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Research article

Mutual interaction of stress concentration and stress intensity factor between semi-circular notch and crack

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Abstract: When cracks are detected during examination, assessments have to be done in order to demonstrate the fitness-for-service (FFS) of the component for continued operation. In order to assess a crack, it has firstly to be characterized as surface or subsurface according to its distance to the free surface of the component. The re-characterization process from subsurface-to-surface crack is addressed in all FFS Codes. The specific criteria for the rules on transforming subsurface cracks to surface cracks differ among the FFS Codes and assume regular free surface of the component. In this article, in order to further improve the subsurface-to-surface flaw proximity rules, a new parameter is investigated: the presence of a geometrical discontinuity at the free surface of the component. The analysis, conducted through extended finite element calculations, considered the interaction between a circular subsurface crack and the free surface with stress concentration induced by a circular notch. After analyzing the stress profiles in the notch-crack neighborhood, the calculations have highlighted that, for a given stress concentration at the free surface of the component, for a given ligament and for a given crack depth, the interaction between the crack and the notch highly depends on the notch radius. These results lead to the proposal of a new subsurface-to-surface proximity rule accounting for the interaction between circular crack and the free surface of the component impacted by a stress concentration due to a circular notch.

Keywords: crack interaction; flaw characterization; fracture mechanics; stress intensity factor; stress concentration; extended finite element method

1. Introduction

From non-destructive examination (NDE) measurements, the flaw characterization aims at determining its size, position and orientation to be used in conjunction with Fracture Mechanics concepts. Flaw characterization is important to assess integrity of a structural component, because it is one of the initial steps for flaw evaluation procedures in fitness-for-service (FFS) Codes and Standards. If the characterization of the crack is not appropriate, the predicted results are unreliable, even if precise calculations for fatigue crack growth, stress corrosion crack growth and fracture criteria are provided.

For the purpose of defining the characteristics of the crack to be used in conjunction with the acceptance standards of the FFS Codes, one of the first steps consists of characterizing the crack as surface or subsurface according to the subsurface-to-surface flaw proximity rule. The re-characterization process from subsurface-to-surface crack is addressed in all FFS Codes. The specific criteria for the rules on transforming subsurface cracks to surface cracks are different among the FFS Codes as introduced in reference [1] and are independent of the aspect ratio of the cracks. Recently, based on experimental data and on the interaction of stress intensity factors with the free surface of the component new subsurface-to-surface flaw proximity rules were proposed [2,3]. These rules were established accounting for the dependence on the aspect ratio of the crack and on the thickness of the component.

Up to now, in order to provide more suitable subsurface-to-surface flaw proximity rules, the analyses have assumed regular free surface of the component. However, as widely observed in engineering structures and experimental specimens, the existence of notches often results in inhomogeneous stress distribution. Stress fields near notch-tip have been broadly addressed in the literature ([4,5] and recently [6]). In damage resistance evaluation, the failure criteria of notched components are of significant importance [7]. Beyond this issue, the stress field induced by a notch can also have an impact on the severity of a subsurface crack located in its neighborhood. In other words, the influence of a geometrical discontinuity at the free surface of the component should be addressed regarding the interaction between the crack and the component. This is precisely the scope of this article: investigate of the interaction between a crack and the free surface with stress concentration due to geometrical discontinuity. This is done through extended finite element method (XFEM) calculations. This work is a first study of the aforementioned interaction. Indeed, a lot of parameters are involved by such an analysis: the crack size, the crack aspect ratio, the type of the geometrical discontinuity, the thickness of the component, etc. Therefore, in this article, only circular notch and circular crack are considered for the calculations. After the analyses, a subsurface-to-surface proximity rule is proposed accounting for the interaction between the crack and the free surface of the component with geometrical discontinuity.

2. Materials and Method

2.1. Proximity Rules for Subsurface Cracks

When a subsurface crack is located near a component surface, it is assessed whether it should be transformed to a surface crack or remained characterized as a subsurface crack. The reason for such a transformation is the high stress level acting in the ligament, which can lead to the ligament failure.

In fact, the stress intensity factor at Point 1 (Figure 1) being higher than that at Point 2 due to the interaction by the component free surface, the initiation of failure of the component is expected to occur at the ligament of the subsurface crack. Therefore, the subsurface crack located near the free surface should be replaced by a surface crack from a safety point of view.

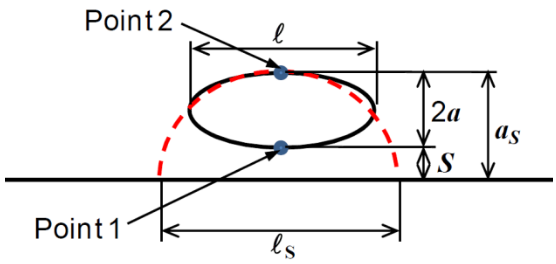

Figure 1. Transformation of a subsurface crack near component free surface.

All fitness-for-service (FFS) codes and standards provide the re-characterization of the subsurface cracks close to the component free surfaces, as rules. The locations at the transformation from subsurface-to-surface cracks and the transformed crack length are described in these rules.

The ASME code [8] provides the proximity rules as follows.

The subsurface crack is treated as a surface crack when the following equation is satisfied:

$$Y = S/a < 0.4 \quad (1)$$

where Y is the flaw-to-surface proximity factor, a is the half crack depth and S is the ligament distance from the subsurface crack to the component free surface (Figure 1).

The depth of the re-characterized crack is described as $a_s = 2a + S$, and surface crack length is given by:

$$\begin{aligned} \ell_s &= \ell && \text{for } a_s/\ell_s \leq 0.5 \\ \ell_s &= 2(2a + S) && \text{for } a_s/\ell_s > 0.5 \end{aligned} \quad (2)$$

where ℓ is the length of the subsurface crack and ℓ_s the length of the transformed surface crack. The meaning of Equation (2) is that when the transformed aspect ratio a_s/ℓ_s is greater than 0.5, the transformed surface crack is always a semi-circular crack.

In order to understand the behavior of the transformation from subsurface-to-surface cracks, fatigue crack growth tests were conducted by using flat plate specimens with subsurface cracks [2]. The idea of these tests was to assess the impact of the crack aspect ratio a/ℓ on the proximity factor Y. The main outcome is that the proximity factor Y is not a constant value, as provided by FFS codes and standards but is increasing with decreasing aspect ratio a/ℓ. This means that the interaction between subsurface crack and plate free surface becomes large when the aspect ratio is small. This observation led to propose a new proximity rule depending on aspect ratio [2].

$$\begin{aligned} &Y < 1.0 - 1.4\,(a/\ell) && \text{for } 0 \leq a/\ell \leq 0.5 \\ &Y < 0.3 && \text{for } 0.5 < a/\ell \end{aligned} \quad (3)$$

In order to validate this new proposed proximity rule, XFEM fatigue crack growth calculations have been performed with various aspect ratios. Stress intensity factors K_1 and K_0 for elliptical subsurface cracks were calculated, where

- K_1 is the stress intensity factor when the crack is close to free surface (left side of Figure 2)
- K_0 is the stress intensity factor for subsurface when the same crack is in the center of the plate (right of Figure 2), i.e., where the interaction with the free surface is negligible.

The stress intensity factor interaction is defined as the ratio K_1/K_0 and gives an indication of the stress intensification when the crack is close to the component free surface.

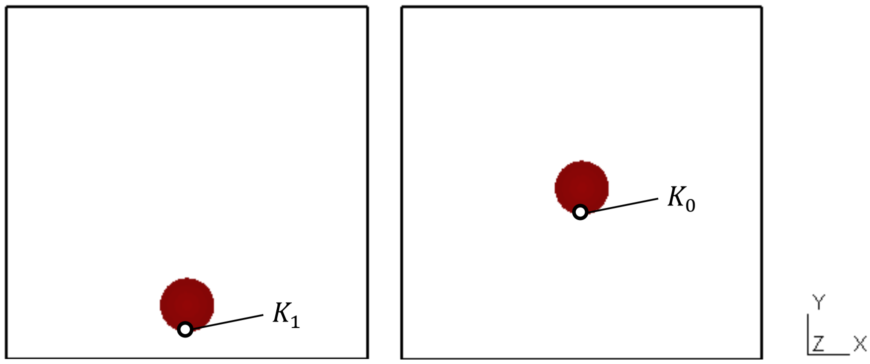

Figure 2. Stress intensity factor locations for assessment of stress intensity factor interaction.

Based on stress intensity factor interaction, equivalent fatigue crack growth rates were obtained for the subsurface cracks. Referring experimental data and the equivalent fatigue crack growth rates, the subsurface cracks are reasonable to be re-characterized to surface cracks when:

$$(\Delta K_1/\Delta K_0)^{4.83} = 1.4 \tag{4}$$

In other words, the subsurface crack should be transformed to a surface crack as soon as the stress intensity factor interaction reaches $K_1/K_0 = 1.072$.

Fatigue crack growth analyses have also highlighted that the subsurface-to-surface proximity rule should be updated according to the thickness of the component. Therefore, an update of the proximity rule has been proposed accounting for the type of component, i.e., piping or vessel [3].

In the work described here above, it has been found that the subsurface-to-surface proximity rule should be considered with a dependence on crack aspect ratio and type of component. On the other hand, the presence of a geometrical discontinuity at the subsurface crack location has not yet been addressed. This is done through XFEM calculations as described in the rest of this article.

2.2. XFEM Method

The XFEM is an extension of the conventional finite element method based on the concept of partition of unity [9]. It allows for the introduction of some knowledge (called enrichment) of the solution into the approximation space. In linear fracture mechanics, the enrichments are the displacement jump across the crack surface, representing the crack opening, and functions spanning the leading term of the asymptotic expansion of the linear elastic solution in the vicinity of the front, accurately capturing the stress singularity. Therefore, the XFEM permits the mesh not to match the crack faces thanks to the addition of a term to the discretization that represents the crack opening.

The description of discontinuities in the context of the XFEM is often realized by the level-set method [10], so as in the Morfeo Crack software. The crack is represented by the intersection of two level-set functions: the tangent level-set ψ and the normal level-set ϕ (Figure 3).

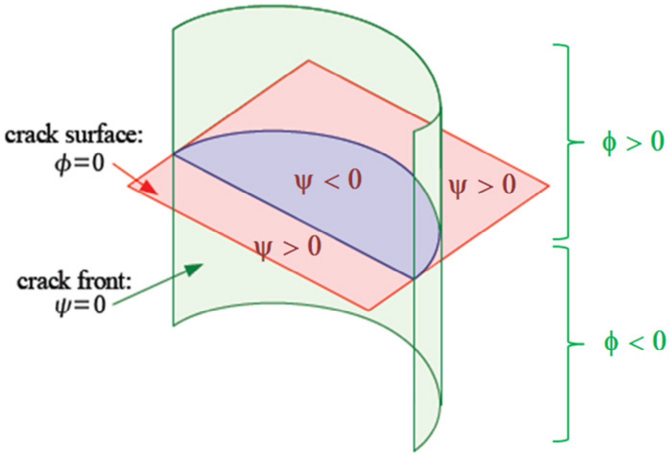

Figure 3. Level-Sets functions for crack representation.

2.3. Notch Geometry and Relevant Parameters

Addressing the impact of a geometrical discontinuity on the subsurface-to-surface proximity rule requires involving many additional parameters related to the geometry of the discontinuity in addition to those already discussed in the paragraph 2.1. Therefore, as a first approach, this article will focus on a simple notch as illustrated in Figure 4. Moreover, the notch geometry is assumed to be semi-circular, i.e., with $b = R$, where b is half the height of the notch, and R is the depth of the notch.

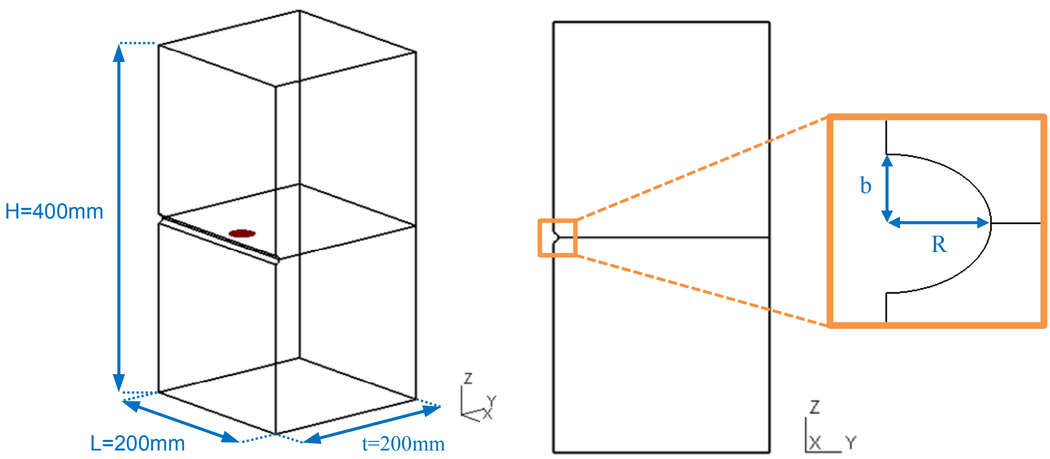

Figure 4. Notch parameters and model dimensions.

As far as the model thickness is concerned, a fixed value of $t = 200\ mm$ is considered, which corresponds to a vessel type component.

As geometrical discontinuities lead to stress concentration, as shown in Figure 5, it is usual to define the stress concentration factor as the ratio of the maximum stress σ_{max} to nominal stress σ_{nom}:

$$\alpha = \sigma_{max}/\sigma_{nom} \tag{5}$$

where the nominal stress is the applied stress σ_{app} corrected with the section reduction due to notch:

$$\sigma_{nom} = \sigma_{app}\, t/(t-R) \tag{6}$$

The stress concentration factor is directly related to the geometrical parameters of the discontinuity. For an elliptical notched geometry, Inglis's solution [11] gives:

$$\alpha = 1 + 2R/b \tag{7}$$

In the present study, the semi-circular notch ($b = R$) leads to a stress concentration factor $\alpha = 3$, as illustrated in Figure 5. The case with $\alpha = 1$ corresponds to a situation without notch.

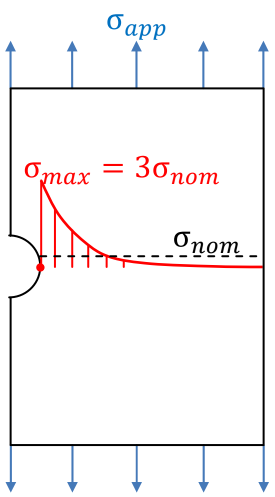

Figure 5. Stress concentration for semi-circular notch.

2.4. Crack Shape and Dimensions

In order to focus on the influence of a notch on the flaw-to-surface proximity rule, the influence of the crack aspect ratio as described in §2.1 will not be considered in this study. Therefore, one single aspect ratio $a/\ell = 0.5$ (circular crack) is addressed.

The crack size $2a = 30\ mm$ is chosen around the depth given in the Table IWB-3510-1 (Allowable Planar Flaws in Ferritic steels) of ASME Code Section XI Acceptance Standards [8] for a thickness of $t = 200\ mm$. The crack is located in the horizontal plane at the level of the notch center. The ligament S corresponds to the shortest distance to the free surface, as illustrated in Figure 6.

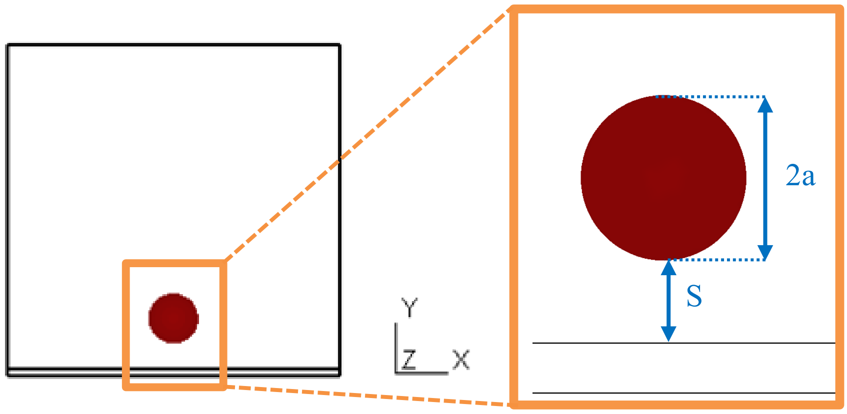

Figure 6. Crack geometry and parameters.

The resulting objective of this study can now be precisely defined: the assessment of the influence of a semi-circular notch on the flaw-to-surface proximity rule of a circular crack in a vessel type component. In order to reach this objective, a sensitivity study will be performed considering three different notch radiuses, i.e., $R/a = 1/3$, $R/a = 1$ and $R/a = 1.5$.

2.5. Loading

The tensile loading σ_{app} is applied at the top of domain so that the crack is solicited in mode I. In order to be able to correctly compare the stress intensity factors of a crack close to notches of different radiuses R, it is important that the nominal stress in the notch section is the same (cf. Figure 5). For that purpose, the applied stress σ_{app} is adapted following Eq. (6) to reach a constant nominal stress of 100 MPa. Figure 7 illustrates this correction for the three cases with notch ($\alpha = 3$) and compares them to the case without notch ($\alpha = 1$).

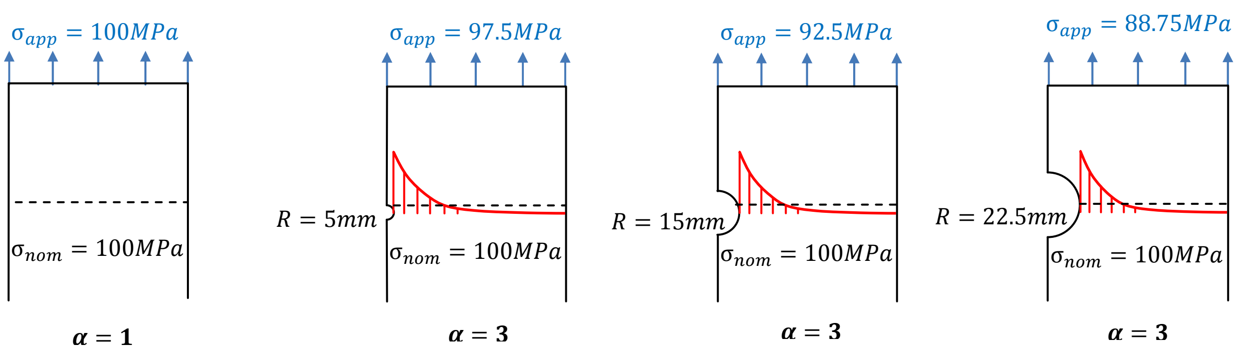

Figure 7. Applied stress correction to reach constant nominal stress without notch and with various notch radiuses.

2.6. XFEM Model

The calculations are performed considering a linear elastic material whose mechanical properties are Young modulus $E = 206$ GPa and Poisson's ratio $\nu = 0.3$.

As depicted in Figure 8 and Figure 9 for the $R/a = 1$ case, the XFEM model is based on a tetrahedral mesh which is refined in the vicinity of the crack as well as in the notch region.

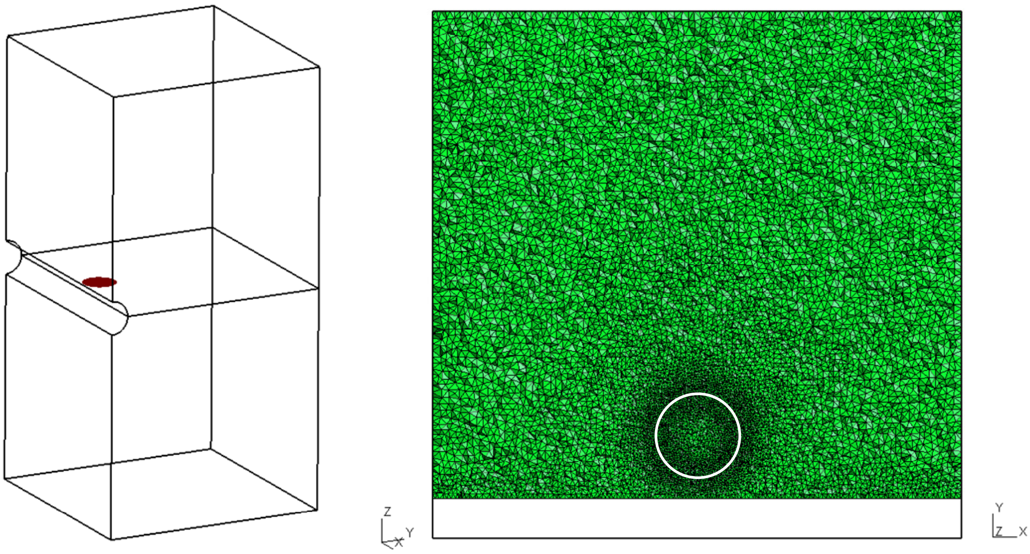

Figure 8. Global view of the R/a = 1 model (left side) with horizontal cross section of the mesh at the crack level (right side).

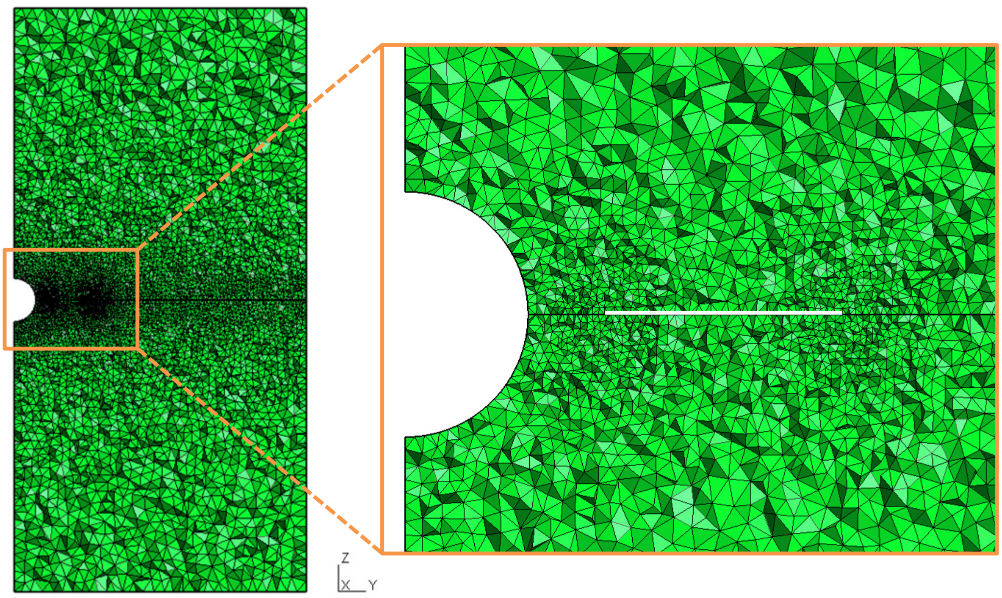

Figure 9. Vertical cross section of the mesh (left side) with zoom on the notch and crack location (right side) for the R/a = 1 case.

3. Results and Discussion

3.1. Stress Profiles

Before calculating the stress intensity factors, the stress profiles in the notch section are analyzed. First the principal stress σ_{zz} is calculated for the three notch radiuses in a model without crack. The results are presented in Figure 10. As expected, the maximum stress is 300 MPa for the three cases. In the left side of Figure 10, the stress gradient in the vicinity of the notch decreases when the notch radius increases. This decrease is proportional to the notch radius R. This is confirmed when looking at the same stress plotted as a function of S/R (cf. right side of Figure 10) where the three stress gradients are the same. Far from the notch, the stress is higher for cases with small notch as the stress profile is there less perturbed than for the case with large notch.

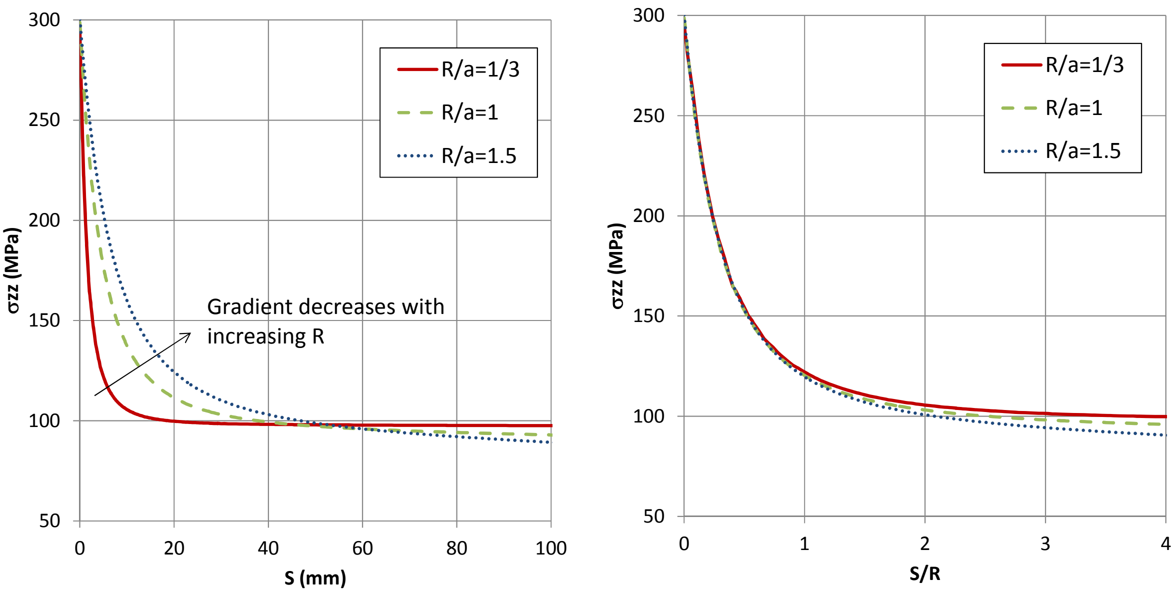

Figure 10. Principal stress in the notch section (without crack) as a function of distance S from free surface, for different values of notch radius R/a.

The second step consists of calculating the stress profiles in the presence of a crack. As linear elastic material behavior is considered, the stress at the crack tip is theoretically infinite. However, the finite element approximation provides a finite value which gives an idea of the stress concentration near the crack tip. An example of the results is presented in Figure 11 with a ligament $S = 6\ mm$. At the notch tip, the stress is not impacted by the presence of the crack except for the case $R/a = 1/3$ (small notch compared to the crack) where the stress value is only slightly above 300 MPa. Near the crack tip P_1 (closest to free surface), the stress level increases with increasing the notch radius R. At the other crack tip P_2 (farthest from free surface), the stress level is lower as it is less impacted by the proximity of the notch.

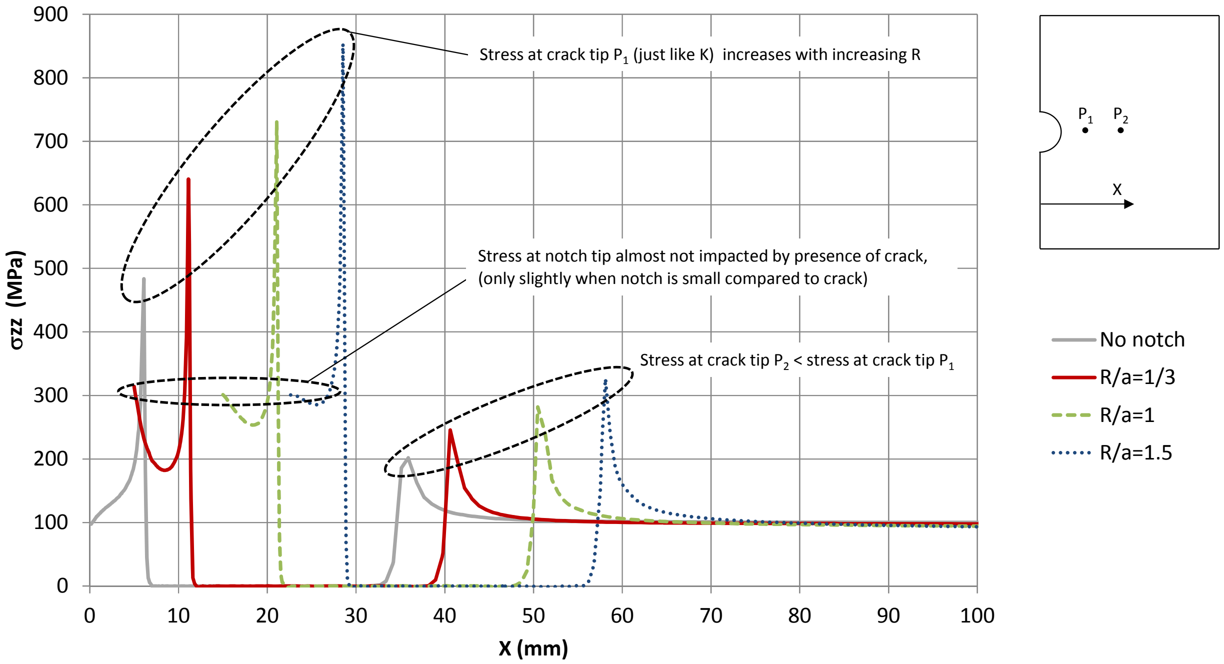

Figure 11. Principal stress in the notch section (with crack) as a function of distance X from model edge, for different values of notch radius R/a.

3.2. Stress Intensity Factor Interaction

The next step of this analysis consists of calculating the Stress Intensity Factors which better represent the stress state at crack tip, and which are required to assess the Stress Intensity Factor interaction K_1/K_0 defined in §2.1.

As explained before, the calculations are performed with only two varying parameters to properly isolate their influence: the ligament S and the notch radius R. As a reminder, the other parameters are set to the constant values reported in Table 1.

Table 1. Value of parameters considered as constant for the analyses.

Crack size	$2a = 30\ mm$
Crack aspect ratio	$a/\ell = 0.5$
Notch shape	$b = R\ (\alpha = 3)$
Wall thickness	$t = 200\ mm$
Nominal stress	$\sigma_{nom} = 100\ MPa$

Figure 12 provides the results of the XFEM analyses expressed as K_1/K_0 as a function of the dimensionless ligament S/a for three dimensionless notch radiuses R/a. As explained before, the transformation from subsurface-to-surface crack is defined as $S/a = 0.4$ in the ASME code. Based on the experimental results described in §2.1, this corresponds to a Stress Intensity Factor interaction of $K_1/K_0 = 1.072$ for a geometry without notch. When a notch is present, K_1/K_0 increases when looking at the same ligament. Moreover, the highest the notch radius, the more K_1/K_0 increases. When looking at the ligament corresponding to the transformation from subsurface-to-surface crack, it also increases in the presence of a notch, and even more when the notch radius is high.

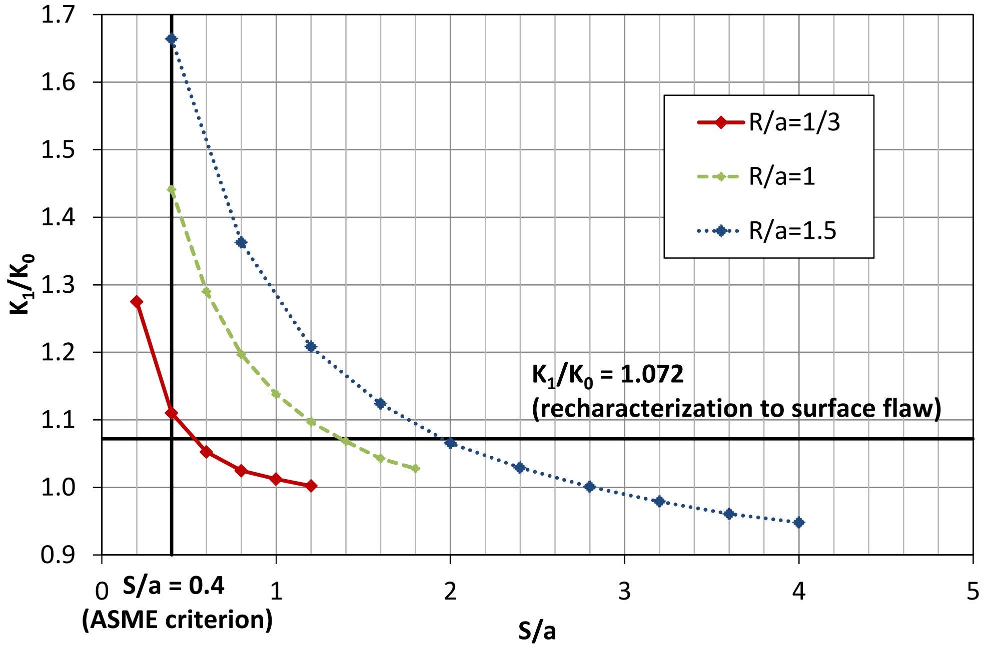

Figure 12. Stress Intensity Factor interaction as a function of the ligament for different notch radiuses.

The points crossing the $K_1/K_0 = 1.072$ horizontal line in Figure 12 are reported in Figure 13 as a function of R/a. These points are considered to be proximity criteria in the presence of a notch. Figure 13 shows the linear relation between the proximity criterion and the dimensionless notch radius R/a, at least in the R/a range covered by this analysis.

The last part of this analysis is dedicated to the influence of the crack size. As explained before, the considered crack size 2a = 30 mm corresponds to allowable depth of ASME Code Section XI. If smaller cracks are reported in the same ferritic materials, the applicability of the linear relation represented in Figure 13 is questioned. To answer this question, two smaller crack sizes ($2a = 20\ mm$ and $2a = 10\ mm$) are considered, keeping the notch radius at $R/a = 1.5$ (which led to the highest proximity criterion). The same analysis as described before for $2a = 30\ mm$ has been conducted and the same proximity criterion has been calculated. Results are presented in Figure 14. It can be observed that the proximity criterion increases when the crack size decreases.

Therefore, an enveloping proximity criterion line can be drawn based on the ASME criterion on one side and on the most penalizing proximity criterion on the other side. This corresponds to the following proximity criterion equation:

$$Y = S/a < 0.4 + 1.3(R/a) \tag{8}$$

Of course, this proximity criterion line is only applicable in the particular range of parameters of this analysis, as reminded in Table 2.

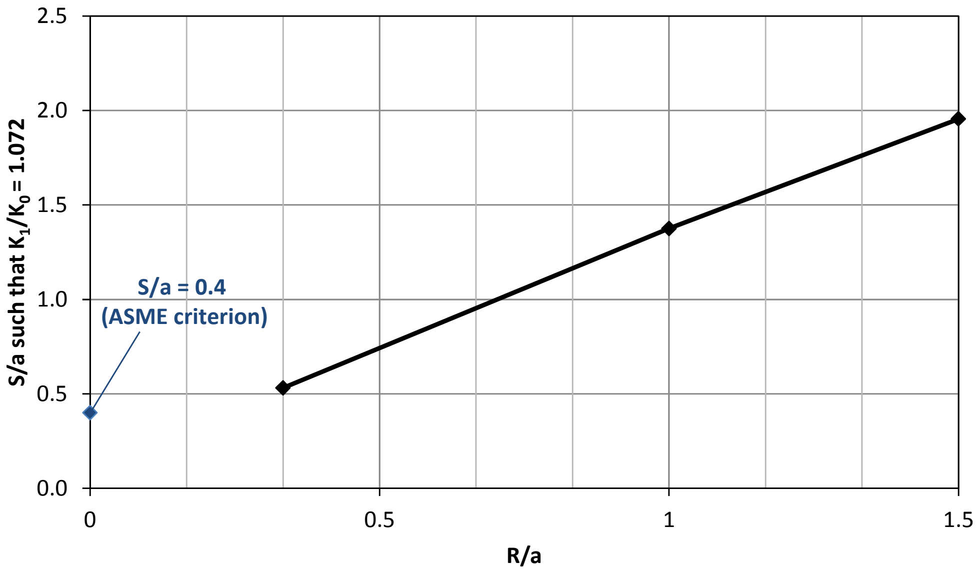

Figure 13. Proximity criterion S/a as a function of dimensionless notch radius R/a.

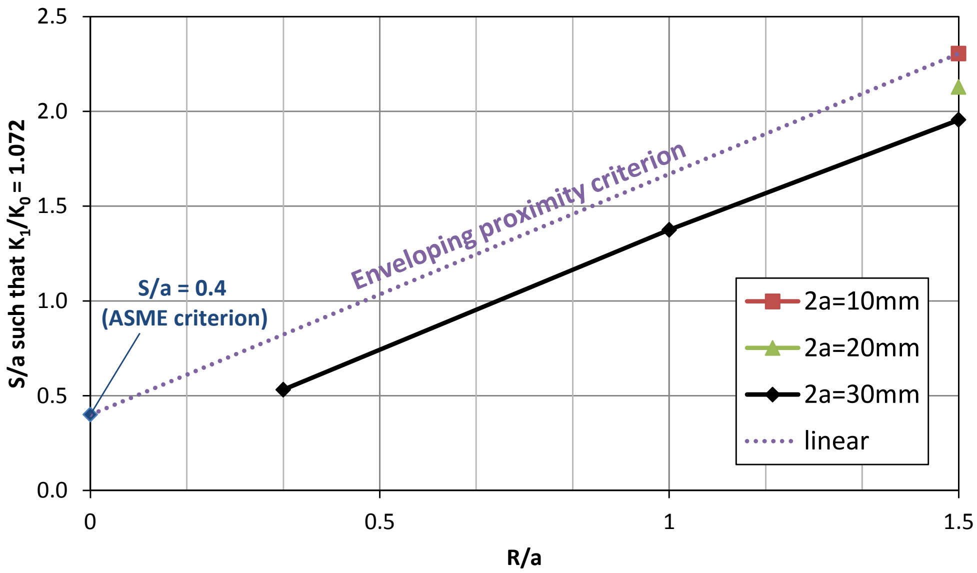

Figure 14. Proximity criterion S/a as a function of dimensionless notch radius R/a considering different crack sizes, and identification of an enveloping proximity criterion.

Table 2. Range of applicability of the proposed proximity criterion line.

For circular cracks	$a/t > 0.025$
For semi-circular notches	from $R/a = 1/3$ to $R/a = 1.5$
In vessel type components	$t = 200\ mm$

4. Conclusion

The size, the orientation and the position of a crack are determined according to characterization rules provided in the FFS Codes. When a crack is located near the free surface of a component, it has to be characterized as surface or subsurface crack. The re-characterization process from subsurface-to-surface crack is addressed in all FFS Codes. However, the presence of a geometrical discontinuity at the free surface of the component corresponding to a stress concentration is not addressed in the FFS Code flaw-to-surface proximity rules.

A first investigation of this purpose has been performed in this paper through XFEM calculations. This first analysis has considered the interaction between a subsurface circular crack and the free surface with stress concentration induced by a circular notch. The calculations have highlighted that, for a given stress concentration at the free surface of the component, for a given ligament and for a given crack depth, the interaction between the crack and the notch highly depends on the notch radius. The results of the analyses have finally led to the proposal of a subsurface-to-surface proximity rule that accounts for the interaction between the crack and the free surface of the component with geometrical discontinuity and that depends on the ratio of the notch radius to the crack depth.

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Conflict of Interest

The authors declare that there are no conflicts of interest in this paper.

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