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Research article

Passivity analysis for Markovian jumping neutral type neural networks with leakage and mode-dependent delay

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Abstract: In this study, we discuss the passivity analysis for Markovian jumping Neural Networks of neural-type. The results are demonstrated using phases of linear matrix inequalities as well as an improved Lyapunov-Krasovskii functional (LKF) of the triple integral terms and quadruple integrals. The information of the mode-dependent of all delays have been taken into account in the constructed Lyapunov–Krasovskii functional and novel stability criterion is derived. The value of selecting as many Lyapunov matrices that are mode-dependent as possible is demonstrated. The effectiveness and decreased conservatism of the aforementioned theoretical results are eventually demonstrated by a numerical example.

Keywords: Markov jump neural networks; Lyapunov-Krasovskii functional; Linear matrix inequality; Passivity theory

1. Introduction

Numerous studies on recurrent neural networks ($RNNs$), including those on bidirectional associative memory neural networks, cellular neural networks, and Hopfield neural networks, among others, have been proposed. In the meantime, neural networks (NNs) have been employed as a tool to address issues that have arisen in associative memory, signal processing, image processing, static image treatment, pattern recognition, and optimization. Additionally, it seems that time delays are crucial to networked control systems, chemical reactions, communication systems, etc. The authors of [1,2] investigated time-delayed artificial neural network electronic implementations. Several studies have looked into different sorts of delays inside NNs in this context; see the papers [38–40] and references therein. The passivity theory is widely applied in many engineering problems. Indeed, it is intimately related to the circuit analysis which is a useful and significant tool to analyze the stability of nonlinear systems, signal processing and chaos control. Thus, it has been employed in various fields of science and engineering [7, 8, 44–48]. In [9], the authors proposed neural adaptive output feedback control based on passivity with adaptive set-point regulation of nonlinear uncertain non-negative and compartmental systems. In the last decade, great attention has been paid to the passivity analysis of RNNs with delay-independent [10, 49] and delay-dependent [11–14, 36]. In [13], the author studied the passivity analysis of Markovian jump neural network with leakage time varying delay, discrete and distributed time varying delays. We can see the discussion of the extended dissipativity of discrete-time neural networks (NNs) with time-varying delay in [55]. However, comparatively less interest has been shown towards the passivity analysis of mode dependent delay on neutral type neural networks ($NNNs$) with Markovian jumping (Mj). On the other hand, a Markov jump system is a special classes of a hybrid system. Indeed, they have great ability to model the dynamical systems and their application can be found in manufacturing systems, economic systems, network control systems, modeling production system, communication systems and so on. In the recent years, several results are reported on the stability analysis for neural networks with Markovian jump parameters, see the references [16–19, 41–43]. In [20], the authors studied global exponential estimates of delayed stochastic NNs with Markovian switching by constructing with positive definite matrices in stochastic Lyapunov functional which are dependent on the system mode and a triple-integral term. The $\mathcal{H}_\infty$ synchronization issue for singularly perturbed coupled neural networks (SPCNNs) affected by both nonlinear constraints and gain uncertainties was explored in [57] using a novel double-layer switching regulation containing Markov chain and persistent dwell-time switching regulation (PDTSR). Convolutional Neural Networks (CNNs) are efficient tools for pattern recognition applications. More on this topic can be seen in [58, 60]. An exponential synchronization problem for the multi-weighted complex dynamical network (MCDN) with hybrid delays on a time scale is investigated in [61]. We can see the establishment of fixed-point and coincidence-point consequences in generalized metric spaces in [59]. The nonfragile $\mathcal{H}_\infty$ synchronization issue for a class of discrete-time Takagi–Sugeno (T–S) fuzzy Markov jump systems was investigated in [56]. In [21], the authors have studied stochastic NNNs with mixed time-delays under adaptive synchronization. Additionally, the problem of state estimation of $RNNs$ with Mj parameters and mixed delays based on mode-dependent approach was investigated in [22].

However, some of the researchers discussed the robust passive filtering for NNNs with delays in [25]. In [26], the authors investigated the global asymptotic stability of NNNs with delays by utilizing the Lyapunov-Krasovskii functional (LKF) and the linear matrix inequality approach. While employing the method of Lyapunov–Krasovskii functional, we necessarily need these three steps for the derivation of a global asymptotic stability criterion: constructing a Lyapunov–Krasovskii functional, estimating the derivative of the Lyapunov–Krasovskii functional, and formulating a global asymptotic stability criterion. You will get an overview of recent developments in each of the above steps if you refer to [54].The author in [52] studied passivity and exponential passivity for NNNs with various delays. In [51], the authors investigated the robust passivity analysis of mixed delayed NNs with distributed time-varying delays. The exponential passivity of discrete-time switched NNs with transmission delay was studied in [53]. Recently, the authors in [5] studied passivity analysis for NNNs with

Mj parameters and time delay in the leakage term. New delay-dependent passivity conditions are derived in terms of LMIs with a proper construction of LKF, and it can be checked easily via standard numerical packages. However, triple and quadruple integrals have not been taken into account to derive the passivity conditions and, moreover, the mode-dependent time delays have not been included in [5]. Recently, a novel Lyapunov functional with some terms involving triple or quadruple integrals are taken into account to study the state estimation problem with mode-dependent approach in [22]. Motivated by the above discussion, the main purpose of this paper is to study the global passivity of Mj for NNNs with leakage and mode-dependent delay terms. By construction of a new LKF involving mode-dependent Lyapunov matrices, some sufficient conditions are derived in terms of LMIs. For the sake of illustration, a numerical example is given to demonstrate the usefulness and effectiveness of the presented results. Unlike previous results, we will introduce an improved Lyapunov–Krasovskii functional with triple and quadruple integrals for deriving the reported stability results in this paper. Based on this discussion, our technique not only provides different approach but also gives less conservative conditions than those studied in [5,22]. The rest of this paper is organized as follows. The problem and some preliminaries are introduced in Section 2. In Section 3, the main results are stated and proved. Some sufficient conditions for global passivity results are developed here. In Section 4, an illustrative example is provided to demonstrate the effectiveness of the proposed criteria. We conclude the results of this paper in Section 5.

Notations: Throughout this paper the following notations are used: $\mathbb{R}^n$ - n-dimensional Euclidean space; $\mathbb{R}^{n\times n}$- the set of all $n \times n$ real matrices; $diag(\cdots)$- a block diagonal matrix; I -the identity matrix with compatible dimensions; C^T - the transpose of C; X and Y are symmetric matrices, where $X \geq Y$ (similarly $X > Y$)- $X - Y$ is a positive semi-definite (similarly positive definite); $(\Omega, \mathfrak{F}, \mathcal{P})$- a complete probability space with a natural filtration $\{\mathfrak{F}_t\}_{t\geq 0}$; $\mathbf{E}[\cdot]$ - expectation operator with respect to the given probability measure $\mathcal{P}$; $C([-d, 0]; \mathbb{R}^n)$ - the family of continuously differentiable function; $\|\varphi\| = \max\{\max_{-\tau\leq\theta\leq 0} |\varphi(\theta)|, \max_{-d\leq\theta\leq 0} |\varphi'(\theta)|\}$; $C^2_{\mathfrak{F}_0}([-d, 0]; \mathbb{R}^n)$ - the family of bounded $\mathfrak{F}_0$-measurable; $C([-d, 0]; \mathbb{R}^n)$-valued stochastic variables $\xi = \{\xi(\theta) : -d \leq \theta \leq 0\}$ such that $\int_{-d}^{0} \mathbf{E}|\xi(\theta)|^2 ds < \infty$; $*$ -the symmetric block in one symmetric matrix.

2. Problem formulation and preliminaries

Let $\{\nabla(t), t \in \mathbb{Z}^+\}$ be right-continuous on $(\Omega, \mathcal{F}, \mathcal{P})$. Here $(\Omega, \mathcal{F}, \mathcal{P})$ is a complete probability space with Markov chain, $\nabla(t)$ takes the values from a finite state space $\mathcal{S} = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\pi_{ij})_{N\times N}$ given by

$$P\{\nabla(t + \Delta t) = j|\nabla(t) = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\Delta t > 0$ and $\lim_{\Delta t\to 0} \frac{o(\Delta t)}{\Delta t} = 0$, $\pi_{ij} \geq 0$ $(i \neq j)$ is the transition rate from i to j, and $\pi_{ii} = -\sum\limits_{j=1,\ j\neq i}^{N} \pi_{ij}$.

We consider Mode-dependent Markov jump $NNNs$ with mixed time-delays:

$$\begin{cases} \dot{x}(t) &= -\mathcal{A}(\nabla(t))x(t-\sigma(\nabla(t))) + \mathcal{B}(\nabla(t))\mathcal{H}(x(t)) + C(\nabla(t))\mathcal{H}(x(t-\tau(t,\nabla(t)))) \\ & \quad +\mathcal{D}(\nabla(t))\dot{x}(t-h(t,\nabla(t))) + \mathcal{E}(\nabla(t)) \int\limits_{t-d(t,\nabla(t))}^{t} \mathcal{H}(x(s))\mathrm{d}s + u(t) \\ y(t) &= \mathcal{H}(x(t)) \end{cases} \tag{2.1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector linked with n neurons. The diagonal matrix $\mathcal{A}(\nabla(t)) = diag(A_1(\nabla(t)), A_2(\nabla(t)), \ldots, A_n(\nabla(t)))$ has positive entries $A_i(\nabla(t)) > 0$ $(i = 1, 2, \ldots, n)$. $\mathcal{B}(\nabla(t))$, $C(\nabla(t))$, $\mathcal{D}(\nabla(t))$, $\mathcal{E}(\nabla(t))$ are known appropriate dimensional constant matrices. Here the neuron activation function is $\mathcal{H}(x(t)) = [\mathcal{H}_1(x_1(t)), \mathcal{H}_2(x_2(t)), \ldots, \mathcal{H}_n(x_n(t))]^T$. $u(t)$ denotes a constant input. $\tau(t, \nabla(t))$, $h(t, \nabla(t))$, $d(t, \nabla(t))$ are mode dependent discrete, neutral and distributed delays, respectively and $\sigma(\nabla(t))$ is the mode dependent leakage delay.

Throughout this paper, we assume the following.
Assumption 1. For any $j = 1, 2, \ldots, n$, $\mathcal{H}_j(0) = 0$ and there exist constants $\widehat{l_j^-}$ and $\widehat{l_j^+}$ such that

$$\widehat{l_j^-} \leq \frac{\mathcal{H}_j(\gamma_1) - \mathcal{H}_j(\gamma_2)}{\gamma_1 - \gamma_2} \leq \widehat{l_j^+}, \tag{2.2}$$

where $\gamma_1, \gamma_2 \in \mathbb{R}$, and $\gamma_1 \neq \gamma_2$.

For the sake of convenience, we denote $\mathcal{A}(\nabla(t) = i) = \mathcal{A}_i, \mathcal{B}(\nabla(t) = i) = \mathcal{B}_i, C(\nabla(t) = i) = C_i$, $\mathcal{D}(\nabla(t) = i) = \mathcal{D}_i$, $\mathcal{E}(\nabla(t) = i) = \mathcal{E}_i$, respectively.

System (2.1) can be rewritten as

$$\begin{cases} \dot{x}(t) &= -\mathcal{A}_i x(t-\sigma_i) + \mathcal{B}_i\mathcal{H}(x(t)) + C_i\mathcal{H}(x(t-\tau_i(t))) + \mathcal{D}_i\dot{x}(t-h_i(t)) \\ & \quad +\mathcal{E}_i \int\limits_{t-d_i(t)}^{t} \mathcal{H}(x(s))\mathrm{d}s + u(t) \\ y(t) &= \mathcal{H}(x(t)) \end{cases} \tag{2.3}$$

and the parameters associated with time delays are assumed to satisfy following:

$$0 \leq \tau_i(t) \leq \tau_i, \;\; \dot{\tau}_i(t) \leq \tau_{\mu_i}, \;\; 0 \leq h_i(t) \leq h_i, \;\; \dot{h}_i(t) \leq h_{\mu_i}, \;\; 0 \leq d_i(t) \leq d_i, \;\; \dot{d}_i(t) \leq d_{\mu_i}, \;\; \sigma_i > 0 \tag{2.4}$$

where $\tau_i, h_i, d_i, \tau_{\mu_i}, h_{\mu_i}$ and d_{μ_i} are some real constants and $\tau = \max_{i\in S}\{\tau_i\}$, $h = \max_{i\in S}\{h_i\}$, $d = \max_{i\in S}\{d_i\}$, $\sigma = \max_{i\in S}\{\sigma_i\}$.

Now we can see a few necessary lemmas and a definition.

Lemma 1. *[33] Let a and b be scalars with $a \leq b$, M be a matrix with $M \geq 0$, and $y(t) : [a, b] \to \mathcal{R}^n$ be a vector function such that, the following integrals are well defined, then the inequality*

$$(b-a)\left[\int_a^b y(s)^T M y(s)\mathrm{d}s\right] \geq \left[\int_a^b y(s)\mathrm{d}s\right]^T M \left[\int_a^b y(s)\mathrm{d}s\right].$$

holds.

Proof. By Schur complement,
$\begin{bmatrix} y(s)^T M y(s) & y(s)^T \\ y(s) & M^{-1} \end{bmatrix} \geq 0, \quad s \in [a, b].$

On integration from a to b yields, $\begin{bmatrix} \int_a^b y(s)^T M y(s) ds & \int_a^b y(s)^T ds \\ \int_a^b y(s) ds & (b-a) M^{-1} \end{bmatrix} \geq 0, \quad s \in [a, b]$. Now using the Schur complement on this inequality, we obtain our desired result. □

Lemma 2. *[50] For any real vectors $x, y \in R^n$ and positive definite matrix $\mathcal{M} = \mathcal{M}^T$ it follows that:*

$$\pm 2x^T y \leq x^T \mathcal{M} x + y^T \mathcal{M}^{-1} y.$$

Lemma 3. *[31] (Schur complement) Given Ω_1, Ω_2 and Ω_3 are constant matrices with appropriate dimensions, where $\Omega_1, \Omega_2 > 0$ are symmetric matrices, then*

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0 \iff \begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0, \quad or \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0.$$

Definition 1. *[32] If there exists a scalar $\nu \geq 0$ such that $\forall t_p \geq 0$ and for all solutions of (2.1), the following inequality holds under zero initial conditions,*

$$2 \int_0^{t_p} E\{y(s)^T u(s)\} \mathrm{d}s \geq -\gamma \int_0^{t_p} E\{u(s)^T u(s)\} \mathrm{d}s, \tag{2.5}$$

then the system (2.3) is said to be passive.

3. Main results

Now, we denote

$$\widehat{L}_1 = \mathrm{diag}\{\widehat{l_1^-}\widehat{l_1^+}, \widehat{l_2^-}\widehat{l_2^+}, \ldots, \widehat{l_m^-}\widehat{l_m^+}\}, \quad \text{and} \quad \widehat{L}_2 = \mathrm{diag}\Big\{\frac{\widehat{l_1^-} + \widehat{l_1^+}}{2}, \frac{\widehat{l_2^-} + \widehat{l_2^+}}{2}, \ldots, \frac{\widehat{l_m^-} + \widehat{l_m^+}}{2}\Big\}.$$

Theorem 1. *For given scalars $\tau_i > 0$, $h_i > 0$, $d_i > 0$, $\tau_{\mu_i} > 0$, $h_{\mu_i} > 0$, $d_{\mu_i} > 0$ and $\sigma_i > 0$, system (2.3) is passive if there exist symmetric positive definite matrices $P_i > 0$, $Q_i = \begin{bmatrix} Q_{1i} & Q_{2i} \\ Q_{2i}^T & Q_{3i} \end{bmatrix} > 0$, $W_i > 0$, $R_i > 0$, $S_i > 0$, $V_i > 0$, $U_i > 0$, $X_i > 0$, $Y_i > 0$, $T_i > 0$, $Z_i > 0$, $L_i > 0$, $K_i > 0$, $Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} > 0$, $W > 0$, $R > 0$, $S > 0$, $V > 0$, $U > 0$, $X > 0$, $Y > 0$, $T > 0$, $Z > 0$, $L > 0$, $M > 0$ and the diagonal matrices $H_{1i} > 0$, $H_{2i} > 0$, $H_{3i} > 0$, $H_{4i} > 0$, $H_{5i} > 0$, $H_{6i} > 0$ and any matrices N_i, J_1, J_2 with appropriate dimensions such that the following LMIs are satisfied for i $= 1, \ldots, N$:*

$$\begin{bmatrix} X_i & N_i \\ * & X_i \end{bmatrix} \geq 0, \tag{3.1}$$

$$\sum_{j=1}^{N} \pi_{ij} V_j - V \leq 0, \tag{3.2}$$

$$\sum_{j=1,j\neq i}^{N} \pi_{ij}\mathcal{G}_j - \mathcal{G} \leq 0, \tag{3.3}$$

$$\Phi = \begin{bmatrix} \Omega & \Gamma^T \\ * & -\frac{1}{\pi_{ij}}K_j \end{bmatrix} < 0, \tag{3.4}$$

where $\mathcal{G}_j$ in (3.3) respectively represents Q_j, W_j, R_j, S_j, U_j, X_j, Y_j, T_j, Z_j, L_j and correspondingly $\mathcal{G}$ represents Q, W, R, S, U, X, Y, T, Z, L (e.q.,when $\mathcal{G}_j$ is Q_j, $\mathcal{G}$ is Q) and

$$\begin{aligned}
\Omega &= (\vartheta_{i,j})_{15\times 15},\\
\vartheta_{1,1} &= -P_i\mathcal{A}_i - \mathcal{A}_i^T P_i + \pi_{ii}P_i + \sum_{j\neq i}\pi_{ij}P_j + \sum_{j\neq i}\pi_{ij}K_j + Q_{1i} + \tau Q_1 + W_i + \tau W + R_i + \sigma R\\
&\quad + \tau_i U_i + \frac{\tau^2}{2}U - \frac{1}{\tau_i}X_i - \frac{1}{h_i}Y_i - 2T_i - 2Z_i - 2L_i - \widehat{L}_1 H_{1i} - \widehat{L}_1 H_{3i} - \widehat{L}_1 H_{4i}\\
&\quad - \widehat{L}_1 H_{5i} - \widehat{L}_1 H_{6i} - 2\sigma^2 M, \quad \vartheta_{1,2} = -\frac{1}{\tau_i}N_i^T + \frac{1}{\tau_i}X_i, \quad \vartheta_{1,3} = \frac{1}{\tau_i}N_i^T + \widehat{L}_2 H_{3i},\\
\vartheta_{1,4} &= \frac{2}{\tau_i}T_i, \quad \vartheta_{1,5} = P_i\mathcal{B}_i + Q_{2i} + Q_2 + \widehat{L}_2 H_{1i} + J_1\mathcal{B}_i, \quad \vartheta_{1,6} = P_i C_i + J_1 C_i,\\
\vartheta_{1,7} &= -J_1\mathcal{A}_i - \mathcal{A}_i^T J_1^T + \widehat{L}_2 H_{5i}, \quad \vartheta_{1,8} = \mathcal{A}_i^T P_i\mathcal{A}_i - \pi_{ii}P_i\mathcal{A}_i - \frac{2}{\sigma_i}L_i, \quad \vartheta_{1,9} = 2\sigma M,\\
\vartheta_{1,10} &= P_i, \quad \vartheta_{1,11} = P_i\mathcal{D}_i + J_1\mathcal{D}_i, \quad \vartheta_{1,12} = \frac{1}{h_i}Y_i + \widehat{L}_2 H_{4i}, \quad \vartheta_{1,13} = \frac{2}{h_i}Z_i,\\
\vartheta_{1,14} &= J_1\mathcal{E}_i, \quad \vartheta_{1,15} = -J_1 + \widehat{L}_2 H_{6i}, \quad \vartheta_{2,2} = -(1-\tau_{\mu_i})Q_{1i} - \widehat{L}_1 H_{2i} - \frac{2}{\tau_i}X_i + \frac{1}{\tau_i}N_i^T,\\
\vartheta_{2,3} &= \frac{1}{\tau_i}X_i - \frac{1}{\tau_i}N_i^T, \quad \vartheta_{2,6} = -(1-\tau_{\mu_i})Q_{1i} + \widehat{L}_2 H_{2i}, \quad \vartheta_{3,3} = -W_i - \frac{1}{\tau_i}X_i - H_{3i},\\
\vartheta_{4,4} &= -\frac{1}{\tau_i}U_i - \frac{2}{\tau_i^2}T_i, \quad \vartheta_{5,5} = Q_{3i} + Q_3 + d_i V_i + \frac{d^2}{2}V - H_{1i}, \quad \vartheta_{5,8} = -\mathcal{B}_i^T P_i\mathcal{A}_i,\\
\vartheta_{5,15} &= \mathcal{B}_i^T J_2^T, \quad \vartheta_{6,6} = -(1-\tau_{\mu_i})Q_{3i} - H_{2i}, \quad \vartheta_{6,8} = -C_i^T P_i\mathcal{A}_i, \quad \vartheta_{6,15} = C_i^T J_2^T,\\
\vartheta_{7,7} &= -R_i - H_{5i}, \quad \vartheta_{7,15} = -\mathcal{A}_i^T J_2^T, \quad \vartheta_{8,8} = \pi_{ii}\mathcal{A}_i^T P_i\mathcal{A}_i - \frac{2}{\sigma_i^2}L_i, \quad \vartheta_{8,10} = -\mathcal{A}_i^T P_i,\\
\vartheta_{8,11} &= -\mathcal{A}_i^T P_i\mathcal{D}_i, \quad \vartheta_{8,14} = -\mathcal{A}_i^T P_i\mathcal{E}_i, \quad \vartheta_{9,9} = \sum_{j\neq i}\pi_{ij}\mathcal{A}_j^T P_j\mathcal{A}_j - 2M, \quad \vartheta_{10,10} = -\gamma I,\\
\vartheta_{10,15} &= J_2^T, \quad \vartheta_{11,11} = -S_i(1-h_{\mu_i}), \quad \vartheta_{11,15} = \mathcal{D}_i^T J_2^T, \quad \vartheta_{12,12} = -\frac{1}{h_i}Y_i - H_{4i},\\
\vartheta_{13,13} &= -\frac{2}{h_i^2}Z_i, \quad \vartheta_{14,14} = -\frac{(1-d_{\mu_i})}{d_i}V_i, \quad \vartheta_{14,15} = \mathcal{E}_i^T J_2^T, \quad \vartheta_{15,15} = S_i + hS + \tau_i X_i + \frac{\tau^2}{2}X\\
&\quad + h_i Y_i + \frac{h^2}{2}Y + \frac{\tau_i^2}{2}T_i + \frac{\tau^3}{6}T + \frac{h_i^2}{2}Z_i + \frac{h^3}{6}Z + \frac{\sigma_i^2}{2}L_i + \frac{\sigma^3}{6}L + \frac{\sigma^4}{2}Z - (J_2 + J_2^T) - H_{6i},\\
\Gamma^T &= [\ \underbrace{0.....0}_{8\ elements}\ \sum_{j\neq i}\pi_{ij}(\mathcal{A}_j^T P_j)^T\ \underbrace{0.....0}_{15\ elements}\ ]^T,
\end{aligned}$$

and the other coefficients are zero.

Proof. Here, we consider LKF candidate:

$$V(x_t, i, t) = \sum_{\kappa=1}^{13} V_\kappa(x_t, i, t), \tag{3.5}$$

where

$$\begin{aligned}
V_1(x_t, i, t) &= \left[x(t) - \mathcal{A}_i \int_{t-\sigma_i}^{t} x(s)\mathrm{d}s\right]^T P_i \left[x(t) - \mathcal{A}_i \int_{t-\sigma_i}^{t} x(s)\mathrm{d}s\right],\\
V_2(x_t, i, t) &= \int_{t-\tau_i(t)}^{t} \zeta^T(s)Q_i\zeta(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t} \zeta^T(s)Q\zeta(s)\mathrm{d}s\mathrm{d}\theta,\\
V_3(x_t, i, t) &= \int_{t-\tau_i}^{t} x^T(s)W_i x(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t} x^T(s)Wx(s)\mathrm{d}s\mathrm{d}\theta,\\
V_4(x_t, i, t) &= \int_{t-\sigma_i}^{t} x^T(s)R_i x(s)\mathrm{d}s + \int_{-\sigma}^{0}\int_{t+\theta}^{t} x^T(s)Rx(s)\mathrm{d}s\mathrm{d}\theta,\\
V_5(x_t, i, t) &= \int_{t-h_i}^{t} x^T(s)S_i x(s)\mathrm{d}s + \int_{-h}^{0}\int_{t+\theta}^{t} x^T(s)Sx(s)\mathrm{d}s\mathrm{d}\theta,\\
V_6(x_t, i, t) &= \int_{-d_i(t)}^{0}\int_{t+\theta}^{t} \mathcal{H}^T(x(s))V_i\mathcal{H}(x(s))\mathrm{d}s\mathrm{d}\theta + \int_{-d}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \mathcal{H}^T(x(s))V\mathcal{H}(x(s))\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta,\\
V_7(x_t, i, t) &= \int_{-\tau_i}^{0}\int_{t+\theta}^{t} x^T(s)U_i x(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} x^T(s)Ux(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta,\\
V_8(x_t, i, t) &= \int_{-\tau_i}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)X_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)X\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta,\\
V_9(x_t, i, t) &= \int_{-h_i}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Y_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)Y\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta,\\
V_{10}(x_t, i, t) &= \int_{-\tau_i}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)T_i\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta + \int_{-\tau}^{0}\int_{\theta}^{0}\int_{\beta}^{0}\int_{t+\alpha}^{t} \dot{x}^T(s)T\dot{x}(s)\mathrm{d}s\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\theta,\\
V_{11}(x_t, i, t) &= \int_{-h_i}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)Z_i\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta + \int_{-h}^{0}\int_{\theta}^{0}\int_{\beta}^{0}\int_{t+\alpha}^{t} \dot{x}^T(s)Z\dot{x}(s)\mathrm{d}s\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\theta,\\
V_{12}(x_t, i, t) &= \int_{-\sigma_i}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)L_i\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta + \int_{-\sigma}^{0}\int_{\theta}^{0}\int_{\beta}^{0}\int_{t+\alpha}^{t} \dot{x}^T(s)L\dot{x}(s)\mathrm{d}s\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\theta,\\
V_{13}(x_t, i, t) &= \sigma^2\int_{-\sigma}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)M\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta,
\end{aligned}$$

where $\zeta^T(t) = [x^T(t),\ \mathcal{H}^T(x(t))]^T$.

From (2.1) , we get

$$\mathcal{L}V(x_t, i, t) \ = \ \sum_{\kappa=1}^{13} \mathcal{L}V_\kappa(x_t, i, t), \tag{3.6}$$

where

$$\mathcal{L}V_1(x_t, i, t) = 2\left[x(t) - \mathcal{A}_i \int_{t-\sigma_i}^{t} x(s)\mathrm{d}s\right]^T P_i \frac{\mathrm{d}}{\mathrm{d}t}\left[x(t) - \mathcal{A}_i \int_{t-\sigma_i}^{t} x(s)\mathrm{d}s\right]$$

$$+\sum_{j=1}^{N}\pi_{ij}\left[x(t)-\mathcal{A}_j\int_{t-\sigma_j}^{t}x(s)\mathrm{d}s\right]^T P_j\left[x(t)-\mathcal{A}_j\int_{t-\sigma_j}^{t}x(s)\mathrm{d}s\right],$$

$$\begin{aligned}
&\leq 2\left[x(t)-\mathcal{A}_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s\right]^T P_i[-\mathcal{A}_i x(t)+\mathcal{B}_i\mathcal{H}(x(t))+C_i\mathcal{H}(x(t-\tau_i(t)))\\
&+\mathcal{D}_i\dot{x}(t-h_i(t))+\mathcal{E}_i\int_{t-d_i(t)}^{t}\mathcal{H}(x(s))\mathrm{d}s+u(t)]\\
&+\pi_{ii}\left[x(t)-\mathcal{A}_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s\right]^T P_i\left[x(t)-\mathcal{A}_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s\right]\\
&+\sum_{j\neq i}\pi_{ij}\Big[x^T(t)P_jx(t)+\int_{t-\sigma_j}^{t}x^T(s)\mathrm{d}s\mathcal{A}_jP_jK_j^{-1}P_j\mathcal{A}_j\int_{t-\sigma_j}^{t}x(s)\mathrm{d}s\\
&+\int_{t-\sigma}^{t}x^T(s)\mathrm{d}s\mathcal{A}_j^TP_j\mathcal{A}_j\int_{t-\sigma}^{t}x(s)\mathrm{d}s\Big],
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}V_2(x_t,i,t)&\leq\zeta^T(t)Q_i\zeta(t)-\zeta^T(t-\tau_i(t))Q_i\zeta(t-\tau_i(t))(1-\tau_{\mu_i})+\sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t}\zeta^T(s)Q_j\zeta(s)\mathrm{d}s\\
&+\tau\zeta^T(t)Q\zeta(t)-\int_{t-\tau}^{t}\zeta^T(s)Q\zeta(s)\mathrm{d}s,\\
\mathcal{L}V_3(x_t,i,t)&=x^T(t)W_ix(t)-x^T(t-\tau_i)W_ix(t-\tau_i)+\sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j}^{t}x^T(s)W_jx(s)\mathrm{d}s\\
&+\tau x^T(t)Wx(t)-\int_{t-\tau}^{t}x^T(s)Wx(s)\mathrm{d}s,\\
\mathcal{L}V_4(x_t,i,t)&=x^T(t)R_ix(t)-x^T(t-\sigma_i)R_ix(t-\sigma_i)+\sum_{j=1}^{N}\pi_{ij}\int_{t-\sigma_j}^{t}x^T(s)R_jx(s)\mathrm{d}s\\
&+\sigma x^T(t)Rx(t)-\int_{t-\sigma}^{t}x^T(s)Rx(s)\mathrm{d}s,\\
\mathcal{L}V_5(x_t,i,t)&\leq\dot{x}^T(t)S_i\dot{x}(t)-\dot{x}^T(t-h_i(t))S_i\dot{x}(t-h_i(t))(1-h_{\mu_i})+\sum_{j=1}^{N}\pi_{ij}\int_{t-h_j(t)}^{t}\dot{x}^T(s)S_j\dot{x}(s)\mathrm{d}s\\
&+h\dot{x}^T(t)S\dot{x}(t)-\int_{t-h}^{t}\dot{x}^T(s)S\dot{x}(s)\mathrm{d}s,\\
\mathcal{L}V_6(x_t,i,t)&=d_i(t)\mathcal{H}^T(x(t))V_i\mathcal{H}(x(t))-(1-d_{\mu_i})\int_{t-d_i(t)}^{t}\mathcal{H}^T(x(s))V_i\mathcal{H}(x(s))\mathrm{d}s\\
&+\sum_{j=1}^{N}\pi_{ij}\int_{-d_j(t)}^{0}\int_{t+\theta}^{t}\mathcal{H}^T(x(s))V_j\mathcal{H}(x(s))\mathrm{d}s\mathrm{d}\theta\\
&+\frac{d^2}{2}\mathcal{H}^T(x(t))V\mathcal{H}(x(t))-\int_{-d}^{0}\int_{t+\theta}^{t}\mathcal{H}^T(x(s))V\mathcal{H}(x(s))\mathrm{d}s\mathrm{d}\theta,
\end{aligned}$$

$$\mathcal{L}V_7(x_t,i,t) = \tau_i x^T(t)U_i x(t) - \int_{t-\tau_i}^{t} x^T(s)U_i x(s)\mathrm{d}s + \sum_{j=1}^{N}\pi_{ij}\int_{-\tau_j}^{0}\int_{t+\theta}^{t} x^T(s)U_j x(s)\mathrm{d}s\mathrm{d}\theta$$
$$+ \frac{\tau^2}{2}x^T(t)Ux(t) - \int_{-\tau}^{0}\int_{t+\theta}^{t} x^T(s)Ux(s)\mathrm{d}s\mathrm{d}\theta,$$

$$\mathcal{L}V_8(x_t,i,t) = \tau_i \dot{x}^T(t)X_i\dot{x}(t) - \int_{t-\tau_i}^{t} \dot{x}^T(s)X_i\dot{x}(s)\mathrm{d}s + \sum_{j=1}^{N}\pi_{ij}\int_{-\tau_j}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)X_j\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$
$$+ \frac{\tau^2}{2}\dot{x}^T(t)X\dot{x}(t) - \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)X\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$
$$\mathcal{L}V_9(x_t,i,t) = h_i \dot{x}^T(t)Y_i\dot{x}(t) - \int_{t-h_i}^{t} \dot{x}^T(s)Y_i\dot{x}(s)\mathrm{d}s + \sum_{j=1}^{N}\pi_{ij}\int_{-h_j}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Y_j\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$
$$+ \frac{h^2}{2}\dot{x}^T(t)Y\dot{x}(t) - \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Y\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$
$$\mathcal{L}V_{10}(x_t,i,t) = \frac{\tau_i^2}{2}\dot{x}^T(t)T_i\dot{x}(t) - \int_{-\tau_i}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)T_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \frac{\tau^3}{6}\dot{x}^T(t)T\dot{x}(t)$$
$$+ \sum_{j=1}^{N}\pi_{ij}\int_{-\tau_j}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)T_j\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta - \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)T\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta,$$
$$\mathcal{L}V_{11}(x_t,i,t) = \frac{h_i^2}{2}\dot{x}^T(t)Z_i\dot{x}(t) - \int_{-h_i}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \frac{h^3}{6}\dot{x}^T(t)Z\dot{x}(t)$$
$$+ \sum_{j=1}^{N}\pi_{ij}\int_{-h_j}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)Z_j\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta - \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)Z\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta,$$
$$\mathcal{L}V_{12}(x_t,i,t) = \frac{\sigma_i^2}{2}\dot{x}^T(t)L_i\dot{x}(t) - \int_{-\sigma_i}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)L_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \frac{\sigma^3}{6}\dot{x}^T(t)L\dot{x}(t)$$
$$+ \sum_{j=1}^{N}\pi_{ij}\int_{-\sigma_j}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)L_j\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta - \int_{-\sigma}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)L\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta,$$
$$\mathcal{L}V_{13}(x_t,i,t) = \frac{\sigma^4}{2}\dot{x}^T(t)M\dot{x}(t) - \sigma^2\int_{-\sigma}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)M\dot{x}(s)\mathrm{d}s\mathrm{d}\theta.$$

Using the upper bounds of discrete, neutral, distributed time-varying delays, leakage delays, Lemma 2 and with $\pi_{ii} < 0$, the following relationship is obtained

$$\sum_{j\neq i}\Big(-2\pi_{ij}x^T(t)P_j\mathcal{A}_j\int_{t-\sigma_j}^{t} x(s)ds\Big)$$
$$\leq \sum_{j\neq i}\pi_{ij}\Big(x^T(t)K_jx(t) + \int_{t-\sigma_j}^{t} x^T(s)ds\mathcal{A}_j^T P_j K_j^{-1}P_j\mathcal{A}_j\int_{t-\sigma_j}^{t} x(s)ds\Big)$$
$$\leq \sum_{j\neq i}\pi_{ij}\Big(x^T(t)K_jx(t) + \int_{t-\sigma}^{t} x^T(s)ds\mathcal{A}_j^T P_j K_j^{-1}P_j\mathcal{A}_j\int_{t-\sigma}^{t} x(s)ds\Big), \tag{3.7}$$

$$\sum_{j\neq i}\pi_{ij}\Big(\int_{t-\sigma_j}^{t}x^T(s)ds\Big)\mathcal{A}_j^TP_j\mathcal{A}_j\Big(\int_{t-\sigma_j}^{t}x(s)ds\Big)$$
$$\leq\sum_{j\neq i}\pi_{ij}\Big(\int_{t-\sigma}^{t}x^T(s)ds\Big)\mathcal{A}_j^TP_j\mathcal{A}_j\Big(\int_{t-\sigma}^{t}x(s)ds\Big). \tag{3.8}$$

Similarly,

$$\begin{aligned}\sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t}\zeta^T(s)Q_j\zeta(s)\mathrm{d}s &\leq \sum_{j=1,j\neq i}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t}\zeta^T(s)Q_j\zeta(s)\mathrm{d}s\\ &\leq \sum_{j=1,j\neq i}^{N}\pi_{ij}\int_{t-\tau}^{t}\zeta^T(s)Q_j\zeta(s)\mathrm{d}s\\ &\leq \int_{t-\tau}^{t}\zeta^T(s)Q\zeta(s)\mathrm{d}s,\end{aligned} \tag{3.9}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j}^{t}x^T(s)W_jx(s)\mathrm{d}s\leq\int_{t-\tau}^{t}x^T(s)Wx(s)\mathrm{d}s, \tag{3.10}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{t-\sigma_j}^{t}x^T(s)R_jx(s)\mathrm{d}s\leq\int_{t-\sigma}^{t}x^T(s)Rx(s)\mathrm{d}s, \tag{3.11}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{t-h_j(t)}^{t}\dot{x}^T(s)S_j\dot{x}(s)\mathrm{d}s\leq\int_{t-h}^{t}\dot{x}^T(s)S\dot{x}(s)\mathrm{d}s, \tag{3.12}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{-d_j(t)}^{0}\int_{t+\theta}^{t}\mathcal{H}^T(x(s))V_j\mathcal{H}(x(s))\mathrm{d}s\mathrm{d}\theta\leq-\int_{-d}^{0}\int_{t+\theta}^{t}\mathcal{H}^T(x(s))V\mathcal{H}(x(s))\mathrm{d}s\mathrm{d}\theta, \tag{3.13}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{-\tau_j}^{0}\int_{t+\theta}^{t}x^T(s)U_jx(s)\mathrm{d}s\mathrm{d}\theta\leq\int_{-\tau}^{0}\int_{t+\theta}^{t}x^T(s)Ux(s)\mathrm{d}s\mathrm{d}\theta, \tag{3.14}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{-\tau_j}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)X_j\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\leq\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)X\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, \tag{3.15}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{-h_j}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Y_j\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\leq\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Y\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, \tag{3.16}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{-\tau_j}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)T_j\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta\leq\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)T\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta, \tag{3.17}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{-h_j}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)Z_j\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta\leq\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)Z\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta, \tag{3.18}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{-\sigma_j}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)L_j\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta\leq\int_{-\sigma}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)L\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta. \tag{3.19}$$

By using Lemma 1, we get

$$-(1-d_{\mu_i})\int_{t-d_i(t)}^{t}\mathcal{H}^T(x(s))V_i\mathcal{H}(x(s))\mathrm{d}s \le -\frac{(1-d_{\mu_i})}{d_i}\int_{t-d_i(t)}^{t}\mathcal{H}^T(x(s))\mathrm{d}sV_i\int_{t-d_i(t)}^{t}\mathcal{H}(x(s))\mathrm{d}s,$$

$$-\int_{t-\tau_i}^{t}x^T(s)U_ix(s)\mathrm{d}s \le -\frac{1}{\tau_i}\int_{t-\tau_i}^{t}x^T(s)\mathrm{d}sU_i\int_{t-\tau_i}^{t}x(s)\mathrm{d}s,$$

$$-\int_{t-h_i}^{t}\dot{x}^T(s)Y_i\dot{x}(s)\mathrm{d}s \le -\frac{1}{h_i}\int_{t-h_i}^{t}\dot{x}^T(s)\mathrm{d}sY_i\int_{t-h_i}^{t}\dot{x}(s)\mathrm{d}s,$$

$$-\int_{-\tau_i}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)T_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \le -\frac{2}{\tau_i^2}\int_{-\tau_i}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\mathrm{d}s\mathrm{d}\theta T_i\int_{-\tau_i}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$-\int_{-h_i}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \le -\frac{2}{h_i^2}\int_{-h_i}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\mathrm{d}s\mathrm{d}\theta Z_i\int_{-h_i}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$-\int_{-\sigma_i}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \le -\frac{2}{\sigma_i^2}\int_{-\sigma_i}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\mathrm{d}s\mathrm{d}\theta Z_i\int_{-\sigma_i}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$\int_{-\sigma}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)M\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \le -\frac{2}{\sigma^2}\int_{-\sigma}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\mathrm{d}s\mathrm{d}\theta Z_i\int_{-\sigma}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta.$$

Note that from (3.1) and using the reciprocally convex technique in [34], we obtain

$$\begin{aligned}-\int_{t-\tau_i}^{t}\dot{x}^T(s)X_i\dot{x}(s)\mathrm{d}s &\le -\int_{t-\tau_i}^{t-\tau_i(t)}\dot{x}^T(s)X_i\dot{x}(s)\mathrm{d}s-\int_{t-\tau_i(t)}^{t}\dot{x}^T(s)X_i\dot{x}(s)\mathrm{d}s\\ &\le -\frac{1}{\tau_i}\varpi^T(t)\begin{bmatrix} R_i & X_i\\ * & R_i\end{bmatrix}\varpi(t),\end{aligned}$$

where $\varpi(t) = [x^T(t-\tau_i(t)) - x^T(t-\tau_i),\ x^T(t) - x^T(t-\tau_i(t))]$. For positive diagonal matrices $H_{1i}, H_{2i}, H_{3i}, H_{4i}, H_{5i}, H_{6i}$, by Assumption 1, we get

$$\begin{bmatrix} x(t)\\ \mathcal{H}(x(t))\end{bmatrix}^T\begin{bmatrix}\widehat{L}_1H_{1i} & -\widehat{L}_2H_{1i}\\ * & H_{1i}\end{bmatrix}\begin{bmatrix} x(t)\\ \mathcal{H}(x(t))\end{bmatrix}\le 0, \tag{3.20}$$

$$\begin{bmatrix} x(t-\tau_i(t))\\ \mathcal{H}(x(t-\tau_i(t)))\end{bmatrix}^T\begin{bmatrix}\widehat{L}_1H_{2i} & -\widehat{L}_2H_{2i}\\ * & H_{2i}\end{bmatrix}\begin{bmatrix} x(t-\tau_i(t))\\ \mathcal{H}(x(t-\tau_i(t)))\end{bmatrix}\le 0, \tag{3.21}$$

$$\begin{bmatrix} x(t)\\ x(t-\tau_i)\end{bmatrix}^T\begin{bmatrix}\widehat{L}_1H_{3i} & -\widehat{L}_2H_{3i}\\ * & H_{3i}\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-\tau_i)\end{bmatrix}\le 0, \tag{3.22}$$

$$\begin{bmatrix} x(t)\\ x(t-h_i)\end{bmatrix}^T\begin{bmatrix}\widehat{L}_1H_{4i} & -\widehat{L}_2H_{4i}\\ * & H_{4i}\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-h_i)\end{bmatrix}\le 0, \tag{3.23}$$

$$\begin{bmatrix} x(t)\\ x(t-\sigma_i)\end{bmatrix}^T\begin{bmatrix}\widehat{L}_1H_{5i} & -\widehat{L}_2H_{5i}\\ * & H_{5i}\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-\sigma_i)\end{bmatrix}\le 0, \tag{3.24}$$

$$\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} \widehat{L}_1 H_{6i} & -\widehat{L}_2 H_{6i} \\ * & H_{6i} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \leq 0. \tag{3.25}$$

Hence, for any matrices J_1, J_2 of appropriate dimensions, we get

$$\begin{aligned} 0 =& 2[x^T(t)J_1 + \dot{x}^T(t)J_2][-\mathcal{A}_i x(t-\sigma_i) + \mathcal{B}_i\mathcal{H}(x(t)) + C_i\mathcal{H}(x(t-\tau_i(t))) \\ &+ \mathcal{D}_i\dot{x}(t-h_i(t)) + \mathcal{E}_i\int_{t-d_i(t)}^{t}\mathcal{H}(x(s))\mathrm{d}s + u(t) - \dot{x}(t)]. \end{aligned} \tag{3.26}$$

Using (3.6) and adding (3.20)-(3.26), we have

$$\mathcal{L}V(x_t, i, t) - 2y^T(t)u(t) - \gamma u^T(t)u(t) \leq \mathcal{L}\Big\{\eta^T(t)\Phi\eta(t)\Big\}, \tag{3.27}$$

where

$$\begin{aligned} \eta^T(t) = & \Big[x^T(t) \;\; x^T(t-\tau_i(t)) \;\; x^T(t-\tau_i) \;\; \int_{t-\tau_i}^{t}x^T(s)\mathrm{d}s \;\; \mathcal{H}^T(x(t)) \;\; \mathcal{H}^T(x(t-\tau_i(t))) \\ & x^T(t-\sigma_i) \;\; \int_{t-\sigma_i}^{t}x^T(s)\mathrm{d}s \;\; \int_{t-\sigma}^{t}x^T(s)\mathrm{d}s \;\; u^T(t) \;\; \dot{x}^T(t-h_i(t)) \;\; x^T(t-h_i) \\ & \int_{t-h_i}^{t}x^T(s)\mathrm{d}s \;\; \int_{t-d_i(t)}^{t}\mathcal{H}^T(x(s))\mathrm{d}s \;\; \dot{x}^T(t)\Big]. \end{aligned}$$

Hence from equation (3.4) we have,

$$\mathcal{L}V(x_t, i, t) - 2y(t)^T u(t) - \gamma u(t)^T u(t) \leq 0.$$

Now, to show the passivity of the delayed NNs in (2.3), we take

$$J(t_p) = \mathbb{E}\left\{\int_0^{t_p}[-\gamma u(t)^T u(t) - 2y(t)^T u(t)]\mathrm{d}t\right\} \tag{3.28}$$

where $t_p \geq 0$.
From Dynkin's formula, we get

$$\mathbb{E}\left[\int_0^{t_p}\mathcal{L}V(x_t, i, t)\mathrm{d}t\right] = \mathbb{E}\left[V(x_{t_p}, i, t_p)\right] - \mathbb{E}\left[V(x_0, \nabla(0), 0)\right].$$

Therefore,

$$J(t_p) = \mathbb{E}\left\{\int_0^{t_p}[-\gamma u(t)^T u(t) - 2y(t)^T u(t) + \mathcal{L}V(x_t, i, t)]\mathrm{d}t\right\} - \mathbb{E}\left[\int_0^{t_p}\mathcal{L}V(x_t, i, t)\mathrm{d}t\right]$$

$$= \mathbb{E}\left\{\int_{0}^{t_p}[-\gamma u(t)^T u(t) - 2y(t)^T u(t) + \mathcal{L}V(x_t, i, t)]\mathrm{d}t\right\}$$
$$-\mathbb{E}\left[V(x_t, i, t)\right] + \mathbb{E}\left[V(x_0, \nabla(0), 0)\right]. \tag{3.29}$$

By applying lemma 3 to (3.4), we have

$$\Phi < 0. \tag{3.30}$$

Thus, if (3.30) holds, then $\mathbb{E}[V(x_{t_p}, i, t_p)] \geq 0$ and $V(x_0, \nabla(0), 0) = 0$ holds with zero initial conditions. From (3.30), it follows that $J(t_p) \leq 0$ for any $t_p \geq 0$, which implies (2.5) is satisfied and hence the delayed NNs (2.3) is locally passive.

Now, we prove the global passivity of the system.

By taking expectation of (3.27) and then integration from 0 to t we get,

$$\int_0^t \mathbb{E}[\mathcal{L}V(x_s, r(s), s)]\mathrm{d}s - 2\int_0^t \mathbb{E}[y^T(s)u(s)]\mathrm{d}s - \gamma\int_0^t \mathbb{E}[u^T(s)u(s)]\mathrm{d}s \leq \int_0^t \mathbb{E}[\eta^T(s)\Phi\eta(s)]\mathrm{d}s.$$

Then by Dynkin's formula,

$$\mathbb{E}[\mathcal{L}V(x_t, i, t)] - \mathbb{E}[\mathcal{L}V(x_0, \nabla(0), 0)] - 2\int_0^t \mathbb{E}[y^T(s)u(s)]\mathrm{d}s - \gamma\int_0^t \mathbb{E}[u^T(s)u(s)]\mathrm{d}s$$
$$\leq \int_0^t \mathbb{E}[\eta^T(s)\Phi\eta(s)]\mathrm{d}s.$$

Hence,

$$\mathbb{E}[\mathcal{L}V(x_t, i, t)] - \int_0^t \mathbb{E}[\eta^T(s)\Phi\eta(s)]\mathrm{d}s \leq \mathbb{E}[\mathcal{L}V(x_0, \nabla(0), 0)] + 2\int_0^t \mathbb{E}[y^T(s)u(s)]\mathrm{d}s$$
$$+ \gamma\int_0^t \mathbb{E}[u^T(s)u(s)]\mathrm{d}s$$
$$< \infty, \quad t \geq 0. \tag{3.31}$$

By Jenson's inequality and (3.6), we get

$$\begin{aligned}\mathbf{E}\left\|\mathcal{A}_i\int_{t-\sigma_i}^t x(s)\mathrm{d}s\right\|^2 &= \mathbf{E}\left[\mathcal{A}_i\int_{t-\sigma_i}^t x(s)\mathrm{d}s\right]^T\left[\mathcal{A}_i\int_{t-\sigma_i}^t x(s)\mathrm{d}s\right]\\ &\leq \lambda_{max}(\mathcal{A}_i^2)\mathbf{E}\left[\int_{t-\sigma_i}^t x(s)\mathrm{d}s\right]^T\left[\int_{t-\sigma_i}^t x(s)\mathrm{d}s\right]\end{aligned}$$

$$
\begin{aligned}
&\le \frac{\lambda_{max}(\mathcal{A}_i^2)}{\lambda_{min}(R_i)}\left[\int_{t-\sigma_i}^{t}\mathbf{E}x(s)\mathrm{d}s\right]^T R_i\left[\int_{t-\sigma_i}^{t}\mathbf{E}x(s)\mathrm{d}s\right]\\
&\le \sigma_i\frac{\lambda_{max}(\mathcal{A}_i^2)}{\lambda_{min}(R_i)}\left\{\int_{t-\sigma_i}^{t}\mathbf{E}x^T(s)R_ix(s)\mathrm{d}s\right\}\\
&\le \sigma_i\frac{\lambda_{max}(\mathcal{A}_i^2)}{\lambda_{min}(R_i)}\mathbf{E}V_4(x_t,i,t)\\
&\le \sigma\frac{\lambda_{max}(\mathcal{A}_i^2)}{\lambda_{min}(R_i)}\mathbf{E}V(x_t,i,t)\\
&\le \sigma\frac{\lambda_{max}(\mathcal{A}_i^2)}{\lambda_{min}(R_i)}\mathbf{E}V(x_0,\nabla(0),0),\quad t\ge 0. \qquad (3.32)
\end{aligned}
$$

Similarly, it follows from the definition of $V_1(x_t, i, t)$ that

$$
\begin{aligned}
\mathbf{E}\left\|x(t)-\mathcal{A}_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s\right\|^2 &= \mathbf{E}\left[\mathcal{A}_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s\right]^T\left[\mathcal{A}_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s\right]\\
&\le \frac{\mathbf{E}V_1(x_t,i,t)}{\lambda_{min}(P_i)}\\
&\le \frac{\mathbf{E}V(x_t,i,t)}{\lambda_{min}(P_i)}\\
&\le \frac{\mathbf{E}V(x_0,\nabla(0),0)}{\lambda_{min}(P_i)},\quad t\ge 0.
\end{aligned}
$$

Hence, it can be obtained that

$$
\begin{aligned}
\mathbf{E}\left\|x(t)\right\|^2 &= \mathbf{E}\left\|x(t)-\mathcal{A}_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s+\mathcal{A}_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s\right\|^2\\
&\le 2\mathbf{E}\left\|A_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s\right\|^2+2\mathbf{E}\left\|x(t)-\mathcal{A}_i\int_{t-\sigma_i}^{t}x(s)\mathrm{d}s\right\|^2\\
&\le 2\sigma\frac{\lambda_{max}(\mathcal{A}_i^2)}{\lambda_{min}(R_i)}\mathbf{E}V(x_0,\nabla(0),0)+2\frac{\mathbf{E}V(x_0,\nabla(0),0)}{\lambda_{min}(P_i)}<\infty,\quad t\ge 0, \qquad (3.33)
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{E}V(x_0,\nabla(0),0)\\
&\le \Big\{2\lambda_{\max_{i\in S}}(P_i)(1+\sigma_i^2\max_{i\in S}\mathcal{A}_i)+\tau\max_{i\in S}\{\lambda_{\max}(Q_i)\}+\tau^2\lambda_{max}(Q)+\tau\max_{i\in S}\{\lambda_{\max}(W_i)\}\\
&+\tau^2\lambda_{max}(W)+\sigma\max_{i\in S}\{\lambda_{\max}(R_i)\}+\sigma^2\lambda_{max}(R)+h\max_{i\in S}\{\lambda_{\max}(S_i)\}+h^2\lambda_{max}(S)\\
&+d^2\max_{i\in S}\{\lambda_{\max}(V_i)\}+d^3\lambda_{max}(V)+\tau^2\max_{i\in S}\{\lambda_{\max}(U_i)\}+\tau^3\lambda_{max}(U)+\tau^2\max_{i\in S}\{\lambda_{\max}(X_i)\}\\
&+\tau^3\lambda_{max}(X)+h^2\max_{i\in S}\{\lambda_{\max}(Y_i)\}+h^3\lambda_{max}(Y)+\tau^3\max_{i\in S}\{\lambda_{\max}(T_i)\}+\tau^4\lambda_{max}(T)\\
&+h^3\max_{i\in S}\{\lambda_{\max}(Z_i)\}+h^4\lambda_{max}(Z)+\sigma^3\max_{i\in S}\{\lambda_{\max}(L_i)\}+\sigma^4\lambda_{max}(L)+\sigma^5\lambda_{max}(M)\Big\}<\infty. \qquad (3.34)
\end{aligned}
$$

From (3.33) and (3.34), we get that the solution of the system (2.3) is locally passive. Then the solutions $x(t) = x(t, 0, \phi)$ of system (2.3) is bounded on $[0, \infty)$. The solution $x(t)$ on $[0, \infty)$ is uniformly continuous because $\dot{x}(t)$ is bounded on $[0, \infty)$. Further, from the equation (3.31), the following holds:

$$\begin{aligned}\lambda_{min}(\Phi)\int_0^t \mathbf{E}[x^T(s)x(s)]\mathrm{d}s &\leq \mathbb{E}[\mathcal{L}V(x_t, i, t)] - \int_0^t \mathbb{E}[\eta^T(s)\Phi\eta(s)]\mathrm{d}s \\ &\leq \mathbb{E}[\mathcal{L}V(x_0, \nabla(0), 0)] + 2\int_0^t \mathbb{E}[y^T(s)u(s)]\mathrm{d}s \\ &+ \gamma \int_0^t \mathbb{E}[u^T(s)u(s)]\mathrm{d}s \\ &< \infty, \quad t \geq 0.\end{aligned}$$

From Barbalats lemma [30], $\mathbb{E}[\|x(t)\|^2] \to 0$ as $t \to \infty$ holds. Hence the proof is complete. □

Remark 1. *Recently, studies on passivity analysis for neural networks of neutral type with Markovian jumping parameters and time delay in the leakage term were conducted in [5]. By constructing proper Lyapunov–Krasovskii functional, new delay-dependent passivity conditions are derived in terms of LMIs and it can easily be checked using standard numerical packages. Moreover, it is well known that the passivity behaviour of neural networks is very sensitive to the time delay in the leakage term. Triple and quadruple integrals have not been taken into account to derive the passivity conditions in [5]. Mode-dependent time delays were not included in [5]. Very recently, a mode-dependent approach is proposed by constructing a novel Lyapunov functional, where some terms involving triple or quadruple integrals are taken into account to study the state estimation problem in [22]. Motivated by this reason, we have introduced improved Lyapunov–Krasovskii functional with triple and quadruple integrals for deriving the reported stability results in this paper. Based on this discussion, our results will give less conservative results than those studied in [5, 22].*

4. Numerical example

In this section, a numerical example is provided to demonstrate the validity of the proposed theorems.

Example 1. *Consider a 2-D Mode-dependent Markov jump NNNs with mixed time-delays (2.3) with the following parameters*

$$\mathcal{A}_1 = \begin{bmatrix} 8.4 & 0 \\ 0 & 9 \end{bmatrix}, \ \mathcal{A}_2 = \begin{bmatrix} 7.8 & 0 \\ 0 & 8.5 \end{bmatrix}, \ \mathcal{B}_1 = \begin{bmatrix} -0.21 & -0.19 \\ -0.24 & 0.1 \end{bmatrix}, \ \mathcal{B}_2 = \begin{bmatrix} 0.9 & -0.9 \\ 0.5 & -0.8 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -0.09 & -0.2 \\ 0.2 & 0.1 \end{bmatrix}, \ C_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}, \ \mathcal{D}_1 = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.09 \end{bmatrix}, \ \mathcal{D}_2 = \begin{bmatrix} 0.1 & 0 \\ 0.5 & -0.1 \end{bmatrix},$$
$$\mathcal{E}_1 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, \ \mathcal{E}_2 = \begin{bmatrix} 0.1 & -0.02 \\ -0.2 & 0.07 \end{bmatrix}, \ \widehat{L}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \widehat{L}_2 = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}.$$

Take $\mathcal{H}_1(s) = \mathcal{H}_2(s) = \tanh(s)$, $\tau_1(t) = \tau_2(t) = h_1(t) = h_2(t) = d_1(t) = d_2(t) = 0.1\cos t + 0.4$,

$\sigma_1 = \sigma_2 = 0.1,\ \tau_{\mu_1} = \tau_{\mu_2} = h_{\mu_1} = h_{\mu_2} = d_{\mu_1} = d_{\mu_2} = 0.1.\ \Gamma = \begin{bmatrix} -7 & 7 \\ 6 & -6 \end{bmatrix}.$

By using the MATLAB LMI toolbox, we can obtain the following feasible solution for the LMIs (3.1) – (3.4)*:*

$$P_1 = \begin{bmatrix} 0.0157 & 0.0008 \\ 0.0008 & 0.0143 \end{bmatrix},\ P_2 = \begin{bmatrix} 0.0068 & 0.0004 \\ 0.0004 & 0.0060 \end{bmatrix},\ Q_{11} = \begin{bmatrix} 0.2401 & 0.0007 \\ 0.0007 & 0.2592 \end{bmatrix},$$

$$Q_{12} = \begin{bmatrix} 0.3770 & -0.0192 \\ -0.0192 & 0.3992 \end{bmatrix},\ Q_{21} = \begin{bmatrix} -0.4101 & 0.0043 \\ 0.0201 & -0.4457 \end{bmatrix},\ Q_{22} = \begin{bmatrix} -0.6428 & 0.0773 \\ 0.0231 & -0.6959 \end{bmatrix},$$

$$Q_{31} = \begin{bmatrix} 1.3786 & 0.0133 \\ 0.0133 & 1.4063 \end{bmatrix},\ Q_{32} = \begin{bmatrix} 1.9101 & -0.0284 \\ -0.0284 & 1.9903 \end{bmatrix},\ W_1 = \begin{bmatrix} 0.1039 & 0.0088 \\ 0.0088 & 0.1018 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 0.1517 & 0.0121 \\ 0.0121 & 0.1487 \end{bmatrix},\ R_1 = \begin{bmatrix} 0.1758 & 0.0160 \\ 0.0160 & 0.1721 \end{bmatrix},\ R_2 = \begin{bmatrix} 0.5675 & 0.0333 \\ 0.0333 & 0.5563 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 1.0817 & -0.0075 \\ -0.0075 & 0.7204 \end{bmatrix},\ S_2 = \begin{bmatrix} 1.2231 & 0.0093 \\ 0.0093 & 0.9256 \end{bmatrix},\ V_1 = \begin{bmatrix} 10.7113 & 0.5692 \\ 0.5692 & 9.9973 \end{bmatrix},$$

$$V_2 = \begin{bmatrix} 10.8483 & 0.5953 \\ 0.5953 & 10.2619 \end{bmatrix},\ U_1 = \begin{bmatrix} 0.5120 & 0.0419 \\ 0.0419 & 0.5056 \end{bmatrix},\ U_2 = \begin{bmatrix} 0.6238 & 0.0492 \\ 0.0492 & 0.6133 \end{bmatrix},$$

$$X_1 = \begin{bmatrix} 21.0470 & -0.1153 \\ -0.1153 & 21.2070 \end{bmatrix},\ X_2 = \begin{bmatrix} 18.5642 & -0.3139 \\ -0.3139 & 18.6620 \end{bmatrix},\ Y_1 = \begin{bmatrix} 27.5946 & -0.0019 \\ -0.0019 & 26.6557 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} 28.4115 & 0.1096 \\ 0.1096 & 27.4131 \end{bmatrix},\ T_1 = \begin{bmatrix} 3.0928 & 0.0943 \\ 0.0943 & 3.0505 \end{bmatrix},\ T_2 = \begin{bmatrix} 3.8182 & 0.0431 \\ 0.0431 & 3.7941 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 3.0867 & 0.1080 \\ 0.1080 & 3.0591 \end{bmatrix},\ Z_2 = \begin{bmatrix} 3.8647 & 0.0504 \\ 0.0504 & 3.8446 \end{bmatrix},\ L_1 = \begin{bmatrix} 0.0068 & 0.0001 \\ 0.0001 & 0.0068 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 0.0059 & 0.0004 \\ 0.0004 & 0.0062 \end{bmatrix},\ K_1 = \begin{bmatrix} 1.3461 & -0.1410 \\ -0.1410 & 1.3280 \end{bmatrix},\ K_2 = \begin{bmatrix} 0.0919 & 0.0135 \\ 0.0135 & 0.0907 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 4.0913 & -0.1850 \\ -0.1850 & 4.4495 \end{bmatrix},\ Q_2 = \begin{bmatrix} -6.6332 & 0.7686 \\ 0.3444 & -7.3948 \end{bmatrix},\ \gamma = 0.2272.$$

Continuing in this way, the remaining feasible matrices are obtained. This shows that the given system (2.3) *is globally passive in the mean square.*

5. Conclusion

In this paper, passivity analysis of Markovian jumping NNNs with time delays in the leakage term is considered. Delay-mode-dependent passivity conditions are derived by taking the inherent characteristic of such kinds of NNs into account. An improved LKF, with the triple integral terms and quadruple integrals, is constructed and the results are derived in terms of linear matrix inequalities. The information of the mode-dependent of all delays have been taken into account in the constructed LKF and derived novel stability criterion. Theoretical results are validated through a numerical example.

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Conflicts of interest

Availability of data and materials: Not applicable. On behalf of all authors, the corresponding author declares that they have no conflict of interest.

Author's contributions

All authors have equally and significantly contributed to the contents of this manuscript.

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