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*Research article*

# Forecasting volatility indices in stock and gold markets: Synergistic effects of the GARCH-MIDAS model and economic policy uncertainty

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## Supplementary

### Appendix A: Risk neutralization

We assume an exponential pricing kernel of the following form:

$$\zeta_{i+1,t} = \frac{M_{i+1,t}}{E_{i,t}[M_{i+1,t}]} \equiv \frac{\exp(\nu_{i,t}\varepsilon_{i+1,t})}{E_t[\exp(\nu_{i,t}\varepsilon_{t,i+1})]}$$
$$= \exp\left(\nu_{i,t}\varepsilon_{i+1,t} - \frac{1}{2}\nu_{i,t}^2\right)$$

Under the no-arbitrage condition

$$E_{i,t}^{Q}[\exp(R_{i+1,t})] = E_{i,t}[\zeta_{i+1,t}\exp(R_{i+1,t})] = \exp(r)$$

Since

$$E_{i,t}[\zeta_{i+1,t}\exp(R_{i+1,t})] = E_{i,t}\left[\zeta_{i+1,t}\exp\left(r-\frac{1}{2}h_{i+1,t}+\lambda\sqrt{h_{i+1,t}}+z_{i+1,t}\right)\right]$$
$$= E_{i,t}\left[\exp\left(\nu_{i,t}\varepsilon_{i+1,t}-\frac{1}{2}\nu_{i,t}^2+r-\frac{1}{2}h_{i+1,t}+\lambda\sqrt{h_{i+1,t}}+\sqrt{h_{i+1,t}}\varepsilon_{i+1,t}\right)\right]$$
$$= \exp\left(\frac{1}{2}(\nu_{i,t}+\sqrt{h_{i+1,t}})^2-\frac{1}{2}\nu_{i,t}^2+r-\frac{1}{2}h_{i+1,t}+\lambda\sqrt{h_{i+1,t}}\right)$$
$$= \exp\left(\nu_{i,t}\sqrt{h_{i+1,t}}+\lambda\sqrt{h_{i+1,t}}+r\right)$$

This implies that

$$\nu_{i,t}\sqrt{h_{i+1,t}}+\lambda\sqrt{h_{i+1,t}}=0$$

In order to determine the form of the risk-neutral distribution of the shocks, we consider the moment-generating function

$$E_{i,t}^{Q}[\exp(u\ \varepsilon_{i+1,t})]$$
$$=\exp\left(u\ \nu_{i+1,t}\right)$$

Under the Q measure, we have

$$\varepsilon_{i+1,t}^{*}=\varepsilon_{i+1,t}-\nu_{i+1,t},\ \varepsilon_{i+1,t}^{*}\overset{iid^{Q}}{\sim}N(0,1)$$

Using the no-arbitrage condition above implies that

$$\nu_{i+1,t}=-\lambda$$

Now we can re-write the returns equation under the risk-neutral measure and condition variance as follows:

$$R_{i+1,t}=r-\frac{1}{2}h_{i+1,t}+\lambda\sqrt{h_{i+1,t}}+\varepsilon_{i+1,t}\sqrt{h_{i+1,t}}$$
$$=r-\frac{1}{2}h_{i+1,t}+\sqrt{h_{i+1,t}}(\lambda+\varepsilon_{i+1,t})$$
$$=r-\frac{1}{2}h_{i+1,t}+\sqrt{h_{i+1,t}}\varepsilon_{i+1,t}^{*}$$

$$g_{i+1,t}=\omega+\beta g_{i,t}+\alpha g_{i,t}\varepsilon_{i,t}^{2}$$
$$=\omega+\beta g_{i,t}+\alpha g_{i,t}(\varepsilon_{i,t}^{*}-\lambda)^{2}.$$

**Appendix B: VIX calculation under the risk-neutral GARCH-MIDAS models**

Under the risk-neutral measure, we have:

$$\frac{1}{252}\left(\frac{VIX_{i,t}^{Model}}{100}\right)^2 \cong \frac{1}{22}\left[\sum_{\mathrm{k=1}}^{\mathrm{M_t\text{-}i}} \mathrm{E}_{\mathrm{i,t}}^{\mathrm{Q}}\left[h_{i+k,t}\right]+\sum_{m=1}^{22-(\mathrm{M_t\text{-}i})} \mathrm{E}_{\mathrm{i,t}}^{\mathrm{Q}}\left[h_{m,t+1}\right]\right]$$
$$=\frac{1}{22}\left[\sum_{\mathrm{k=1}}^{\mathrm{M_t\text{-}i}} \mathrm{E}_{\mathrm{i,t}}^{\mathrm{Q}}\left[\tau_{\mathrm{t}} g_{i+k,t}\right]+\sum_{m=1}^{22-(\mathrm{M_t\text{-}i})} \mathrm{E}_{\mathrm{i,t}}^{\mathrm{Q}}\left[\tau_{\mathrm{t+1}} g_{m,t+1}\right]\right]$$
$$=\frac{1}{22}\left[\sum_{\mathrm{k=0}}^{\mathrm{M_t\text{-}1\text{-}i}} \tau_{\mathrm{t}} \mathrm{E}_{\mathrm{i,t}}^{\mathrm{Q}}\left[g_{i+k+1,t}\right]+\sum_{m=0}^{21-(\mathrm{M_t\text{-}i})} \hat{\tau}_{\mathrm{t+1}} \mathrm{E}_{\mathrm{i,t}}^{\mathrm{Q}}\left[g_{m+1,t+1}\right]\right]$$

Since

$$\begin{aligned}
E_{i,t}^{Q}[g_{i+k+1,t}] &= E_{i,t}^{Q}E_{i+k,t}^{Q}[g_{i+k+1,t}] \\
&= E_{i,t}^{Q}[\omega+\beta g_{i+k,t}+\alpha g_{i+k,t}(\varepsilon_{i+k,t}^{*}-\lambda)^2] \\
&= \Omega^{*}+\Gamma^{*}E_{i,t}^{Q}[g_{i+k,t}]
\end{aligned}$$

$$\begin{aligned}
E_{i,t}^{Q}[g_{m+1,t+1}] &= E_{i,t}^{Q}E_{m,t}^{Q}[g_{m+1,t+1}] \\
&= E_{i,t}^{Q}[\omega+\beta g_{m,t+1}+\alpha g_{m,t+1}(\varepsilon_{m,t+1}^{*}-\lambda)^2] \\
&= \Omega^{*}+\Gamma^{*}E_{i,t}^{Q}[g_{m,t+1}]
\end{aligned}$$

We can deduce that

$$\begin{aligned}
\frac{1}{22}\sum_{k=1}^{M_t-i}\tau_t E_{i,t}^{Q}[g_{i+k,t}] &= \frac{1}{22}\sum_{k=0}^{M_t-1-i}\tau_t E_{i,t}^{Q}[g_{i+k+1,t}] \\
&= \frac{1}{22}\sum_{k=0}^{M_t-1-i}\tau_t\left\{\Omega^{*}+\Gamma^{*}E_{i,t}^{Q}[g_{i+k,t}]\right\} \\
&= \frac{1}{22}\sum_{k=0}^{M_t-1-i}\tau_t\left\{\Omega^{*}+\Gamma^{*}[\Omega^{*}+\Gamma^{*}E_{i,t}^{Q}[g_{i+k-1,t}]]\right\} \\
&= \frac{1}{22}\sum_{k=0}^{M_t-1-i}\tau_t\left\{\Omega^{*}(1+\Gamma^{*})+\Gamma^{*2}E_{i,t}^{Q}[g_{i+k-1,t}]\right\} \\
&= \frac{1}{22}\sum_{k=0}^{M_t-1-i}\tau_t\left\{\Omega^{*}(1+\Gamma^{*}+\cdots+\Gamma^{*k-1})+\Gamma^{*k}g_{i+1,t}\right\} \\
&= \frac{1}{22}\sum_{k=0}^{M_t-1-i}\tau_t\left\{\frac{\Omega^{*}(1-\Gamma^{*k})}{1-\Gamma^{*}}+\Gamma^{*k}g_{i+1,t}\right\} \\
&= \tau_t\left[\frac{\Omega^{*}}{1-\Gamma^{*}}\left(\frac{M_t-i}{22}-\frac{(1-\Gamma^{*M_t-i})}{22(1-\Gamma^{*})}\right)+\frac{(1-\Gamma^{*M_t-i})}{22(1-\Gamma^{*})}g_{i+1,t}\right] \\
&= \tau_t[A+Bg_{i+1,t}]
\end{aligned}$$

$$\frac{1}{22}\sum_{m=1}^{22-(M_t-i)}\hat{\tau}_{t+1}E_{i,t}^{Q}\left[g_{m,t+1}\right]=\frac{1}{22}\sum_{m=0}^{21-M_t+i}\hat{\tau}_{t+1}E_{i,t}^{Q}[g_{m+1,t+1}]$$
$$=\frac{1}{22}\sum_{m=0}^{21-M_t+i}\hat{\tau}_{t+1}\left\{\Omega^{*}+\Gamma^{*}E_{i,t}^{Q}[g_{m,t+1}]\right\}$$
$$=\frac{1}{22}\sum_{m=0}^{21-M_t+i}\hat{\tau}_{t+1}\left\{\Omega^{*}+\Gamma^{*}[\Omega^{*}+\Gamma^{*}E_{i,t}^{Q}[g_{m-1,t+1}]]\right\}$$
$$=\frac{1}{22}\sum_{m=0}^{21-M_t+i}\hat{\tau}_{t+1}\left\{\Omega^{*}(1+\Gamma^{*})+\Gamma^{*2}E_{i,t}^{Q}[g_{m-1,t+1}]\right\}$$
$$=\frac{1}{22}\sum_{m=0}^{21-M_t+i}\hat{\tau}_{t+1}\left\{\Omega^{*}(1+\Gamma^{*}+\cdots+\Gamma^{*M_t-1-i+m})+\Gamma^{*M_t-i+m}g_{i+1,t}\right\}$$
$$=\frac{1}{22}\sum_{m=0}^{21-M_t+i}\hat{\tau}_{t+1}\left\{\Omega^{*}\frac{(1-\Gamma^{*M_t-i+m})}{(1-\Gamma^{*})}+\Gamma^{*M_t-i+m}g_{i+1,t}\right\}$$
$$=\hat{\tau}_{t+1}\left[\frac{\Omega^{*}}{1-\Gamma^{*}}\left(\frac{22-M_t+i}{22}-\Gamma^{*M_t-i}\frac{(1-\Gamma^{*22-M_t+i})}{22(1-\Gamma^{*})}\right)+\Gamma^{*M_t-i}\frac{(1-\Gamma^{*22-M_t+i})}{22(1-\Gamma^{*})}g_{i+1,t}\right]$$
$$=\hat{\tau}_{t+1}[C+D\,g_{i+1,t}]$$

Finally, we get the expression for VIX as follows:

$$\frac{1}{252}\left(\frac{VIX_{i,t}^{Model}}{100}\right)^{2}\cong\frac{1}{22}\left[\sum_{\mathrm{k=1}}^{M_t\text{-i}}\mathrm{E}_{\mathrm{i,t}}^{\mathrm{Q}}[h_{i+k,t}]+\sum_{m=1}^{22-(M_t-i)}\mathrm{E}_{\mathrm{i,t}}^{\mathrm{Q}}[h_{m,t+1}]\right]$$
$$=\tau_{\mathrm{t}}\left[A+Bg_{i+1,t}\right]+\hat{\tau}_{\mathrm{t+1}}\left[C+Dg_{i+1,t}\right]$$

where $M_t$ denotes the number of days in the t month, $A$, $B$, $C$, and $D$ are functions of the parameters, satisfying $A=\frac{\Omega^{*}}{1-\Gamma^{*}}\left(\frac{M_t-i}{22}-B\right)$, $B=\frac{1-\Gamma^{*M_t-i}}{22(1-\Gamma^{*})}$, $C=\frac{\Omega^{*}}{1-\Gamma^{*}}\left(\frac{22-M_t+i}{22}-D\right)$, and $D=\Gamma^{*M_t-i}\frac{1-\Gamma^{*22-M_t+i}}{22(1-\Gamma^{*})}$, $\Omega^{*}>0,\Omega^{*}=\omega$. The persistence of the conditional variance satisfies $\Gamma^{*}=\beta+\alpha(1+\lambda^{2})<1$.