain rate (Equation 30).

$$F(\dot{\varepsilon}) = F(\dot{\varepsilon}_0)\left(m\log_{10}\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} + 1\right) \tag{30}$$

where F = strength (Yt, Yc, Sl), m = 0.057.

As a result of the strain-rate effects, the new stresses are given by Equation 31:

$$\sigma_i^* = \sigma_i(m\log_{10}\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} + 1)^{-1} \tag{31}$$

where σi = σ22, τ12.

By replacing such new stress to the respective shown in Equation 31 it is possible to consider the effect of the strain rate on the failure mechanisms.

3.4. Numerical modelling of Progressive Failure analysis

In a finite element model, failures can be predicted approximatively by means of the aforementioned criteria. If failures occur in a modelled lamina, the material properties of that lamina must be adjusted in order to model the degradation of its stiffness. As a result, several post-failure degradation models have been developed for Progressive Failure Analysis PFA purposes. The main intent is to try to model the structural behavior of a component affected by the onset of a damage. Such PFA models can be mainly classified in three domains: instantaneous unloading, gradual unloading (linearly, exponentially, etch.) and constant stress, when a failure occurs (Figure 5).

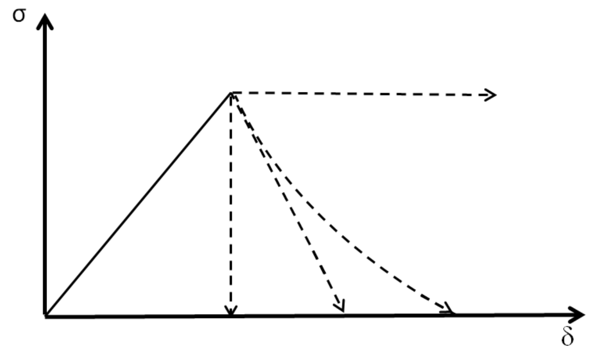

Figure 5. Progressive Failure Analysis: four different unloading patterns.

According to the former, the material properties are degraded instantly to zero, whilst according to the second model, the material properties are degraded linearly or exponentially to zero. Finally, according to the last model, material properties remain constant after the failure occurs. It means that the material will not be able to sustain additional loads. The choice of the post-failure phase strictly depends on chosen initiation failure criterion.

By a numerical point of view, when failures occur, it is necessary to re-compute the stiffness matrix coefficient. In such a way, it is possible to take into account local failures, local stress and deformation.

Several examples of developed finite element models performing the PFA can be found in literature.

Caputo et al. [10,11,12] developed several FE procedures able to predict damage mechanisms occurring in composite laminate under impact loading conditions. Hashin criteria and linear/exponential unloading phase have been implemented as initiation and post-failure criteria, respectively. PFA has been performed also to predict delamination damages onset and evolution.

Reddy et al [56–59] developed a FE procedure to predict failures in laminate under in-plane and/or transverse load. The maximum stress, Tsai-Hill, Tsai-Wu and Hoffman failure criteria have been used to predict damages. In such papers, the authors investigate deeply on the implementation of failure criteria inside the presented FE models. The PFA was carried out also for notched plate under uniaxial and transverse pressure. Also an original three-dimensional progressive failure algorithm for composite laminates under axial tension was presented in [24]. The finite element analysis used Reddy's Layerwise Laminated Plate theory (LWLT) and predicted both in-plane and inter-laminar stresses at integration Gauss points. Reddy et al [56–59] investigated also on the prediction of failure mechanisms in flat composite unstiffened panel under buckling loading conditions. Newton-Raphson method was used to solve the nonlinear analysis and the maximum stress and Tsai-Wu criteria were used. A numerical-experimental comparison of the results was provided.

Other authors such as Ochoa and Emblem [60] investigated on composite failure mechanisms, by using a higher-order plate theory with shear deformable elements. In such work, Hashin and Lee criteria were used to predict inter-laminar and intra-laminar damages, respectively.

Hwang and Sun [61] performed the PFA on composite laminate by employing an iterative three-dimensional finite element method. The authors implemented a modified Tsai-Wu failure criterion to predict fiber failure and Lee and Chang equation [62,63] was used to predict delaminations. By a computational point of view, the mechanical properties of the material are degraded depending on the failure mode.

Huang et al. [64] carried out with finite element models by employing triangular elements including transverse shear effects. A new method was introduced for the calculation of the shear correction factors using a parabolic function.

Riccio et al. [65,66] carried out a study on the shear behavior of a stiffened composite panel with a notch. According to such paper, the structural behavior of the panel has been numerically simulated by means of a three-dimensional progressive degradation model, able to take into account the intra-laminar gradual degradation of composite laminates. As a relevant added value with respect to State of the Art progressive damage models, the proposed methodology, based on energy balance considerations, on three-dimensional failure criteria and on gradual material degradation laws, allows to take into account the real three-dimensional stress distribution, failure onset, and gradual propagation in laminated composites in a Finite Element Analysis without any mesh dependency. A User Defined Material Subroutine (USERMAT) has been used to implement the proposed methodology within the FE commercial code ANSYS©. The same authors proposed other FE techniques for the failure modelling [67,68].

Interesting results are also provided by Lamanna et al. [69,70] about the structural behavior of bolted hybrid joints under tensile loads. In particular, FE techniques have been developed in order to simulate the failure mechanisms in presence of notch. FE models have been validate against experimental tests, allowing the investigation of other laminate stacking sequences, without the need to carry out other experimental tests.

4. Conclusions

A review on failure mechanisms for composite materials has been presented in such paper. Nowadays, the State of the Art is full of failure criteria that can be adopted to predict damage onset and evolution. The efficiency of such criteria is being significantly increasing in the last years thanks to the failure mechanisms investigation [71].

Dependent and not dependent failure mode criteria have been investigated in such paper. Contrary to the former, the latter allow modelling the non-homogeneity of the composite materials, by considering the effects of the different failure modes. Such criteria can be classified in two other sub-groups: non-interactive failure criteria, which do not consider the interaction between stresses/strains acting on each lamina and interactive failure criteria, which consider the interaction between stress/strains acting on each lamina. Such dependent failure criteria can be used also for Progressive Failure Analysis—PFA purposes, allowing the modelling of the post-failure phase. Hence, after damage onset, the progressive lamina stiffness degradation, occurring during the damage evolution phase, can be modelled. Also numerical procedures for the modelling of both intra-laminar and inter-laminar damages have been investigated. In particular, attention has been paid on delamination damages by presenting analytical equations for the prediction of the damage size.

Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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