

Research article

Constant price input-output and productivity surplus

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Appendix A. Single deflation price indices

Table A.1. 2019 Price indices (base 100 = 2010).

NACE Rev.2	Description	Z	f(fd)	e	m	f(i)	k	l
A1	Products of agriculture, hunting and related services	121.4	113.4	121.4	100.0	108.1	107.6	110.3
A2	Products of forestry, logging and related services	121.4	112.4	121.4	100.0	108.1	116.7	110.3
A3	Fish and other fishing products; aquaculture products; etc.	121.4	113.2	121.4	100.0	108.1	103.2	110.3
B5	Mining and quarrying	109.5	96.5	112.0	117.7	101.0	103.0	121.8
C10-12	Food products; beverages; tobacco products	114.0	116.1	114.9	119.4	102.0	110.1	110.5
C13-15	Textiles; wearing apparel; leather and related products	106.9	104.1	113.8	116.4	101.9	104.4	110.5
C16	Wood and of products of wood and cork, except furniture; etc.	112.8	106.5	112.3	121.1	101.3	107.9	110.5
C17	Paper and paper products	113.6	120.0	116.3	112.8	102.6	108.6	110.5
C18	Printing and recording services	113.2	113.8	98.7	100.0	102.6	103.5	110.5
C19	Coke and refined petroleum products	106.5	113.5	89.2	107.5	103.4	108.1	110.5
C20	Chemicals and chemical products	110.9	105.8	114.7	114.6	103.9	105.7	110.5
C21	Basic pharmaceutical products and pharmaceutical preparations	103.4	102.7	101.2	94.3	102.6	103.1	110.5
C22	Rubber and plastics products	109.1	107.5	108.0	111.8	101.6	105.2	110.5
C23	Other non-metallic mineral products	105.8	106.9	108.5	111.7	101.6	105.1	110.5
C24	Basic metals	111.8	108.9	110.5	109.9	102.3	107.0	110.5
C25	Fabricated metal products, except machinery and equipment	105.4	105.1	103.4	109.6	102.3	105.8	110.5
C26	Computer, electronic and optical products	100.1	75.4	68.1	90.7	102.6	100.3	110.5
C27	Electrical equipment	106.3	96.9	103.7	109.8	104.5	104.0	110.5
C28	Machinery and equipment n.e.c.	107.3	98.0	108.4	111.7	104.3	104.1	110.5
C29	Motor vehicles, trailers and semi-trailers	103.7	114.5	99.8	103.6	104.9	103.2	110.5
C30	Other transport equipment	114.2	114.0	118.6	113.4	104.9	106.9	110.5
C31-32	Furniture; other manufactured goods	108.0	106.0	106.4	109.2	102.6	103.4	110.5
C33	Repair and installation services of machinery and equipment	103.3	76.6	104.6	100.0	102.6	104.0	110.5
D35	Electricity, gas, steam and air conditioning	117.4	117.0	59.2	78.2	101.9	107.9	117.1
E36	Natural water; water treatment and supply services	117.4	116.2	117.4	100.0	103.1	107.3	110.6
E37-39	Sewerage services; sewage sludge; waste collection, etc.	109.4	115.8	109.4	100.0	102.2	105.5	110.3

F	Constructions and construction works	107.1	100.7	107.1	100.0	100.6	103.7	105.7
G45	Wholesale and retail trade and repair services of motor vehicles	94.2	113.3	94.2	100.0	97.0	100.7	108.8
G46	Wholesale trade services, except of motor vehicles and motorcycles	94.2	101.6	94.2	100.0	103.3	100.3	108.8
G47	Retail trade services, except of motor vehicles and motorcycles	94.2	101.6	94.2	100.0	97.0	99.1	108.8
H49	Land transport services and transport services via pipelines	106.0	116.5	106.0	100.0	97.0	101.1	107.6
H50	Water transport services	94.2	116.9	94.2	100.0	97.0	99.2	107.6
H51	Air transport services	110.4	116.9	110.4	100.0	97.0	100.0	107.6
H52	Warehousing and support services for transportation	98.7	113.7	98.7	100.0	97.0	101.5	107.6
H53	Postal and courier services	113.9	122.7	113.9	100.0	97.0	108.5	107.6
I55-56	Accommodation and food services	94.2	110.8	94.2	100.0	102.0	104.7	112.0
J58	Publishing services	94.2	101.1	94.2	100.0	101.2	98.9	109.1
J59-60	Motion picture, video and television production services, etc.	94.2	101.4	94.2	100.0	101.2	96.4	109.1
J61	Telecommunications services	60.7	68.1	60.7	100.0	102.0	90.2	109.1
J62	Computer programming, consultancy and related services; inf. services	108.4	108.0	108.4	100.0	73.7	99.8	109.1
K64	Financial services, except insurance and pension funding	94.2	113.8	94.2	100.0	73.7	96.8	109.9
K65	Insurance, pension funding services, except social security	94.2	113.8	94.2	100.0	73.7	94.8	109.9
K66	Services auxiliary to financial services and insurance services	94.2	113.8	94.2	100.0	73.7	96.1	109.9
L68	Real estate services	94.2	104.5	94.2	100.0	86.5	101.3	114.6
L68a	Imputed rents of owner-occupied dwellings	94.2	104.5	94.2	100.0	97.0	100.3	114.6
M69-70	Legal and accounting services; services of head offices; etc.	103.1	113.8	103.1	100.0	103.4	101.4	114.8
M71	Architectural and engineering services; technical analysis services	102.9	113.4	102.9	100.0	104.3	102.1	114.8
M72	Scientific research and development services	94.2	101.6	94.2	100.0	101.2	101.5	114.8
M73	Advertising and market research services	92.9	113.2	92.9	100.0	101.0	96.7	114.8
M74-75	Other professional, scientific and technical services; etc.	94.2	110.0	94.2	100.0	106.2	99.9	114.8
N77	Rental and leasing services	94.2	104.0	94.2	100.0	102.2	99.1	114.8
N78	Employment services	110.2	113.8	110.2	100.0	102.2	101.4	112.5
N79	Travel agency, tour operator and other reservation services	94.2	122.5	94.2	100.0	102.2	96.8	112.5
N80-82	Security and investigation services; etc.	103.7	115.3	103.7	100.0	102.2	103.3	112.5
O84	Public administration and defence services; etc.	94.2	113.2	94.2	100.0	102.2	99.3	112.5
P85	Education services	94.2	118.1	94.2	100.0	103.6	100.5	101.6
Q86	Human health services	94.2	105.3	94.2	100.0	103.3	100.5	101.1
Q87-88	Residential care services; social work services without accommodation	94.2	113.7	94.2	100.0	97.0	100.6	101.1
R90-92	Creative, arts and entertainment services; etc.	94.2	107.3	94.2	100.0	102.5	97.0	114.0
R93	Sporting services and amusement and recreation services	94.2	108.0	94.2	100.0	102.2	99.7	114.0
S94	Services furnished by membership organisations	94.2	105.7	94.2	100.0	102.2	96.3	106.8
S95	Repair services of computers and personal and household goods	94.2	98.3	94.2	100.0	102.2	98.4	106.8
S96	Other personal services	94.2	105.7	94.2	100.0	99.5	99.8	106.8
T97-98	Services of households as employers; etc.	94.2	108.9	94.2	100.0	97.0	100.0	106.8
U99	Services provided by extraterritorial organisations and bodies	94.2	101.6	94.2	100.0	97.0	100.0	106.8

Legend: Z: production prices; f(fd): consumer prices; e: export prices; m: import prices; f(i): GCF deflator; k: capital deflator; l: labor prices.

Appendix B. Nominal and constant multipliers and TFPS results

Table B.2. Nominal and constant multipliers and TFPS (in million euros) by Nace Rev. 2. classification.

NACE Rev.2	Description	L	L ^f	L ^e	L ^m	TFPS
A1	Products of agriculture, hunting and related services	1.74	1.83	1.74	1.84	-6017.6
A2	Products of forestry, logging and related services	2.08	2.09	2.05	2.15	-49.5
A3	Fish and other fishing products; aquaculture products; etc.	1.61	1.68	1.61	1.71	-395.7
B5	Mining and quarrying	1.11	1.12	1.12	1.11	2264.5
C10-12	Food products; beverages; tobacco products	2.21	2.30	2.24	2.29	-4370.0
C13-15	Textiles; wearing apparel; leather and related products	1.34	1.36	1.37	1.35	1443.0
C16	Wood and of products of wood and cork, except furniture; etc.	2.16	2.23	2.20	2.23	-116.4
C17	Paper and paper products	2.05	2.15	2.08	2.12	-887.2
C18	Printing and recording services	2.01	2.12	2.05	2.12	-327.2
C19	Coke and refined petroleum products	1.91	1.86	1.92	1.90	2877.5
C20	Chemicals and chemical products	1.78	1.84	1.83	1.83	-1088.4
C21	Basic pharmaceutical products and pharmaceutical preparations	1.45	1.45	1.44	1.46	-1269.1
C22	Rubber and plastics products	1.85	1.89	1.88	1.89	-251.8

C23	Other non-metallic mineral products	2.03	2.05	2.06	2.04	31.7
C24	Basic metals	2.00	2.06	2.02	2.06	-1234.5
C25	Fabricated metal products, except machinery and equipment	2.08	2.08	2.10	2.08	1010.7
C26	Computer, electronic and optical products	1.16	1.14	1.15	1.16	1412.7
C27	Electrical equipment	1.75	1.75	1.78	1.76	819.3
C28	Machinery and equipment n.e.c.	1.64	1.66	1.67	1.66	723.3
C29	Motor vehicles, trailers and semi-trailers	1.88	1.89	1.90	1.89	63.7
C30	Other transport equipment	1.89	1.98	1.92	1.96	-983.3
C31-32	Furniture; other manufactured goods	1.53	1.55	1.56	1.56	110.4
C33	Repair and installation services of machinery and equipment	2.06	2.06	2.09	2.06	613.5
D35	Electricity, gas, steam and air conditioning	2.07	2.17	2.07	2.19	-5094.4
E36	Natural water; water treatment and supply services	1.95	2.06	1.97	2.06	-770.2
E37-39	Sewerage services; sewage sludge; waste collection, etc.	2.02	2.11	2.04	2.07	-1162.5
F	Constructions and construction works	2.10	2.09	2.12	2.14	2310.6
G45	Wholesale and retail trade and repair services of motor vehicles	1.81	1.84	1.84	1.75	-497.8
G46	Wholesale trade services, except of motor vehicles and motorcycles	1.84	1.81	1.87	1.78	8240.3
G47	Retail trade services, except of motor vehicles and motorcycles	1.59	1.57	1.62	1.55	5694.1
H49	Land transport services and transport services via pipelines	1.92	1.99	1.95	1.96	-3150.4
H50	Water transport services	2.34	2.33	2.37	2.26	19.7
H51	Air transport services	2.25	2.40	2.28	2.37	-1351.3
H52	Warehousing and support services for transportation	1.97	1.97	1.99	1.94	1023.4
H53	Postal and courier services	1.96	2.04	1.96	2.03	-322.3
I55-56	Accommodation and food services	1.79	1.82	1.81	1.71	-1221.8
J58	Publishing services	1.70	1.69	1.73	1.66	467.8
J59-60	Motion picture, video and television production services, etc.	1.76	1.76	1.80	1.74	348.5
J61	Telecommunications services	1.91	1.65	2.00	1.61	15180.8
J62	Computer programming, consultancy and related services; inf. services	1.78	1.85	1.83	1.85	-2168.8
K64	Financial services, except insurance and pension funding	1.42	1.44	1.45	1.41	116.8
K65	Insurance, pension funding services, except social security	1.94	2.06	1.99	1.92	-1310.2
K66	Services auxiliary to financial services and insurance services	1.71	1.74	1.75	1.69	-202.4
L68	Real estate services	1.40	1.39	1.41	1.37	3917
L68a	Imputed rents of owner-occupied dwellings	1.15	1.15	1.15	1.14	-3477.4
M69-70	Legal and accounting services; services of head offices; etc.	1.68	1.70	1.72	1.70	764.3
M71	Architectural and engineering services; technical analysis services	2.00	2.02	2.06	2.01	679.1
M72	Scientific research and development services	1.61	1.61	1.64	1.56	752.1
M73	Advertising and market research services	1.89	1.86	1.95	1.85	1558.5
M74-75	Other professional, scientific and technical services; etc.	1.74	1.71	1.78	1.70	1231.9
N77	Rental and leasing services	1.69	1.66	1.71	1.65	1141.4
N78	Employment services	1.19	1.21	1.21	1.21	-50.6
N79	Travel agency, tour operator and other reservation services	2.33	2.56	2.37	2.26	-1811.0
N80-82	Security and investigation services; etc.	1.70	1.71	1.73	1.70	1554.0
O84	Public administration and defence services; etc.	1.40	1.46	1.45	1.38	-3454.4
P85	Education services	1.22	1.25	1.22	1.20	-8343.0
Q86	Human health services	1.48	1.50	1.49	1.45	-2743.7
Q87-88	Residential care services; social work services without accommodation	1.49	1.56	1.51	1.46	-2773.7
R90-92	Creative, arts and entertainment services; etc.	1.74	1.81	1.78	1.71	-1070.2
R93	Sporting services and amusement and recreation services	1.63	1.65	1.67	1.59	274.2
S94	Services furnished by membership organizations	1.54	1.56	1.57	1.52	69.5
S95	Repair services of computers and personal and household goods	1.81	1.80	1.85	1.77	167.5
S96	Other personal services	1.37	1.39	1.38	1.35	-401.4
T97-98	Services of households as employers; etc.	1.00	1.00	1.00	1.00	-176.9
U99	Services provided by extraterritorial organizations and bodies	0.00	0.00	0.00	0.00	0.00

Appendix C. A 2-sector application of the Productivity Gains Distribution Matrix

C.1. Algebraic formalization

Let us consider a two-sector economy (2x2 matrix). Equation (7) can be specified as follows:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \bar{l}_{11}^c & \bar{l}_{12}^c \\ \bar{l}_{21}^c & \bar{l}_{22}^c \end{bmatrix} \right\}^{-1} \begin{bmatrix} \bar{l}_{11}^c & \bar{l}_{12}^c \\ \bar{l}_{21}^c & \bar{l}_{22}^c \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (C.1)$$

In order to remove the curly braces, we solve to obtain the following:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \frac{1-\bar{l}_{22}^c}{S} & \frac{\bar{l}_{12}^c}{S} \\ \frac{\bar{l}_{21}^c}{S} & \frac{1-\bar{l}_{11}^c}{S} \end{bmatrix} \begin{bmatrix} \bar{l}_{11}^{rc} & \bar{l}_{12}^{rc} \\ \bar{l}_{21}^{rc} & \bar{l}_{22}^{rc} \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (C.2)$$

where $S = (1 - \bar{l}_{11}^c)(1 - \bar{l}_{22}^c) - \bar{l}_{12}^c - \bar{l}_{21}^c \leq 0$. S is the determinant of the resulting matrix and can take positive or negative values depending on the \bar{l}^c coefficients. By multiplying the matrices, we obtain the TFPS of each sector:

$$\begin{aligned} t_1 &= \left(\frac{1 - \bar{l}_{22}^c}{S} \bar{l}_{11}^{rc} + \frac{\bar{l}_{12}^c}{S} \bar{l}_{21}^{rc} \right) \bar{y}_1 + \left(\frac{1 - \bar{l}_{22}^c}{S} \bar{l}_{12}^{rc} + \frac{\bar{l}_{12}^c}{S} \bar{l}_{22}^{rc} \right) \bar{y}_2 \\ t_2 &= \left(\frac{1 - \bar{l}_{11}^c}{S} \bar{l}_{21}^{rc} + \frac{\bar{l}_{21}^c}{S} \bar{l}_{11}^{rc} \right) \bar{y}_1 + \left(\frac{1 - \bar{l}_{11}^c}{S} \bar{l}_{22}^{rc} + \frac{\bar{l}_{21}^c}{S} \bar{l}_{12}^{rc} \right) \bar{y}_2 \end{aligned} \quad (C.3)$$

Given this result, we can obtain the effect of a demand shock by deriving the TFPS of a sector in terms of y in both sectors.

$$\begin{aligned} \frac{\partial t_1}{\partial \bar{y}_1} &= \frac{1 - \bar{l}_{22}^c}{S} \bar{l}_{11}^{rc} + \frac{\bar{l}_{12}^c}{S} \bar{l}_{21}^{rc} \\ \frac{\partial t_1}{\partial \bar{y}_2} &= \frac{1 - \bar{l}_{22}^c}{S} \bar{l}_{12}^{rc} + \frac{\bar{l}_{12}^c}{S} \bar{l}_{22}^{rc} \end{aligned} \quad (C.4)$$

Adding the two derivatives together, we obtain the impact on the whole economy. In order to avoid sector-specific results, we generalize the equations for two sectors c and r where both indices can take values 1 and 2. This new specification gives two elements representing the following 4 options:

$$\begin{aligned} \frac{\partial t_i}{\partial \bar{y}_i} &= \frac{\partial t_1}{\partial \bar{y}_1} + \frac{\partial t_2}{\partial \bar{y}_2} \\ \frac{\partial t_i}{\partial \bar{y}_j} &= \frac{\partial t_1}{\partial \bar{y}_2} + \frac{\partial t_2}{\partial \bar{y}_1} \end{aligned} \quad (C.5)$$

Finally, we can then obtain the generalized effect of a demand shock for a two-sector economy by adding the two effects together:

$$\frac{\partial t_i}{\partial \bar{y}_i} + \frac{\partial t_i}{\partial \bar{y}_j} = \frac{1 - \bar{l}_{jj}^c}{S} \bar{l}_{ii}^{rc} + \frac{\bar{l}_{ij}^c}{S} \bar{l}_{ji}^{rc} + \frac{1 - \bar{l}_{ii}^c}{S} \bar{l}_{ji}^{rc} + \frac{\bar{l}_{ji}^c}{S} \bar{l}_{ii}^{rc} \quad (C.6)$$

Rearranging the factors, we obtain the final generalized effect of a demand shock:

$$\frac{\partial t_i}{\partial \bar{y}_i} + \frac{\partial t_i}{\partial \bar{y}_j} = \left[\frac{1 - \bar{l}_{jj}^c + \bar{l}_{ji}^c}{S} \bar{l}_{ii}^{rc} + \frac{1 - \bar{l}_{ii}^c + \bar{l}_{ij}^c}{S} \bar{l}_{ji}^{rc} \right] \quad (C.7)$$

C.2. Numerical application

For a better understanding of the algebra detailed above, let us consider a numerical 2-sector example, with the following I-O table:

Table C.1. Example of a nominal IO table.

	S1	S2	Household consumption	Government expenditure	GCF	Exports	Output
S1	30,00	25,00	20,00	20,00	15,00	40,00	150
S2	35,00	50,00	40,00	35,00	25,00	15,00	200,00
Comp. of employees	35,00	20,00					
Net Operative Surplus	20,00	45,00					
Depreciation	20,00	10,00					
Indirect taxes subsidies	-10,00	25,00					
Imports	20,00	25,00					
Output	150,00	200,00					

Once deflated by single deflation with ad-hoc price indices, we obtain the following constant I-O table:

Table C.2. Example of a constant IO table.

	S1	S2	Household consumption	Government expenditure	GCF	Exports	Output
S1	27,27	22,73	18,69	19,61	13,39	36,36	138,06
S2	33,33	47,62	39,60	34,31	23,36	14,56	192,80
Comp. of employees	34,31	19,23					
Net Operative Surplus	19,23	42,06					
Depreciation	19,09	9,48					
Indirect taxes subsidies	-9,55	23,69					
Imports	20,00	25,00					
Output	143,70	189,80					

According to the definition of TFP, the disequilibrium between output and input evaluated in constant prices for each sector is the source of productivity gains:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} -5,64 \\ 2,99 \end{bmatrix} \quad (\text{C.8})$$

With these data we can now obtain the two new sets of technical coefficients and constant multipliers:

$$\overline{A}^r = \begin{bmatrix} 0,20 & 0,12 \\ 0,24 & 0,25 \end{bmatrix}; \overline{A}^c = \begin{bmatrix} 0,19 & 0,12 \\ 0,23 & 0,25 \end{bmatrix} \quad (\text{C.9})$$

$$\overline{L}^r = \begin{bmatrix} 1,31 & 0,20 \\ 0,42 & 1,39 \end{bmatrix}; \overline{L}^c = \begin{bmatrix} 1,29 & 0,21 \\ 0,40 & 1,40 \end{bmatrix} \quad (\text{C.10})$$

Let us now specify Equation (7) applying the data provided by Table C.2:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1,29 & 0,21 \\ 0,40 & 1,40 \end{bmatrix} \right\}^{-1} \begin{bmatrix} 0,014 & -0,002 \\ 0,019 & -0,005 \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (\text{C.11})$$

We solve the algebra to remove the curly braces:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} -11,645 & 6,035 \\ 11,692 & -8,566 \end{bmatrix} \begin{bmatrix} 0,014 & -0,002 \\ 0,019 & -0,005 \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (\text{C.12})$$

By multiplying the results matrices, we obtain the productivity gains distribution matrix:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} -0,053 & -0,008 \\ 0,007 & 0,022 \end{bmatrix} \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} \quad (\text{C.13})$$

Finally, we shock the economy by the constant final demand¹ and observe that the productivity gains match those obtained from Equation (C.8):

$$\begin{bmatrix} -5,64 \\ 2,99 \end{bmatrix} = \begin{bmatrix} -0,053 & -0,008 \\ 0,007 & 0,022 \end{bmatrix} \begin{bmatrix} 88,06 \\ 111,85 \end{bmatrix} \quad (\text{C.14})$$

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} -5,64 \\ 2,99 \end{bmatrix} \quad (\text{C.15})$$



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¹Sum of household consumption, government expenditure, GCF and exports of the constant I-O table.