Environmental Science

AIMS Environmental Science, 10(5): 644–676.
DOI: 10.3934/environsci.2023036
Received: 09 January 2023
Revised: 09 October 2023
Accepted: 17 October 2023
Published: 30 October 2023

http://www.aimspress.com/journal/environmental

Research article

An environmental decision support system for manufacturer-retailer within a closed-loop supply chain management using remanufacturing

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Abstract: Industries face many challenges when emergencies arise. In emergency, there is an increasing demand for self-administered products that are easy to use. The decay rate of these products decreases with time. Moreover, the lack of disposal of used products increases waste and carbon emissions. By observing the scenario, this study develops a closed-loop supply chain management that considers the collection and remanufacturing of used products. The manufacturing rate is linear and the demand is ramp-type and carbon emissions dependent. The model is solved by a classical optimization and calculates the optimal total cost. The results show that the retailer can handle a shortage situation when the demand becomes stable (Case 2) and the total cost increases with the production rate. A sensitivity analysis shows the changes in the total cost with respect to the parameters.

Keywords: supply chain management; cost and cost analysis; decision making; reverse logistics; emission reduction

1. Introduction

Nowadays, people utilize products that easily self-administered. For example, health-conscious people use automated machines to monitor their health status. A supply chain management (SCM) that deals with those kinds of products differs from a general SCM that supplies a general product type. Time is one of the greatest factors when dealing with an emergency. For example, in the recent scenario of COVID-19, oximeters and thermometers are two products in a high demand. A patient requires an oximeter at the right moment to measure their blood oxygen levels. A delay in this measurement can produce a life-threatening situation for the patient. Such products are normally in high demand at first, which gradually increases until a certain point in time. When the high demand subsides, the rush and demand for the product reduces. These scenarios highlight the importance of time. For example, Rani et al. [1] researched an electronic industry that used refurbished electronic products. Those items were then sold for profit at a lower price. The cannibalization impact of refurbished product sales on new product sales was taken into consideration . Self-administered medical products have a limited time to be used; after this time period, these products are not safe to use. Used products can accumulate in the environment, thereby increasing waste, carbon footprint, and pollution. The Illinois Environmental Protection Agency (EPA) Medication Takeback Program provides a method to collect and dispose of used products and medications (https://epa.illinois.gov/topics/waste-management/waste-disposal/medication-disposal.html, 22nd September 2023; 21.30 KST). The main goal of this initiative is to reduce waste and emissions. Such products can either be collected from tackback centers and retailers or they can be tracked through various tracking devices [2]. Remanufacturing provides an avenue for both the reuse of products and the reduction of emissions from waste products. Several emissions reduction policies exist in the literature [3] and are often used by various industries. These industries use several policies to reduce environmental effects, such as eco-design products [4], carbon tax, and green investments [2].

The collection of these used products from customers through reverse logistics (RL) forms a closed-loop supply chain management (CLSCM) for an emergency. One of the important facts about the reuse of products is that the product which is supplied through the SCM should be a reusable type. For example, used oximeters can be reused, which helps to reduce solid waste. To reuse an oximeter, a reusable type of oximeter (i.e., not a disposable type) is needed. CLSCM supplies the product through forwarding logistics and returns the used product for remanufacturing through RL. In RL, the used products flow from customers to the manufacturer. Generally, companies focus on setting up a reverse supply chain either because of environmental regulations or to reduce their operating cost by remanufacturing products and components. From an economic point of view, this process is quite advantageous, thereby reducing the environmental load by returning used items in the manufacturing process.

1.1. Research gaps

- There is a lack of literature in CLSCM [5] under remanufacturing when the market demand increases with time. Most previous studies that consider remanufacturing or similar conditions do not permit shortages with partial backlogging.In an emergency situation, the manufacturer faces sudden challenges. Because of this, the demand for various products, such as oximeters and thermometers, increases rapidly, for which the manufacturer has to increase their manufacturing rate. So far, this situation has not been considered in previous stuidies [6].
- CLSCM needs RL to collect and remanufacture the returned products, which must support the manufacturing system [7]. However, implementing a manufacturing-remanufacturing combination has not been considered to aid the integrated strategies for ramp-type demand under two different

situations based on partial backlogging.

1.2. Research fulfilled

- To the best of the authors' knowledge, no reverse logistics model has been developed that considers linear production and emission-dependent ramp-type demand rates. Therefore, this study attempts to develop a CLSCM with linear production and emissions-dependent ramp-type demand rates. The business world is becoming more competitive daily, and the related issue is an environmental concern. This research fills the supply chain gap under remanufacturing when the market demand increases with time. Here, the shortages are permitted with partial backlogging [8].
- There is no model of CLSCM under a specific situation such as COVID-19.This research incorporates elements of the COVID-19 pandemic. The manufacturer faces additional challenges and set-backs during a pandemic. Due to this, the demand for specific products, such as oximeters and thermometers, increases rapidly, for which the manufacturer has to increase their manufacturing rate. Here, this condition has been considered [9].
- CLSCM uses RL to collect and remanufacture returned products, which, in turn, supports the manufacturing system [10]. As a result, this research takes implementing a manufacturing-remanufacturing combination into account, which helps the integrated strategies capture and retrieve them.This study designs an environmental decision support for a manufacturer-retailer within a CLSCM and considers products with a sudden increase in demand. The rate of production is a linear function. The customers desire can be met swiftly, and the demand can be calculated with an emergency, thereby raising demand for specific items. The demand is considered to be ramped and carbon-emissions dependent. As a result, partial backlogs of unmet demands are considered.

1.3. Orientation of the research

The model is organized in the following manner. Section 2 provides a literature review, and Section 3 provides the problem description, assumptions, and notation used in the models. Section 4 examines the model description with a mathematical formulation of the inventory model. The best response policy is determined in Section 5. Section 6 contains numerical examples, findings, and a comparative study. Section 7 discusses the managerial insights, and Section 8 includes a sensitivity analysis with tables and charts. Section 9 provides the conclusions of the paper.

2. Literature review

This section illustrates a detailed literature analysis based on the key features of the study. Discussions are divided into subsections.

2.1. Demand pattern within an SCM under different circumstances

Over time, researchers have accepted that inventory management for different types of products and situations can vary. From the demand pattern to the SCM, everything changes based on the situations and products. In a subsequent report, Cheng et al. [11] identified an effective replenishment strategy for a declining inventory model. In that model, a trapezoidal demand trend was considered for seasonal goods; shortages were permitted, and there was a partial backlog. Rini et al. [12] studied a model for determining the best manufacturing schedule for a deteriorating item. Kawakatsu [13] looked into an economic order quantity (EOQ) model with a seasonal impact. The demand was assumed to be ramp-type and was a function of time and an on-hand inventory. In the COVID-19 environment, Kumar et al. [8] developed an inventory model with ramp-type demand and explained the important role of preservation technology. Panda et al. [14] examined an inventory model for the time-dependent demand rate of trendy products. By assuming a constant rate of deterioration, they derived an ideal replenishment strategy for deteriorating products. Sarkar et al. [15] developed a concept for creating innovative products that inherited certain risks for market demand. The higher the risks were, the higher the market demand for innovative products was. Saha et al. [16] discovered the ideal solution for a dynamic investment-dependent demand within an SCM. Geometric programming could help to find the global optimum solutions. Skouri et al. [17] proposed an inventory model of ramp-type demand for allowable shortages and partial backlogs, where products were naturally deteriorating in nature. Wu et al. [18] proposed an inventory model assuming the seasonal impact of declining goods. Wee and Chang [19] created a product life cycle that included remanufacturing and deterioration. That being said, there is a tendency within the literature to achieve sustainability and sustainable development goals (SDG), either fully or partially. Miramontes-Viña et al. [20] studied an energy model which established a sustainable energy resource in rural areas. In a similar context, Monteiro et al. [21] analyzed a few business policies for SDG. They found that SDG 8, SDG 13, and SDG 12 were the top three prioritized SDG goals among different countries. Carbon emissions for industries and trades were based on production and energy consumption [22]. Zhou et al. [23] discussed different tariffs for carbon emissions under a global SCM. The discussions show that the demand pattern plays an important role, and the use of ramp-type demand for a CLSCM with remanufacturing is not a widely discussed research area.

2.2. Product return and remanufacturing through RL

Returning and reusing used products is becoming more and more popular. The increasing demand for specific products is one of the concerns for non-renewable raw materials resources. Likewise, the increasing the use of products increases waste and, therefore, increases pollution. RL helps to collect the returned products, and researchers pay interest for different policies for a CLSCM. Rossit et al. [24] surveyed an RL model with remanufacturing, which could be solved by goal programming and multi-criteria decision-making (MCDM). Jaber and Saadany [25] investigated an RL inventory model which assumed that the demand for manufactured products differed from remanufactured (repaired) products. Konstantaras et al. [26] extended the work of Koh et al. [27] by adding the inspection and sorting of returned products. Liao [28] built a non-linear programming model for a logistics network architecture. For multi-echelons, the RL definition was considered. As a result, either the net value of products returned for repair, remanufacturing, recycling, reuse, or incineration/landfill was increased. Maric and Opazo [29] looked at green services to foster a more flexible and effective SCM. They spoke about the motivations and challenges that computer and electronics industry experts face during RL. Simultaneously, they aimed to show how practitioners in the electronics industry can use green technology and RL services to achieve sustainable end-of-life item recovery. Weng and McClurg [30] created a product life cycle with a fuzzy distribution time and demand, and Ahmadi et al. [31] produced a case study on a customer-centric reverse logistics model for electronic devices. Ahmed and Sarkar [32] looked for a next-generation biofuel production system manufactured from the waste of a non-edible biomass. Through the integration of production, distribution, inventory management, recycling, and locational decisions, Goodarzian et al. [33] explored the citrus supply chain design challenge and addressed the triple bottom lines of sustainability and a circularity strategy. Momenitabar et al. [34] examined the effects of the backup suppliers and lateral transshipment/resupply while constructing a sustainable closed-loop supply chain (CLSC) network at the same time to reduce the shortage that may occur during the transmission of manufactured items within the network. Thus, it was discovered that the use of ramp-type demand for emergency products within a CLSCM and RL is not properly discussed.

2.3. Carbon emissions due to various activities within an SCM

The transportation, warehousing, and storage of rotting goods all contribute to carbon emissions. Mashud et al. [35] proposed a sustainable inventory system with greenhouse facilities by controlling carbon emissions through green investment. They included investments in energy-efficient green technologies in a two-warehouse inventory system to reduce carbon emissions during commodity transportation from owned to leased warehouses and final customers. Sarkar et al. [36] developed an uncertain advertisement-dependent SCM model. Mishra et al. [37] used a linear and non-linear price-dependent market to create a carbon limit and tax-regulated sustainable inventory management for a buyer. They investigated how to handle inventory in a greenhouse farm with managed carbon emissions. Kugele and Sarkar [38] proposed a multi-stage smart production system with a newly developed emissions reduction policy from the transportation sector. Mridha et al. [39] investigated a sustainable SCM for a green product and investigated the carbon emissions from the production system. Teng and Feng [40] established a remanufacturing system under the cap-and-trade policy to reduce defective products and carbon emissions, which is the manufacturer's main methodology to ensure sustainability. Tiwari et al. [41] proposed an integrated single-vendor-single-buyer inventory model for declining goods of poor quality that took carbon emissions into account. In various backorder circumstances, investments into preservation and green technology efforts were needed. Yadav and Khanna [42] addressed an inventory model for perishable products with an expiration date under a carbon tax policy. Mishra et al. [43] studied a model that invested in the reduction of carbon emissions from the production system. Babaeinesami et al. [44] investigated the network design for a CLSC, which considered suppliers, assembly centers, retailers, customers, collection centers, refurbishing centers, disassembly centers, and disposal centers. In order to simultaneously reduce the overall cost and carbon emissions, the CLSC tried to create a distribution network around the demands of the consumer. Goodarzian et al. [45] concentrated on the network of supply chains for various citrus fruits. The originality of the study was in the mathematical model for a three-echelon supply chain (SC) that took carbon emissions , and time frame into consideration at the same time. The model depicted the impact of a carbon reduction investment strategy on the inventory system. From these aforementioned studies, no CLSCM model discussed the ramp-type demand and linear production rate for a remanufacturing center.

Some recent research works are shown in Table 1. It is seen that few concepts of this study can be matched with other research articles, partly but terminologically. For example, the CLSC is a well-discussed topic. However, few studies have discussed the collection of electronics products [10], and few have discussed agricultural products and methodological development [33]. However, this study examines a CLSCM with remanufacturing based on the properties of a self-care monitoring medical device. Thus, a direct similarity does not lie in the line of the aforementioned research.

Table 1. Authors' contribution in CLSCM with remanufacturing.

Author (s)	Model	Production rate	Demand rate	Carbon emissions	Partial backlogging
Dey et al. [9]	SCM	Variable	Price	No	Yes
Ullah and Sarkar [10]	SCM with RP	Constant	Random	No	No
Skouri et al. [17]	Inventory	No	Ramp-type	No	Yes
Koh et al. [27]	Production with RP	Constant	Constant	No	No
Goodarzian et al. [33]	SCM	No	Constant	No	No
Alamri [46]	SCM with RP	Time-dependent	Time-dependent	No	No
Chung and Wee [47]	Prod-inv with	Constant	Constant	No	No
Motla et al. [48]	SCM with RP	Constant	Constant	No	No
Rani et al. [49]	SCM with RP	Constant	Constant	Yes	No
Rani et al. [50]	SCM	Constant	Linear and constant	No	No
Garai and Sarkar [51]	CLSCM	Constant	Constant	Yes	No
Safdar et al. [52]	Networking with RP	Constant	Constant	CAPT	No
Sarkar et al. [53]	Two-stage SCM	Constant	Constant	Yes	No
Singh and Sharma [54]	SCM with RP	Variable	Price-dependent	No	No
Wang and Huang [55]	Prod-inv	Constant	Ramp-type	No	No
Yang et al. [56]	SCM	No	Price sensitive	No	No
This study	SCM with RP	Linear type	Em-dep ramp-type	Yes	Yes

Notes: SCM – Supply chain management, RP – Remanufacturing process, Yes – Include, No – Exclude, CAPT – Cap-and-trade policy, Prod-inv – Production-inventory, Em-dep – Emission-dependent.

3. Problem description, assumptions, and notation

The following problem description, assumptions, and notation are considered in this study.

3.1. Problem description

This study describes a CLSCM that involves self-care monitoring products. The market demand for these types of products is preferably time-dependent. The waiting time and shortage greatly affect peoples, and the situation can be life-threatening. As the demand for these types of products skyrockets, shortage and lost sale scenarios are very common. Interestingly, even though the products have an exponentially-increasing demand, there will eventually be a time at which the demand is stable and follows a constant pattern (ramp-type demand). The problem arises when a shortage occurs. Now, the products are reusable. After using the products, customers return those products to a retailer. Due to the lack of proper awareness, all sold-out products will not be returned by customers after use. The return rate of used products is δ and the recovery rate from returned products is σ. These reusable products are handled with a lot of care due to of its impact on human life. Making remanufactured products using the returned products is faster than making new products. These remanufactured products can help to fulfill the shortage quantity for the retailer. This study aims to utilize remanufacturing in two ways: i) support the retailer to overcome shortage quantities by utilizing reusable medical devices and ii) reduce solid waste and carbon emissions from waste. For a single type of product, a single manufacturer and a single retailer participate in this CLSCM. The decision is taken in a centralized way to optimize the time of the inventory level. As medical devices are required on an emergency basis, time is one of the prime issues to be solved in a centralized way.

3.2. Assumptions

The following assumptions are used to formulate the proposed mathematical model:

- In recent days, the demand for self-care products has risen at an increasing rate in the early stages of their existence. However, when a saturation point is achieved, it remains constant. In this situation, it is important to pay special attention to the issue of carbon emissions. The demand rate for a single type of product is ramp type, and carbon emissions affect the retailer's market demand. These products have a rate of decay.
- The manufacturer runs the manufacturing and remanufacturing processes within the same cycle. After collecting the used products from customers, the manufacturer remanufactured these products and sends them back to the retailer. The demand for the product from the manufacturer is $\alpha_1 + \beta_1 t \ (\alpha_1 > \beta_1 > 0)$.
- The market demand rate $f(t)$ [8] of the product for the retailer is a function of time t and carbon emission ϕ, which is denoted by $f(t) = \begin{cases} D(t) - \phi\varphi \ t < u \\ D_0 - \phi\varphi \ t > u \end{cases}$, where $D(t) = x + yt \ (x > y > 0)$ is the linear function and D_0 is a constant function of the retailer. The unit carbon emission [10,49] in the cycle is denoted by ϕ, while the carbon emission demand parameter is denoted by φ.
- Due to the ramp nature of the market demand, which always varies with time t until a certain time u, the manufacturer considers a linear type of time-dependent manufacturing rate $(\alpha + \beta t)$ during the manufacturing cycle to control the situation [9]. Similarly, the remanufacturing rate of the manufacturer is $a + bt \ (a > b > 0)$, which is a time-dependent remanufacturing rate.
- Products have a decay rate. θ_1 is the decay rate of products for manufacturers and θ_2 is the decay rate of products of the retailer. Then, a shortage is allowed, and an unsatisfied demand is partially backlogged for the retailer. The backlogging rate for the retailer is $\Delta(\mathrm{t}) = e^{-\rho t}$, where $\rho \geq 0$ and t is the waiting time of customers to receive a product. $\Delta(\mathrm{t})$is a decreasing function of the waiting time t [8].
- The study is presented with two strategies for the retailer. In the first strategy, it is assumed that $t_r < u$, i.e., the time when the inventory level reaches zero after completion of the shortage is less than changing the time point from linear demand to constant demand (Figure 1). In the second strategy, $t_r > u$ (Figure 2).
- There are many retailing cycles for each manufacturing and remanufacturing cycle. The lead time is negligible [49].
- The manufacturer receives used products and those returned products are remanufactured. Returned products are received at a rate δ and recovered at the rate σ. Waste products are recycled [10].

3.3. Notation

The following notation is used throughout the model.

Decision variables	
t_{m1}	maximum inventory level time of the manufacturer (month)
t_r	time when the inventory level of the retailer reaches zero after the completion of the shortage (month)
t_R	maximum inventory level time of remanufacturing (month)
Manufacturer's parameters	
α_1	basic demand (unit/unit time)
β_1	time-dependent demand (unit/unit time)
θ_1	decay rate
Manufacturing process	
α	basic manufacturing rate (units/unit time)
β	time-dependent manufacturing rate (units/unit time)
t_{m2}	time when the inventory level is zero (month)
L	total number of manufacturing cycles in t (integer)
S_M	inventory level when $0 \leq t \leq t_{m1}$ (units/month)
S_{2M}	inventory level when $t_{m1} \leq t \leq t_{m2}$ (units/month)
S_{CM}	setup cost ($/setup)
P_{CM}	manufacturing cost ($/unit)
H_{CM}	holding cost ($/unit/unit time)
D_{CM}	decay cost ($/unit)
Remanufacturing process	
a	basic remanufacturing rate (units/unit time)
b	time-dependent remanufacturing rate (units/unit time)
T	cycle time when inventory level reaches zero (month)
S_R	inventory level during the period $0 \leq t \leq t_R$ (units)
S_{R2}	inventory level during the period $t_R \leq t \leq T$ (units)
S_{CR}	setup cost ($/setup)
P_{CR}	remanufacturing cost ($/unit)
H_{CR}	holding cost ($/unit/unit time)
D_{CR}	decay cost ($/unit)
C_{CR}	collection cost ($/unit)
δ, σ	returned rate and recovery rate of used products, respectively
Retailer's parameters	
ϕ	carbon emissions from demand
φ	demand parameter associated with carbon emission (units)
Z_r	retailer's cycle time (month)
x	market size (units/unit time)
y	time-dependent demand (units/unit time)
D_0	constant demand (units/month)
θ_2	decay rate

N	highest inventory level at the cycle period (units/month)
u	changing point from linear demand to constant demand (time unit)
t_{r2}	time when the inventory level reaches zero after the completion cycle (time unit)
$S_rB(t)$	backlogged quantity (units/month)
$S_r(t)$	inventory level at the cycle period $[t_r, t_{2r}]$ (units/month)
O_{cr}	ordering cost ($/order)
P_{cr}	purchasing cost ($/unit)
H_r	holding cost ($/unit/unit time)
D_r	decay cost ($/unit)
L_s	opportunity cost per unit due to the lost sales ($/unit)
B_r	shortage cost ($/unit)
$\Delta(\mathrm{t})$	backlogging rate
ρ	scaling parameter
$f(t)$	ramp-type demand

4. Model description with a mathematical formulation

The manufacturer produces a single type of product. The product has a natural decay type. Products decrease their functionality with respect to time. The manufacturer sends these products to the retailer. Now, due to the deterioration of products, the retailer faces a shortage of products. Due to the shortage, the retailer faces a partial backlogging situation. Customers return a percentage of the used products which the manufacturer collects. Then, the manufacturer runs a remanufacturing center to remanufacture the used products. Those remanufactured products help to fulfill the shortage of the retailer.

4.1. Manufacturer's model: Manufacturing process

The manufacturing process starts at time t = 0 and continues until time t_{m1}. The manufacturing rate is linear and given by $(\alpha + \beta t)$. Due to the demand $(\alpha_1 + \beta_1 t)$ and the decay rate θ_1, the inventory level decreases. At $t = t_{m1}$, enough inventory has been accumulated, and the manufacturing process is paused. The inventory that has accumulated continues to meet the demand t_{m2}. At this stage, the inventory level drops to zero, and the manufacturing cycle restarts (Figure 1).

$$\frac{dS_M}{dt} + \theta_1 S_M = (\alpha + \beta t) - (\alpha_1 + \beta_1 t), 0 \leq t \leq t_{m1} \tag{1}$$

with the initial condition $S_M(0) = 0$.

$$\frac{dS_{2M}}{dt} + \theta_1 S_{2M} = -(\alpha_1 + \beta_1 t), t_{m1} \leq t \leq t_{m2} \tag{2}$$

with the boundary condition $S_{2M}(t_{m2}) = 0$.

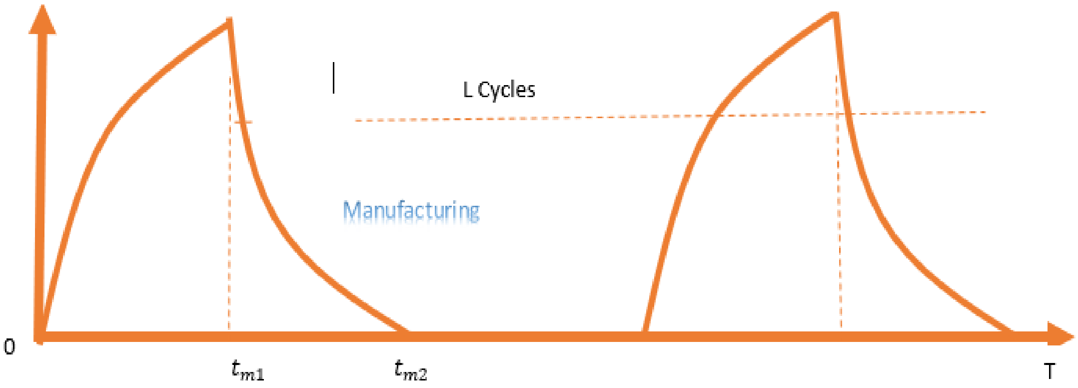


Figure 1. The behavior of the manufacturing process.

Then, the solutions of both equations are as follows:

$$S_M = \frac{e^{-\theta_1 t}}{\theta_1^2}[\{((\alpha+\beta t) - (\alpha_1+\beta_1 t))\theta_1 - \beta + \beta_1\}e^{\theta_1 t} + \{(\beta - \beta_1) - (\alpha - \alpha_1)\theta_1\}] \tag{3}$$

$$S_{2M} = \frac{e^{-\theta_1 t}}{\theta_1^2}[(\beta_1 - (\alpha_1+\beta_1 t)\theta_1)e^{\theta_1 t} + ((\alpha_1+\beta_1 t_{m2})\theta_1 - \beta_1)e^{\theta_1 t_{m2}}] \tag{4}$$

Assuming that there exist L manufacturing cycles in one overall cycle, t_{m2} is given by $t_{m2} = \frac{T}{L}$. The associative costs are given below.

4.1.1. Setup cost (SC_M)

The setup cost refers to the expense of getting the equipment ready to process. The setup cost for the manufacturing system is $SC_M = S_{CM}$.

4.1.2. Production cost (PC_M)

Production costs include labor, raw materials, consumable manufacturing supplies, and general overhead expenses. Hence, the production cost is as follows:

$$PC_M = P_{CM}\int_0^{t_{m1}}(\alpha+\beta t)dt = P_{CM}(\alpha t_{m1} + \frac{\beta}{2}{t_{m1}}^2).$$

4.1.3. Holding cost (HC_M)

Holding expenses include the cost of damaged or spoilt items, as well as storage space, labor, and insurance. As a result, the holding cost is determined as follows:

$$HC_M = H_{CM}[\int_0^{t_{m1}} S_M(t)dt + \int_{t_{m1}}^{t_{m2}} S_{2M}(t)dt]$$

$$= H_{CM}\frac{e^{-t_{\mathrm{m}1}\theta_1}}{2\theta_1^3}[2(-\beta+\beta_1+(\alpha-\alpha_1)\theta_1)+e^{t_{\mathrm{m}1}\theta_1}(2(\beta-\beta_1)+2(-\alpha+\alpha_1+t_{\mathrm{m}1}(-\beta+\beta_1))\theta_1+t_{\mathrm{m}1}(2(\alpha-\alpha_1)+t_{\mathrm{m}1}(\beta-\beta_1))\theta_1^2)]+\frac{1}{2\theta_1^3}H_{CM}[2\alpha_1\theta_1(-1+e^{(-t_{\mathrm{m}1}+t_{\mathrm{m}2})\theta_1}+(t_{\mathrm{m}1}-t_{\mathrm{m}2})\theta_1)+\beta_1(2-2e^{(-t_{\mathrm{m}1}+t_{\mathrm{m}2})\theta_1}+\theta_1(2(-1+e^{(-t_{\mathrm{m}1}+t_{\mathrm{m}2})\theta_1})t_{\mathrm{m}2}+(t_{\mathrm{m}1}-t_{\mathrm{m}2})(-2+(t_{\mathrm{m}1}+t_{\mathrm{m}2})\theta_1)))].$$

4.1.4. Decay cost (DC_M)

The cost of decay is comprised of the costs of product valuation, quality of production, the functionality of the product, working hours, expiration, and other factors that reduce the stock's value. The cost of decay is calculated as follows:

$$DC_M = D_{CM}[\int_0^{t_{m1}}\theta_1 S_M(t)dt \; + \; \int_{t_{m1}}^{t_{m2}}\theta_1 S_{2M}(t)dt]$$

$$= D_{CM}\frac{e^{-t_{\mathrm{m}1}\theta_1}}{2\theta_1^2}[2(-\beta+\beta_1+(\alpha-\alpha_1)\theta_1)+e^{t_{\mathrm{m}1}\theta_1}(2(\beta-\beta_1)+2(-\alpha+\alpha_1+t_{\mathrm{m}1}(-\beta+\beta_1))\theta_1+t_{\mathrm{m}1}(2(\alpha-\alpha_1)+t_{\mathrm{m}1}(\beta-\beta_1))\theta_1^2)]+\frac{1}{2\theta_1^2}D_{CM}[2\alpha_1\theta_1(-1+e^{(-t_{\mathrm{m}1}+t_{\mathrm{m}2})\theta_1}+(t_{\mathrm{m}1}-t_{\mathrm{m}2})\theta_1)+\beta_1(2-2e^{(-t_{\mathrm{m}1}+t_{\mathrm{m}2})\theta_1}+\theta_1(2(-1+e^{(-t_{\mathrm{m}1}+t_{\mathrm{m}2})\theta_1})t_{\mathrm{m}2}+(t_{\mathrm{m}1}-t_{\mathrm{m}2})(-2+(t_{\mathrm{m}1}+t_{\mathrm{m}2})\theta_1)))].$$

The total cost of manufacturing per cycle is

$$\frac{L}{T}(SC_M+PC_M+HC_M+DC_M) \tag{5}$$

4.2. Retailer's model

There are multiple retailer cycles in each manufacturing and remanufacturing period. The mathematical model begins with shortages that arise during the time interval [0, t_r] and are partially backlogged. A replenishment carries the inventory level up to a higher N at the time t_r. Due to the demand and decay of products, the inventory level of the products decreases throughout the duration $[t_r, t_{r2}]$ until it reaches zero at the time t_{r2}. We have looked at the model in two different cases. Now, the retailer has two cases: i) Case 1: $t_r < u$ (shortage occurs before the demand reaches the stable state) and ii) Case 2: $t_r > u$ (shortage occurs after the demand reaches the stable state).

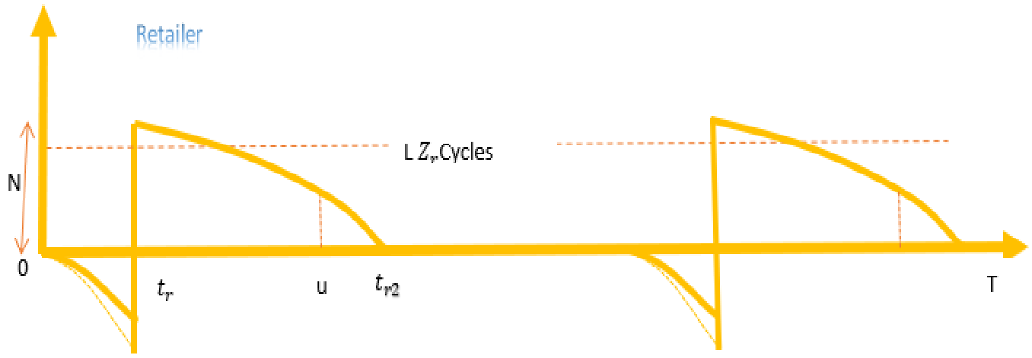


Figure 2. The behavior of retailer process for Case 1.

i) Case 1: When $t_r < u$ (shortage occurs before the demand reaches the stable state)

In this case, the inventory is replenished before the demand reaches a stable state at $t = u$, from when the demand follows a constant demand pattern. Figure 2 illustrates the actions of Case 1. The inventory model's differential equations at time t over the duration $[0, t_{r2}]$ can be written as follows:

$$\frac{S_rB(t)}{dt} = -\Delta(t_r - t)F(t), 0 \le t < t_r, SB(0) = 0 \tag{6}$$

$$\frac{S_r(t)}{dt} + \theta_2 S_r(t) = -F(t), t_r \le t < u, S(u^-) = S\,(u^+) \tag{7}$$

$$\frac{S_r(t)}{dt} + \theta_2 S_r(t) = -(D_0 - \phi\varphi), u \le t \le t_{r2}, S(\,t_{r2}) = 0. \tag{8}$$

After solving the above differential equations, the inventories are calculated as follows:

$$S_rB(t) = -\frac{e^{-\rho t_r}}{\rho^2}[\{(x + yt)\rho - y\}e^{\rho t} - (x\rho - y)]\ 0 \le t < t_r \tag{9}$$

$$\begin{aligned} S_r(t) = -\frac{1}{{\theta_2}^2}\Big[\{(x + yt)\theta_2 - y\} + [\{f_0\ \theta_2(1 - e^{-\theta_2(u - t_{r2})}\Big)\} \\ - \{(x + yu)\theta_2 - y\}\,]e^{-\theta_2(t-u)}\ ], t_r \le t < u \end{aligned} \tag{10}$$

$$S_r(t) = -\frac{(D_0 - \phi\varphi)e^{-\theta_2 t}}{\theta_2}\left\{e^{\theta_2 t} - e^{\theta_2 t_{r2}}\right\}, u \le t \le t_{r2} \tag{11}$$

$$\begin{aligned} N = \frac{1}{{\theta_2}^2}\Big[\{(x - \phi\varphi + yt_r)\theta_2 - y\} + [\{(D_0 - \phi\varphi)\ \theta_2(1 - e^{-\theta_2(u - t_{r2})}\Big)\} \\ + \{(x - \phi\varphi + yu)\theta_2 - y\}\,]e^{-\theta_2(t_r - u)}\ ] \end{aligned} \tag{12}$$

The associative costs of the retailer for Case 1 are given below.

4.2.1. Ordering cost $(\mathrm{OC_r})$

The costs of placing and processing an order with a supplier are defined as $OC_r = O_{cr}$.

4.2.2. Purchasing cost (PC_r)

This is an expenditure associated with purchasing and transporting produts to their destination. The cost of purchasing is determined as follows:

$$PC_r = P_{cr}N$$

$$= \frac{P_{cr}}{{\theta_2}^2}[\{(x - \phi\varphi + yt_r)\theta_2 - y\} + [\{f_0\,\theta_2(1 - e^{-\theta_2(u-t_{r2})})\} + \{(x - \phi\varphi + yu)\theta_2 - y\}]e^{-\theta_2(t_r-u)}].$$

4.2.3. Holding cost (HC_r)

Holding costs include the cost of damaged or spoiled items, as well as storage space, labor, and insurance. The following formula is used to calculate the cost of holding:

$$HC_r = H_r[\int_{t_r}^{u} S_r(t)dt + \int_{u}^{t_{r2}} S_r(t)dt]$$

$$= -\frac{H_r}{{\theta_2}^2}[\Big\{\Big(x - \phi\varphi + \frac{y}{2}(u - t_r)\Big)(u - t_r)\theta_2 - y(u - t_r)\Big\} - (D_0 - \phi\varphi)\big(e^{\theta_2 u} - e^{\theta_2 t_{r2}}\big)\big(e^{-\theta_2 u} - e^{-\theta_2 t_r}\big) + \Big\{(x - \phi\varphi + yu) - \frac{y}{\theta_2}\Big\}(1 - e^{-\theta_2(t_r-u)})] - \frac{(D_0 - \phi\varphi)}{{\theta_2}^2}[(e^{-\theta_2 t_{r2}} - e^{-\theta_2 u}) + (t_{r2} - u)\theta_2].$$

4.2.4. Decay cost (DC_r)

The cost of decay is calculated as follows:

$$DC_r = D_r[\int_{t_r}^{u} \theta_2 S_r(t)dt + \int_{u}^{t_{r2}} \theta_2 S_r(t)dt]$$

$$= -\frac{D_r}{\theta_2}[\Big\{\Big(x - \phi\varphi + \frac{y}{2}(u - t_r)\Big)(u - t_r)\theta_2 - y(u - t_r)\Big\} - (D_0 - \phi\varphi)\big(e^{\theta_2 u} - e^{\theta_2 t_{r2}}\big)\big(e^{-\theta_2 u} - e^{-\theta_2 t_r}\big) + \Big\{(x - \phi\varphi + yu) - \frac{y}{\theta_2}\Big\}(1 - e^{-\theta_2(t_r-u)})] - \frac{D_r(D_0 - \phi\varphi)}{\theta_2}[(e^{-\theta_2 t_{r2}} - e^{-\theta_2 u}) + (t_{r2} - u)\theta_2].$$

4.2.5. Backlogging cost (BC_r)

Market shortages happen due to the stock being unavailable in warehouses. The backlogging cost refers to the non-shipped products to a consumer. Backlogging costs are estimated in the following manner:

$$BC_r = B_r \int_0^{t_r} S_r B(t)dt$$

$$= -\frac{B_r e^{-\rho t_r}}{\rho^3}[\{(x - \phi\varphi + yt_r)\alpha - 2y\}e^{\alpha t_r} - ((x - \phi\varphi)\rho - y)\rho t_r - ((x - \phi\varphi)\rho - 2y)].$$

4.2.6. Lost sale cost (LC_r)

When a product is unavailable, i.e., a customer wants to buy a product but there is no inventory, the buyer is compelled to change their buying behavior. The change may cause the intended product to either be purchased later or not at all. In this study, the partial backorder for shortages is taken into consideration. The supply chain loses a small portion of demand due to the partial backorder. In this duration, the rate of loss sales is calculated as follows:

$$LC_r = L_s \int_0^{t_r} \{1 - \Delta(t_r - t)\} f(t)dt$$

$$= \int_0^{t_r} \left[1 - e^{-\alpha(t_r - t)}\right](x - \phi\varphi + yt)\, dt$$

$$= (x - \phi\varphi + \frac{y}{2}t_r)t_r - \frac{1}{\rho^2}[(x - \phi\varphi + yt_r)\rho - (x - \phi\varphi)\rho e^{-\rho t_r} - y(1 - e^{-\rho t_r})].$$

Therefore, the total cost of the retailer per cycle is calculated as follows:

$$\frac{LZ_r}{T}[OC_r + PC_r + DC_r + HC_r + BC_r + LC_r] \tag{13}$$

Assuming that one manufacturing cycle includes the retailer's cycle, Z_r, the cycle time is given by $t_{r2} = \frac{T}{LZ_r}$ and $t_{r2} = 2t_{r1}$.

ii) Case 2: When $t_r > u$ (shortage occurs after the demand reaches to the stable state)

In this case, the demand reaches a stable state at $t = u$ before the inventory replenishes at a time $t = t_r$. Figure 3 depicts the actions of the inventory model for Case 2. The following differential equations represent this model's inventory level:

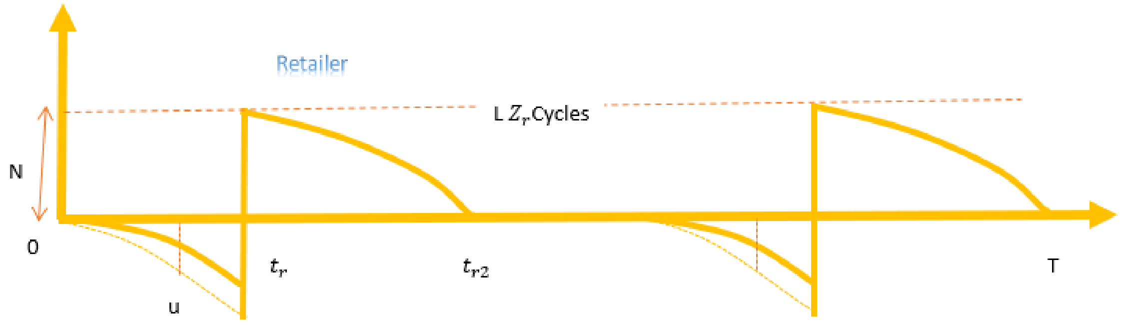


Figure 3. The behavior of retailer process for Case 2.

$$\frac{S_rB(t)}{dt} = -\Delta(t_r - t)F(t), 0 \le t < u, SB(0) = 0 \tag{14}$$

$$\frac{S_rB(t)}{dt} = -\Delta(t_r - t)(D_0 - \phi\varphi), u \le t < t_r, S(u^-) = S\,(u^+) \tag{15}$$

$$\frac{S_r(t)}{dt} + \theta_2 S_r(t) = -(D_0 - \phi\varphi), t_r \le t \le t_{r2}, S(T) = 0\ \ S_r(t_r) = N \tag{16}$$

Solving the differential equations above yields the following:

$$S_rB(t) = -\frac{e^{-\rho t_r}}{\rho^2}\left[\{(x - \phi\varphi + yt)\rho - y\}e^{\rho t} - \left((x - \phi\varphi)\rho - y\right)\right],\ 0 \le t < u \tag{17}$$

$$\begin{aligned} S_rB(t) = &-\frac{(D_0 - \phi\varphi)}{\rho}\left[e^{-\rho(t_r - t)} - e^{-\rho(t_r - u)}\right] \\ &-\frac{e^{-\rho t_r}}{\rho^2}[((x - \phi\varphi + yu)\rho - y)e^{\rho u} - ((x - \phi\varphi)\rho - y)], \\ &u \le t < t_r \end{aligned} \tag{18}$$

$$S_r(t) = -\frac{(D_0 - \phi\varphi)}{\theta_2}[1 - e^{\theta_2(t_{r2} - t)}],\ t_r \le t \le T \tag{19}$$

$$S_r(t_r) = N = -\frac{(D_0 - \phi\varphi)}{\theta_2}[1 - e^{\theta_2(t_{r2} - t_r)}]. \tag{20}$$

The corresponding costs of the retailer for Case 2 are described below.

4.2.7. Ordering cost (OC_r)

The costs of placing and processing an order with a supplier are defined as $OC_r = O_{cr}$.

4.2.8. Purchasing cost (PC_r)

The purchasing cost includes all costs related to purchasing and transporting produts to their final destination. The cost of purchasing is determined as follows:

$$PC_r = P_{cr}N$$
$$= -\frac{P_{cr}(D_0 - \phi\varphi)}{\theta_2}[1 - e^{\theta_2(t_{r2} - t_r)}]$$

4.2.9. Holding cost (HC_r)

Holding expenses include damaged or spoilt commodities, as well as storage space, labor, and insurance. The cost of holding is calculated as follows:

$$= H_r \int_{t_r}^{t_{r2}} S_r(t)dt$$

$$= H_r(D_0 \ - \ \phi\varphi)(\frac{-1+e^{\theta_2(t_{r2}-t_s)}-t_{r2}\theta_2+\theta_2 t_s}{{\theta_2}^2})$$

4.2.10. Decay cost (DC_r)

The following formula is used to calculate the cost of decay:

$$DC_r = D_r \int_{t_r}^{t_{r2}} \theta_2 S_r(t)dt$$

$$= \frac{D_r(D_0 - \phi\varphi)(-1+e^{\theta_2(t_{r2}-t_r)}-t_{r2}\theta_2+\theta_2 t_r)}{\theta_2}.$$

4.2.11. Backlogging cost (BC_r)

The backlogging cost refers to the non-shipped products to a consumer. The backlogging cost is determined as follows:

$$BC_r = B_r \int_0^{t_r} SB(t)dt \ = B_r(\int_0^u -SB(t)dt + \int_u^{t_r} -SB(t)dt)$$

$$= B_r(-\frac{e^{-t_r\rho}\left((x - \phi\varphi)\alpha(1-e^{u\rho}+u\rho)-y\left(2+u\rho+e^{u\rho}(-2+u\rho)\right)\right)}{\rho^3} - \frac{e^{-t_r\rho}D_0\left(-e^{\rho t_r}+e^{v\rho}(1-v\rho+\rho t_r)\right)}{\rho^2} +$$
$$\frac{e^{-t_r\rho}(y-(x - \phi\varphi)\rho+e^{u\rho}(-y+(x - \phi\varphi+yu)\rho))(-u+t_r)}{\rho^2}).$$

4.2.12. Lost sale (LC_r)

When a product is not accessible when a consumer wants to buy it, the customer is forced to adjust their purchasing habits. Because of the modification, the desired product may either be acquired later or not at all. In this case, the lost sale cost is determined as follows:

$$LC_r = \ L_s(\int_0^u \{1 - \Delta(\,t_r - t)\}f(t)dt + \ \int_u^{t_r}\{1 - \Delta(\,t_r - t)\}(D_0 \ - \ \phi\varphi)dt]$$

$$= L_s\left[(x \ - \ \phi\varphi)u + \frac{yu^2}{2} - \frac{(x - \phi\varphi)e^{-t_r^2\rho}(-1+e^{u\rho})}{\rho} - \frac{ye^{-t_r\rho}\left(1+e^{u\rho}(-1+u\rho)\right)}{\rho^2} +\right.$$
$$\left.\frac{\left(-1+e^{(-t_r+u)\rho}+t_r\rho-u\rho\right)(D_0 - \phi\varphi)}{\rho}\right).$$

Therefore, the total cost of the retailer is

$$\frac{LZ_r}{T}[OC_r + \ PC_r + DC_r + HC_r + BC_r + \ LC_r] \qquad (21)$$

Assuming that one manufacturing cycle includes the retailer's cycle, Z_r, the cycle time is given by

$t_{r2} = \frac{T}{LZ_r}$ and $t_{r2} = 2t_{r1}$.

4.3. Manufacturer's model: Remanufacturing process

Used product collection takes place during the manufacturing and remanufacturing processes. Products that have been used or returned by customers are collected and returned to the manufacturer for remanufacturing. The collected inventory of used products at time t is given by

$$S_C(t) = \delta\sigma(D_o - \phi\varphi) \tag{22}$$

Where D_o is the constant market demand and δ is the returned rate.

The producer uses this inventory to remanufacture items while also recycling waste. The reproduction process starts at time $t = 0$ and lasts until the time t_R. Due to the demand $(\alpha_1 + \beta_1 t)$ and the decay rate θ_1, the inventory level decreases. At t_R, enough inventory has been collected, and the development is stopped. Due to θ_1 deterioration and demand, the inventory level continues to decrease until time T, when it reaches zero (as shown in Figure 4).

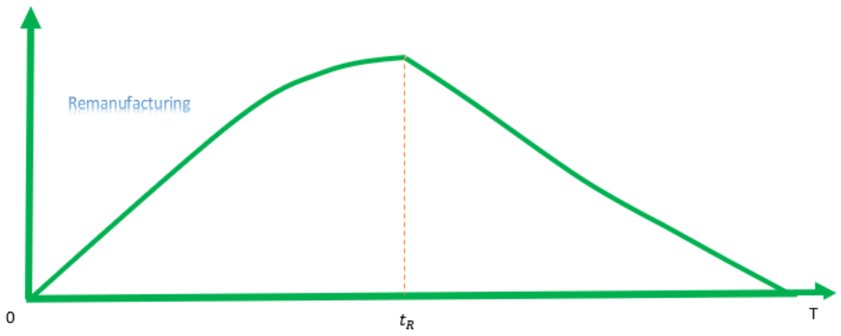


Figure 4. Behavior of the remanufacturing process.

$$\frac{dS_R}{dt} + \theta_1 S_R = a + bt - (\alpha_1 + \beta_1 t),\ 0 \leq t \leq t_R \tag{23}$$

with the initial condition $V_R(0) = 0$.

$$\frac{dS_{R2}}{dt} + \theta_1 S_{R2} = -(\alpha_1 + \beta_1 t), t_R \leq t \leq T \tag{24}$$

with the boundary condition $V_{2R}(T) = 0$ .Then, the solutions of both equations are as follows:

$$S_R = \frac{e^{-\theta_1 t}}{\theta_1^2}\left[\{((a + bt) - (\alpha_1 + \beta_1 t))\theta_1 - b + \beta_1\}e^{\theta_1 t} + \{(b - \beta_1) - (a - \alpha_1)\theta_1\}\right] \tag{25}$$

$$S_{R2} = \frac{e^{-\theta_1 t}}{\theta_1^2}\left[(\beta_1 - (\alpha_1 + \beta_1 t)\theta_1)e^{\theta_1 t} + ((\alpha_1 + \beta_1 T)\theta_1 - \beta_1)e^{\theta_1 T}\right]. \tag{26}$$

The costs for the remanufacturing process are given below.

4.3.1. Setup cost (SC_R)

The cost of getting the equipment ready to process is referred to as the setup cost. During the remanufacturing process, the setup cost is $SC_R = S_{CR}$.

4.3.2. Remanufacturing cost (PC_R)

The production cost is the most significant cost in any remanufacturing process. The remanufacturing cost of the remanufactured products includes the extra cost of returning the used and waste products [49,50]. Therefore, the remanufacturing cost is calculated as follows:

$$PC_R = P_{CR}\int_0^{t_R} P_R\,dt = P_{CR}(at_R + \frac{bt_R^2}{2}).$$

4.3.3. Holding cost (HC_R)

The system necessitates the storage of remanufactured items. As a result, some expenses are required. The remanufactured products' holding cost can be expressed as follows:

$$HC_R = H_{CR}[\int_0^{t_R} S_R(t)dt \ + \ \int_{t_R}^{T} S_{R2}(t)dt]$$

$$= H_{CR}\frac{e^{-t_\mathrm{R}\theta_1}}{2\theta_1^3}[(2(-b+\beta_1+(a-\alpha_1)\theta_1)+e^{t_R\theta_1}(2(b-\beta_1)+2(-a+\alpha_1+t_R(-b+\beta_1))\theta_1+ t_R(2(a-\alpha_1)+t_R(b-\beta_1))\theta_1^2))]+\frac{1}{2\theta_1^3}\ H_{CR}[2\alpha_1\theta_1(-1+e^{(-t_\mathrm{R}+\mathrm{T})\theta_1}+(t_\mathrm{R}-\mathrm{T})\theta_1)+\beta_1(2- 2e^{(-t_\mathrm{R}+\mathrm{T})\theta_1}+\theta_1(2(-1+e^{(-t_\mathrm{R}+\mathrm{T})\theta_1})\mathrm{T}+(t_\mathrm{R}-\mathrm{T})(-2+(t_\mathrm{R}+\mathrm{T})\theta_1)))].$$

4.3.4. Decay cost (DC_R)

The cost of decay of products is calculated using the following formula:

$$DC_R = D_{RC}[\int_0^{t_R} \theta_1 S_R(t)dt \ + \ \int_{t_R}^{T} \theta_1 S_{R2}(t)dt]$$

$$= D_{CR}\frac{e^{-t_\mathrm{R}\theta_1}}{2\theta_1^2}[(2(-b+\beta_1+(a-\alpha_1)\theta_1)+e^{t_R\theta_1}(2(b-\beta_1)+2(-a+\alpha_1+t_R(-b+\beta_1))\theta_1+ t_R(2(a-\alpha_1)+t_R(b-\beta_1))\theta_1^2))]+\frac{1}{2\theta_1^2}\ D_{CR}[2\alpha_1\theta_1(-1+e^{(-t_\mathrm{R}+\mathrm{T})\theta_1}+(t_\mathrm{R}-\mathrm{T})\theta_1)+\beta_1(2- 2e^{(-t_\mathrm{R}+\mathrm{T})\theta_1}+\theta_1(2(-1+e^{(-t_\mathrm{R}+\mathrm{T})\theta_1})\mathrm{T}+(t_\mathrm{R}-\mathrm{T})(-2+(t_\mathrm{R}+\mathrm{T})\theta_1)))].$$

4.3.5. Collection cost (CC_R)

The collection cost is the expense of collecting the returned items. The cost of collection is calculated as follows:

$CC_R = C_{CR} \int_0^T S_C(t)\, dt = C_{CR} \sigma \delta T (D_o - \phi\varphi)$.

The total cost of remanufacturing is

$$\frac{1}{T}(SC_R + PC_R + HC_R + DC_R + CC_R). \tag{27}$$

5. Solution procedure

The total cost of the CLSCM $(TS_{Cj}, j = 1{,}2)$ = [total cost of manufacturing + total cost of retailer + total cost of remanufacturing]. $TS_{C1}, j = 1$ is the total cost of the system of Case 1 and $TS_{C1}, j = 2$ gives the total cost for Case 2. Thus,

$$TS_{Cj} = \frac{L}{T}(SC_M + PC_M + HC_M + DC_M) + \frac{LZ_r}{T}(OC_r + PC_r + DC_r + HC_r + BC_r + LC_r) + \frac{1}{T}(SC_R + PC_R + HC_R + DC_R + CC_R), j = 1{,}2, \tag{28}$$

Where

$$TS_{C1} = \frac{L}{T}\Big[S_{CM} + P_{CM}\int_0^{t_{m1}}(\alpha + \beta t)dt + H_{CM}\left[\int_0^{t_{m1}} S_M(t)dt + \int_{t_{m1}}^{t_{m2}} S_{2M}(t)dt\right] + D_{CM}\Big[\int_0^{t_{m1}} \theta_1 S_M(t)dt + \int_{t_{m1}}^{t_{m2}} \theta_1 S_{2M}(t)dt\Big]\Big] + \frac{LZ_r}{T}\Big[O_{cr} + P_{cr}N + H_r\left[\int_{t_r}^{u} S_r(t)dt + \int_u^{t_{r2}} S_r(t)dt\right] + D_r\left[\int_{t_r}^{u} \theta_2 S_r(t)dt + \int_u^{t_{r2}} \theta_2 S_r(t)dt\right] + B_r\int_0^{t_r} S_r B(t)dt + L_s\int_0^{t_r}\{1 - \Delta(t_r - t)\}f(t)dt\Big] + \frac{1}{T}\Big[S_{CR} + P_{CR}\int_0^{t_R} P_R\, dt + H_{CR}\Big[\int_0^{t_R} S_{R1}(t)dt + \int_{t_R}^{T} S_{R2}(t)dt\Big] + D_{RC}\Big[\int_0^{t_R} \theta_1 S_{R1}(t)dt + \int_{t_R}^{T} \theta_1 S_{R2}(t)dt\Big] + C_{CR}\int_0^T S_C(t)\, dt\Big], \tag{29}$$

$$TS_{C2} = \frac{L}{T}\Big[S_{CM} + P_{CM}\int_0^{t_{m1}}(\alpha + \beta t)dt + H_{CM}\left[\int_0^{t_{m1}} S_M(t)dt + \int_{t_{m1}}^{t_{m2}} S_{2M}(t)dt\right] + D_{CM}\Big[\int_0^{t_{m1}} \theta_1 S_M(t)dt + \int_{t_{m1}}^{t_{m2}} \theta_1 S_{2M}(t)dt\Big]\Big] + \frac{LZ_r}{T}\Big[O_{cr} + P_{cr}N + H_r\int_{t_r}^{t_{r2}} S_r(t)dt + D_r\int_{t_r}^{t_{r2}} \theta_2 S_r(t)dt + B_r\int_0^{t_r} SB(t)dt + L_s\Big(\int_0^{u}\{1 - \Delta(t_r - t)\}f(t)dt + \int_u^{t_r}\{1 - \Delta(t_r - t)\}(D_0 - \phi\varphi)dt\Big]\Big] + \frac{1}{T}\Big[S_{CR} + P_{CR}\int_0^{t_R} P_R\, dt + H_{CR}\Big[\int_0^{t_R} S_{R1}(t)dt + \int_{t_R}^{T} S_{R2}(t)dt\Big] + D_{RC}\Big[\int_0^{t_R} \theta_1 S_{R1}(t)dt + \int_{t_R}^{T} \theta_1 S_{R2}(t)dt\Big] + C_{CR}\int_0^T S_C(t)\, dt\Big]. \tag{30}$$

(Detail calculations are in Appendix.)

The objective function is solved for a centralized system. As this study aims to solve an emergent problem and products such as medical devices are delivered through a CLSC, centralized time

minimization for the inventory is prioritized rather than prioritizing the importance of each CLSCM player in a decentralized system. Therefore, optimal solutions to Eq 28 are given by solving the following equations:
$\frac{\partial TS_{Cj}(t_{m1},t_r,t_R)}{\partial t_{m1}}=0, \frac{\partial TS_{Cj}(t_{m1},t_r,t_R)}{\partial t_r}=0, \frac{\partial TS_{Cj}(t_{m1},t_r,t_R)}{\partial t_R}=0$, for $j=1,2$, which gives the optimal values of $t_{m1}{}^{*}, t_r{}^{*}, t_R{}^{*}$. To minimize the total cost, sufficient conditions are as follows:

$\frac{\partial TS_{Cj}{}^{2}(t_{m1},t_r,t_R)}{\partial t_{m1}{}^{2}}=A>0$, $\frac{\partial TS_{Cj}{}^{2}(t_{m1},t_r,t_R)}{\partial t_{m1}\,\partial t_r}=H$, $\frac{\partial TS_{Cj}{}^{2}(t_{m1},t_r,t_R)}{\partial t_r{}^{2}}=B>0$, $\frac{\partial TS_{Cj}{}^{2}(t_{m1},t_r,t_R)}{\partial t_{m1}\,\partial t_R}=G$,
$\frac{\partial TS_{Cj}{}^{2}(t_{m1},t_r,t_R)}{\partial t_R{}^{2}}=C>0$, $\frac{\partial TS_{Cj}{}^{2}(t_{m1},t_r,t_R)}{\partial t_r\,\partial t_R}=F$.

The principal minors of the Hessian matrix = $\begin{bmatrix} A & H & G \\ H & B & F \\ G & F & C \end{bmatrix}$ are $A, \begin{vmatrix} A & H \\ H & B \end{vmatrix}=A_1$, and $\begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix}=A_2$. The principal minors' values are $A>0$, $A_1>0$, and $A_2>0$; hence, the total cost during the cycle is a global minimum at (t_{m1},t_r,t_R). The sufficient conditions are numerically proven.

5.1. Algorithm

This study's fundamental goal is to minimize $(TS_{Cj}, j=1,2)$. The following procedure is used to calculate the optimum strategy.
Algorithm
Step 1 Input all the parameters for $TS_{Cj}, j=1,2$.
Step 2 Necessary condition: Find the optimum values of decision variables $t_{m1}{}^{*}, t_r{}^{*}$, and $t_R{}^{*}$ from $\frac{\partial TS_{Cj}(t_{m1},t_r,t_R)}{\partial t_{m1}}=0, \frac{\partial TSj_{C}(t_{m1},t_r,t_R)}{\partial t_r}=0, \frac{\partial TS_{Cj}(t_{m1},t_r,t_R)}{\partial t_R}=0$, for $j=1,2$.
Step 3 Sufficient condition: If the optimal values are the satisfied principal minors of the Hessian matrix A > 0, $A_1>0$, and $A_2>0$, then the total cost ($TS_{Cj}, j=1,2$) during the cycle is minimized.
Step 4 Then, the values $t_{m1}{}^{*}, t_r{}^{*}$, and $t_R{}^{*}$ are calculated.
Step 5 Finally, the total cost is determined.

6. Numerical experiments and discussions

Numerical examples are provided to numerically establish the model. Associative data for the numerical examples are taken from Ullah and Sarkar [10] and Rani et al. [49].

i) Case 1 $t_r<u$ (shortage occurs before the demand reaches the stable state)

The results of this study are based on the following parameters: $\alpha=22$ units/unit time, $\beta=0.3$ units/unit time, $\alpha_1=101$ units/unit time, $\beta_1=0.2$ units/unit time, $\theta_1=0.5$, L = 3 cycles, $S_{CM}=\$41/\text{setup}$, $P_{CM}=\$42/\text{unit}$, $H_{CM}=\$0.6$/units/unit time, $D_{CM}=\$7/\text{unit}$, $x=75$ units, $y=0.2$ units, $\rho=2$, $D_0=71$ units, $\theta_2=0.6$, $O_{cr}=\$35/\text{order}$, $P_{cr}=\$31/\text{unit}$, $H_r=\$0.5$/units/unit time, $D_r=\$6/\text{unit}$, Z_r= 2 cycles, a = 15 units/unit time, b = 0.1 units/unit time, $B_r=\$4/\text{unit}$, $L_s=\$35/\text{unit}$, $\delta=0.1$, $\sigma=0.3$, $S_{CR}=\$35/\text{setup}$, $P_{CR}=\$45/\text{unit}$, $H_{CR}=\$0.3/$

unit/unit time, $D_{RC} = \$5$/unit, $C_{CR} = \$33$/unit, $\phi = 0.2$, $\varphi = 0.3$ units, $u = 1.2$ months, and $T = 10$ months.

Then, using the Wolfram Mathematica 11.0 software, the system's optimal result is determined, as shown in Table 2. The results satisfy the condition that the principal minors of the Hessian matrix are $A = 145.214 > 0$, $A_1 = 86{,}389.260 > 0$, and $A_2 = 4{,}112{,}128.81 > 0$ for all optimal values. Thus, the total cost is a global minimum, and the minimum total cost is \$6215.08/cycle. The retailer's cycle time is $t_{r2} = 1.66$ months and the manufacturer's cycle time for the manufacturing process is $t_{m2} = 3.33$ months.

Table 2. Optimal results for Case 1 ($t_r < u$) and Case 2 ($t_r > u$).

Cases	Case 1	Case 2
$\boldsymbol{t_{m1}}$ (months)	2.38	2.38
$\boldsymbol{t_{m2}}$ (months)	3.33	3.33
$\boldsymbol{t_r}$ (months)	0.68	0.23
$\boldsymbol{t_{r2}}$ (months)	1.66	1.66
$\boldsymbol{t_R}$ (months)	9.34	9.34
Retailer's holding cost (\$/cycle)	\$52.822	\$29.809
Manufacturing's cost (\$/cycle)	\$557.38	\$557.38
Retailer's cost (\$/cycle)	\$5282.13	\$280.07
Remanufacturing's cost (\$/cycle)	\$375.56	\$375.56
Total cost (\$/cycle)	\$6215.08	\$1213.03

ii) Case 2 $t_r > u$ (shortage occurs after the demand reaches the stable state)

All parameters are the same as Example 1 except that $u = 0.2$ months**.** Then, using the Wolfram Mathematica 11.0 software, the system's optimal result is determined, as shown in Table 2.

The results of the sufficient condition for the principal minors of the Hessian matrix are $A = 145.214 > 0$, $A_1 = 638{,}368.00 > 0$, and $A_2 = 25{,}560{,}254.9 > 0$ for the optimal values. Thus, the total cost of the system is a global minimum, and the total cost is \$1213.03/cycle. The manufacturer and retailer cycle time is the same as in Case 1. Therefore, in Figure 5, the given examples demonstrate the convexity of TS_{C1} and TS_{C2}. These figures confirm that the solution is globally optimal. The total cost performance of Case 1 and Case 2 with respect to L are shown in Figure 6.

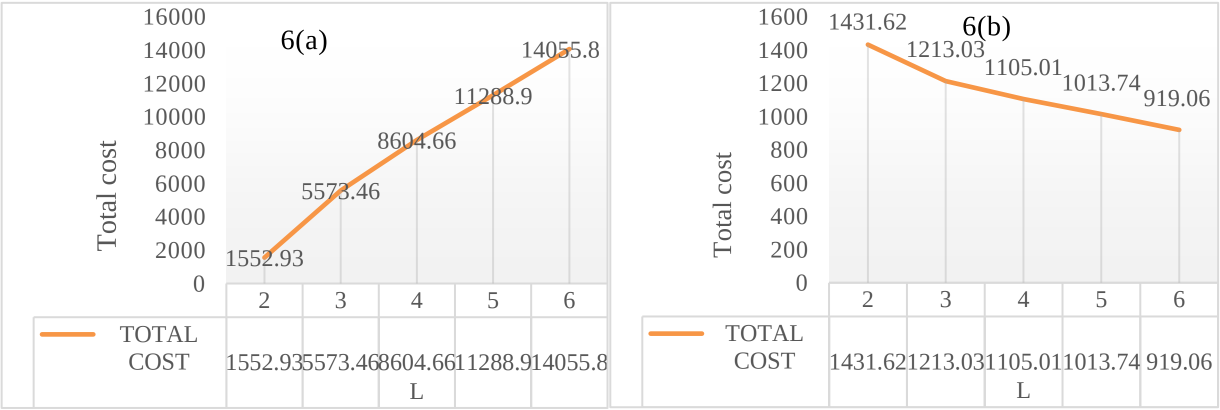


Figure 6. Total cost performance of (a) Case 1 and (b) Case 2 with respect to L.

6.1. Comparative study

Some more comparative results are provided below:

- The retailer's holding cost is higher in Case 1 than in Case 2, which indicates that the holding capacity in Case 1 is better than in Case 2.
- The results show that Case 2 has a lower total cost of the CLSC than in Case 1. This implies that the inventory turnover in Case 1 is more than in Case 2. This phenomenon increases the total cost of the CLSC for Case 1.
- For comparison, the costs of manufacturing and remanufacturing of Rani et al. [49], the total cost of Kumar et al. [10], and the total cost of the retailer of Kumar et al. [8] are higher than the cost of this study. Therefore, this study is more beneficial than the previous studies.
- Apart from a direct comparison, the study can be compared with other studies based on different concepts. Goodarzian et al. [33,45] discussed a CLSCM for agricultural products, whereas the present study discussed a self-care monitoring medical device with a remanufacturing facility. Goodarzian et al. [33] developed a methodological model (Pareto-based solutions and meta-heuristic approaches), though this study develops a theoretical model.
- Momenitabar et al. [34] developed a CLSC network and solved it by a fuzzy methodology, whereas this study developed a CLSC management problem and solved it with a crisp approach. Similarly, Babaeinesami et al. [44] derived a CLSC network model and solved it with a non-dominated sorting genetic algorithm II (NSGA-II).

7. Managerial insights

This section provides a few insights about this study. It discusses the academic and practical contributions of the study in CLSCM.

- The study analyzes the remanufacturing possibilities of a self-care monitoring product and the corresponding total cost of the CLSCM. The self-care monitoring product is one of the essential products whose market demand is ramp type. As self-care medical products are highly sensitive, remanufacturing these products from collected waste products is very risky. Due to the shelf-life of the medical device, the used products' accumulation rate (in the market) is higher than traditional products. This is a realistic situation that people face in everyday life. Thus, the cost estimation and remanufacturing possibilities for these types of products are always necessary.
- In Case 1, the increasing demand maintains a longer duration along with the replenishment time. The market demand increases until 1.2 months; the shortage period ends at 1.08 months. As the market demand increases for a long time, the shortage cannot be fulfilled quickly. It takes a long time to fulfill the partial backlog. After that, the inventory reaches a positive index. The retailer's holding cost is \$52.822/cycle. When the demand rate x of the customer increases up to 20%, the overall cost increases by approximately 19.20%.
- Case 2 implies that during the partial backlogging period $0 \leq t < t_r$, the market demand reaches its maximum level such that, at time u, the time duration for increasing demand is u = 0.2 months. After that, the demand follows a constant pattern D_0. Almost at the same time (i.e., at 0.23 months), the inventory replenishes. The shortage period is almost over at 0.23 months. This implies that the retailer does not face any loss from partial backlogging. The retailer's holding cost is \$29.809/cycle. When the demand rate x of the customer increases up to 20%, the overall cost of decreases by approximately 0.39%. Thus, the total cost of the CLSC is reduced.

- A CLSC that deals with self-care products is very time-sensitive. Delays in the delivery of products throughout the supply chain cause delays in the delivery of products to the customer. In an emergency, people want to save time to and decrease the chanses of any adverse affects. Following this, people also value the ability to save money.
- The longer the backlogging period, the more risk continues within the supply chain. A shorter partial backlog period delivers products faster to the customers. In the case of a medical emergency, customers do not want to wait long periods of time to receive products. Thus, it can be assumed that this situation can be possible if customers have some alternatives or if the product is not mandatory right now.
- Customers for partial backlogging can be considered loyal to that retailer. The longer shortage period now increases the lost sales, which adds other costs into the system. For a CLSC that deals with an emergency, it is recommended to replenish inventory as early as possible to prevent a huge cost (maybe sometimes loss).
- A huge amount of used products is generated at the end of the forward supply chain. In this study, products are considered reusuable and not disposable. Thus, the collection of used products serves two purposes: reduce the waste in the environment and reduce the manufacturing of new products.
- The reduction of new product manufacturing mainly reduces the use of raw materials, energy reserves, and labor. Remanufacturing reduces the manufacturing cost of the product. Thus, a remanufacturing process is beneficial for management in both ways (i.e., economically and environmentally).

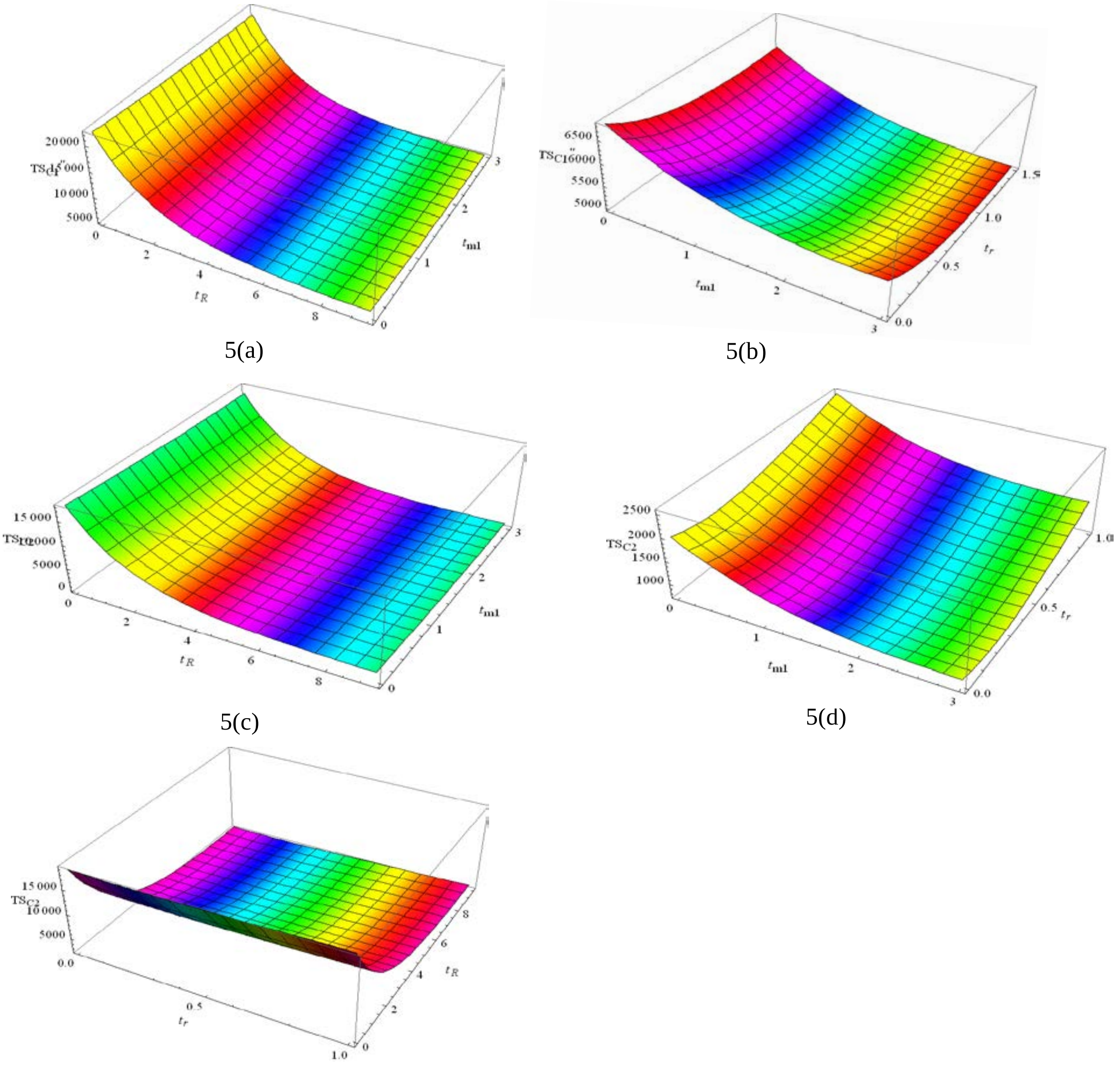
8. Sensitivity analysis

The percentage changes in some parameters, such as setup cost, production cost, order cost, deterioration cost, carbon emission, collection cost, deterioration cost, and demand rate of manufacturing, remanufacturing, and retailer are discussed in this section. These parameters are exposed to a sensitivity analysis to observe how those parameters affect the expected total cost.

Case 1's results are presented in Tables 3, 4 and Figure 7, while Case 2 results are presented in Table 3 and Figure 7. The following are the findings of this study:

- Table 3 shows that changing the cost parameters S_{CM} and O_{cr} by a percentage change increases the overall cost. As a result, the overall cost decreases as a percentage change in the carbon emissions parameter. Therefore, it is cost-effective as well as environmentally friendly.
- Table 4 shows that when the customer's demand increases, the retailer's optimal time increases, the optimal cycle time is ineffective, and the overall cost increases. Table 4 shows that the overall cost is reduced when the constant demand parameter D_0 increases.
- The total cost amount increases due to the percentage increase (–20%, –10%, 0%, +10%, and +20%) in the output parameters P_{CM}, as shown in Table 3. When the percentage changes (–20%, –10%, 0%, +10%, and +20%) in the decoy cost parameters, D_{CM} and D_r, from negative to positive, the overall cost decreases.
- Table 4 illustrates that as the production rate parameter, α increases, the manufacturing's optimal time decreases, the optimal cycle time becomes ineffective, and the overall cost increases. When the parameters P_{CR}, S_{CR} and C_{CR} are changed by a percentage (–20%, –10%, 0%, +10%, and +20%), the overall cost is increased (Table 3).
- Table 3 shows that changing the first demand parameter *x* by a percentage change reduces the

overall cost. The overall cost increases when the constant demand parameter D_0 is changed by a percentage.



5(a)

5(b)

5(c)

5(d)

5(e)

Figure 5. (a) Convexity of TS_{C1} with respect the t_R & t_{m1}; (b) Convexity of TS_{C1} with respect the t_{m1} & t_r; (c) Convexity of TS_{C2} with respect the t_R & t_{m1}; (d) Convexity of TS_{C2} with respect the t_{m1} & t_r; (e) Convexity of TS_{C2} with respect the t_R & t_r.

Table 3. Change in total cost due to percentage change in parameters in Case 1 and Case 2.

Cases	Parameters	Total cost				
		–20%	–10%	0%	+10%	+20%
Case 1	S_{CM}	6212.62	6213.85	6215.08	6216.31	6217.54
	P_{CM}	6070.76	6145.48	6215.08	6280.00	6340.60
	D_{CM}	6228.96	6224.09	6215.08	6202.65	6187.35
	O_{cr}	6210.88	6212.98	6215.08	6217.18	6219.28
	D_r	6166.20	6190.71	6215.08	6239.34	6263.49
	ϕ	6215.93	6215.51	6215.08	6214.66	6214.23
	P_{CR}	6082.31	6149.40	6215.08	6279.48	6342.71
	S_{CR}	6214.38	6214.73	6215.08	6215.43	6215.78
	C_{CR}	6201.04	6208.06	6215.08	6222.11	6229.13
	x	5005.97	5613.12	6215.08	6813.21	7408.39
	D_0	6338.25	6278.57	6215.08	6146.90	6076.99
Case 2	S_{CM}	1210.57	1211.80	1213.03	1214.26	1215.49
	P_{CM}	1068.70	1143.33	1213.03	1277.94	1338.54
	D_{CM}	1226.90	1222.04	1213.03	1200.60	1185.29
	O_{cr}	1208.83	1210.93	1213.03	1215.13	1217.23
	D_r	1167.54	1191.43	1213.03	1234.35	1255.4
	ϕ	1213.08	1213.05	1213.03	1213.00	1212.98
	P_{CR}	1080.26	1147.35	1213.03	1277.93	1240.66
	S_{CR}	1211.33	1212.68	1213.03	1213.38	1213.73
	C_{CR}	1198.98	1206.00	1213.03	1220.05	1227.07
	x	1218.31	1215.82	1213.03	1209.97	1208.31

Table 4. Total cost and optimal time performance effect of linear demand parameter x and manufacturing rate parameter α for Case 1.

		t_{m1}	t_{m2}	t_r	t_{r2}	t_R	Total cost ($/cycle)
x	60	2.38	3.33	0.57	1.66	9.34	5005.97
	67.5	2.38	3.33	0.63	1.66	9.34	5613.12
	75	2.38	3.33	0.68	1.66	9.34	6215.08
	82.5	2.38	3.33	0.72	1.66	9.34	6813.21
	90	2.38	3.33	0.75	1.66	9.34	7408.39
α	17.60	2.78	3.33	0.68	1.66	9.34	6059.94
	19.8	2.57	3.33	0.68	1.66	9.34	6140.67
	22	2.38	3.33	0.68	1.66	9.34	6215.08
	24.2	2.21	3.33	0.68	1.66	9.34	6283.83
	26.4	2.05	3.33	0.68	1.66	9.34	6347.43

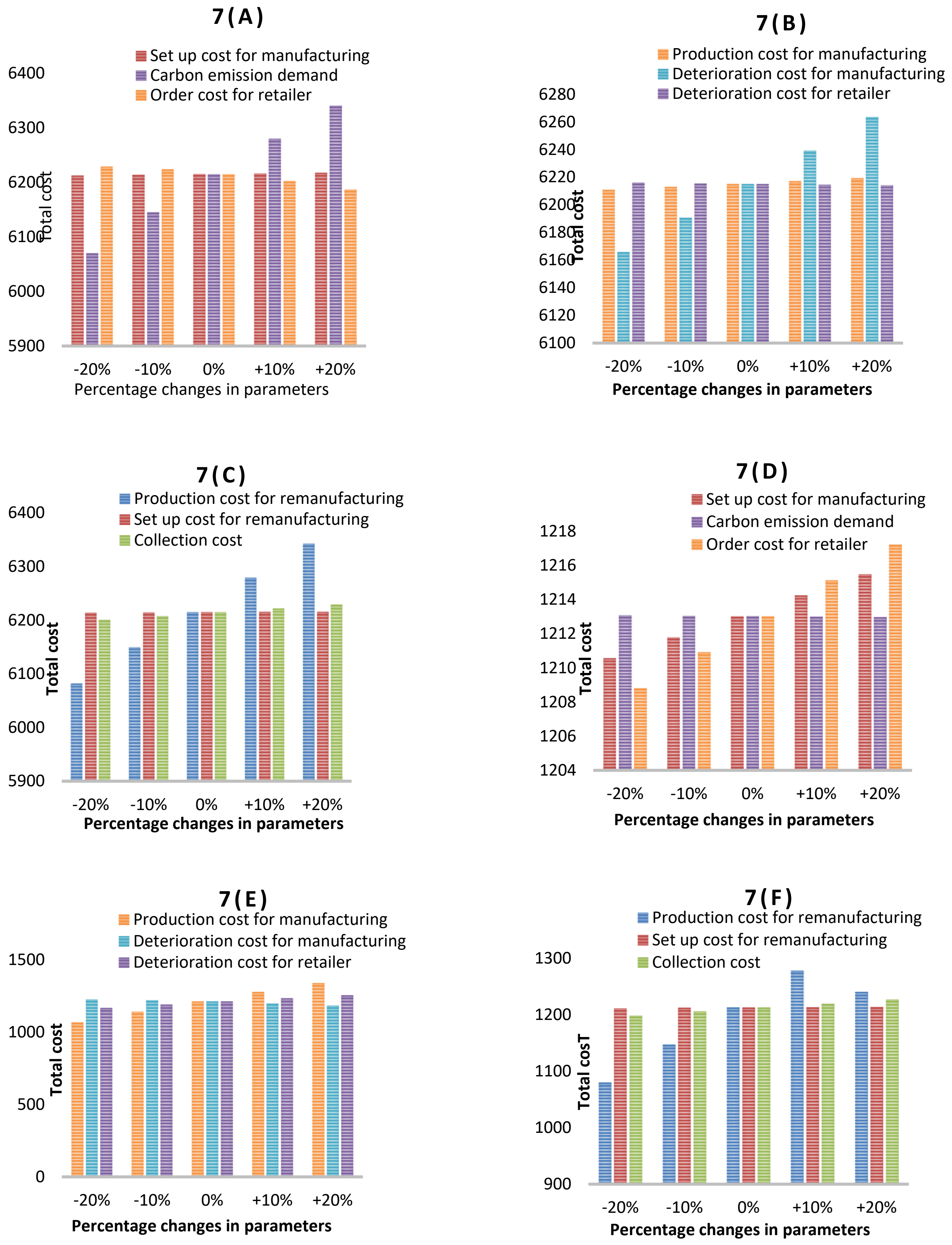


Figure 7. Total cost changes regarding the percentage change in the various parameters of (a), (b), (c) Case 1 and (d), (e), (f) Case 2.

9. Conclusions

A CLSCM was designed for the rapid demand and production rate for self-care products. Within production, remanufacturing policies existed, and manufacturing and remanufacturing occurred simultaneously. The flaw was immediately apparent and products were backordered since there were defective products. Through remanufacturing, the proposed research aimed to generate zero waste. RL collected used products from customers. This helped to reduce waste in the environment, and remanufactured products helped improve the retailer's partial backlog situation. Thus, the necessity of RL was both environmental and economical. As the remanufactured products fulfilled the shortage, it kept consumers happy. As a result, RL contributed to green SCM by reusing items, which is a component of environmental management. This study solved the rapid production rate, carbon emissions with a ramp-type demand rate, and the remanufacturing process. Case 1 indicated that the products were purchased quickly due to the initial linear demand rate. The total cost increased as several cycles L increased (Table 4). According to Case 2, the products were purchased less than the stock, owing to a non-starting linear type demand, resulting in a low profit. The total cost similarly reduced as several cycle L increased (Table 5). The results showed that as the system's production rate increased, the system's total cost increased. The situation fitted well for emergencies such as the COVID-19 pandemic, where the production rate of some products increased due to higher demand. This was a real-life example of this study. The numerical result showed that the remanufacturing cost ($375.56/cycle) was less than the manufacturing cost ($557.38/cycle) in both cases. That implied that remanufacturing medical devices reduced the production cost; then, the manufacturer, the retailer, or both could earn an increased profit. Likewise, the retailer had a decreased holding cost in Case 2 ($29.809/cycle) compared to Case 1 ($52.822/cycle). This indicated that the holding capacity in Case 1 was better than in Case 2. That indicated the retailer could handle a shortage when the demand became stable. Therefore, the retailer's total cost in Case 2 ($280.07/cycle) was less than in Case 1 ($5282.13/cycle).

This study had some limitations. In this, it is necessary to coordinate production and demand; otherwise, it may cause some problems. Smart manufacturing can be used in a CLSCM to minimize waste by allowing for variable production rates. It can be applied to a problem in which many parameters are stochastic [57]. The impact of machine failure on this model can be further investigated [58]. The medical devices that were explained in this study face an uncertain situation. This study can be extended by using uncertainty [59,60] and can be solved using different methodologies such as metaheuristics [61] and reverse logistics [62]. Another real perspective of emergencies is that the products are outsourced to another country to subside the medical condition or remanufacturing [63]. Investments into emissions reduction [64,65] and the improvement for emissions from the production system [66] to find the optimum investment for medical devices are another extension for this study. A global supply chain with a waste reduction perspective can be an immediate extension of this study [67]. Besides waste reduction, this study can be extended by using a multi-stage complex production model [68] for multiple products with multi-objective optimization by utilizing the carbon minimization from the system [69,70]. The random decay rate and random demand can make the model more realistic, which can be studied further.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work was supported by the Yonsei University Research Fund (Post Doc. Researcher Supporting Program) of 2022 (Project number: 2022-12-0040).

Conflict of interest

There is no conflict of interest.

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