

Research article
Constant price input-output and productivity surplus
Davide Rognini^{1,*}, Patrizio Lecca^{1,2} and Jorge Díaz-Lanchas^{1,2}
¹ Department of Economics, Comillas Pontifical University, 23 Alberto Aguilera St., 28015 Madrid, Spain

² Institute for Research in Technology, Comillas Pontifical University, 4 Rey Francisco St., 28008 Madrid, Spain

* **Correspondence:** Email: drognini@comillas.edu.

Appendix A. Single deflation price indices
Table A.1. 2019 Price indices (base 100 = 2010).

| NACE Rev.2 | Description | Z | f(fd) | e | m | f(i) | k | l |
|------------|---|-------|-------|-------|-------|-------|-------|-------|
| A1 | Products of agriculture, hunting and related services | 121.4 | 113.4 | 121.4 | 100.0 | 108.1 | 107.6 | 110.3 |
| A2 | Products of forestry, logging and related services | 121.4 | 112.4 | 121.4 | 100.0 | 108.1 | 116.7 | 110.3 |
| A3 | Fish and other fishing products; aquaculture products; etc. | 121.4 | 113.2 | 121.4 | 100.0 | 108.1 | 103.2 | 110.3 |
| B5 | Mining and quarrying | 109.5 | 96.5 | 112.0 | 117.7 | 101.0 | 103.0 | 121.8 |
| C10-12 | Food products; beverages; tobacco products | 114.0 | 116.1 | 114.9 | 119.4 | 102.0 | 110.1 | 110.5 |
| C13-15 | Textiles; wearing apparel; leather and related products | 106.9 | 104.1 | 113.8 | 116.4 | 101.9 | 104.4 | 110.5 |
| C16 | Wood and of products of wood and cork, except furniture; etc. | 112.8 | 106.5 | 112.3 | 121.1 | 101.3 | 107.9 | 110.5 |
| C17 | Paper and paper products | 113.6 | 120.0 | 116.3 | 112.8 | 102.6 | 108.6 | 110.5 |
| C18 | Printing and recording services | 113.2 | 113.8 | 98.7 | 100.0 | 102.6 | 103.5 | 110.5 |
| C19 | Coke and refined petroleum products | 106.5 | 113.5 | 89.2 | 107.5 | 103.4 | 108.1 | 110.5 |
| C20 | Chemicals and chemical products | 110.9 | 105.8 | 114.7 | 114.6 | 103.9 | 105.7 | 110.5 |
| C21 | Basic pharmaceutical products and pharmaceutical preparations | 103.4 | 102.7 | 101.2 | 94.3 | 102.6 | 103.1 | 110.5 |
| C22 | Rubber and plastics products | 109.1 | 107.5 | 108.0 | 111.8 | 101.6 | 105.2 | 110.5 |
| C23 | Other non-metallic mineral products | 105.8 | 106.9 | 108.5 | 111.7 | 101.6 | 105.1 | 110.5 |
| C24 | Basic metals | 111.8 | 108.9 | 110.5 | 109.9 | 102.3 | 107.0 | 110.5 |
| C25 | Fabricated metal products, except machinery and equipment | 105.4 | 105.1 | 103.4 | 109.6 | 102.3 | 105.8 | 110.5 |
| C26 | Computer, electronic and optical products | 100.1 | 75.4 | 68.1 | 90.7 | 102.6 | 100.3 | 110.5 |
| C27 | Electrical equipment | 106.3 | 96.9 | 103.7 | 109.8 | 104.5 | 104.0 | 110.5 |
| C28 | Machinery and equipment n.e.c. | 107.3 | 98.0 | 108.4 | 111.7 | 104.3 | 104.1 | 110.5 |
| C29 | Motor vehicles, trailers and semi-trailers | 103.7 | 114.5 | 99.8 | 103.6 | 104.9 | 103.2 | 110.5 |
| C30 | Other transport equipment | 114.2 | 114.0 | 118.6 | 113.4 | 104.9 | 106.9 | 110.5 |
| C31-32 | Furniture; other manufactured goods | 108.0 | 106.0 | 106.4 | 109.2 | 102.6 | 103.4 | 110.5 |
| C33 | Repair and installation services of machinery and equipment | 103.3 | 76.6 | 104.6 | 100.0 | 102.6 | 104.0 | 110.5 |
| D35 | Electricity, gas, steam and air conditioning | 117.4 | 117.0 | 59.2 | 78.2 | 101.9 | 107.9 | 117.1 |
| E36 | Natural water; water treatment and supply services | 117.4 | 116.2 | 117.4 | 100.0 | 103.1 | 107.3 | 110.6 |
| E37-39 | Sewerage services; sewage sludge; waste collection, etc. | 109.4 | 115.8 | 109.4 | 100.0 | 102.2 | 105.5 | 110.3 |

| F | Constructions and construction works | 107.1 | 100.7 | 107.1 | 100.0 | 100.6 | 103.7 | 105.7 |
|--------|---|-------|-------|-------|-------|-------|-------|-------|
| G45 | Wholesale and retail trade and repair services of motor vehicles | 94.2 | 113.3 | 94.2 | 100.0 | 97.0 | 100.7 | 108.8 |
| G46 | Wholesale trade services, except of motor vehicles and motorcycles | 94.2 | 101.6 | 94.2 | 100.0 | 103.3 | 100.3 | 108.8 |
| G47 | Retail trade services, except of motor vehicles and motorcycles | 94.2 | 101.6 | 94.2 | 100.0 | 97.0 | 99.1 | 108.8 |
| H49 | Land transport services and transport services via pipelines | 106.0 | 116.5 | 106.0 | 100.0 | 97.0 | 101.1 | 107.6 |
| H50 | Water transport services | 94.2 | 116.9 | 94.2 | 100.0 | 97.0 | 99.2 | 107.6 |
| H51 | Air transport services | 110.4 | 116.9 | 110.4 | 100.0 | 97.0 | 100.0 | 107.6 |
| H52 | Warehousing and support services for transportation | 98.7 | 113.7 | 98.7 | 100.0 | 97.0 | 101.5 | 107.6 |
| H53 | Postal and courier services | 113.9 | 122.7 | 113.9 | 100.0 | 97.0 | 108.5 | 107.6 |
| I55-56 | Accommodation and food services | 94.2 | 110.8 | 94.2 | 100.0 | 102.0 | 104.7 | 112.0 |
| J58 | Publishing services | 94.2 | 101.1 | 94.2 | 100.0 | 101.2 | 98.9 | 109.1 |
| J59-60 | Motion picture, video and television production services, etc. | 94.2 | 101.4 | 94.2 | 100.0 | 101.2 | 96.4 | 109.1 |
| J61 | Telecommunications services | 60.7 | 68.1 | 60.7 | 100.0 | 102.0 | 90.2 | 109.1 |
| J62 | Computer programming, consultancy and related services; inf. services | 108.4 | 108.0 | 108.4 | 100.0 | 73.7 | 99.8 | 109.1 |
| K64 | Financial services, except insurance and pension funding | 94.2 | 113.8 | 94.2 | 100.0 | 73.7 | 96.8 | 109.9 |
| K65 | Insurance, pension funding services, except social security | 94.2 | 113.8 | 94.2 | 100.0 | 73.7 | 94.8 | 109.9 |
| K66 | Services auxiliary to financial services and insurance services | 94.2 | 113.8 | 94.2 | 100.0 | 73.7 | 96.1 | 109.9 |
| L68 | Real estate services | 94.2 | 104.5 | 94.2 | 100.0 | 86.5 | 101.3 | 114.6 |
| L68a | Imputed rents of owner-occupied dwellings | 94.2 | 104.5 | 94.2 | 100.0 | 97.0 | 100.3 | 114.6 |
| M69-70 | Legal and accounting services; services of head offices; etc. | 103.1 | 113.8 | 103.1 | 100.0 | 103.4 | 101.4 | 114.8 |
| M71 | Architectural and engineering services; technical analysis services | 102.9 | 113.4 | 102.9 | 100.0 | 104.3 | 102.1 | 114.8 |
| M72 | Scientific research and development services | 94.2 | 101.6 | 94.2 | 100.0 | 101.2 | 101.5 | 114.8 |
| M73 | Advertising and market research services | 92.9 | 113.2 | 92.9 | 100.0 | 101.0 | 96.7 | 114.8 |
| M74-75 | Other professional, scientific and technical services; etc. | 94.2 | 110.0 | 94.2 | 100.0 | 106.2 | 99.9 | 114.8 |
| N77 | Rental and leasing services | 94.2 | 104.0 | 94.2 | 100.0 | 102.2 | 99.1 | 114.8 |
| N78 | Employment services | 110.2 | 113.8 | 110.2 | 100.0 | 102.2 | 101.4 | 112.5 |
| N79 | Travel agency, tour operator and other reservation services | 94.2 | 122.5 | 94.2 | 100.0 | 102.2 | 96.8 | 112.5 |
| N80-82 | Security and investigation services; etc. | 103.7 | 115.3 | 103.7 | 100.0 | 102.2 | 103.3 | 112.5 |
| O84 | Public administration and defence services; etc. | 94.2 | 113.2 | 94.2 | 100.0 | 102.2 | 99.3 | 112.5 |
| P85 | Education services | 94.2 | 118.1 | 94.2 | 100.0 | 103.6 | 100.5 | 101.6 |
| Q86 | Human health services | 94.2 | 105.3 | 94.2 | 100.0 | 103.3 | 100.5 | 101.1 |
| Q87-88 | Residential care services; social work services without accommodation | 94.2 | 113.7 | 94.2 | 100.0 | 97.0 | 100.6 | 101.1 |
| R90-92 | Creative, arts and entertainment services; etc. | 94.2 | 107.3 | 94.2 | 100.0 | 102.5 | 97.0 | 114.0 |
| R93 | Sporting services and amusement and recreation services | 94.2 | 108.0 | 94.2 | 100.0 | 102.2 | 99.7 | 114.0 |
| S94 | Services furnished by membership organisations | 94.2 | 105.7 | 94.2 | 100.0 | 102.2 | 96.3 | 106.8 |
| S95 | Repair services of computers and personal and household goods | 94.2 | 98.3 | 94.2 | 100.0 | 102.2 | 98.4 | 106.8 |
| S96 | Other personal services | 94.2 | 105.7 | 94.2 | 100.0 | 99.5 | 99.8 | 106.8 |
| T97-98 | Services of households as employers; etc. | 94.2 | 108.9 | 94.2 | 100.0 | 97.0 | 100.0 | 106.8 |
| U99 | Services provided by extraterritorial organisations and bodies | 94.2 | 101.6 | 94.2 | 100.0 | 97.0 | 100.0 | 106.8 |

Legend: Z: production prices; f(fd): consumer prices; e: export prices; m: import prices; f(i): GCF deflator; k: capital deflator; l: labor prices.

Appendix B. Nominal and constant multipliers and TFPS results

Table B.2. Nominal and constant multipliers and TFPS (in million euros) by Nace Rev. 2. classification.

| NACE Rev.2 | Description | L | \bar{L}^e | \bar{L}^c | \bar{L} | TFPS |
|------------|---|------|-------------|-------------|-----------|---------|
| A1 | Products of agriculture, hunting and related services | 1.74 | 1.83 | 1.74 | 1.84 | -6017.6 |
| A2 | Products of forestry, logging and related services | 2.08 | 2.09 | 2.05 | 2.15 | -49.5 |
| A3 | Fish and other fishing products; aquaculture products; etc. | 1.61 | 1.68 | 1.61 | 1.71 | -395.7 |
| B5 | Mining and quarrying | 1.11 | 1.12 | 1.12 | 1.11 | 2264.5 |
| C10-12 | Food products; beverages; tobacco products | 2.21 | 2.30 | 2.24 | 2.29 | -4370.0 |
| C13-15 | Textiles; wearing apparel; leather and related products | 1.34 | 1.36 | 1.37 | 1.35 | 1443.0 |
| C16 | Wood and of products of wood and cork, except furniture; etc. | 2.16 | 2.23 | 2.20 | 2.23 | -116.4 |
| C17 | Paper and paper products | 2.05 | 2.15 | 2.08 | 2.12 | -887.2 |
| C18 | Printing and recording services | 2.01 | 2.12 | 2.05 | 2.12 | -327.2 |
| C19 | Coke and refined petroleum products | 1.91 | 1.86 | 1.92 | 1.90 | 2877.5 |
| C20 | Chemicals and chemical products | 1.78 | 1.84 | 1.83 | 1.83 | -1088.4 |
| C21 | Basic pharmaceutical products and pharmaceutical preparations | 1.45 | 1.45 | 1.44 | 1.46 | -1269.1 |
| C22 | Rubber and plastics products | 1.85 | 1.89 | 1.88 | 1.89 | -251.8 |

| | | | | | | |
|--------|---|------|------|------|------|---------|
| C23 | Other non-metallic mineral products | 2.03 | 2.05 | 2.06 | 2.04 | 31.7 |
| C24 | Basic metals | 2.00 | 2.06 | 2.02 | 2.06 | -1234.5 |
| C25 | Fabricated metal products, except machinery and equipment | 2.08 | 2.08 | 2.10 | 2.08 | 1010.7 |
| C26 | Computer, electronic and optical products | 1.16 | 1.14 | 1.15 | 1.16 | 1412.7 |
| C27 | Electrical equipment | 1.75 | 1.75 | 1.78 | 1.76 | 819.3 |
| C28 | Machinery and equipment n.e.c. | 1.64 | 1.66 | 1.67 | 1.66 | 723.3 |
| C29 | Motor vehicles, trailers and semi-trailers | 1.88 | 1.89 | 1.90 | 1.89 | 63.7 |
| C30 | Other transport equipment | 1.89 | 1.98 | 1.92 | 1.96 | -983.3 |
| C31-32 | Furniture; other manufactured goods | 1.53 | 1.55 | 1.56 | 1.56 | 110.4 |
| C33 | Repair and installation services of machinery and equipment | 2.06 | 2.06 | 2.09 | 2.06 | 613.5 |
| D35 | Electricity, gas, steam and air conditioning | 2.07 | 2.17 | 2.07 | 2.19 | -5094.4 |
| E36 | Natural water; water treatment and supply services | 1.95 | 2.06 | 1.97 | 2.06 | -770.2 |
| E37-39 | Sewerage services; sewage sludge; waste collection, etc. | 2.02 | 2.11 | 2.04 | 2.07 | -1162.5 |
| F | Constructions and construction works | 2.10 | 2.09 | 2.12 | 2.14 | 2310.6 |
| G45 | Wholesale and retail trade and repair services of motor vehicles | 1.81 | 1.84 | 1.84 | 1.75 | -497.8 |
| G46 | Wholesale trade services, except of motor vehicles and motorcycles | 1.84 | 1.81 | 1.87 | 1.78 | 8240.3 |
| G47 | Retail trade services, except of motor vehicles and motorcycles | 1.59 | 1.57 | 1.62 | 1.55 | 5694.1 |
| H49 | Land transport services and transport services via pipelines | 1.92 | 1.99 | 1.95 | 1.96 | -3150.4 |
| H50 | Water transport services | 2.34 | 2.33 | 2.37 | 2.26 | 19.7 |
| H51 | Air transport services | 2.25 | 2.40 | 2.28 | 2.37 | -1351.3 |
| H52 | Warehousing and support services for transportation | 1.97 | 1.97 | 1.99 | 1.94 | 1023.4 |
| H53 | Postal and courier services | 1.96 | 2.04 | 1.96 | 2.03 | -322.3 |
| I55-56 | Accommodation and food services | 1.79 | 1.82 | 1.81 | 1.71 | -1221.8 |
| J58 | Publishing services | 1.70 | 1.69 | 1.73 | 1.66 | 467.8 |
| J59-60 | Motion picture, video and television production services, etc. | 1.76 | 1.76 | 1.80 | 1.74 | 348.5 |
| J61 | Telecommunications services | 1.91 | 1.65 | 2.00 | 1.61 | 15180.8 |
| J62 | Computer programming, consultancy and related services; inf. services | 1.78 | 1.85 | 1.83 | 1.85 | -2168.8 |
| K64 | Financial services, except insurance and pension funding | 1.42 | 1.44 | 1.45 | 1.41 | 116.8 |
| K65 | Insurance, pension funding services, except social security | 1.94 | 2.06 | 1.99 | 1.92 | -1310.2 |
| K66 | Services auxiliary to financial services and insurance services | 1.71 | 1.74 | 1.75 | 1.69 | -202.4 |
| L68 | Real estate services | 1.40 | 1.39 | 1.41 | 1.37 | 3917 |
| L68a | Imputed rents of owner-occupied dwellings | 1.15 | 1.15 | 1.15 | 1.14 | -3477.4 |
| M69-70 | Legal and accounting services; services of head offices; etc. | 1.68 | 1.70 | 1.72 | 1.70 | 764.3 |
| M71 | Architectural and engineering services; technical analysis services | 2.00 | 2.02 | 2.06 | 2.01 | 679.1 |
| M72 | Scientific research and development services | 1.61 | 1.61 | 1.64 | 1.56 | 752.1 |
| M73 | Advertising and market research services | 1.89 | 1.86 | 1.95 | 1.85 | 1558.5 |
| M74-75 | Other professional, scientific and technical services; etc. | 1.74 | 1.71 | 1.78 | 1.70 | 1231.9 |
| N77 | Rental and leasing services | 1.69 | 1.66 | 1.71 | 1.65 | 1141.4 |
| N78 | Employment services | 1.19 | 1.21 | 1.21 | 1.21 | -50.6 |
| N79 | Travel agency, tour operator and other reservation services | 2.33 | 2.56 | 2.37 | 2.26 | -1811.0 |
| N80-82 | Security and investigation services; etc. | 1.70 | 1.71 | 1.73 | 1.70 | 1554.0 |
| O84 | Public administration and defence services; etc. | 1.40 | 1.46 | 1.45 | 1.38 | -3454.4 |
| P85 | Education services | 1.22 | 1.25 | 1.22 | 1.20 | -8343.0 |
| Q86 | Human health services | 1.48 | 1.50 | 1.49 | 1.45 | -2743.7 |
| Q87-88 | Residential care services; social work services without accommodation | 1.49 | 1.56 | 1.51 | 1.46 | -2773.7 |
| R90-92 | Creative, arts and entertainment services; etc. | 1.74 | 1.81 | 1.78 | 1.71 | -1070.2 |
| R93 | Sporting services and amusement and recreation services | 1.63 | 1.65 | 1.67 | 1.59 | 274.2 |
| S94 | Services furnished by membership organizations | 1.54 | 1.56 | 1.57 | 1.52 | 69.5 |
| S95 | Repair services of computers and personal and household goods | 1.81 | 1.80 | 1.85 | 1.77 | 167.5 |
| S96 | Other personal services | 1.37 | 1.39 | 1.38 | 1.35 | -401.4 |
| T97-98 | Services of households as employers; etc. | 1.00 | 1.00 | 1.00 | 1.00 | -176.9 |
| U99 | Services provided by extraterritorial organizations and bodies | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Appendix C. A 2-sector application of the Productivity Gains Distribution Matrix

C.1. Algebraic formalization

Let us consider a two-sector economy (2x2 matrix). Equation (7) can be specified as follows:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \bar{l}_{11}^c & \bar{l}_{12}^c \\ \bar{l}_{21}^c & \bar{l}_{22}^c \end{bmatrix} \right\}^{-1} \begin{bmatrix} \bar{l}_{11}^{rc} & \bar{l}_{12}^{rc} \\ \bar{l}_{21}^{rc} & \bar{l}_{22}^{rc} \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (C.1)$$

In order to remove the curly braces, we solve to obtain the following:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \frac{1 - \bar{l}_{22}^c}{S} & \frac{\bar{l}_{12}^c}{S} \\ \frac{\bar{l}_{21}^c}{S} & \frac{1 - \bar{l}_{11}^c}{S} \end{bmatrix} \begin{bmatrix} \bar{l}_{11}^c & \bar{l}_{12}^c \\ \bar{l}_{21}^c & \bar{l}_{22}^c \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (C.2)$$

where $S = (1 - \bar{l}_{11}^c)(1 - \bar{l}_{22}^c) - \bar{l}_{12}^c \bar{l}_{21}^c \leq 0$. S is the determinant of the resulting matrix and can take positive or negative values depending on the \bar{L}^c coefficients. By multiplying the matrices, we obtain the TFPS of each sector:

$$\begin{aligned} t_1 &= \left(\frac{1 - \bar{l}_{22}^c}{S} \bar{l}_{11}^c + \frac{\bar{l}_{12}^c}{S} \bar{l}_{21}^c \right) \bar{y}_1 + \left(\frac{1 - \bar{l}_{22}^c}{S} \bar{l}_{12}^c + \frac{\bar{l}_{12}^c}{S} \bar{l}_{22}^c \right) \bar{y}_2 \\ t_2 &= \left(\frac{1 - \bar{l}_{11}^c}{S} \bar{l}_{21}^c + \frac{\bar{l}_{21}^c}{S} \bar{l}_{11}^c \right) \bar{y}_1 + \left(\frac{1 - \bar{l}_{11}^c}{S} \bar{l}_{22}^c + \frac{\bar{l}_{21}^c}{S} \bar{l}_{12}^c \right) \bar{y}_2 \end{aligned} \quad (C.3)$$

Given this result, we can obtain the effect of a demand shock by deriving the TFPS of a sector in terms of y in both sectors.

$$\begin{aligned} \frac{\partial t_1}{\partial \bar{y}_1} &= \frac{1 - \bar{l}_{22}^c}{S} \bar{l}_{11}^c + \frac{\bar{l}_{12}^c}{S} \bar{l}_{21}^c \\ \frac{\partial t_1}{\partial \bar{y}_2} &= \frac{1 - \bar{l}_{22}^c}{S} \bar{l}_{12}^c + \frac{\bar{l}_{12}^c}{S} \bar{l}_{22}^c \end{aligned} \quad (C.4)$$

Adding the two derivatives together, we obtain the impact on the whole economy. In order to avoid sector-specific results, we generalize the equations for two sectors c and r where both indices can take values 1 and 2. This new specification gives two elements representing the following 4 options:

$$\begin{aligned} \frac{\partial t_i}{\partial \bar{y}_i} &= \frac{\partial t_1}{\partial \bar{y}_1} + \frac{\partial t_2}{\partial \bar{y}_2} \\ \frac{\partial t_i}{\partial \bar{y}_j} &= \frac{\partial t_1}{\partial \bar{y}_2} + \frac{\partial t_2}{\partial \bar{y}_1} \end{aligned} \quad (C.5)$$

Finally, we can then obtain the generalized effect of a demand shock for a two-sector economy by adding the two effects together:

$$\frac{\partial t_i}{\partial \bar{y}_i} + \frac{\partial t_i}{\partial \bar{y}_j} = \frac{1 - \bar{l}_{jj}^c}{S} \bar{l}_{ii}^c + \frac{\bar{l}_{ij}^c}{S} \bar{l}_{ji}^c + \frac{1 - \bar{l}_{ii}^c}{S} \bar{l}_{ji}^c + \frac{\bar{l}_{ji}^c}{S} \bar{l}_{ii}^c \quad (C.6)$$

Rearranging the factors, we obtain the final generalized effect of a demand shock:

$$\frac{\partial t_i}{\partial \bar{y}_i} + \frac{\partial t_i}{\partial \bar{y}_j} = \left[\frac{1 - \bar{l}_{jj}^c + \bar{l}_{ji}^c}{S} \bar{l}_{ii}^c + \frac{1 - \bar{l}_{ii}^c + \bar{l}_{ij}^c}{S} \bar{l}_{ji}^c \right] \quad (C.7)$$

C.2. Numerical application

For a better understanding of the algebra detailed above, let us consider a numerical 2-sector example, with the following I-O table:

Table C.1. Example of a nominal IO table.

| | S1 | S2 | Household consumption | Government expenditure | GCF | Exports | Output |
|--------------------------|--------|--------|-----------------------|------------------------|-------|---------|--------|
| S1 | 30,00 | 25,00 | 20,00 | 20,00 | 15,00 | 40,00 | 150 |
| S2 | 35,00 | 50,00 | 40,00 | 35,00 | 25,00 | 15,00 | 200,00 |
| Comp. of employees | 35,00 | 20,00 | | | | | |
| Net Operative Surplus | 20,00 | 45,00 | | | | | |
| Depreciation | 20,00 | 10,00 | | | | | |
| Indirect taxes subsidies | -10,00 | 25,00 | | | | | |
| Imports | 20,00 | 25,00 | | | | | |
| Output | 150,00 | 200,00 | | | | | |

Once deflated by single deflation with ad-hoc price indices, we obtain the following constant I-O table:

Table C.2. Example of a constant IO table.

| | S1 | S2 | Household consumption | Government expenditure | GCF | Exports | Output |
|--------------------------|--------|--------|-----------------------|------------------------|-------|---------|--------|
| S1 | 27,27 | 22,73 | 18,69 | 19,61 | 13,39 | 36,36 | 138,06 |
| S2 | 33,33 | 47,62 | 39,60 | 34,31 | 23,36 | 14,56 | 192,80 |
| Comp. of employees | 34,31 | 19,23 | | | | | |
| Net Operative Surplus | 19,23 | 42,06 | | | | | |
| Depreciation | 19,09 | 9,48 | | | | | |
| Indirect taxes subsidies | -9,55 | 23,69 | | | | | |
| Imports | 20,00 | 25,00 | | | | | |
| Output | 143,70 | 189,80 | | | | | |

According to the definition of TFP, the disequilibrium between output and input evaluated in constant prices for each sector is the source of productivity gains:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} -5, 64 \\ 2, 99 \end{bmatrix} \quad (C.8)$$

With these data we can now obtain the two new sets of technical coefficients and constant multipliers:

$$\bar{A}^r = \begin{bmatrix} 0, 20 & 0, 12 \\ 0, 24 & 0, 25 \end{bmatrix}; \bar{A}^c = \begin{bmatrix} 0, 19 & 0, 12 \\ 0, 23 & 0, 25 \end{bmatrix} \quad (C.9)$$

$$\bar{L}^r = \begin{bmatrix} 1, 31 & 0, 20 \\ 0, 42 & 1, 39 \end{bmatrix}; \bar{L}^c = \begin{bmatrix} 1, 29 & 0, 21 \\ 0, 40 & 1, 40 \end{bmatrix} \quad (C.10)$$

Let us now specify Equation (7) applying the data provided by Table C.2:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1, 29 & 0, 21 \\ 0, 40 & 1, 40 \end{bmatrix} \right\}^{-1} \begin{bmatrix} 0, 014 & -0, 002 \\ 0, 019 & -0, 005 \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (C.11)$$

We solve the algebra to remove the curly braces:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} -11, 645 & 6, 035 \\ 11, 692 & -8, 566 \end{bmatrix} \begin{bmatrix} 0, 014 & -0, 002 \\ 0, 019 & -0, 005 \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (C.12)$$

By multiplying the results matrices, we obtain the productivity gains distribution matrix:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} -0, 053 & -0, 008 \\ 0, 007 & 0, 022 \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (C.13)$$

Finally, we shock the economy by the constant final demand¹ and observe that the productivity gains match those obtained from Equation (C.8):

$$\begin{bmatrix} -5, 64 \\ 2, 99 \end{bmatrix} = \begin{bmatrix} -0, 053 & -0, 008 \\ 0, 007 & 0, 022 \end{bmatrix} \begin{bmatrix} 88, 06 \\ 111, 85 \end{bmatrix} \quad (\text{C.14})$$

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} -5, 64 \\ 2, 99 \end{bmatrix} \quad (\text{C.15})$$



© 2025 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (<https://creativecommons.org/licenses/by/4.0>)

¹Sum of household consumption, government expenditure, GCF and exports of the constant I-O table.