Supplementary

Table A1. Some well-known G families based on the T–X criteria.

No.	Well-known families	Range of T	CDF of the family
1	Kumaraswamy-G (Kw-G) [1]	$\mathbb{I}$	$1-\{1-[G(x;\psi)]^{a}\}^{b}, a,b>0$
2	Beta-G (B-G) [2]	$\mathbb{I}$	$I_{G(x;\psi)}(a,b)\,, a,b>0$
3	McDonald-G (Mc-G) [3]	$\mathbb{I}$	$I_{[G(x;\psi)]^{c}}(a,b)\,, a,b>0$
4	Topp-Leone-G (TL-G) [4]	$\mathbb{I}$	$[1-[\bar{G}(x;\psi)]^{2}]^{b}\,, b>0$
5	Odd gamma-G (OGa-G) [5]	$\mathbb{R}^{+}$	$\dfrac{\gamma\left(a, \dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)}{\Gamma(a)}, a>0$
6	ZBgamma-G (ZBGa-G) [6]	$\mathbb{R}^{+}$	$\dfrac{\gamma\left(a, -\log\left(\bar{G}(x;\psi)\right)\right)}{\Gamma(a)}, a>0$
7	Odd additive Weibull-G (OAddW-G) [7]	$\mathbb{R}^{+}$	$1-\exp\left\{-c\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{d}-a\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{b}\right\}, a,b,c,d>0$
8	Additive Weibull-G (AddW-G) [8]	$\mathbb{R}^{+}$	$1-\exp\{-c(-\log\bar{G}(x;\psi))^{d}-a\,(-\log\bar{G}(x;\psi))^{b}\}$, $a,b,c,d>0$
9	Odd Xgamma-G (OXGa-G) [9]	$\mathbb{R}^{+}$	$1-\dfrac{1+\lambda+\lambda\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)+\dfrac{\lambda^{2}}{2}\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{2}}{1+\lambda}$ $\times\exp\left\{-\lambda\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)\right\}, \lambda>0$
10	Xgamma-G (XGa-G) [10]	$\mathbb{R}^{+}$	$1-\dfrac{1+\lambda-\log\left(\bar{G}(x;\psi)\right)^{\lambda}+\dfrac{\lambda^{2}}{2}(-\log\bar{G}(x;\psi))^{2}}{1+\lambda}$ $\times\left(\bar{G}(x;\psi)\right)^{\lambda}, \lambda>0$
11	Odd Weibull-G (OW-G) [11]	$\mathbb{R}^{+}$	$1-\exp\left\{-a\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{b}\right\}, a,b>0$
12	Weibull-G (W-G) [12]	$\mathbb{R}^{+}$	$1-\exp\{-a\,[-\log\bar{G}(x;\psi)]^{b}\}, a,b>0$
13	Odd half-logistic-G (OHL-G) [13]	$\mathbb{R}^{+}$	$\dfrac{1-\exp\left\{-\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right\}}{1+\exp\left\{-\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right\}}$
14	Half-logistic-G (HL-G) [14]	$\mathbb{R}^{+}$	$\dfrac{1-\bar{G}(x;\psi)}{1+\bar{G}(x;\psi)}$
15	Odd Burr-G (OBr-G) [15]	$\mathbb{R}^{+}$	$1-\left\{1+\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{c}\right\}^{-k}, c,k>0$
16	Burr-G (Br-G) [16]	$\mathbb{R}^{+}$	$1-\{1+(-\log\bar{G}(x;\psi))^{c}\}^{-k}, c,k>0$
17	Odd Burr Hatke-G (OBrH-G) [17]	$\mathbb{R}^{+}$	$1-\left[1+\lambda\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)\right]^{-1}\exp\left\{-\lambda\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)\right\}, \lambda>0$
18	Burr Hatke-G (BrH-G) [18]	$\mathbb{R}^{+}$	$1-\left(\bar{G}(x;\psi)\right)^{\lambda}[1-\log\bar{G}(x;\psi)]^{-1}\,, \lambda>0$
19	Odd Burr X-G (OBrX-G) [19]	$\mathbb{R}^{+}$	$\left[1-\exp\left\{-\left(\dfrac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{2}\right\}\right]^{\theta}, \theta>0$

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No.	Well-known families	Range of T	CDF of the family
20	Burr X-G (BrX-G) [20]	$\mathbb{R}^+$	$[1-\exp\{-(-\log\bar{G}(x;\psi))^2\}]^\theta, \theta>0$
21	Odd Burr 3-G (OBr3-G) [21]	$\mathbb{R}^+$	$\left\{1+\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{-c}\right\}^{-k}, c,k>0$
22	Burr 3-G (Br3-G) [22]	$\mathbb{R}^+$	$\{1+(-\log\bar{G}(x;\psi))^{-c}\}^{-k}, c,k>0$
23	Odd Lomax-G (OLx-G) [23]	$\mathbb{R}^+$	$1-\left\{1+\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right\}^{-k}, k>0$
24	Lomax-G (Lx-G) [24]	$\mathbb{R}^+$	$1-\{1-\log\bar{G}(x;\psi)\}^{-k}, k>0$
25	Odd log-logistic-G (OLL-G) [25]	$\mathbb{R}^+$	$\frac{[G(x;\psi)]^\alpha}{[G(x;\psi)]^\alpha+[\bar{G}(x;\psi)]^\alpha}, \alpha>0$
26	Log-logistic-G (LL-G) [26]	$\mathbb{R}^+$	$\frac{[-\log\bar{G}(x;\psi)]^\alpha}{1+[-\log\bar{G}(x;\psi)]^\alpha}, \alpha>0$
27	Odd Lindley-G (OLi-G) [27]	$\mathbb{R}^+$	$1-\frac{\lambda+\bar{G}(x;\psi)}{(1+\lambda)\bar{G}(x;\psi)}\exp\left\{-\lambda\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)\right\}, \lambda>0$
28	Lindley-G (Li-G) [28]	$\mathbb{R}^+$	$1-\left[1-\frac{\lambda}{1+\lambda}\log\bar{G}(x;\psi)\right]\left(\bar{G}(x;\psi)\right)^\lambda, \lambda>0$
29	Odd Power Lindley-G (OPLi-G) [29]	$\mathbb{R}^+$	$1-\left[1+\frac{\theta}{1+\theta}\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^\alpha\right]\exp\left\{-\theta\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^\alpha\right\}, \alpha,\theta>0$
30	Power Lindley-G (PLi-G) [26]	$\mathbb{R}^+$	$1-\left[1+\frac{\theta(-\log\bar{G}(x;\psi))^\alpha}{1+\theta}\right]\exp\{-\theta(-\log\bar{G}(x;\psi))^\alpha\}, \alpha,\theta>0$
31	Odd Gompertz-G (OGz-G) [26]	$\mathbb{R}^+$	$1-\exp\left\{-\frac{\theta}{\gamma}\left[\exp\left(\frac{\gamma G(x;\psi)}{\bar{G}(x;\psi)}\right)-1\right]\right\}, \theta,\gamma>0$
32	Gompertz-G (Gz-G) [30]	$\mathbb{R}^+$	$1-\exp\left\{\frac{\theta}{\gamma}\left[1-\left(\bar{G}(x;\psi)\right)^{-\gamma}\right]\right\}, \theta,\gamma>0$
33	Odd Muth-G (OMu-G) [26]	$\mathbb{R}^+$	$1-\exp\left\{\alpha\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)-\frac{1}{\alpha}\left\{\exp\left[\alpha\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)\right]-1\right\}\right\}, \alpha>0$
34	Muth-G (Mu-G) [26]	$\mathbb{R}^+$	$1-\left(\bar{G}(x;\psi)\right)^{-\alpha}\exp\left\{\frac{1}{\alpha}-\frac{\left(\bar{G}(x;\psi)\right)^{-\alpha}}{\alpha}\right\}, \alpha>0$
35	Odd Nadarajah-Haghighi-G (ONH-G) [31]	$\mathbb{R}^+$	$1-\exp\left\{1-\left(1+\lambda\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)\right)^\alpha\right\}, \lambda,\alpha>0$
36	Nadarajah-Haghighi-G (NH-G) [32]	$\mathbb{R}^+$	$1-\exp\left\{1-\left(1-\lambda(\log\bar{G}(x;\psi))\right)^\alpha\right\}, \lambda,\alpha>0$
37	Odd Maxwell-G (OMax-G) [26]	$\mathbb{R}^+$	$\frac{2}{\sqrt{\pi}}\gamma\left(\frac{2}{3},\frac{1}{2}\left[\frac{G(x;\psi)}{\lambda\bar{G}(x;\psi)}\right]^2\right), \gamma,\lambda>0$
38	Maxwell-G (Max-G) [33]	$\mathbb{R}^+$	$\frac{2}{\sqrt{\pi}}\gamma\left(\frac{2}{3},\frac{1}{2}\left[-\log\left(\bar{G}(x;\psi)\right)^{1/\lambda}\right]^2\right), \gamma,\lambda>0$
39	Odd Fréchet-G (OFr-G) [34]	$\mathbb{R}^+$	$\exp\left\{-\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{-\theta}\right\}, \theta>0$
40	Fréchet-G (Fr-G) [26]	$\mathbb{R}^+$	$\exp\left\{-(-\log\bar{G}(x;\psi))^{-\theta}\right\}, \theta>0$
41	Odd inverse Pareto-G (OIPa-G) [35]	$\mathbb{R}^+$	$\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^\alpha\left[\beta+\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]^{-\alpha}, \beta,\alpha>0$

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No.	Well-known families	Range of T	CDF of the family
42	Inverse Pareto-G (IPa-G) [26]	$\mathbb{R}^+$	$\left[\frac{-\log \bar{G}(x;\psi)}{\beta-\log \bar{G}(x;\psi)}\right]^{\alpha}, \beta,\alpha>0$
43	Odd generalized exponential -G (OGE-G) [36]	$\mathbb{R}^+$	$1-\exp\left\{-\alpha\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right\}, \alpha>0$
44	Odd Power Cauchy-G (OPCa-G) [37]	$\mathbb{R}^+$	$\frac{2}{\pi}\tan^{-1}\left(\left\{\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right\}^{\alpha}\right), \alpha>0$
45	Odd Half Cauchy-G (OHCa-G) [38]	$\mathbb{R}^+$	$\frac{2}{\pi}\tan^{-1}\left(\frac{\left(G(x;\psi)\right)^{\alpha}}{1-\left(G(x;\psi)\right)^{\alpha}}\right), \alpha>0$
46	Log-odd logistic-G (LOLo-G) [39]	$\mathbb{R}$	$\left\{1+\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{-\lambda}\right\}^{-1}, \lambda>0$
47	Logistic-G (Lo-G) [40]	$\mathbb{R}$	$\left\{1+\left[-\log \bar{G}(x;\psi)\right]^{-\lambda}\right\}^{-1}, \lambda>0$
48	Odd Chen-G (OC-G) [41]	$\mathbb{R}^+$	$1-\exp\left(-\alpha\left\{\exp\left[\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]^{\beta}-1\right\}\right), \alpha,\beta>0$
49	Type II Exponentiated Half Logistic-G (TIIEHL-G) [42]	$\mathbb{R}^+$	$1-\left[\frac{1-[G(x;\psi)]^{\lambda}}{1+[G(x;\psi)]^{\lambda}}\right]^{\alpha}, \alpha,\lambda>0$
50	Marshall-Olkin odd Burr III-G (MOOB-G) [43]	$\mathbb{R}^+$	$\frac{\left\{1+\left[\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]^{-c}\right\}^{-b}}{1-(1-\phi)\left(1-\left\{1+\left[\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]^{-c}\right\}^{-b}\right)}, \phi,b,c>0$
51	Log-logistic tan-G (LLT-G) [44]	$\mathbb{R}^+$	$1-\left(1+s^{-c}\left\{\tan\left[\frac{\pi}{2}\left(G(x;\psi)\right)^{\alpha}\right]\right\}^{c}\right)^{-1}, \alpha,c,s>0$
52	Kumaraswamy Transmuted-G (Kw-TG) [45]	$\mathbb{R}^+$	$1-\left[1-\left\{(1+\lambda)G(x;\psi)-\lambda\left(G(x;\psi)\right)^{2}\right\}^{a}\right]^{b}, \lambda,a,b>0$
53	Transmuted Burr X-G (TBX-G) [46]	$\mathbb{R}^+$	$\left(1+\rho-\rho\left\{1-\exp\left(-\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{2}\right)\right\}^{\varphi}\right)\times\left\{1-\exp\left(-\left(\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right)^{2}\right)\right\}^{\varphi}, \varphi>0, \lvert\rho\rvert\le 1$
54	Fréchet Topp Leone-G (FTL-G) [47]	$\mathbb{R}$	$\exp\left\{-\beta^{\alpha}\left(\frac{1-\left[1-\left(\bar{G}(x;\psi)\right)^{2}\right]^{\lambda}}{\left[1-\left(\bar{G}(x;\psi)\right)^{2}\right]^{\lambda}}\right)^{\alpha}\right\}, \alpha,\lambda,\beta>0$
55	Generalized Marshall-Olkin transmuted-G (GMOT-G) [48]	$\mathbb{R}$	$\left[\frac{\alpha[1-G(x;\psi)\{1+\lambda-\lambda\, G(x;\psi)\}]}{1-\bar{\alpha}[1-G(x;\psi)\{1+\lambda-\lambda\, G(x;\psi)\}]}\right]^{\theta}, \alpha,\theta>0, \lvert\lambda\rvert\le 1$
56	Type-I extended-G (TIEx-G) [49]	$\mathbb{R}$	$\frac{\exp\left(G(x;\psi)\right)-[1-\eta G(x;\psi)]}{e-\bar{\eta}}, \eta>0, \eta\ne 1-e$
57	Exponentiated reduced-Kies-G (ERKi-G) [50]	$\mathbb{R}^+$	$\left\{1-\exp\left(-\left[\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]^{\rho}\right)\right\}^{\eta}, \rho,\eta>0$
58	Kumaraswamy tan generalized-G (KwT-G) [51]	$\mathbb{R}$	$1-\left\{1-\exp\left(-\alpha\left[\tan\left(\frac{\pi\left(\bar{G}(x;\psi)\right)}{2}\right)\right]\right)\right\}^{b}, a,b>0$

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No.	Well-known families	Range of T	CDF of the family
59	Generalized Kavya-Manoharan-G (GKM-G) [52]	$\mathbb{R}$	$p^{\alpha}\left[1-\exp\left(-G(x;\psi)\right)\right]^{\alpha}, \alpha>0, p=\frac{e}{e-1}$
60	New exponential-G (NEx-G) [53]	$\mathbb{R}^{+}$	$1-\exp\left\{-\alpha\left[\frac{G(x;\psi)\left(2-G(x;\psi)\right)}{\bar{G}(x;\psi)}\right]\right\}, \alpha>0$
61	Lambert-G (L-G) [54]	$\mathbb{R}^{+}$	$1-\exp(-\{-\ln[\bar{G}(x;\psi)]\}^{\upsilon}\exp\{-\omega\ln[\bar{G}(x;\psi)]\}),$ $\upsilon,\omega>0$
62	Burr-X Marshall–Olkin-G (BXMO-G) [55]	$\mathbb{R}$	$\left\{1-\exp\left(-\left\{\frac{G(x;\psi)}{\alpha\bar{G}(x;\psi)}\right\}^{2}\right)\right\}^{\theta}, \alpha,\theta>0$
63	Inverse-power Burr–Hatke-G (IPBH-G) [56]	$\mathbb{R}$	$\frac{\exp\left\{-\alpha\left[\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]^{-\eta}\right\}}{1+\left[\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]^{-\eta}}, \alpha,\eta>0$
64	Modified Kies-Kavya-Manoharan-G (MKKM-G) [57]	$\mathbb{R}^{+}$	$1-\exp\left\{-\left[\frac{e-\exp\left(\bar{G}(x;\psi)\right)}{\exp\left(\bar{G}(x;\psi)\right)-1}\right]^{\alpha}\right\}, \alpha>0$
65	New Lomax-G (NLx-G) [58]	$\mathbb{R}$	$1-\left(1+\beta\left\{\frac{G(x;\psi)}{\bar{G}(x;\psi)}[2-G(x;\psi)]\right\}\right)^{-\alpha}, \alpha,\beta>0$
66	New Weibull-G (NW-G) [59]	$\mathbb{R}$	$1-\exp\left\{-\alpha\left[\frac{G(x;\psi)\left(2-G(x;\psi)\right)}{\bar{G}(x;\psi)}\right]^{\beta}\right\}, \alpha,\beta>0$
67	Marshall-Olkin Extended Weibull-Truncated Weibull (MOEW-TW-G) [60]	$\mathbb{I}$	$\frac{1-\exp\left\{-\alpha\left(\frac{1-\exp[-\gamma\, G(x;\psi)]}{1-\bar{\delta}\exp[-\gamma\, G(x;\psi)]}\right)^{\beta}\right\}}{1-\exp\{-\alpha\}}, \alpha,\beta,\gamma,\delta>0$
68	Extended Odd Weibull-Weibull-G (ExOW-W-G) [60]	$\mathbb{R}^{+}$	$1-\left\{1+\beta\left[\frac{1-\exp\left\{-\left[\frac{-\log\left(\bar{G}(x;\psi)\right)}{\bar{G}(x;\psi)}\right]^{\lambda}\right\}}{\exp\left\{-\left[\frac{-\log\left(\bar{G}(x;\psi)\right)}{\bar{G}(x;\psi)}\right]^{\lambda}\right\}}\right]^{\alpha}\right\}^{\frac{-1}{\beta}},$ $\alpha,\beta,\lambda>0$
69	Odd-Generalized Rayleigh-Reciprocal Weibull-G (OGR-RW-G) [61]	$\mathbb{R}$	$1-\exp\left\{-\left[\frac{\exp\left(-2\alpha\left[\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]^{-\beta}\right)}{\left(1-\exp\left(-2\alpha\left[\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]^{-\beta}\right)\right)^{2}}\right]\right\},$ $\alpha,\beta>0$
70	Odd Weibull-G (OW-G) [62]	$\mathbb{R}^{+}$	$1-\exp\left(-\alpha\left\{-\left[\frac{G(x;\psi)}{\bar{G}(x;\psi)}\right]\log\left[\frac{1}{\bar{G}(x;\psi)}\right]\right\}^{\beta}\right), \alpha,\beta>0$

Table A2. NFx G-families generated from our introduced generator that fulfill the T–X criteria.

No.	NFx-G	Range of T	CDF of the NFx G-family
1	NFxKw-G	$\mathbb{I}$	$1-\{1-[p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)]^{a}\}^{b}, a,b>0$
2	NFxB-G	$\mathbb{I}$	$I_{p\,(1-exp\{-[1-\bar{G}(x)^{G(x)}]\})}(a,b), a,b>0$
3	NFxMc-G	$\mathbb{I}$	$I_{[p\,(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\})]^{c}}(a,b), a,b>0$
4	NFxTL-G	$\mathbb{I}$	$\left(1-\left[1-p\left(1-exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right]^{2}\right)^{b}, b>0$
5	NFxOGa-G	$\mathbb{R}^{+}$	$\frac{1}{\Gamma(a)}\gamma\left(a, \frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right), a>0$
6	NFxZBGa-G	$\mathbb{R}^{+}$	$\frac{1}{\Gamma(a)}\gamma\left(a, -\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right), a>0$
7	NFxOAddW-G	$\mathbb{R}^{+}$	$1-\exp\left\{-c\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{d} - a\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{b}\right\},$ $a,b,c,d>0$
8	NFxAddW-G	$\mathbb{R}^{+}$	$1-\exp\left\{-c\left(-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)^{d} - a\left(-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)^{b}\right\},$ $a,b,c,d>0$
9	NFxOXGa-G	$\mathbb{R}^{+}$	$1-\left[1+\lambda+\lambda\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right) + \frac{\lambda^{2}}{2}\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{2}\right]/(1+\lambda)$ $\times\exp\left\{-\lambda\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)\right\}, \lambda>0$
10	NFxXGa-G	$\mathbb{R}^{+}$	$1-\left[1+\lambda+\frac{\lambda^{2}}{2}\left(-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)^{2} - \log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{\lambda}\right]/(1+\lambda)$ $\times\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{\lambda}, \lambda>0$
11	NFxOW-G	$\mathbb{R}^{+}$	$1-\exp\left\{-a\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{b}\right\}, a,b>0$
12	NFxW-G	$\mathbb{R}^{+}$	$1-\exp\left\{-a\left[-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]^{b}\right\}, a,b>0$
13	NFxOHL-G	$\mathbb{R}^{+}$	$\frac{1-\exp\left\{-\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right\}}{1+\exp\left\{-\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right\}}$
14	NFxHL-G	$\mathbb{R}^{+}$	$\frac{p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)}{2-p\,(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\})}$

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No.	NFx-G	Range of T	CDF of the NFx G-family
15	NFxOBr-G	$\mathbb{R}^+$	$1-\left\{1+\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{c}\right\}^{-k}, c,k>0$
16	NFxBr-G	$\mathbb{R}^+$	$1-\left\{1+\left(-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)^{c}\right\}^{-k}, c,k>0$
17	NFxOBrH-G	$\mathbb{R}^+$	$1-\left[1+\lambda\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)\right]^{-1}$ $\times\exp\left\{-\lambda\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)\right\}, \lambda>0$
18	NFxBrH-G	$\mathbb{R}^+$	$1-\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{\lambda}$ $\times\left[1-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]^{-1}, \lambda>0$
19	NFxOBrX-G	$\mathbb{R}^+$	$\left[1-\exp\left\{-\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{2}\right\}\right]^{\theta}, \theta>0$
20	NFxBrX-G	$\mathbb{R}^+$	$\left[1-\exp\left\{-\left(-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)^{2}\right\}\right]^{\theta}, \theta>0$
21	NFxOBr3-G	$\mathbb{R}^+$	$\left\{1+\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{-c}\right\}^{-k}, c,k>0$
22	NFxBr3-G	$\mathbb{R}^+$	$\left\{1+\left(-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)^{-c}\right\}^{-k}, c,k>0$
23	NFxOLx-G	$\mathbb{R}^+$	$1-\left\{1+\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right\}^{-k}, k>0$
24	NFxLx-G	$\mathbb{R}^+$	$1-\left\{1-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right\}^{-k}, k>0$
25	NFxOLL-G	$\mathbb{R}^+$	$\frac{\left[p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right]^{\alpha}}{[p\,(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\})]^{\alpha}+[1-p\,(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\})]^{\alpha}}$ $,\alpha>0$
26	NFxLL-G	$\mathbb{R}^+$	$\frac{\left[-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]^{\alpha}}{1+\left[-\log\left(1-p\,(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\})\right)\right]^{\alpha}}, \alpha>0$
27	NFxOLi-G	$\mathbb{R}^+$	$1-\frac{\lambda+1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)}{(1+\lambda)[1-p\,(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\})]}$ $\times\exp\left\{-\lambda\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)\right\}, \lambda>0$
28	NFxLi-G	$\mathbb{R}^+$	$1-\left[1-\frac{\lambda}{1+\lambda}\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]$ $\times\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{\lambda}, \lambda>0$
29	NFxOPLi-G	$\mathbb{R}^+$	$1-\left[1+\frac{\theta}{1+\theta}\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{\alpha}\right]$ $\times\exp\left\{-\theta\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{\alpha}\right\}, \alpha,\theta>0$

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No.	NFx-G	Range of T	CDF of the NFx G-family
30	NFxPLi-G	$\mathbb{R}^+$	$1-\left[1+\frac{\theta\left(-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)^{\alpha}}{1+\theta}\right]$ $\times\exp\left\{-\theta\left(-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)^{\alpha}\right\},\alpha,\theta>0$
31	NFxOGz-G	$\mathbb{R}^+$	$1-\exp\left\{-\frac{\theta}{\gamma}\left[\exp\left(\frac{\gamma p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-\gamma\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)-1\right]\right\},\theta,\gamma>0$
32	NFxGz-G	$\mathbb{R}^+$	$1-\exp\left\{\frac{\theta}{\gamma}\left[1-\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{-\gamma}\right]\right\},\theta,\gamma>0$
33	NFxOMu-G	$\mathbb{R}^+$	$1-\exp\left\{\alpha\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)-\frac{1}{\alpha}\left\{\exp\left[\alpha\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)\right]-1\right\}\right\},$ $\alpha>0$
34	NFxMu-G	$\mathbb{R}^+$	$1-\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{-\alpha}$ $\times\exp\left\{\frac{1}{\alpha}-\frac{\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{-\alpha}}{\alpha}\right\},\alpha>0$
35	NFxONH-G	$\mathbb{R}^+$	$1-\exp\left\{1-\left(1+\lambda\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)\right)^{\alpha}\right\},\lambda,\alpha>0$
36	NFxNH-G	$\mathbb{R}^+$	$1-\exp\left\{1-\left(1-\lambda\left(\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)\right)^{\alpha}\right\},$ $\lambda,\alpha>0$
37	NFxOMax-G	$\mathbb{R}^+$	$\frac{2}{\sqrt{\pi}}\gamma\left(\frac{2}{3},\frac{1}{2}\left[\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{\gamma(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+\gamma p}\right]^{2}\right),\gamma,\lambda>0$
38	NFxMax-G	$\mathbb{R}^+$	$\frac{2}{\sqrt{\pi}}\gamma\left(\frac{2}{3},\frac{1}{2}\left[-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{1/\lambda}\right]^{2}\right),\gamma,\lambda>0$
39	NFxOFr-G	$\mathbb{R}^+$	$\exp\left\{-\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{-\theta}\right\},\theta>0$
40	NFxFr-G	$\mathbb{R}^+$	$\exp\left\{-\left(-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)^{-\theta}\right\},\theta>0$
41	NFxOIPa-G	$\mathbb{R}^+$	$\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{\alpha}$ $\times\left[\beta+\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]^{-\alpha},\beta,\alpha>0$
42	NFxIPa-G	$\mathbb{R}^+$	$\left[\frac{-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)}{\beta-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)}\right]^{\alpha},\beta,\alpha>0$
43	NFxOGE-G	$\mathbb{R}^+$	$1-\exp\left\{-\alpha\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right\},\alpha>0$

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No.	NFx-G	Range of T	CDF of the NFx G-family
44	NFx OPCa-G	$\mathbb{R}^+$	$\frac{2}{\pi}\tan^{-1}\left(\left\{\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right\}^{\alpha}\right), \alpha>0$
45	NFxOHCa-G	$\mathbb{R}^+$	$\frac{2}{\pi}\tan^{-1}\left(\frac{\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{\alpha}}{1-\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{\alpha}}\right), \alpha>0$
46	NFxLOLo-G	$\mathbb{R}$	$\left\{1+\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{-\lambda}\right\}^{-1}, \lambda>0$
47	NFxLo-G	$\mathbb{R}$	$\left\{1+\left[-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]^{-\lambda}\right\}^{-1}, \lambda>0$
48	NFxOC-G	$\mathbb{R}^+$	$1-\exp\left(-\alpha\left\{\exp\left[\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]^{\beta}-1\right\}\right), \alpha,\beta>0$
49	NFxTIIEHL-G	$\mathbb{R}^+$	$1-\left[\frac{1-\left[p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right]^{\lambda}}{1+\left[p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right]^{\lambda}}\right]^{\alpha}, \alpha,\lambda>0$
50	NFxMOOB-G	$\mathbb{R}^+$	$\frac{\left\{1+\left[\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]^{-c}\right\}^{-b}}{1-(1-\phi)\left(1-\left\{1+\left[\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]^{-c}\right\}^{-b}\right)}$, $\phi,b,c>0$
51	NFxLLT-G	$\mathbb{R}^+$	$1-\left(1+s^{-c}\left\{\tan\left[\frac{\pi}{2}\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{\alpha}\right]\right\}^{c}\right)^{-1}, \alpha,c,s>0$
52	NFxKw-TG	$\mathbb{R}^+$	$1-\left[1-\left\{(1+\lambda)\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right) - \lambda\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{2}\right\}^{a}\right]^{b}, \lambda,a,b>0$
53	NFxTBX-G	$\mathbb{R}^+$	$\left(1+\rho-\rho\left\{1-\exp\left(-\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{2}\right)\right\}^{\varphi}\right) \times\left\{1-\exp\left(-\left(\frac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)^{2}\right)\right\}^{\varphi}, \varphi>0, \lvert\rho\rvert\leq 1$
54	NFxFTL-G	$\mathbb{R}$	$\exp\left\{-\beta^{\alpha}\left(\frac{1-\left[1-\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{2}\right]^{\lambda}}{\left[1-\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)^{2}\right]^{\lambda}}\right)^{\alpha}\right\}$, $\alpha,\lambda,\beta>0$
55	NFxGMOT-G	$\mathbb{R}$	$\left[\alpha\left[1-\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\left\{1+\lambda - \lambda\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right\}\right]\right]^{\theta} \times\left[1-\bar{\alpha}\left[1-\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\left\{1+\lambda - \lambda\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right\}\right]\right]^{-\theta}$, $\alpha,\theta>0, \lvert\lambda\rvert\leq 1$

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No.	NFx-G	Range of T	CDF of the NFx G-family
56	NFxTIEx-G	$\mathbb{R}$	$\dfrac{\exp\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)-\left[1-\eta\left[p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right]\right]}{e-\bar{\eta}}$, $\eta>0,\eta\neq 1-e$
57	NFxERKi-G	$\mathbb{R}^{+}$	$\left\{1-\exp\left(-\left[\dfrac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]^{\rho}\right)\right\}^{\eta}, \rho,\eta>0$
58	NFxKwT-G	$\mathbb{R}$	$1-\left\{1-\exp\left(-a\left[\tan\left(\dfrac{\pi\left(1-p\left(1-exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)}{2}\right)\right]\right)\right\}^{b}$, $a,b>0$
59	NFxGKM-G	$\mathbb{R}$	$p^{\alpha}\left[1-\exp\left(-\left[p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right]\right)\right]^{\alpha}, \alpha>0$
60	NFxNEx-G	$\mathbb{R}^{+}$	$1-\exp\left\{-a\left[\left(\left[p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right]\left(2-\left[p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right]\right)\right)/\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]\right\}, a>0$
61	NFxL-G	$\mathbb{R}^{+}$	$1-\exp\left(-\{-\ln[1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)]\}^{\upsilon}\exp\{-\omega\ln[1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)]\}\right)$, $\upsilon,\omega>0$
62	NFxBXMO-G	$\mathbb{R}$	$\left\{1-\exp\left(-\left\{\dfrac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{\alpha[(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p]}\right\}^{2}\right)\right\}^{\theta}, \alpha,\theta>0$
63	NFxIPBH-G	$\mathbb{R}$	$\dfrac{\exp\left\{-\alpha\left[\dfrac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]^{-\eta}\right\}}{1+\left[\dfrac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]^{-\eta}}, \alpha,\eta>0$
64	NFxMKKM-G	$\mathbb{R}^{+}$	$1-\exp\left\{-\left[\dfrac{e-\exp\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)}{\exp\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)-1}\right]^{a}\right\}, a>0$
65	NFxNLx-G	$\mathbb{R}$	$1-\left(1+\beta\left\{\left(\dfrac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right)\left[2-\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]\right\}\right)^{-\alpha}, \alpha,\beta>0$
66	NFxNW-G	$\mathbb{R}$	$1-\exp\left\{-\alpha\left[\left(\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\left(2-\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right)\right)/\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]^{\beta}\right\}\alpha,\beta>0$
67	NFxMOEW-TW-G	$\mathbb{I}$	$\dfrac{1-\exp\left\{-\alpha\left(\dfrac{1-\exp\left[-\gamma\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]}{1-\bar{\delta}\exp\left[-\gamma\left(p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)\right]}\right)^{\beta}\right\}}{1-\exp\{-\alpha\}}$, $\alpha,\beta,\gamma,\delta>0$

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No.	NFx-G	Range of T	CDF of the NFx G-family
68	NFxExOW-W-G	$\mathbb{R}^{+}$	$1-\left\{1+\beta\left[\dfrac{1-\exp\left\{-\left[\dfrac{-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)}{1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)}\right]^{\lambda}\right\}}{\exp\left\{-\left[\dfrac{-\log\left(1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)\right)}{1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)}\right]^{\lambda}\right\}}\right]^{\alpha}\right\}^{\frac{-1}{\beta}},$ $\alpha,\beta,\lambda>0$
69	NFxOGR-RW-G	$\mathbb{R}$	$1-\exp\left\{-\left[\dfrac{\exp\left(-2\alpha\left[\dfrac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]^{-\beta}\right)}{\left(1-\exp\left(-2\alpha\left[\dfrac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]^{-\beta}\right)\right)^{2}}\right]\right\},$ $\alpha,\beta>0$
70	NFxOW-G	$\mathbb{R}^{+}$	$1-\exp\left(-\alpha\left\{-\dfrac{\left[\dfrac{p\left(\exp\{1-\bar{G}(x)^{G(x)}\}-1\right)}{(1-p)\exp\{1-\bar{G}(x)^{G(x)}\}+p}\right]}{\left(\log\left[\dfrac{1}{1-p\left(1-\exp\{-[1-\bar{G}(x)^{G(x)}]\}\right)}\right]\right)^{-1}}\right\}^{\beta}\right),$ $\alpha,\beta>0$

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