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Research article

Price-induced stable matching in shared healthcare

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A. Appendix: Proofs of analytical results

This appendix provides the rigorous mathematical proofs for the theorems and propositions stated in the main text.

A.1. Proof of Theorem 1

Proof. For every $p \in \mathcal{P}_\Delta$, Eqs (4.2)–(4.5) and the stated zero-surplus convention determine the acceptable graph $\mathcal{E}(p)$. The fixed exogenous tiebreaker converts all preferences on that graph into strict lists. Nurse-proposing deferred acceptance therefore returns a unique selected matching μ_p^N, which gives a well-defined finite quantity $Q(p, \mu_p^N)$ and real-valued profit $\Pi(p)$.

Because $\mathcal{P}_\Delta = \{p_1, \ldots, p_K\}$ is finite and nonempty, the finite set $\{\Pi(p_1), \ldots, \Pi(p_K)\}$ has a largest element. Any price attaining that element is a global optimizer. This argument does not require continuity of Q or Π between grid points. □

A.2. Proof of Proposition 1

Proof. For fixed p and pair (i, j), Eq (4.3) implies

$$\pi_{ij}(p; \tau_j') - \pi_{ij}(p; \tau_j) = -(\tau_j' - \tau_j)h(t_{ij}) \leq 0.$$

Patient utility and clinical compatibility are unchanged. Thus every pair that is mutually acceptable under $\boldsymbol{\tau}'$ is mutually acceptable under $\boldsymbol{\tau}$, proving $\mathcal{E}(p; \boldsymbol{\tau}') \subseteq \mathcal{E}(p; \boldsymbol{\tau})$. Deleting edges cannot increase the maximum cardinality over all feasible matchings. The proposition makes no monotonicity claim about the optimizer p^*, so no further demand shape assumption is required. □

A.3. Proof of Corollary 1

Proof. For a fixed price, the model is a one-to-one stable matching problem with incomplete strict preference lists. The rural hospitals result implies that every stable matching matches the same set of patients and the same set of nurses. Therefore, all stable matchings have the same $Q(p)$. In the tiered model, the matched patient set is also invariant, so the number of matched patients of each service type is invariant. Because platform margin depends only on price and service type, profit is identical across stable matchings for every feasible price. Maximizing identical pointwise profit functions yields the same set of optimal prices. Pair-specific travel and attribute terms in Eq (5.5) need not be identical, so welfare and surplus distribution can differ. □

A.4. Proof of Proposition 2

Proof. Because $p^V < \bar{p}$, a right neighborhood of p^V lies within $\mathcal{P}$. Under Assumption 1, continuous fluid expected profit on that neighborhood is

$$\bar{\Pi}(p) = [(1-\eta)p - k]q(p).$$

Its right derivative at p^V equals the left-hand side of Eq (5.6), which is positive by assumption. Hence, there exists $\varepsilon > 0$ such that $\bar{\Pi}(p^V + \varepsilon) > \bar{\Pi}(p^V)$, so p^V is not a maximizer. If a global maximizer lay weakly to the left of p^V, single-peakedness on $\mathcal{P}$ would require $\bar{\Pi}$ to be weakly decreasing at and immediately to the right of p^V, contradicting this strict local increase. Thus, every continuous-envelope expected-profit maximizer p^{Π} satisfies $p^{\Pi} > p^V$. If the derivative condition fails, this reasoning does not apply, which is why the proposition is conditional. □

A.5. Proof of Proposition 3

Proof. The result is immediate when $\mathcal{T} = \emptyset$. Suppose, therefore, that $\mathcal{T} \neq \emptyset$. Condition (6.3) implies, for every compatible triplet (j, i_H, i_L),

$$\tau_j[h(t_{i_H j}) - h(t_{i_L j})] \leq \Delta w + c_{jL} - c_{jH}.$$

By Eq (6.2), i_L cannot be strictly preferred to i_H. Under the tiebreaking condition stated in Proposition 3, an equality is resolved in favor of i_H, proving priority separation. Without that condition, the strict version of Eq (6.3) rules out equality and yields the same conclusion under any fixed deterministic tiebreaker.

Priority separation changes order but not compatibility. An advanced nurse rejected by, or incompatible with, all remaining advanced requests can still propose to an acceptable basic request. If the high-tier-only submarket matches every advanced nurse who has an acceptable basic alternative, no such nurse remains available for a cross-tier match, so downward substitution is zero. If that condition fails, an unmatched advanced nurse with an acceptable basic alternative may remain available for a cross-tier match; priority separation alone therefore does not guarantee zero downward substitution. This establishes the capacity-absorption condition as sufficient, not necessary, and shows why the ranking bound by itself is insufficient to guarantee complete market separation. □